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Development Of Metaheuristic Algorithms For The Efficient Allocation Of Power Flow Control Devices

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DEVELOPMENT OF METAHEURISTIC ALGORITHMS FOR THE EFFICIENT ALLOCATION OF POWER FLOW CONTROL DEVICES

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DEVELOPMENT OF METAHEURISTIC ALGORITHMS FOR THE EFFICIENT ALLOCATION OF POWER FLOW CONTROL DEVICES

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Chapter 1: Introduction

1.1. Background Information

The American electric grids are facing increasing issues, from the now yearly phenomenon of wildfires caused by fallen PG&E lines in California to the weeks-long outages in Texas in early 2021. While some of these issues are attributable to unexpected weather conditions, other issues such as congestion occur as supply and demand grow. Transmission congestion is a major issue in U.S. power grids. While there have been significant investments in congestion reduction (around US$40 billion in 2018), the congestion costs are still measured in the billions of dollars each year (US DOE, 2020). As a way of mitigating the problem, variable-impedance series flexible AC transmission systems (FACTS) can help provide effective power flow control as part of smart transmission systems (Li, et al., 2010). FACTS devices can thus help improve the utilization of an existing network and provide a more reliable and sustainable power delivery network (Gotham & Heydt, 1998).

As an extension of the FACTS devices, and in order to improve deployability, Distributed FACTS (D-FACTS) are a lightweight version of FACTS. These have lower costs and possess the capacity of being re-allocated throughout their life to better respond to shifting needs. While traditional FACTS devices are installed at buses, D-FACTS can be installed throughout transmission lines or towers in a modular fashion (Sang & Sahraei-Ardakani, Effective power flow control via distributed FACTS considering future uncertainties, 2019). Thanks to these properties, D-FACTS devices are slowly becoming popular solutions to reduce line congestion in numerous electric grid improvement projects throughout the country (Kakkar & Agarwal, 2010). The capacity of FACTS devices to better integrate renewable energies into new grids has been previously demonstrated as detailed by Gandoman, et al. (2018). D-FACTS, however, haven't been studied in such detail, although the expectation is that they will similary be very useful in the integration of renewable energies into existing grids.

1.2. Research Objective

Although arguably more versatile and effective, the allocation of D-FACTS modules rather than traditional FACTS devices introduces nonlinearities to the model which can be computationally exhausting to solve (Sang & Sahraei-Ardakani, 2018). Thus, the challenge is now not necessarily to optimally allocate the modules, but to do so in a computationally-efficient fashion. One of the main objectives of this research is to create algorithms which can allocate the modules in a quick, computationally-efficient way in order to optimize one or more objectives.

For this purpose, metaheuristic algorithms will be used in conjunction with other exact and heuristic approaches in order to minimize the computational time. After all, the benefits and applications of D-FACTS and FACTS devices have already been thoroughly proven (Gandoman, et al., 2018), so what is crucial now is to improve the optimization algorithms in order to hasten the allocation and implementation of these new technologies.

Thus, the main objective of this work is to create effective and efficient metaheuristic algorithms to optimally allocate D-FACTS modules on transmission systems based on improving one or more objectives to be studied, including operational costs and environmental impact metrics. These objectives may be optimized individually, that is, one at a time and with different results for each, or concurrently, at which point the allocation becomes a multiple-objective problem. There are multiple procedures for solving a multi-objective problem, such as using a utility function or with methods such as the NSGA algorithms. For this study, a non-dominated Pareto-optimal approach over Multiple Objective Evolutionary Algorithms will be implemented.

1.3. Scope and Limitations

The present work will analyze a number of electrical test systems, some of which are IEEE test systems. The systems may have undergone some slight modifications to be more suitable for studying the effects of D-FACTS modules on the system. If modifications are made, they will be specified and detailed as needed. Stochasticity is added to the systems in the forms of load scenarios, renewable energy capacity scenarios, and generation scenarios with the purpose of demonstrating the flexibility of the devices under different operating conditions.

This work aims to minimize total operational costs over the multiple scenarios for each case study, resulting in an increased profit for the relevant utility companies. Additionally, environmental impacts, in the form of Global Warming Potential (GWP) and Human Toxicity Potential (HTP) are to be minimized, resulting in better living conditions for surrounding communities. Other objectives, such as renewable energy integration will also be considered.

For the optimization process, this work will focus on the combination of evolutionary and linear programming algorithms to minimize computation time and improve the quality of the solutions. However, the scope of this research is limited to the generation of Pareto-Optimal solutions in multi-objective case studies, as the selection of the ideal solution is best left for an experienced decision-maker. Still, some methods for pruning the Pareto Set will be studied. In the case of single-objective optimizations, sensitivity analyses will be conducted on relevant variables.

The problems are studied under the assumption of static fuel costs and static investment costs and limits during an optimization run. Costs and emissions associated with energy generation and device installation are obtained from test systems data and not modified for the study.

1.4. Dissertation Outline

The remainder of this work will be structured as follows:

Chapter 2 will consist on a comprehensive literature review, to include topics such as FACTS and D-FACTS devices, optimization algorithms with an emphasis on those commonly used for FACTS and D-FACTS allocation, multi-objective methods, and relevant mathematical models.

Chapter 3 will show previous publications by the author in the field, while chapter 4 will include mainly the mathematical formulations to be used in the study, including formulations found in research, for illustrative purposes. Chapter 5 will carefully detail the optimization algorithm used in the case study presented in chapter 6. Finally, chapter 7 provides some concluding remarks as well as future work directions.

Chapter 2: Literature Review

2.1. FACTS AND D-FACTS

Flexible AC Transmission Systems (FACTS) and Distributed FACTS (D-FACTS) are thyristor-based controllers designed to manage series impedance, shunt impedance, phase angle, or some other parameter in electric transmission systems (Hingorani, 1993). Some of the most common types of FACTS devices are: Static Var Compensator (SVC) which are used to control the voltage of electric power systems; Thyristor Controlled Series Capacitor (TCSC) which is used to increase transfer capacity and system stability; Static Synchronous Series Compensator (SSSC), which is used for power transmission series compensation as a source of synchronous voltage; and Unified Power Flow Controller (UPFC), which can be used for enhancing steady state, dynamic, and transient stability (Murali, Rajaram, & Reka, 2010).

It has been repeatedly demonstrated that the installation of FACTS devices can not only improve the stability of the transmission networks, but also reduce operational costs and open the possibility for increased sales by utilities (Habur & O'Leary, 2004). They can also be installed with the more specific objectives of congestion relief and voltage stability (Wibowo, Yorino, Eghbal, Zoka, & Sasaki, 2011), in order to integrate different energy sources into the grid (De Oliveira, Marangon Lima, & De Almeida, 2000), or in order to improve security in the network (Yorino, El-Araby, Sasaki, & Harada, 2003).

2.1.1. FACTS Allocation

The allocation of traditional FACTS has been thoroughly studied and algorithms for this purpose include Particle Swarm Optimization (PSO) (Jordehi, 2015), with some studies optimizing not only location but also device type and settings (Chansareewittaya & Jirapong, 2014). Additionally, the aspect of the network being optimized can vary from maximizing voltage stability during outages (Srivastava, Dixit, & Agnihotri, 2014), optimizing power system loadability and minimizing installation costs (Malathy, Shunmugalatha, & Thaineesh, 2015), and total operation and installation cost (Mohamed, Rama Rao, & Hasan, 2010). Genetic Algorithms (GA) have also been used in order to minimize cost (Cai, Erlich, & Stamtsis, 2004), simultaneous maximization of system security and minimization of installation costs (Radu & Besanger, 2006), optimization of branch loading, voltage stability and loss minimization (Surender Reddy, Sailaja Kumari, & Sydulu, 2010), power system security (Baghaee, *et al*. 2008a), among other objectives. Other optimization methods include Khan *et al*'s (2021) modified lightning attachment procedure optimization (MLAPO), a fairly novel metaheuristic algorithm; and the Firefly algorithm for reducing power loss, voltage deviations, fuel costs, and branch loading (Gundavarapu & Bathina, 2015). The results are fairly promising in reducing power losses and achieving quick convergence. Further details on optimization algorithms and methods are described in section 2.2.

2.1.2. D-FACTS Allocation and Benefits

Distributed FACTS, or D-FACTS, are a smaller, light-weight version of traditional FACTS. D-FACTS were proposed in 2005 with the objective of dealing with some of the obstacles that traditional FACTS have for deployment, namely the investment cost, space requirements, system stress, and reliability requirements (Divan & Johal, 2005). They have the added conveniences of being modular and re-deployable, not needing large spaces for installation at each bus, but their adjustance ranges are lower and the computational burden to optimally allocate them is larger due to the added variable of how many to allocate. Still, the potential economic benefit of D-FACTS is larger when compared to traditional FACTS (Sang and Sahraei-Ardakani 2018), not to mention the long-term benefit of re-deployability, which has not yet been studied in research, but promises reduced costs if a situation arises in which the network configuration changes and re-allocation becomes necessary.

