Interval and Symmetry Approaches to Uncertainty – Pioneered by Wiener – Help Explain Seemingly Irrational Human Behavior

Joe Lorkowski and Vladik Kreinovich
Department of Computer Science
University of Texas at El Paso
El Paso, Texas 79968, USA
lorkowski@computer.org, vladik@utep.edu

***Abstract*—It has been observed that in many cases, when we present a user with three selections od different price (and, correspondingly, different quality), then the user selects the middle selection. This empirical fact – known as a *compromise effect* – seems to contradicts common sense. Indeed, when a rational decision-maker selects one of the two alternatives, and then we add an additional option, then the user will either keep the previous selection or switch to a new option, but he/she will *not* select a previously rejected option. However, this is exactly what happens under the compromise effect. If we present the user with three options $a < a' < a''$, then, according to the compromise effect, the user will select the middle option a', meaning that between a' and a'', the user will select a'. However, if instead we present the user with three options $a' < a'' < a'''$, then, according to the same compromise effect, the use will select a previously rejected option a''. In this paper, we show that this seemingly irrational behavior actually makes sense: it can be explained by an application of a *symmetry approach*, an approach whose application to uncertainty was pioneered by N. Wiener (together with interval approach to uncertainty).**

I. Compromise Effect: A Particular Case of Seemingly Irrational Human Behavior

Customers make decisions. A customer shopping for an item usually has several choices. Some of these choices have better quality, lead to more possibilities, etc. – but are, on the other hand, more expensive. For example, a customer shopping for a photo camera has plenty of choices ranging from the cheapest one whose photos are good to very professional cameras enabling the user to make highest-quality photos even under complex circumstances. A person planning to spend a night at a different city has a choice from the cheapest motels which provide a place to sleep to luxurious hotels providing all kinds of comfort, etc. A customer selects one of the alternatives by taking into account the additional advantages of more expensive choices versus the need to pay more money for these choices.

It is important to understand customer's decisions. Whether we are motivated by a noble goal of providing alternatives which are the best for the customers – or whether a company wants to make more money by providing what is wanted by the customers – it is important to understand how customers make decisions.

Experimental studies. In many real-life situations, customers face numerous choices. As usual in science, a good way to understand complex phenomena is to start by analyzing the simplest cases. In line with this reasoning, researchers provided customers with two alternatives and recorded which of these two alternatives a customer selected. In many particular cases, these experiments helped better understand the customer's selections – and sometimes even predict customer selections.

At first glance, it seems like such pair-wise comparisons are all we need to know: if a customer faces several choices $a_1, a_2, \ldots, a_n$, then a customer will select an alternative a_i if and only if this alternative is better in pair-wise comparisons that all other possible choices. To confirm this common-sense idea, in the 1990s, several researchers asked the customers to select one of the three randomly selected alternatives.

What was expected. The experimenters expected that since the three alternatives were selected at random, a customers would:

- sometimes select the cheapest of the three alternative (of lowest quality of all three),
- sometimes select the intermediate alternative (or intermediate quality), and
- sometimes select the most expensive of the three alternatives (of highest quality of all three).

What was observed. Contrary to the expectations, the experimenters observed that in the overwhelming majority of cases, customers selected the intermediate alternative; see, e.g., [12], [13], [16]. In all these cases, the customer selected an alternative which provided a compromise between the quality and cost; because of this, this phenomenon was named *compromise effect*.

Why is this irrational? At first glance, selecting the middle alternative is reasonable. However, it is not.

For example, let us assume that we have four alternative $a_1 < a_2 < a_3 < a_4$ ordered in the increasing order of price and at the same time, increasing order of quality. Then:

- if we present the user with three choices $a_1 < a_2 < a_3$, in most cases, the user will select the middle choice a_2; this means, in particular, that, to the user, a_2 better than the alternative a_3;

- on the other hand, if we present the user with three other choices $a_2 < a_3 < a_4$, in most cases, the same user will select the middle choice a_3; but this means that, to the user, the alternative a_3 better than the alternative a_2.

If in a pair-wise comparison, a_2 is better, then the second choice is wrong. If in a par-wise comparison, the alternative a_3 is better, then the first choice is wrong. In both cases, one of the two choices is irrational.

This is not just an experimental curiosity, customers' decisions have been manipulated this way. At first glance, the above phenomena may seem like one of optical illusions or logical paradoxes: interesting but not that critically important. Actually, it is serious and important, since, according to anecdotal evidence, many companies have tried to use this phenomenon to manipulate the customer's choices: to make the customer buy a more expensive product.