The use of FACTS and D-FACTS devices also helps the integration of renewable energy sources into the power grid. Analysis has shown that FACTS devices can improve voltage profile at buses and reduce power loss in lines (Suresh & Sreejith, 2017). Smart grids are using FACTS devices in order to improve power quality levels (Liao, Abdelrahman, & Milanović, 2016). It is estimated that by 2050 20-25% of energy in global grids will come from renewable sources such as solar, wind, etc. (Jha, Bilalovic, Jha, Patel, & Zhang, 2017). As such, power flow control devices will become more relevant in managing distribution networks and grid congestion. More than FACTS, D-FACTS are more attractive control devices to dynamically manage voltage, reactive power, and power quality (Gupta and Kumar 2016; Gaigowal and Renge 2016). It is important to note that due to the uncertainty that comes with the incorporation of renewable energy sources, precise and dynamic management of microgrids becomes more important still. D-FACTS-based green plug-switched filter capacitor filters have been developed to improve the energy use and dynamic voltage stabilization of wind energy-connected systems with load changes and temporary fault conditions, promising high-speed controllability and maintained power factor correction capability (Gandoman, Sharaf, Abdel Aleem, & Jurado, 2017). FACTS and D-FACTS technologies thus play a key role in the improvement of energy management as grids transition towards smart, dynamic control schemes (Gandoman, et al., 2018).

2.1.3. Mathematical Optimization Models for FACTS and D-FACTS

Similarly to the objectives being optimized, the formulations for the optimization models have also varied greatly between studies. Some studies have focused more on studying only what happens at transmission lines, thus disregarding some other aspects of the transmission systems such as spinning reserves or even costs, while other studies may be more interested in testing the integration of renewable energies, omitting things such as generator or line reliability.

Elmetwaly *et al*. (2020) modeled the integration of Adaptive Switched Filter Compensator (ASFC) and D-STATCOM type devices into a microgrid to improve power quality, specifically harmonic distortion and voltage stability in renewable energy sources, considering only the power sources and battery banks in their constraints as that was their focus of study, which had only 5 constraints in their optimization plus an objective function, with most of the article being devoted to describing the renewable energy sources and the specifications used in the devices for their simulation. On the other hand, the study by Sang and Sahraei-Ardakani (2019) aimed to minimize total system operating costs considering power reserves and multiple stochastic scenarios, which needed to consider constraints for transmission capacities and voltage stability limits, thus resulting in a model with 21 constraints and an objective function, which resulted in a fairly realistic model for how a transmission network may operate.

Ultimately, the number of constraints in the model serves mainly to determine the computational burden that can result from using a solver and is no direct reflection of the quality of the model. The number of constraints does also help to estimate how many elements are being considered into the optimization but as every problem is different so will every study have a different formulation.

2.1.4. Types of D-FACTS devices

Some of the most common types of D-FACTS are the following: Distributed Static Compensator (D-STATCOM), which is useful for voltage regulation, compensation of current harmonics, control of reactive power, and uninterrupted supply from storage devices (Divan & Johal, Distributed FACTS - A New Concept for Realizing Grid Power Control, 2005); Distributed Static Series Compensator (DSSC), which allows for control of active line power flow, are smaller and cheaper than other types of devices, and help minimize real power losses (Divan, et al. 2004; Divan, 2005); Distributed Thyristor Controlled Series Compensator (D-TCSC), which helps improve system stability and development of cyber-secure control methods as well as controlling system voltages (Gandoman, et al., 2018); Distriubted Series Impedance (DSI) which can adjust line impedance for improving power flow (Divan & Johal, 2005); and Distributed Power Flow Controller (DPFC), a distributed version of the Unified Power Flow Controller (UPFC), which can control all parameters in a network including line impedance, power angle, and voltage magnitude, with the added advantage of lower installation and maintenance costs and much higher reliability (Yuan, de Haan, & Ferreira, 2007). Out of these, DSI type devices will be used as the focus of the research, as the devices will be used to modify the line impedances to improve transmission capacity in order to optimize the various objectives studied.

2.2. Optimization Methods in FACTS and D-FACTS Allocation

The optimal allocation of indivisible items with connectivity constraints is considered to be at least an NP-hard problem (Igarashi, 2019). While there is no research studying the computational complexity of FACTS or D-FACTS allocation, it can be deduced based on the problem formulation that the computational time cannot be easily estimated on a polynomial time scale, and so we assume that the FACTS and D-FACTS allocation problem to also be at least NP-hard, if not NP-complete. Additionally, FACTS and D-FACTS allocation methods generally allocate the devices based on one of the following methods based on the objective to optimize: Sensitivity-based methods, cost-benefit analysis-based methods, voltage security margin-based methods, and optimization-based methods (Gupta & Kumar, 2019). While formulations such as linear programming or mixed-integer programming can be used to optimally solve complex optimization problems, they are faced with the drawback of large computational times. A popular alternative to reduce the computational burden is the use of heuristic and metaheuristic optimization methods. These may not be capable of guaranteeing an optimal solution due to their nature, but the solutions found are almost always at least very close to the true optimums. This section will focus on reviewing some of the most common optimization methods for FACTS and D-FACTS allocation problems, their base formulation, and modifications used in case studies.

2.2.1. Linear Programming

Linear programming (LP) is a method of achieving an optimal solution from a mathematical model which is expressed in linear relationships. While similar methods date back to Fourier, modern interpretation of linear programming are mainly attributed to George Dantzig, who designed the simplex method in 1947 (Chvatal & Chvatal, 1983). The standard form of a linear programming problem is expressed as follows:

$$\max\{\boldsymbol{c}^T\boldsymbol{x} \mid \boldsymbol{x} \in \mathbb{R}^n \wedge \boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b} \wedge \boldsymbol{x} \geq 0\}$$

Where **c** represents the vector of cost coefficients, **x** represents the vector of variables to be optimized, and **A** and **b** are the constraint coefficients and right-hand-sides in matrix/vector form.

In addition to linear programming, there are integer programming (ILP) and mixed-integer programming (MILP), where all or some of the variables in **x** are also constrained to integer space, increasing the computational complexity of the problem.

Many of the LP-based approaches to optimal FACTS and D-FACTS allocation use mixed-integer programming. Sahraei-Ardakani and Hedman's 2015 study proposed a mixed-integer reformulation of the nonlinear program in order to make the problem computationally solvable when optimizing FACTS allocation to improve system transfer capacity. MILP has also been used in order to optimize allocation and settings of FACTS devices to maximize system loadability in large networks, with simulations for networks up to 904 buses (Lima, Galiana, Kockar, & Munoz, 2003). Other authors chose to linearize the allocation problem into a standard LP formulation to relieve overloads and voltage violations (Shao & Vittal, 2006). Overall, it can be argued that different types of LP formulations are effective in solving problems in FACTS and D-FACTS allocation as well as Optimal Power Flow (OPF) problems. However, the fact remains that these are at least NP-hard problems, and thus exact optimization methods such as this are not very efficient in finding a solution.

2.2.2. Branch and Bound Method

Branch and Bound is a heuristic search method. It was proposed in 1960 by Ailsa Land and Alison Doig as a method for solving discrete programming problems (Land & Doig, 1960). In essence, the algorithm consists of an enumeration of possible solutions in the form of a tree with the full set of possible solutions at the root and subsets at the branches. The algorithm then explores the branches and discards them based on their upper or lower bounds.

Due to the difficulty in finding upper and lower bounds for solutions of complex problems, branch and bound is not always a very popular choice for some combinatorial problems. However, it has been used in combination with Mixed-Integer Non-Linear Programming (MINLP) to find optimal allocation of SVC devices based on an Optimal Reactive Power Flow (ORPF) model. The study used branch and bound to restrict the solution space and then solved each sub-problem using the MINLP. By branching the problem, it was then possible to reduce the computational time of each solution (Alves Silva & Belati, 2016).

2.2.3. Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a metaheuristic search method. Metaheuristics are high-level procedures designed to find or generate a lower-level search method which may provide a good solution to an optimization problem with limited information or resources (Bianchi, Gorigo, Gambardella, & Gutjahr, 2009).

Particle Swarm Optimization is a popular search algorithm thanks to its simplicity. It was originally proposed by Kennedy and Eberhart in the 1995 IEEE International Conference on Neural Networks. Its basis is to search the solution space by having a set of solutions (particles) which are, initially, randomly placed across it, and which then move across it based on simple mathematical formulas, converging on an optimum. Particle Swarm Optimization has been commonly used for FACTS and D-FACTS optimization, since it is a very convenient method for graphics-based problems. Since the allocation of power flow control devices can be represented graphically, PSO is an easily applicable. Some of the applications of PSO in the allocation of FACTS devices are covered by (Jordehi, 2015).

In addition to the examples provided in section 2.1, PSO has been used in optimizing location and size of STATCOM-type devices performing sensitivity analysis on the inertia weight of the optimization process (Ravi & Rajaram, 2013). Multi-objective PSO is used in the form of a non-dominated sorting PSO to provide pareto fronts in the allocation of TCSC and SVC units (Sedighizadeh, Faramarzi, Mahmoodi, & Sarvi, 2014). Another modification to the PSO algorithm applied to this field is an Enhanced Leader PSO (ELPSO) algorithm for allocating distributed TCSC devices (Resaee Jordehi, Jasni, Abd Wahab, Kadir, & Javadi, 2015). The key advance in the ELPSO algorithm is a mutation strategy for the "swarm leader" or the solution with the best objective function value, with the purpose of moving this leader out of what may be a local optimum. Overall, it has been found that the application of PSO methods highly improves the computational time of power flow control optimization problems while still yielding solutions that are close enough to the real optimum that any difference is negligible.

2.2.4. Genetic and Evolutionary Algorithms

Genetic Algorithms (GAs) were first proposed by J. H. Holland in his 1975 publication "Adaptation in Natural and Artificial Systems." They were designed to mimic natural evolutionary processes by manipulating potential solutions in an optimization problem. Their basic procedure is to initialize a population of possible solutions, evaluate them, and, while the termination criteria is not reached, select solutions for the next population and perform crossover and mutation before evaluating this new population (Srinivas & Patnaik, 1994).

Genetic algorithms have also been used (although not as much as PSO methods) in solving power flow control problems with specific objectives in mind via the implementation of FACTS and DFACTS devices. Baghaee *et al*. (2008b) used GAs to allocate multi-type FACTS devices (specifically TSCS, SVC and UPFC types) in order to improve voltage stability and reduce losses across the IEEE 30 bus system, considering multiple scenarios, showing that installation of multiple types of FACTS devices do improve both of their objectives, although economic aspects were not considered. Fuzzy logic-based approaches in which some uncertainty exists for some parameters is used by Phadke, Fozdar, and Niazi (2012) to also allocate FACTS to optimize maximum distance to saddle-node bifurcation and minimum voltage deviation, applying their method of adding shunt compensation to the weakest bus over the IEEE 14-bus and 57-bus systems.

Other GA-inspired algorithms such as the Bees Algorithm is also used for optimal FACTS allocation in deregulated markets by Idris, Khairuddin, and Mustafa (2009) in order to optimize available transfer capacity considering TCSC, SVC, UPFC, and TCPST-type FACTS devices, comparing both the bees algorithm and traditional GAs, finding both algorithms are able to find the optimal solution and settings of the devices, although the bees algorithm has a slight advantage on convergence speed. Similarly, Cai, Erlich, and Stamtsis (2004) also studied optimal allocation of FACTS in deregulated markets using GAs, but focused on minimizing costs, with their findings falling in line with other studies and showing that the installation of FACTS devices can reduce total system costs. Genetic Algorithms are widely used in optimization methods thanks to their fast convergence speed and ease of use.

The advantages of GAs as well as many of their applications are described and detailed by Vikhar (2016), who described the key advantages as:

- Being conceptually simple and flexible
- Uses prior information (considers already-known data)
- Is independent of the numeric representation of the formulation
- Can use parallel processes within each iteration
- Are more robust and can adapt the solution to a changing environment
- Does not require human expertise

Still, there are some drawbacks to the use of genetic algorithms, namely that, like every other metaheuristic, it has a risk of converging towards local optima in the search space and thus not finding the most optimal solution.

2.3. Multiple Objective Optimization Methods

For years after the formulation of traditional single-objective optimization methods, problems were solved by one objective at a time, the objective being cost minimization or reliability maximization. However, single objective optimization is not a very realistic framing for most problems as improving one parameter will usually worsen others. While for most problems considering only one objective can reduce the complexity and computational burden, it also reduces their capacity of describing and explaining many of the nuances that would be present in real-world problems. This reasoning resulted in the understanding that single-objective optimization is no longer suitable for solving problems, and the framework for multiple-objective optimization was laid down, as it allows for more information to be taken into account (Caramia & Dell'Olmo, 2008).

As so, a basic single-objective problem can be formulated as follows:

$$\min\{f(x)\}$$

$$subject\ to\ x \in S$$

Where f is a scalar function and S is the set of constraints.

While a single-objective optimization problem is expressed by a single objective function, a multi-objective problem is expressed by a number of objective functions, as well as several equality and inequality constraints. The notation for this can be written as follows:

$$\min f_i(x)$$

$$subject\ to:$$

$$g(x) \leq 0$$

$$h(x) = 0$$

Where $f_i(x)$ stands for each of the objective functions ($i = 1, \ldots, n$) that is being optimized, and the set of constraints $g(x)$ and $h(x)$ define the feasible region, with the decision variables represented by x.

Several methods exist for solving both single and multiple-objective optimization problems. Given that single-objective optimization methods are fairly straight-forward and some have already been discussed, the remainder of this sub-section will be dedicated to discussing some of the methodologies used for solving multi-objective optimization problems. These can be classified in two main groups based on whether the expected output is a single solution or a set of solutions in the form of a non-dominated Pareto set.

2.3.1. Single Solution Approaches

The first set of methods involve combining all objective functions into a singular aggregated objective function, essentially transforming the problem back into a single-objective problem in order to simplify the algorithm. While this type of approach has the benefit of being very straight-forward in its implementation and less intensive in computational burden, it suffers from the drawback of not allowing as much exploration of possible solutions compared to Pareto-optimal methods.

Goal Programming

The Goal Programming approach was first developed by Charnes *et al*. in 1955. The aim was to find specific goal values for each of the objective functions considered in the problem. In their method, each objective function *F(x)* is assigned a goal b_j. The aggregated objective function becomes reducing the total deviation from the goals, d_j. The deviations are separated into positive and negative values, to consider under and overachievement, where achievement or zero values imply reached goals. The optimization problem is then formulated as follows:

$$\min_{x \in Z, d^-, d^+} \sum_{i=1}^{k} (d_i^+, d_i^-)$$

$$Subject\ to:$$

$$F_j(x) + d_j^+ - d_j^- = b_j$$

$$d_j^+, d_j^- \geq 0$$

Where j = 1, …, k. This approach can simplify a multi-objective problem and is useful if the solver's objective is to approximate a specific result, but it does not actually guarantee a Pareto-optimal solution and can become more computationally exhausting in larger problems due to the increased number of variables. Some variations to this method include Weighted Goal Programming, Preemptive Goal Programming, Multi-Goal Programming, and Goal Attainment Method.

Weighted Sum or Scalarization

The Weighted Sum method is one of the most common approaches to multi-objective optimization. This strategy converts a multi-objective problem into a single-objective one by constructing a weighted sum *F(x)* of all objectives in the vector of criteria functions (Marler & Arora, 2010). More specifically, the Weighted Sum method minimizes a positively weighted convex sum of all objectives, and can be represented as follows:

$$F(x) = \sum_{i=1}^{n} w_i f_i(x)$$

Where n represents the total number of objectives I and wi their respective weights. An advantage to this method is its simplicity, transforming a multi-objective problem into a single-objective one by simple addition, while also allowing the decision maker to assign priority to the objectives through the weights. Its disadvantage, however, lies precisely in assigning the weight coefficients, which can be difficult to choose. Determining the adequate weights for this type of problem has been the subject of much research, with some researchers arguing that weight functions be more appropriate for better representing preferences. Some approaches to weight assignment include ranking, categorization, rating, and eigenvalues.

Lexicographic Method

Yet another way to address multiple objectives is through the lexicographic approach proposed by Fishburn (1974). This method requires the decision maker to establish a priority for each objective. In this method, the objective functions are arranged in order of importance, and solutions are compared in respect to the most important objective. In the event of a tie, the next most important functions are compared, and so on until there are no ties or objectives remaining. This method can be described with the following formulation:

$$\min x \in X \;\; F_i(x)$$

$$subject\ to$$

$$F_j(x) \leq F_j x * j$$

$$j = 1, 2, \dots, i-1; i = 2, 3, \dots, k$$

Where i represents a function's position in the preferred sequence, and $F_j\ x{*}j$ represents the optimum of the j^th^ objective function, found in the j^th^ iteration. After the first iteration (where j=1), $F_j\ x{*}j$ is not necessarily the same as the independent minimum of $Fj(x)$, since new constraints have been introduced from the results of the previous iteration.

Multi-Attribute Utility Theory

In this case, utility refers to the satisfaction that each attribute or objective function provides to the decision maker. This way, the utility theory approach assumes that any decision is made on the basis of the utility maximization principle. This principle suggests that the best choice is the one that would provide the most satisfaction to the decision maker. In multi-attribute utility analysis, the total utility of a design solution is a scalar on the interval between 0 and 1, where 0 represents no utility and 1 represents the highest possible utility (Dyer, 2005). According to utility theory, if X_i is the measure of satisfaction provided by an attribute *I*, and there are *n* attributes, then the joint utility function for all attributes can be expressed as:

$$U(X_1, X_2, \dots, X_n) = f\big(U_1(X_1), U_2(X_2), \dots, U_n(X_n)\big)$$

In this case, $U_i(X_i)$ is the utility of the i^{th} attribute. The overall utility function is the sum of all individual utilities if the attributes are independent. This function can be expressed as

$$U(X_1, X_2, \dots, X_n) = \sum_{i=1}^{n} U_i(X_i)$$

And after assigning weights to each attribute, the function becomes

$$U(X_1, X_2, \dots, X_n) = \sum_{i=1}^{n} W_i U_i(X_i)$$

While utility optimization is effective and widely used, it requires extensive interviews with the decision maker to determine appropriate utility functions and weights. Once the utility function has been constructed, however, optimization can occur and the design alternative with the maximum utility can be determined.