For example, if there are two possible types of a certain product, a company can make sure that most customers select the most expensive type – simply by offering, as the third option, an even more expensive type of the same product.

Manipulation possibility has been exaggerated. Recent research shows that manipulation is not very easy: the compromise effect only happens when a customer has no additional information – and no time (or no desire) to collect such information. In situations when customers were given access to additional information, they selected – as expected from rational folks – one of the three alternatives with almost equal frequency, and their pairwise selections, in most cases, did not depend on the presence of any other alternatives; see, e.g., [15].

Compromise effect: mystery remains. The new experiment shows that the compromise effect is not as critical and not as wide-spread as it was previously believed. However, in situation when decisions need to be made under major uncertainty, this effect is clearly present – and its seemingly counterintuitive, inconsistent nature is puzzling.

How can we explain such a seemingly irrational behavior?

What we do in this paper. In this paper, we show that it is possible to find a rational explanation for such a behavior.

Interesting, this explanations is related to two ideas promoted by N. Wiener – interval uncertainty (which later encouraged fuzzy uncertainty) and symmetry.

II. Before We Explain the Mystery of Seemingly Irrational Behavior, Let Us First Recall How Rational Behavior Is Usually Described

Traditional decision theory is based on the assumption that a decision maker can always make a definite choice. Traditional decision theory (see, e.g., [1], [5], [8], [11]) is based on the assumption that if we present a decision maker with two alternatives A and A', then the decision maker will always make a definition decision about which of this alternatives is better for him/her. In other words, the decision maker will always select one of the following three options:

- the alternative A is better than the alternative A'; we will denote this option by $A > A'$;
- the alternative A' is better than the alternative A; we will denote this option by $A < A'$;
- the alternatives A and A' are of equal value; we will denote this by $A = A'$.

Resulting numerical description of preferences. The above assumption enables us to provide a numerical scale for describing quality of different alternatives.

To describe this scale, let us select two fixed outcomes:

- we select a very bad outcome, which is worse than any of the alternatives that we encounter in decision making; we will denote this situation by A_0;
- we also select a very good outcome, which is better than any of the alternatives that we encounter in decision making; we will denote this situation by A_1.

Then, for each number p from the interval $[0, 1]$, we can form a lottery in which we get A_1 with probability p and A_0 with the remaining probability $1 - p$. This lottery will be denoted by $L(p)$.

- When $p = 1$, the corresponding lottery $L(1)$ means that we select a very good outcome with probability 1, i.e., we have $L(1) = A_1$.
- When $p = 0$, the corresponding lottery $L(0)$ means that we select a very bad outcome with probability 1, i.e., we have $L(0) = A_0$.

When the probability p in strictly between 0 and 1, the resulting lottery is better than A_0 but worse than A_1:

$$A_0 < L(p) < L(1).$$

The larger p, the larger the probability p of the very good outcome A_1 and the smaller the probability $1 - p$ of the very bad outcome. Thus, the larger the probability p, the better the lottery $L(p)$. In precise terms, if $p < p'$, then $L(p) < L(p')$; in this sense, the scale is *monotonic*.

Intuitively, if we change p a little bit, the quality of the resulting lottery $L(p)$ will also change only slightly. In this sense, the scale if *continuous*.

In other words, the lotteries $L(p)$ corresponding to different values $p \in [0, 1]$ form a monotonic scale which changes continuously from a very bad outcome A_0 to a very good outcome A_1. We can use this scale to describe the user's preferences in numerical terms.

Namely, suppose that we have an alternative A. Because of our selection of the very bad alternative A_0 and the very good alternative A_1, we have $A_0 < A < A_1$. In other words, we have $L(0) < A < L(1)$. As we increase p from 0 up, we will first have still $L(p) < A$. However, as we increase p even more, we will at some point switch to $A < L(p)$ – since this is what we have for $p = 1$. Thus, there is a threshold value u at which this switch happens, i.e., for which:

- $L(p) < A$ for all $p < u$, and
- $A < L(p)$ for all $p > u$.

This threshold value of probability is called the *utility* of the alternative A. We will denote this utility by $u(A)$. The above threshold property will be denoted by $A \sim L(u(A))$, meaning that for every $\varepsilon > 0$, we have $L(u(A)-\varepsilon) < A < L(u(A)+\varepsilon)$. We will say that the original alternative A is *equivalent* to the lottery $L(p)$.