ε-Constraint Approach

This procedure originally proposed by Chankong and Haimes (1983) overcomes some of the convexity problems that the Weighted Sum technique faces. In this approach, the decision maker chooses one objective to be optimized, and constraints the remaining objective to be within a target range (equal or less than a target in a maximization objective, or equal or larger than a target in a maximization objective). For example, in a bi-objective minimization problem, the problem is divided into two problems, $P_1(\varepsilon_2)$ and $P_2(\varepsilon_1)$, which are the following:

$$\min f_1(\vec{x})$$
$$subject\ to$$
$$\vec{x} \in X$$
$$f_2(\vec{x}) \leq \epsilon_2$$

And

$$\min f_2(\vec{x})$$
$$subject\ to$$
$$\vec{x} \in X$$
$$f_1(\vec{x}) \leq \epsilon_1$$

Respectively.

Contrary to other aggregation methods, the ε-constraints approach is able to identify a number of non-inferior solutions within a nonconvex boundary. However, it has the drawback of its hard constraints not being adequate for representing real design objectives.

2.3.2. Pareto-based Optimization Approaches

In Pareto-optimality-based approaches, there is no single optimal solution, but instead a set of non-dominated alternative solutions. These solutions are considered “Pareto optimal”, since none of the other solutions are dominated by other solutions. The concept of dominance in optimization means that there is no other solution in the feasible region which is quantifiably better than the pareto-optimal solutions in all the objectives considered. This makes it possible for there to be tradeoffs between objectives from the decision-maker’s point of view (Zitzler & Thiele, 1998; Zitzler *et al.*, 2002)

These optimal solutions are called Pareto-optimal solutions and the set of these is denoted as the Pareto-optimal set. Figure 2.1 shows an illustration of the terminologies in Pareto-based optimization:

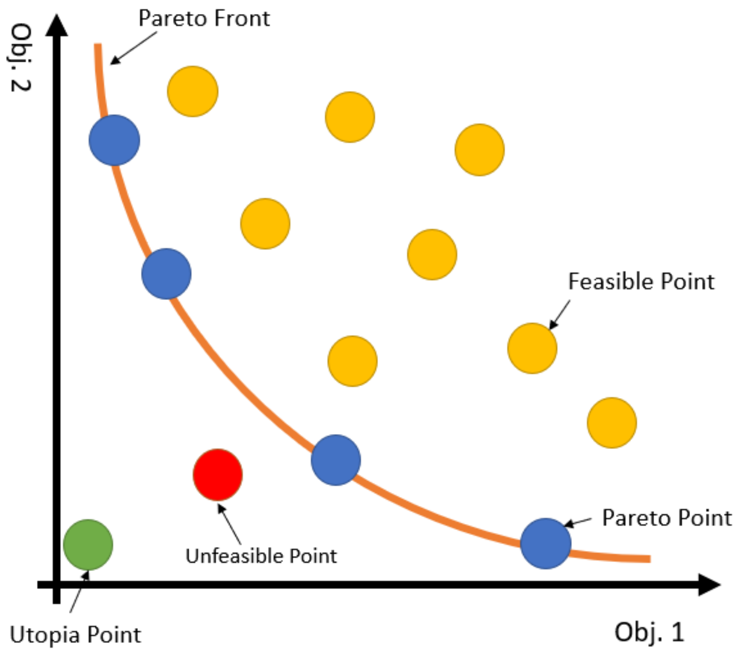

Figure 2.1: Pareto-based optimization illustration and terms

In the figure above, each axis represents an objective being minimized, with an ideal or utopia solution marked in green, an unfeasible solution in red (where the objective function values would not be achievable under existing constraints), pareto-dominant or nondominated solutions in blue over the orange line representing the Pareto-front, and feasible but non-optimal solutions in yellow.

As an example of conflicting objectives, such as when maximizing an objective involves increasing another that we want to minimize, we could consider a situation in which cost and efficiency are to be optimized. Cost is an objective that is almost always being minimized, while efficiency is almost always maximized. However, it is also true that increasing efficiency involves increasing costs, creating a conflict in the objectives. The final decision will rely on which of the two objectives is more important to the designer.

Formulating, analyzing, and solving problems with conflicting objectives usually requires a decision maker to express preference relations between alternative solutions. The decision maker must have some expertise in the field for which the problem is solved, as well as knowledge on the resulting set of solutions, in order to make an educated choice as to which solution would be best, as well as to give weights to the objectives being evaluated. The Pareto-optimal set can help reduce the design alternatives from a feasible region into optimal trade-offs (Yancang, Lina, & Shujing, 2010).

Multiple-Objective Evolutionary Algorithms

Optimizing multi-objective problems can be a challenging task, since one of the characteristics of these problems is that the objectives tend to conflict with each other, and the multi-dimensional search space tends to be very complex. As a solution, researchers have proposed several different models to obtain Pareto-Optimal solutions. Much like in single-objective optimization, evolutionary algorithms are some of the more popular models for solving these types of problems due to their capacity to be adapted to different types of problems.

Genetic Algorithms (GAs) optimize a desired objective by altering its encoded variables. Comparing it to biological evolution, the solutions arise from a set of possible “genetic” sequences. Hence, the best solutions result from organisms that were able to survive and reproduce within the environment, which is to say, the solutions with the best objective function values. Genetic Algorithms are a variety of Evolutionary Algorithms (EAs), which apply techniques inspired by evolutionary biology such as inheritance, mutation, and crossover or combination. A set of random solutions represented by a data structure is generated. In technical GA terms, these solutions are considered the chromosomes that constitute the individuals in a population. The chromosomes consist of a sequence of genes, or specific data characteristics which will be used during the evaluation of the objective function or fitness value for the individual, and can be in the form of bits, digits, or letters (Kumar, Hussian, Upreti, & Gupta, 2010).

A genetic algorithm simulates the best individuals in successive generations, where a set of individuals composes each generation's population. Each generation's population is evaluated to continually identify the best solutions. The general methodology followed in the formulation of a genetic algorithm is showed below in figure 2.2. The process for a genetic algorithm will be more thoroughly explained in section 5.2.

In multiple objective optimization, Evolutionary Algorithms follow similar processes as Genetic Algorithms. Still, all evolutionary algorithms have different techniques when attempting to achieve diversity in their population set. In multiple-objective evolutionary optimization, it is crucial to achieve diversity in their population, as it means that it's possible to deliver a varied set of alternatives to the decision maker. For this, different fitness assignment methodologies have been explored. Pareto-based approaches can be divided into elitist and non-elitist approaches, depending on whether the stored non-dominated solutions are included in the crossover process. Some of the most common algorithms are presented in the following subsections.

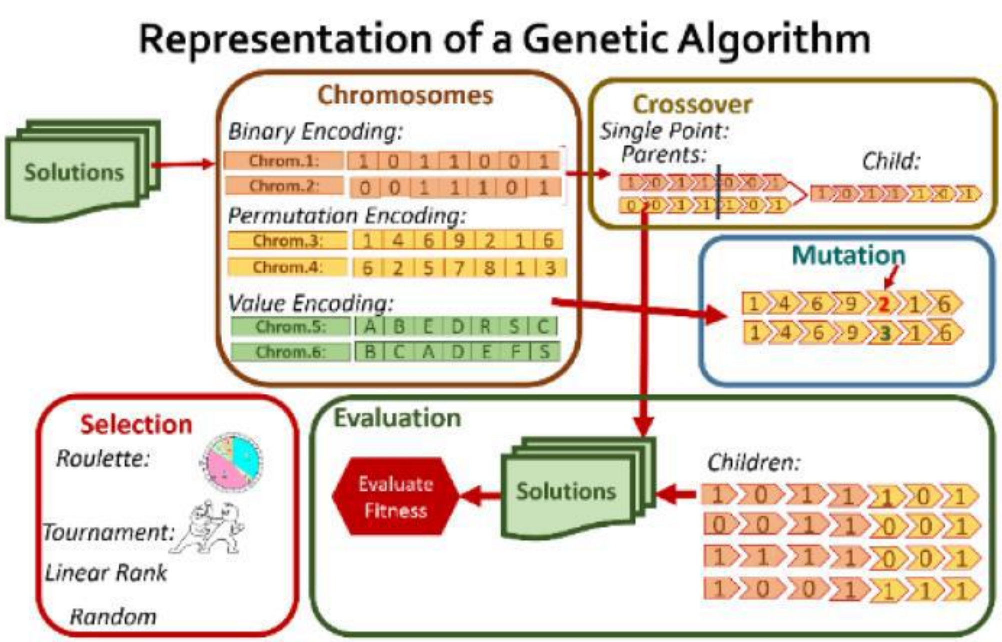

Figure 2.2: Graphic representation of a GA (Delgado, 2016)

2.4. PARETO-BASED NON-ELITIST APPROACHES

1. Multiple Objective Genetic Algorithm (MOGA) in an approach in which individuals are assigned ranks corresponding to the number of individuals in the current population by which they are dominated. The non-dominated individuals are ranked and ties are averaged to maintain the sampling rate (Murata & Ishibuchi, 1995).
2. Niched Pareto Genetic Algorithm (NPGA) proposed by Horn et al. in 1994, has a Pareto dominance-based tournament selection with a sample of the population to determine the winner between two candidate solutions. A subset of individuals is used to determine dominance between the two solutions competing, and the non-dominated individual is selected for reproduction. In the event of a tie, the winner is decided through fitness sharing.
3. Non-Dominated Sorting Genetic Algorithm (NSGA) in an approach in which all non-dominated individuals are classified into one category, and collectively assigned a fitness value proportional to population size. This group is removed, and the remaining population is repeatedly classified until all the population has been classified (Deb, Pratap, Agarwal, & Meyarivan, 2002)

Pareto-based Elitist Approaches

1. Strength Pareto Evolutionary Algorithm (SPEA), developed by Zitzler and Thiele in 1999, has many similarities with other algorithms in terms of storing previously-obtained Pareto-optimal solutions, as well as the use of the dominance concept and the use of clustering to reduce the number of stored solutions. However, what makes it stand apart from the rest is the use of all three of these concepts in a single algorithm. It determines a fitness function out of the stored solutions, avoiding dominance from the existing population, and uses all the stored Pareto solutions for selection

2. Strength Pareto Evolutionary Algorithm 2 (SPEA2), formulated as an improvement of the original SPEA by Zitzler et al. in 2001. This iteration of the algorithm intends to avoid the situations in which individuals in the population can have the same fitness value according to their dominance by stored solutions. This time, the fitness function is calculated using both the stored solutions and the current population, adopting a new scheme to prevent the loss of boundary solutions during archive updating. Diversity is maintained in this approach with a density-based cluster on the kth nearest neighbor.
3. Pareto Archived Evolutionary Strategy (PAES) is a single-parent single-child EA, similar to a (1+1) evolutionary strategy. The PAES method has two main objectives: The first is that the algorithm should be strictly confined to local search, moving from one solution to a nearby neighbor. The second is for the algorithm to be a true Pareto optimizer by treating all nondominated solutions as equal in terms of their value to the decision maker. This can be troublesome when comparing a pair of solutions that do not dominate each other. This is addressed by keeping a record of previous non-dominated solutions, which can be used to estimate a dominance ranking for this pair of solutions. The authors concluded that in a multi-objective routing problem, the PAES method provided competitive results compared to a traditional MOEA (Knowles & Corne, 1999)
4. Pareto Envelope-based Selection Algorithm (PESA) is an algorithm that uses a smaller internal (or primary) population and a larger external (secondary) population than that adopted by PAES. PESA uses the same hypergrid division of objective space from PAES to maintain diversity, but its selection mechanism is based on a hypergrid crowding measure. This crowding measure is used to decide which solutions are to be introduced into the external population (Corne, Knowles, & Oates, The Pareto Envelope-Based Selection Algorithm for Multiobjective Optimization, 2000)
5. Pareto Envelope-based Selection Algorithm II (PESA-II), developed by Corne et al. in 2011. This revised version of PESA proposes the use of region-base selection, in which the

unit to be selected is a hyperbox rather than an individual. The procedure in this method is to select a hyperbox and then randomly choose an individual within that box.

6. Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) is the revised method for the NSGA, proposed by Deb *et al*. in 2002. This method was devised to eliminate some of the weaknesses of the original NSGA, particularly its non-elitist nature and the specification of the sharing parameter. In this version, the individuals still undergo non-dominated sorting as in the first version, with the individuals receiving ranks based on this sorting. However, a new selection technique called crowded tournament selection is proposed. This selection method chooses individuals based on crowding distance (representing neighborhood density). Elitism is implemented by not allowing dominated solutions to pass on to the next generations

The multi-objective algorithm method used in this study is similar in some ways to the NSGA method, but with a different fitness assigning metric, proposed by Taboada *et al* in 2007. The fitness metric in this approach combines separate fitness assignments for proximity and diversity in the Pareto set and allows for weights to be assigned depending on which of the two is preferred.

Post-Pareto Optimality

An aspect of multi-objective optimization is that the selection of the best solution is often left to a decision maker's judgement. There are three approaches to incorporate the decision maker's opinions into the optimization method: Firstly, an a priori method, in which the decision-maker's preferences are incorporated into an algorithm before generating solution points. Then, there is the a posteriori approach, which first generates the solution points to make up the Pareto-optimal set before a decision is made. Lastly, there is the interactive method, in which the decision maker's preferences are incorporated during the search. The objective of post-pareto analyses is to make the final decision easier for the decision maker, who can often be found struggling to make sense of hundreds or thousands of data points.

As an example of an *a priori* approach, Pinchera *et al*. (2017) introduced a function which they named Quantized Lexicographic Weighted Sum (QLWS), based on the definition of a Global Cost Function, which required the decision maker to define priorities among the targets to optimize in order to more quickly generate a set of solutions by avoiding the evaluation of the Pareto front. For *a posteriori* methods, Reynoso-Meza *et al*. (2010) used a Differential Evolution algorithm to generate a set of non-dominated, Pareto-optimal solutions, and then introduced spherical pruning, which is less sensitive to the loss of non-dominated solutions, in order to reduce the Pareto set for the decision-maker. Lastly, Gong *et al* (2014) used an interactive method, which they call an Interval Multiple-objective Optimization Problem, which has the goal of finding the decision-maker's preferred solution, by having them input the importance relations between the objectives during the evolution.

The methods that this study will focus on are those which deal with the reduction of the Pareto-optimal set after the Pareto frontier has been established, i.e., *a posteriori* methods. These methods are intended to alleviate that part of the decision-making stage by delivering a more feasible to analyze amount of solutions. Still, it can be difficult to visualize the Pareto-optimal set when there are more than two objectives, not to mention the challenge of presenting this set to the decision maker. Below are described some *a posteriori* methodologies for Pareto-optimal set analysis.

The non-uniform weight generator with pseudo-ranking scheme, developed by Carrillo and Taboada (2012) uses a weight generating algorithm to generate a set of weights for the solutions. The algorithm then performs a weighted sum of the normalized objectives, where the best solution for each set of weights is marked, and after repeating the process with all different weight sets, the solutions that are marked becomes the new pareto set, while the unmarked solutions are deleted.

Another method for reducing the Pareto set was recently developed by Fernandez (2017), using the concept of Nash Dominance. This approach uses game theory to reduce the number of Pareto-optimal solutions. The approach considers each solution a strategy and each objective function a player, and has the objective of maximizing the number of players that benefits from a change in selected solutions, basically choosing to keep solutions that excel in a larger number of objectives compared to others. This generally resulted in the elimination of extreme solutions, which only yield good results in one objective while sacrificing others.

2.5. State of the Art

As shown in the literature, this is a very active research area in the topic of power systems and transmission systems optimization, but there is still much to be studied in the field. For one, no studies currently consider quantified environmental impact metrics as an objective to optimize, which becomes a much more significant topic as climate change effects worsen. Additionally, while there have been studies such as Sang and Sahraei-Ardakani (2019) that consider varying load scenarios, no study was found that would consider generator or line failure scenarios. As such, most research has opted for less computationally-burdensome deterministic approaches, which only considers what would be the most likely scenario or standard operating conditions. In this research, D-FACTS devices will be allocated on electric transmission systems to verify the benefits of installing D-FACTS devices on existing grids as well as show the improvements brought to the optimization process by the developed algorithms.

The present research proposes the development of Single- and Multi-Objective Metaheuristic Algorithms to solve stochastic D-FACTS allocation problems. Moreover, this thesis can provide the industry with an initial analysis of both the economic and environmental advantages to the installation of these devices.

Chapter 3: Mathematical Models

The D-FACTS allocation problem can be considered at least NP-hard, if not NP-complete in terms of its computational complexity. This chapter will present the basic formulation of the D-FACTS allocation problem as has been found in literature, as well as a number of modifications in track of the problems being solved in this dissertation. Since sections 4.1 and 4.2 are reviewing previous models, equation numbers are not used in them.

3.1. Early Models

The earliest formulation of this problem can be attributed to Li *et al*. (2009), who used a linearized DC power flow model with a voltage of 1.0 and no real power loss in the lines. Using this model, the flow at the k^{th} line (connecting buses *i* and *j*) is obtained from the equation

$$F_k = F_{ij} = \frac{\delta_{ij}}{x_{ij}} = \frac{\delta_i - \delta_j}{x_{ij}} = b_{ij}(\delta_i - \delta_j)$$

Where

F_k	Flow at line *k* (from bus *i* to *j*)	x_{ij}	Reactance of line *k*
δ_i, δ_k	Voltage angles at bus *i,j*	b_{ij}	Susceptance of line *k* = $\frac{1}{x_{ij}}$

With a simple optimization formulation given as:

$$Min \sum_{k=1}^{M} \left(F_k^2 - F_k^{max\,2}\right)^2$$

Subject to

$$[\boldsymbol{B}] \cdot \delta = \boldsymbol{P}$$

$$F_k^2 = F_{ij}^2 = \left(b_{ij}\left(\delta_i - \delta_j\right)\right)^2 \leq F_k^{max\,2}$$

$$b_{ij} = b_{ij,0} + b_{ij,c}^{max}$$

$$b_{ij,c}^{min} \leq b_{ij,c} \leq b_{ij,c}^{max}$$

Where

N	Number of buses	M	Number of lines

F_k	Flow through line k	F_k^{max}	Flow limit at line k
[**B**]	Nodal admittance matrix	δ	Vector of bus voltage angle
P	Vector of net nodal injection	$b_{ij,0}$	Susceptance of original line
$b_{ij,c}$	Equivalent susceptance of series compensator	$b_{ij,c}^{min}, b_{ij,c}^{max}$	Min/Max susceptance of line with series compensator

This basic formulation assumes the only unknowns to be the variables in $b_{ij,c}$, and its goal is to obtain the lowest congestion in the system. In such a simplified system, however, the model fails to adequately model more complex aspects of the system it tries to describe. This issue is addressed in later studies.