The higher the utility, the better the alternative: indeed, if $u(A) < u(B)$, then $L(u(A)) < L(u(B))$, so

$$A \sim L(u(A)) < L(u(B)) \sim B$$

and $A < B$.

Utilities depend on the selection of A_0 and A_1. The numerical value of the utility $u(A)$ depends on the selection of the fixed outcomes A_0 and A_1. If we fix two different outcomes A'_0 and A'_1, then we will get different numerical values of the utility $u'(A) \neq u(A)$. The above definition enables us to describe the relation between different possible utility scales.

Let us first consider the case when $A'_0 \le A_0 < A_1 \le A'_1$. In this case, both outcomes A_0 and A_1 are in between A'_0 and A'_1. Thus, each of the outcomes A_0 and A_1 is equivalent to an appropriate lottery: $A_0 \sim L'(u'_0)$ and $A_1 \sim L'(u'_1)$, for some values $u'_0, u'_1 \in [0, 1]$. For each alternative A, this alternative is equivalent to the lottery $L(u(A))$ in which we get the outcome A_1 with probability $u(A)$ and the outcome A_0 with probability $1 - u(A)$. Here:

- The outcome A_1 is, in its turn, equivalent to the lottery $L'(u'_1)$ in which we get A'_1 with probability u'_1 and A'_0 with probability $1 - u'_1$.
- Similarly, the outcome A_0 is, in its turn, equivalent to the lottery $L'(u'_0)$ in which we get A'_1 with probability u'_0 and A'_0 with probability $1 - u'_0$.

Thus, the original alternative A is equivalent to a complex lottery in which:

- first, we select A_1 with probability $u(A)$ and A_0 with probability $1 - u(A)$;
- then, depending on which the the outcomes A_i we selected, we select A'_1 with probability u'_i and A'_0 with probability $1 - u'_i$.

In this complex lottery, we end up with either A'_0 or with A'_1. The probability of getting A'_1 can be computed as

$$u'(A) \stackrel{\text{def}}{=} u(A) \cdot u'_1 + (1 - u(A)) \cdot (1 - u'_1).$$

Thus, the original alternative A is equivalent to a lottery in which we get A'_1 with probability $u'(A)$ and A'_0 with the remaining probability $1 - u'(A)$. By definition of utility, this means that the value $u'(A)$ is a utility of the alternative A with respect to the fixed outcomes A'_0 and A'_1. The above formula shows that the utility $u'(A)$ is a linear function of the original utility $u(A)$.

Thus, when $A'_0 < A_0 < A_1 < A'_1$, the utilities $u(A)$ and $u'(A)$ are related by a linear dependence. In the general case of pairs $A_0 < A_1$ and $A'_0 < A'_1$, let us define a new pair (A''_0, A''_1):

- as A''_0, we take the worst of the two outcomes A_0 and A'_0 (remember that we are operating under an assumption that a user can always confidently decide which of the two alternatives is better), and
- as A''_1, we take the best of the two outcomes A_1 and A'_1.

In this case, $A''_0 \le A_0 < A_1 \le A''_1$ and $A''_0 \le A'_0 < A'_1 \le A''_1$. Thus, each of the scales $u(A)$ and $u'(A)$ are linearly related to the scale $u''(A)$. We can therefore get from $u(A)$ to $u'(A)$ as follows:

- first, we apply a linear transformation to get from $u'(A)$ to $u''(A)$;
- then, we apply another linear transformation to get from $u''(A)$ to $u'(A)$.

A composition of two linear transformations is also linear, so we conclude that every two utility scales $u(A)$ and $u'(A)$ are related to each other by a linear transformation: $u'(A) = a \cdot u(A) + b$ for some real numbers $a > 0$ and b (we need to have $a > 0$ to preserve the fact that the higher the utility, the better the alternative).

From utility to expected utility. In many practical situations, we need to decide between several possible actions. Let us consider the situation in which we know all possible outcomes $S_1, \ldots, S_n$, and for each action a, we know the probabilities $p_1, \ldots, p_n$ of different outcomes.

Let $u(A_i)$ be the utilities of different outcomes. This means that each outcome S_i is equivalent to a lottery $L(u(S_i))$ in which we get A_1 with probability $u(S_i)$ and A_0 with the remaining probability $1-u(S_i)$. Thus, the action is equivalent to a complex lottery, in which:

- first, we select one of the outcomes $S_1, \ldots, S_n$; each alternative S_i is selected with the corresponding probability p_i;
- then, depending on the selected outcome S_i, we select either A_1 (with probability $u(S_i)$) or A_0 (with the remaining probability $1 - u(S_i)$).