Das *et al.* (2009) used a Particle Swarm Optimization (PSO) algorithm. While they don't go into too much detail of the mathematical model they use, their objective function was to minimize cost as a linear function of a line utilization factor (LUF) and a module price:

$$Cost = LUF + \frac{1}{W} Price$$

$$LUF = \frac{1}{37} \sum_{i=1}^{37} \left(\frac{I_i}{I_{max}} \right)^{100}$$

$$Price = \frac{1}{37} \sum_{i=1}^{37} NM_i$$

Where

W	Max number of modules in network	I_{max}	Thermal limit for lines
I_i	Flow in line *i*	NM_i	Number of modules installed in line *i*

3.2. More Modern and Comprehensive Models

A more recent model by Dorostkar-Ghamsari *et al* (2015) used DSSC-type modules to maximize (1) system loadability factor and (2) system reliability via the following models:

(1)

$$Max\ \alpha$$

Subject to

$$\boldsymbol{G} - \alpha \times \boldsymbol{D} = \boldsymbol{A} \times \boldsymbol{L}$$

$$\boldsymbol{L} = \boldsymbol{B_L}\left(\boldsymbol{A}^T \times \boldsymbol{\delta} + \boldsymbol{V}_q\right)$$

$$N_k \leq \overline{N}_k \times u_k \qquad k \in \Omega_l$$

$$\underline{N_k} \times u_k \leq N_k \qquad k \in \Omega_l$$

$$3 \times \sum_{k \in \Omega_l} N_k \leq \overline{N}$$

$$\underline{\boldsymbol{G}} \leq \boldsymbol{G} \leq \overline{\boldsymbol{G}}$$

$$-\pi \leq \boldsymbol{\delta} \leq \pi$$

$$N_k \underline{V_{q_k}} \leq V_{q_k} \leq N_k \overline{V_{q_k}} \quad k \in \Omega_l$$

(2)

$$Min\ EDC$$

Subject to

$$\boldsymbol{G^S} + \boldsymbol{C^S} - \boldsymbol{D} = \boldsymbol{A} \times \boldsymbol{L^S}, \quad s \in \boldsymbol{\Omega_S}$$

$$\boldsymbol{L^S} = \boldsymbol{B_S}\left(\boldsymbol{A^T} \times \boldsymbol{\delta^S} + \boldsymbol{V_q^S}\right), \quad s \in \boldsymbol{\Omega_S}$$

$$N_k \leq \overline{N}_k \times u_k \qquad k \in \Omega_l$$

$$\underline{N_k} \times u_k \leq N_k \qquad k \in \Omega_l$$

$$3 \times \sum_{k \in \Omega_l} N_k \leq \overline{N}$$

$$\underline{\boldsymbol{G^S}} \leq \boldsymbol{G^S} \leq \overline{\boldsymbol{G^S}}, \quad s \in \boldsymbol{\Omega}_S$$

$$\left|\boldsymbol{L^S}\right| \leq \underline{\boldsymbol{L}^S}, \quad s \in \boldsymbol{\Omega}_S$$

$$-\pi \leq \boldsymbol{\delta^S} \leq \pi, \quad s \in \Omega_{\boldsymbol{S}}$$

$$N_k \underline{V^S}_{q_k} \leq V_{q_k}^S \leq N_k \overline{V_{q_k}^S}, \quad k \in \Omega_l\ , s \in \Omega_S$$

$$EDC = \sum_{s \in \Omega_S} \sum_{i \in \Omega_d} \left(8760.\overset{s}{\mathrm{Pr}}.C_i^S.IEAR_i\right)$$

Where

Ω_d	Set of demand buses	Ω_l	Set of transmission lines
Ω_s	Set of Scenarios	A	Network node incidence matrix
$\boldsymbol{B}_l$	Diagonal Matrix of line susceptances	D	Vector of bus demands
$\overline{G}, \underline{G}$	Vectors of upper and lower limits on active generation	u_k	Binary parameter indicating whether DSSCs are allowed on line k
$IEAR_i$	Interrupted energy assessment rate of load point i	$\overline{L}$	Vector of max flow limits
$\overline{N}$	Max number of DSSCs allowed in system	$\overline{N_k}, \underline{N_k}$	Upper and lower limits of DSSCs per conductor in line k
Pr^s	Probability of scenario s	$\overline{V_{q_k}}, \underline{V_{q_k}}$	Upper and lower limits on injected voltage per conductor of line k
$EENS_i$	Expected energy not supplied at point i	G	Vector of active power generations
L	Vector of active power flows	$\mathbf{C}^S$	Vector of load curtailment in scenario s
N_k	Number of DSSCs installed in line k	V_q	Vector of injected voltages
$\boldsymbol{V_{q_k}}$	Injected voltage on each conductor of line k	$\boldsymbol{\alpha}$	System loadability factor

$\boldsymbol{\delta}$	Vector of bus voltage angles		

These formulations presented one of the first explorations into optimization of these devices into multiple stochastic scenarios. However, this formulation is only valid for DSSC-type devices, which according to Sang and Sahraei-Ardakani (2019) are more expensive and have a lower market prospect. In Response, they proposed the following model to allocate DSI-type devices, which accounts not only for multiple scenarios, but also accounts for reserve requirements which is a requirement of many modern transmission systems and serve to account for fluctuations on load, and renewable energy integration, which is becoming a more common subsystem in many electrical networks.

Due to power flow constraints, the optimization model is split into two parts, as they depend on power flow direction:

$$If\ \theta_{fr,k,s} - \theta_{to,k,s} \geq 0, \left(\theta_{fr,k,s} - \theta_{to,k,s}\right)/X_k^{\max} \leq F_{k,s} \leq \left(\theta_{fr,k,s} - \theta_{to,k,s}\right)/X_k^{\min}$$

$$If\ \theta_{fr,k,s} - \theta_{to,k,s} \leq 0, \left(\theta_{fr,k,s} - \theta_{to,k,s}\right)/X_k^{\min} \leq F_{k,s} \leq (\theta_{fr,k,s} - \theta_{to,k,s})/X_k^{max}$$

As so, the formulation first solves the system without considering D-FACTS devices to obtain the flow directions of each line and then uses this information in solving for the allocation of D-FACTS at each line. The first step uses the following formulation:

$$\min\left(\sum_{s=1}^{N_s} Ps\left(\sum_{g=1}^{N_g}\left(\sum_{seg}^{N_{seg}} c_{g,seg}^{linear} P_{g,s}^{seg} + c_g^U R_{g,s}^U + c_g^D R_{g,s}^D + C_g^{NL}\right) + \sum_{r=1}^{N_r} c_r P_{r,s}^C\right)\right)$$

Subject to

$$P_{g,s} = \sum_{seg=1}^{N_{seg}} P_{g,s}^{seg}$$

$$P_g^{\min} \leq P_{g,s} \leq P_g^{max}$$

$$-F_k^{max} \leq F_{k,s} \leq F_k^{maz}$$

$$\left(\theta_{fr,k,s} - \theta_{to,k,s}\right)/X_k \ = F_{k,s}$$

$$\sum_{k\in\sigma^+(n)} F_{k,s} - \sum_{k\in\sigma^-(n)} F_{k,s} + \sum_{g\in g(n)} P_{g,s} + \sum_{r\in r(n)} \left(P_{r,s} - P_{r,s}^C\right) = L_{n,s}$$

$$\sum_{g=1}^{N_g} R_{g,s}^{U} \geq S^{U}$$

$$\sum_{g=1}^{N_g} R_{g,s}^{D} \geq S^{D}$$

$$R_{g,s}^{U} \leq P_{g}^{max} - P_{g,s}$$

$$R_{g,s}^{D} \leq P_{g,s} - P_{g}^{min}$$

$$R_{g,s}^{U} \geq 0$$

$$R_{g,s}^{D} \geq 0$$

$$\Delta\theta_{k}^{min} \leq \theta_{fr,k,s} - \theta_{to,k,s} \leq \Delta\theta_{k}^{max}$$

$$\theta_{1,s} = 0$$

The second step uses the same formulation, after obtaining the values for $f_{k,s}$ encoded as 0/1 for positive/negative flows, and changing the objective function to the one given below and adding new constraints:

$$\min\left(\sum_{s=1}^{N_s} Ps\left(\sum_{g=1}^{N_g}\left(\sum_{seg}^{N_{seg}} c_{g,seg}^{linear} P_{g,s}^{seg} + c_{g}^{U} R_{g,s}^{U} + c_{g}^{D} R_{g,s}^{D} + C_{g}^{NL}\right) + \sum_{r=1}^{N_r} c_r P_{r,s}^{C}\right) + c_{inv}^{D}\right)$$

$$\sum_{i=1}^{i_{max}} x_{k,i}^{D} \leq 1$$

$$(1 + i\eta_L) X_k F_{k,s} + \left(1 - x_{k,i}^{D}\right) M + \left(1 - f_{k,s}\right) M \geq \theta_{fr,k,s} - \theta_{to,k,s}$$

$$(1 - i\eta_C) X_k F_{k,s} - \left(1 - x_{k,i}^{D}\right) M - \left(1 - f_{k,s}\right) M \geq \theta_{fr,k,s} - \theta_{to,k,s}$$

$$(1 + i\eta_L) X_k F_{k,s} - \left(1 - x_{k,i}^{D}\right) M - f_{k,s} M \geq \theta_{fr,k,s} - \theta_{to,k,s}$$

$$(1 - i\eta_C) X_k F_{k,s} + \left(1 - x_{k,i}^{D}\right) M + f_{k,s} M \geq \theta_{fr,k,s} - \theta_{to,k,s}$$

$$X_k F_{k,s} + M \sum_{i=1}^{i_{max}} x_{k,i}^{D} \geq \theta_{fr,k,s} - \theta_{to,k,s}$$

$$X_k F_{k,s} - M \sum_{i=1}^{i_{max}} x_{k,i}^{D} \leq \theta_{fr,k,s} - \theta_{to,k,s}$$

$$c_{inv}^{D} = \sum_{k=1}^{N_{br}} \sum_{i=1}^{i_{max}} 3 u l_k i C_{sh}^{D} x_{k,i}^{D}$$

$$C_{inv}^{D} \leq C_{inv}^{max}$$

Allocating a number *i* of modules in each line *k* per phase per mile, and where the following nomenclature is used:

g	Generator	i	Number of D-FACTS pr phase per mile
k	Line	n	Node
r	Renewable generator	seg	Segment of linearized generator cost func.
$\sigma^{+}(n)$	Transmission lines connected 'to' bus n	$\sigma^{-}(n)$	Transmission lines connected 'from' bus n
g(n)	Generators connected to bus n	r(n)	Renewable generators connected to bus n
C_{inv}^{D}	Total investment in D-FACTS	$F_{k,s}$	Power flow through line k in scen. s
$P_{g,s}$	Power from generator g in scen. s	$P_{g,s}^{seg}$	Power from gen. g in scen. s in segment seg
$P_{r,s}$	Power from ren. gen. r in scen. s	$P_{r,s}^{C}$	Curtailed power from ren. Gen. r in scen. s
$R_{g,s}^{D}$, $R_{g,s}^{U}$	Down/Up reserves from gen. g in scen. s	$x_{k,i}^{D}$	Binary integer indicating the number of D-FACTS i installed in line k
$\theta_{b,s}$	Voltage angle at bus b in scen. s	$\theta_{fr,k,s}, \theta_{to,k,s}$	Voltage angle at the from/to node of line k in scen. s
C_{g}^{NL}	No load cost of generator g	$C_{g,seg}^{linear}$	Linear cost of generator g in segment seg

C_g^D, C_g^U	Down/up reserve cost of generator g	$f_{k,s}$	Binary integer indicating flow direction in line k in scen. s
C_{sh}^D	Cost of a D-FACTS unit converted to an hourly value	F_k^{max}	Thermal capacity/ voltage drop limit of line k
i_{max}	Max. num. of D-FACTS to be allocated per mile per phase	l_k	Length of line k
$L_{n,s}$	Load at bus n in scen. n	M	A very large positive number
N	Expected lifespan of D-FACTS devices	N_g	Number of generators
N_r	Number of renewable generators	N_s	Number of scenarios
N_{seg}	Number of segments in cost function	p_s	Probability of scenario s
P_g^{max}	Upper limit for generator g	P_g^{min}	Lower limit for generator g
S^U, S^D	Down/Up reserve requirements	X_k	Reactance of line k
u	Unit distance for allocation per line	X_k^{max}, X_k^{min}	Max/min reactance of line k if D-FACTS are installed on it

η_C, η_L	Max adjustment percentage of the line's reactance in capacitive/inductive mode that a single D-FACT can achieve	$\Delta\theta_k^{max}, \Delta\theta_k^{min}$	Max/min value of bus voltage angle difference to maintain stability in line k

In addition to these, other studies have focused on environmental emissions. One such study by Santacruz and Sang (2021) aims to track renewable energy penetration into grids using a Marginal Emissions Factor (MEF). Here, they track emissions and power sources in order to minimize total costs to serve the load at each bus. The eventual objective of increasing more accurately handling power flow control to reduce fossil fuel reliance and make the grid more sustainable. A further study incorporated the use of electric vehicles as batteries and part of the grid to further reduce system costs, congestion, and emissions (Santacruz & Sang, 2022).

3.3. NEW PROPOSED MODEL

The formulation used for our study is similar to the one given above, but some modifications were made as the use of metaheuristic algorithms can transform the nonlinearities into linear equations by pre-determining the number of modules in the chromosome. Additionally, new indices and variables were introduced to account for the calculations of environmental impact metrics and multi-objective dominance metrics. Thus, we are able to not only reduce the computational time but also achieve a reduced model for optimization. The following nomenclature is used in our optimization model. Note that this nomenclature will be used throughout the remainder of this dissertation paper to identify relevant variables in the model.

Indices			
a, b	Solutions	r	Renewable Generator.

c	Contaminant	s	Scenario.
k	Transmission line.	seg	Segment of linearized generator cost function.
g	Generator.	i	Objective or Fitness Function
n	Node.		
Sets			
$\sigma^{+}(n)$	Transmission lines with their "to" bus connected to node n.	$g(n)$	Generators connected to node n.
$\sigma^{-}(n)$	Transmission lines with their "from" bus connected to node n.	$r(n)$	Renewable generators connected to node *n*.
Variables			
C_{inv}^{D}	Total investment in D-FACTS ($).	$P_{g,s}^{seg}$	Real power generation of generator g in scenarios s in segment seg.
$D_{a,b}$	Dominance of solution *a* over solution *b*	$R_{g,s}^{D}$	Spinning down reserve available through generator g in scenario s.
$F_{k,s}$	Real power flow through transmission line k in scenarios s.	$R_{g,s}^{U}$	Spinning up reserve available through generator g in scenario s.

$FM_{i,a}, FM_{i,a}^{n}$	Value of fitness metric, normalized F.M. i for solution a	x_k^D	Integer indicating the number of D-FACTS installed on transmission line k
$OF_{i,a}, OF_{i,a}^{n}$	Value of objective function, normalized O.F. i for solution a	$\theta_{b,s}$	Voltage angle at bus b in scenarios s.
$P_{g,s}$	Real power generation of generator g in scenarios s.	$\theta_{fr,k,s}$	Voltage angle at the "from" node of line k in scenarios s.
$P_{r,s}^{C}$	Curtailed renewable generation from renewable generator r in scenario s	$\theta_{to,k,s}$	Voltage angle at the "to" node of line k in scenarios s.
Parameters			
C_g^{NL}	No load cost of generator g.	N_k	Total number of lines.
$C_{g,seg}^{linear}$	Linear cost of generator g in segment seg.	N_{obj}	Number of Objective Functions
C_g^{D}	Down reserve cost of generator g.	N_s	Number of scenarios.
C_g^{U}	Up reserve cost of generator g.	N_{seg}	Number of segments for the linearized generator cost function.

C_{single}^{D}	Cost a of single D-FACTS unit ($).	N_{pop}	Population size for the algorithm.
C_{sh}^{D}	Cost a of single D-FACTS unit converted to an hourly figure ($/h).	N_r	Number of renewable generators.
C_{inv}^{max}	Maximum investment allowed for D-FACTS.	p_s	Probability of scenario s.
$f_{k,s}$	Flow direction for line k in scenario s	P_g^{max}	Upper generation limit of generator g.
F_k^{max}	Thermal capacity/voltage drop limit of transmission line k.	P_g^{min}	Lower generation limit of generator g.
$H_{g,seg}^{linear}$	Linearized Heat production of generator g in generation segment seg (MMBTU/MW)	$P_{r,s}$	Renewable generation produced by renewable generator r in scenario s
$G_{g,c}$	Gaseous contaminant c released by generator g (kg/MMBTU)	$P_{r,s}^{C}$	Renewable energy curtailed from renewable generator r in scenario s.
$GWP_{g,c,s}$	Global Warming Potential caused by contaminant c from	S^D	Spinning down reserve requirement g.

	generator g in scenario s (1kg CO_2 eq.)		
i_k^{max}	Maximum number of D-FACTS that can be allocated per line.	S^U	Spinning up reserve requirement g.
I	Interest rate/discount rate.	X_k	The reactance of transmission line k.
l_{max}^{alloc}	Maximum number of lines in which D-FACTS devices may be allocated	X_k^{max}	The maximum reactance of line k if D-FACTS are installed on this line.
l_k	Length of line k	X_k^{min}	The minimum reactance of line k if D-FACTS are installed on this line.
$L_{n,s}$	Load at bus n in scenario s.	W_c	GWP factor for contaminant c (1 kg CO_2 eq.)
N	Lifespan of D-FACTS.	η_C	The maximum adjustment percentage of the line's reactance in the capacitive mode that a single D-FACTS module (1 device/phase/mile) can achieve.
N_c	Total number of contaminants considered	η_L	The maximum adjustment percentage of the line's reactance in the inductive mode that a single D-FACTS module (1 device/phase/mile) can achieve.

N_g	Total number of generators.	$\Delta\theta_k^{max}$	Maximum value of bus voltage angle difference to maintain stability for line k.
N_{fit}	Number of fitness metrics (non-aggregated)	$\Delta\theta_k^{min}$	Minimum value of bus voltage angle difference to maintain stability for line k.