As a result of this complex lottery, we get either A_1 or A_0. The probability of getting A_1 in situation in which we selected S_i is equal to the product $p_i \cdot u(S_i)$. Thus, the total probability of selecting A_1 is equal to the sum $u \stackrel{\text{def}}{=} \sum\limits_{i=1}^{n} p_i \cdot u(S_i)$. Thus, the action a is equivalent to the lottery $L(u)$. This means that the utility of each action is equal to the corresponding sum – and so, when selecting between different actions, we need to select an action for which this sum is the largest.

From the mathematical viewpoint, the sum $\sum_{i=1}^{n} p_i \cdot u(S_i)$ corresponding to each action is simply the mathematical expectation of the utility. So, in these terms, we must select the action for which the expected utility is the largest.

III. Need to Take Uncertainty into Account: from Probabilistic to Interval and Fuzzy Uncertainty

Uncertainty is ubiquitous. In practice, we never know the exact consequences of an action, there is always some uncertainty in our predictions.

Traditional approach to uncertainty: statistical. Certainty means that we know the exact consequence of an action. Uncertainty means that we can have several possible consequences of an action. Based on our prior experiences, we sometimes know that consequences of a certain type were more frequent in the past and consequences of other types were less frequent. If we have a sufficient number of past records, we can determine the frequency with which different types of consequences occurred – and consider these frequencies as god approximations to probabilities (= limit values of these frequencies).

For example, if we are designing a building in a seismic zone for which we have a century of seismic records, we can estimate the probability of earthquakes of different magnitude and thus, take the possibility of these earthquakes into account.

This idea underlies the traditional statistical approach to dealing with uncertainty in science and engineering; see, e.g., [10], [14].

Need for interval uncertainty: Wiener's idea and its current state. Predictions of consequences of different actions are usually obtained by using known formulas for describing the systems' dynamics, starting from the simpler formulas of Newton's mechanics to more complex formulas describing more complex physical phenomena. These formulas use parameters which need to be determined experimentally, based on measurements – and measurements are never 100% accurate. The measurement result $\widetilde{x}$ is, in general, different from the actual (unknown) value of the corresponding physical quantity. Traditional statistical approach to processing measurement uncertainty assumes that we know the probability of different values of measurement error $\Delta x \stackrel{\text{def}}{=} \widetilde{x} - x$.

These probabilities are usually obtained by comparing the results of measurements $\widetilde{x}$ by a current measuring instruments with the results $\widetilde{x}^{\,\text{st}}$ of measuring the same quantity by a much more accurate ("standard") measuring instrument. When the standard measuring instrument is much more accurate, i.e., if for $\Delta x^{\text{st}} = \widetilde{x}^{\,\text{st}} - x$, we have $|\Delta x^{\text{st}}| \ll |\Delta x|$, then we can safely ignore Δx^{st} in comparison to Δx and thus, take the difference $\widetilde{x} - \widetilde{x}^{\,\text{st}} \approx \widetilde{x} - x$ as a reasonable approximation to the measurement error $\Delta x = \widetilde{x} - x$.

Norbert Wiener was the first to notice ([17], [18]) that in many practical situations – e.g., in cutting-edge measurements, when no more accurate measuring instrument is available – we do not know these probabilities. In such situations, at best, we know the upper bound Δ on the measurement error Δx: $|\Delta x| \le \Delta$ (and if we do not even know the upper bound, this means that the value $\widetilde{x}$ is not a measurement, it is a wild guess which can be as far away from the actual value as possible). In this case, once we know the measurement result $\widetilde{x}$, the only think that we can conclude about the actual (unknown) value of the corresponding quantity x is that this value is somewhere in the interval $[\widetilde{x} - \Delta, \widetilde{x} + \Delta]$. This situation is known as *interval uncertainty.*

Another case when we have interval uncertainty is manufacturing. In this case, in principle, it is possible to calibrate every single sensor, but such a calibration is usually several orders of magnitude more expensive than the actual measurement – so it is not done unless really necessary.

In general, we make predictions by using a known relation $y = f(x_1, \ldots, x_n)$ between the desired value y and the quantities $x_1, \ldots, x_n$ which we need to measure. by applying an appropriate algorithm f to the the values of the relevant quantities $x_1, \ldots, x_n$. In practice, as we have just mentioned, we often only know the intervals $[\underline{x}_i, \overline{x}_i]$ that containing the values x_i. Different values x_i from these intervals lead, in general, to different predictions $y = f(x_1, \ldots, x_n)$. In such situations, it is desirable to find the range of all possible values of $y = f(x_1, \ldots, x_n)$ when x_i are in the corresponding intervals. Computation of this range is known as *interval computation*. Interval computations have indeed been very useful in solving many practical problems; see, e.g., [2], [6].

From interval to fuzzy uncertainty. In the traditional statistical approach, we know the probabilities of different values of measurement error Δx. In the interval approach, we only know the range $[-\Delta, \Delta]$ of possible values of measurement error – and we have no information about which values are more frequent and/or more reasonable to expect.

There are two extreme situations: either we have full information about probabilities, or we have no information whatsoever. In many practical situations, while we do not have enough statistics to make definite conclusions, we have an intuitive feeling of which values are more probable and which values are less probable. A natural way to describe this intuitive feeling is to use Zadeh's idea of *fuzzy logic* [20] (see also [3], [9]), where we ask an expert to describe his or her degree of confidence about the possibility of each value $\Delta x \in [-\Delta, \Delta]$ by selecting a number on a scale from 0 to 1, so that:

- the value 0 means that the value Δx is definitely not possible;
- the value 1 means that the value Δx is definitely possible; and
- values between 0 and 1 describe different degrees of confidence.

Such situations are known as *fuzzy uncertainty*.

Interval uncertainty can be viewed as a particular case of fuzzy uncertainty, when an expert assigns the degree 1 to all the values inside the interval $[-\Delta, \Delta]$ (and degree 0 to all the values Δx outside this interval).

IV. How to Deal with Major Uncertainty: Symmetry Idea

Situations with major uncertainty: a problem. When uncertainty is relatively small, we can use interval and fuzzy approaches, and get reasonable results. However, when the uncertainty is large, the traditional interval and fuzzy methods are not always helpful: e.g., if for each possible action a, we have the same huge range of all possible values of the utility u, then which of these actions should we choose?

Symmetry approach: main idea. In situations with major uncertainty, what is often helpful is a *symmetry approach* – whose application to uncertainty can also be largely traced to N. Wiener; see, e.g., [19].

The main idea behind this approach is that if the situation is invariant with respect to some natural symmetries, then it is reasonable to select an action which is also invariant with respect to all these symmetries.

This approach has indeed been helpful in dealing with uncertainty. There have been many applications of this approach. In particular, it has been shown that for many empirically successful techniques related to neural networks, fuzzy logic, and interval computations, their empirical success can be explained by the fact that these techniques can be deduced from the appropriate symmetries; see, e.g., [7]. In particular, this explains the use of a sigmoid activation function $s(z) = \dfrac{1}{1 + \exp(-z)}$ in neural networks, the use of the most efficient t-norms and t-conorms in fuzzy logic, etc.

What we do in this paper. In this paper, we show that the use of symmetry approach can explain the compromise effect.

V. Symmetry Approach Explains the Compromise Effect

Description of the situation. We have three alternative a, a' and a'':

- the alternative a is the cheapest – and is, correspondingly, of the lowest quality among the give there alternatives;
- the alternative a' is intermediate in terms of price – and is, correspondingly, intermediate in terms of quality;
- finally, the alternative a'' is the most expensive – and is, correspondingly, of the highest quality among the give there alternatives.

What do we know about the utility of each alternative. The utility of each alternatives comes from two factors:

- the first factor comes from the quality: the higher the quality, the better – i.e., larger the corresponding component u_1 of the utility;
- the second factor comes from price: the lower the price, the better for the user – i.e., the larger the corresponding component u_2 of the utility.

In the fast experiments which established the compromise effect, the users do not have enough time and/r information to find the corresponding utility values u_i, u_i', and u_i'' corresponding to different alternatives. Also, we do not know how, for each alternative, how the corresponding components u_1 and u_2 are combined into a single utility value characterizing this alternative – we do not even know which of the two components is more important.

Since we do not know how utility components are combined, a reasonable way to represent each alternative is by assigning to it a par consisting of the two component utilities:

- to the alternative a, we assign the pair of values (u_1, u_2);
- to the alternative a', we assign the pair of values (u_1', u_2'); and
- to the alternative a'', we assign the pair (u_1'', u_2'').

We do not know the actual values of the component utilities, all we know is the relative order of the corresponding values: namely, we know that $u_1 < u_1' < u_1''$ and $u_2'' < u_2' < u_2$. Since we do not know the actual values of each utility component, the only we know about each of these values is whether this value is:

- the lowest of the three value; we will denote such a value by L;
- the intermediate (median) value; we will denote such a value by M; and
- the highest of the three values; we will denote such a value by H.

In these terms, we have:

- for the first utility component, $u_1 = L$, $u_1' = M$, and $u_1'' = H$;
- for the second utility component: $u_2 = H$, $u_2' = M$, and $u_2'' = L$.

In these terms, the above description of each alternative by the corresponding pair of utility values takes the following form:

- the alternative a is characterized by the pair (L, H);
- the alternative a' is characterized by the pair (M, M); and
- the alternative a'' is characterized by the pair (H, L).

Natural transformations and natural symmetries. As we have mentioned, we do not know a priori which of the utility components is more important. As a result, it is reasonable to treat both components equally. So, swapping the two components is a reasonable transformation, in the sense that we should select the same of three alternatives before and after swap:

- if we are selecting an alternative based on the pairs (L, H), (M, M), and (H, L),
- then we should select the exact same alternative if the pairs were swapped, i.e., if:
  - the alternative a was characterized by the pair (H, L);
  - the alternative a' was characterized by the pair (M, M); and

- ◦ the alternative a'' was characterized by the pair (L, H).

Similarly, there is no reason to a priori prefer one alternative versus the other. So, the selection should not depend on which of the alternatives we name mark as a, which we mark as a', and which we mark as a''. In other words, any permutation of the three alternatives is a reasonable transformation. For example, if, in our case, we select an alternative a which is characterized by the pair (L, H), then, after we swap a and a'' and get the choice of the following three alternatives:

- the alternative a which is characterized by the pair (H, L);
- the alternative a' is characterized by the pair (M, M); and
- the alternative a'' is characterized by the pair (L, H),

then we should select the same alternative – which is now denoted by a''.

What can be conclude based on these symmetries. Now, we can observe the following: that if we *both* swap u_1 and u_2 *and* swap a and a'', then you get the exact same characterization of all alternatives:

- the alternative a is still characterized by the pair (L, H);
- the alternative a' is still characterized by the pair (M, M); and
- the alternative a'' is still characterized by the pair (H, L).

The only difference is that:

- now, a indicates an alternative which was previously denoted by a'', and
- a'' now denotes the alternative which was previously denoted by a.

As we have mentioned, it is reasonable to conclude that:

- if in the original triple selection, we select the alternative a,
- then in the new selection – which is based on the exact same pairs of utility values – we should also select an alternative denoted by a.

But this "new" alternative a is nothing else but the old a''. So, we conclude that:

- if we selected a,
- then we should have selected a different alternative a'' in the original problem.

This is clearly a contradiction:

- we started by assuming that, to the user a was better than a'' (because otherwise a would not have been selected in the first place), and
- we ended up concluding that to the same user, the original alternative a'' is better than a.

This contradiction shows that, under the symmetry approach, we cannot prefer a.

Similarly:

- if in the original problem, we preferred an alternative a'',
- then this would mean that in the new problem, we should still select an alternative which marked by a''.

But this "new" a'' is nothing else but the old a. So, this means that:

- if we originally selected a'',
- then we should have selected a different alternative a in the original problem.

This is also a contradiction:

- we started by assuming that, to the user a'' was better than a (because otherwise a'' would not have been selected in the first place), and
- we ended up concluding that to the same user, the original alternative a is better than a''. This contradiction shows that, under the symmetry approach, we cannot prefer a''.

We thus conclude that out of the three alternatives a, a', and a'':

- we cannot select a, and
- we cannot select a''.

This leaves us only once choice: to select the intermediate alternative a'. This is exactly the compromise effect that we planned to explain.

Conclusion. Experiments show when people are presented with three choices $a < a' < a''$ of increasing price and increasing quality, and they do not have detailed information about these choices, then in the overwhelming majority of cases, they select the intermediate alternative a'.

This "compromise effect" is, at first glance, irrational: selecting a' means that, to the user, a' is better than a'', but in a similar situation when the user is presented with $a' < a'' < a'''$, the same principle would indicate that the user will select a'' – meaning that a'' is better than a'.

In this paper, we show that, somewhat surprisingly, a natural symmetry approach explains this seemingly irrational behavior.

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