As discussed by Sahraei-Ardakani and Hedman (2015), the flow direction in the lines is relevant when adjusting their impedance. As so, we must use the following DC power flow constraints:

$$If\ \theta_{fr,k,s} - \theta_{to,k,s} \geq 0, (\theta_{fr,k,s} - \theta_{to,k,s})/X_k^{\max} \leq F_{k,s} \leq \ (\theta_{fr,k,s} - \theta_{to,k,s})/X_k^{\min} \quad (3.1)$$

$$If\ \theta_{fr,k,s} - \theta_{to,k,s} \leq 0, (\theta_{fr,k,s} - \theta_{to,k,s})/X_k^{\min} \leq F_{k,s} \leq (\theta_{fr,k,s} - \theta_{to,k,s})/X_k^{max} \quad (3.2)$$

Having considered flow direction constraints, the following model is created to simultaneously optimize (1) total expected system costs, (2) expected environmental impact in the form of global warming potential, (3) Line Utilization Factor as described previously by Das *et al* (2009), and (4) renewable energy curtailment, subject to the following constraints:

$$\min OF_1 = \sum_{s=1}^{N_s} P_s \left(\sum_{g=1}^{N_g} \left(\sum_{seg=1}^{N_{seg}} C_{g,seg}^{linear} P_{g,s}^{seg} + C_g^U R_{g,s}^U + C_g^D R_{g,s}^D + C_g^{NL} \right) + \sum_{r=1}^{N_r} c_r P_{r,s}^C \right) + C_{inv}^D \quad (3.3)$$

$$\min OF_2 = \sum_{s=1}^{N_s} P_s \left(\sum_{g=1}^{N_g} \sum_{c=1}^{N_c} GWP_{g,c,s} \right) \quad (3.4)$$

$$\min OF_3 = \frac{1}{N_k} \sum_{s=1}^{N_s} \sum_{k=1}^{N_k} P_s \left(\frac{F_{k,s}}{F_k^{\max}} \right)^{100} \quad (3.5)$$

$$\min OF_4 = \sum_{s=1}^{N_s} P_s \frac{\sum_{r=1}^{N_r} P_{r,s}^C}{\sum_{r=1}^{N_r} P_{r,s}} \quad (3.6)$$

$$P_{g,s} = \sum_{seg=1}^{N_{seg}} P_{g,s}^{seg} \quad (3.7)$$

$$P_g^{min} \leq P_{g,s} \leq P_g^{max} \tag{3.8}$$

$$-F_k^{max} \leq F_{k,s} \leq F_k^{max} \tag{3.9}$$

$$\sum_{k\in\sigma^+(n)} F_{k,s} - \sum_{k\in\sigma^-(n)} F_{k,s} + \sum_{g\in g(n)} P_{g,s} + \sum_{r\in r(n)} \left(P_{r,s} - P_{r,s}^C\right) = L_{n,s} \tag{3.10}$$

$$\sum_{g=1}^{N_g} R_{g,s}^U \geq S^U \tag{3.11}$$

$$\sum_{g=1}^{N_g} R_{g,s}^D \geq S^D \tag{3.12}$$

$$R_{g,s}^U \leq P_g^{max} - P_{g,s} \tag{3.13}$$

$$R_{g,s}^D \leq P_{g,s} - P_g^{min} \tag{3.14}$$

$$R_{g,s}^U, R_{g,s}^D \geq 0 \tag{3.15}$$

$$\Delta\theta_k^{min} \leq \theta_{fr,k,s} - \theta_{to,k,s} \leq \Delta\theta_k^{max} \tag{3.16}$$

$$\theta_{1,s} = 0 \tag{3.17}$$

$$f_{k,s}\left(1 + \frac{x_k^D}{l_k}\eta_L\right) X_k F_{k,s} \geq f_{k,s}\left(\theta_{fr,k,s} - \theta_{to,k,s}\right) \tag{3.18}$$

$$f_{k,s}\left(1 + \frac{x_k^D}{l_k}\eta_C\right) X_k F_{k,s} \leq f_{k,s}\left(\theta_{fr,k,s} - \theta_{to,k,s}\right) \tag{3.19}$$

$$0 \leq x_k^D \leq i_k^{max} \tag{3.20}$$

$$\sum_{k=1}^{N_k} \frac{x_k^D}{\max(x_k^D, 1)} \leq l_{max}^{alloc} \tag{3.21}$$

$$GWP_{g,c,s} = \sum_{seg}^{N_{seg}} H_{g,seg}^{linear} P_{g,s}^{seg} G_{g,s} W_c \tag{3.22}$$

$$C_{inv}^D = \sum_{k=1}^{N_k} 3x_k^D C_{sh}^D \tag{3.23}$$

$$C_{inv}^D \leq C_{inv}^{max} \tag{3.24}$$

$$C_{sh}^D = C_{single}^D \frac{I(1+I)^N}{8760((1+I)^N - 1)} \tag{3.25}$$

$$0 \leq P_{r,s}^C \leq P_{r,s} \tag{3.26}$$

However, as described in Sang and Sahraei-Ardakani's (2019) model, due to the way in which the devices affect the reactances, it is necessary to know the flow directions in the transmission lines. For this, a reduced model is first solved consisting solely of equations (3.1), (3.4-4.10), and (3.16-4.19), for the sole purpose of determining power flow directions, which are then obtained from the output of this reduced linear model using the following equation:

$$f_{k,s} = \frac{F_{k,s}}{|F_{k,s}|} \tag{3.27}$$

For this, the variables $f_{k,s}$ and x_k^D are both initialized to have the values of 1 and 0, respectively, at each possible index, essentially turning both of these equations into the equality in (3.28), which is the DC power flow equation:

$$X_k F_{k,s} = \theta_{fr,k,s} - \theta_{to,k,s} \tag{3.28}$$

The minutiae of the solving algorithm will be discussed later in section 6.2. The remainder of this subsection will be devoted to detailing the significance of each of the above equations.

Firstly, (3.3) is the first objective function to be minimized. This function is the sum of all segmentized generation costs, up and down reserve costs, no-load generation costs and renewable energy curtailment costs of all the traditional and renewable generators across all scenarios plus the total D-FACTS investment costs, which does remain constant over the scenarios. The second objective function in (3.4) is to minimize the total Global Warming Potential of the power generation of the system, for all the contaminants known to be emitted by every traditional generator over all scenarios. The third objective function in (3.5) is the minimization of the Line Utilization Factor, a ratio of the current power flow in a line and its thermal capacity, as a measure of long-term health for the line and system flexibility (as less-used lines can pick up new transmission demands more easily). Equation (3.6) is the objective function with the objective of minimizing the amount of renewable energy curtailed from the system, in order to better integrate renewable energies into the power grid.

Equation (3.7) then served to define the total power generated by a generator as a sum of all its segments, divided due to the shape of their associated cost function; while the associated generation limits are described in (3.8). Constraint (3.9) serves to define the upper and lower limits to power flow within each line. Based on previous models, the capacity of short lines (0-50 miles) is set to their thermal limits, the capacity of medium lines (50-156 miles) is determined by their voltage drop limit and the capacity of long lines (156+ miles) is given by their voltage stability limits.

The load at each bus is defined in (3.10) to be equal to the sub of all incoming flows minus the sum of all outgoing flows plus the generation, both traditional and renewable, attached to said bus. Equations (3.11) and (3.12) define the up and down (respectively) reserve requirements for the system, with (3.13-3.15) defining the upper and lower limits for each generator's reserves. In (3.16), the upper and lower voltage angle limits are established for the purpose of maintaining stability in the system, with (3.17) defining the angle at the first bus to be zero as a reference value.

Equations (3.18) and (3.19) are the modified DC power flow equations, augmented to account for the impedance adjustments brought by the D-FACTS modules and considering the flow directions by the use of the $f_{k,s}$ variable which would flip the inequality signs when the flow directions are negative. Equation (3.20) defines the number of D-FACTS modules which can be installed at each line, while (3.21) constraints the number of lines in which D-FACTS modules can be installed, for feasibility purposes.

In (3.22), the Global Warming Potential (our chosen environmental impact metric, although the equation holds true for any metric by just adjusting the values for the parameter W_c) is calculated for each generator as the sum over all its generation segments of the heat (BTU/MW) produced at each segment times the power generated in each segment (MW), multiplied by the amount of each contaminant released (kg/BTU) and finally applying to this the corresponding GWP factor (kg CO2 equivalent/kg).

Equation (3.23) is used to define the total investment cost as 3 times the cost of a single device times the number of allocated devices (for 3 phases); with (3.24) limiting the total investment cost to a preset limit. Equation (3.25) converts the device cost to an hourly figure to be in line with all other costs in the system. Finally, (3.26) defines the curtailment for each renewable generator to be between 0 and the amount of renewable energy produced.

Mathematically, the formulation of the problem may appear to be a non-linear optimization problem. However, the use of a metaheuristic approach to pre-assign the values of x_k^D and pre-calculate the values of $f_{k,s}$ to ± 1, means that all the constraints become linear.

Chapter 4: Optimization Methods

An optimization method is a mathematical algorithm that will, upon completion, return the combination of values for the decision variables that will maximize or minimize a desired output. Among these, metaheuristic approaches are some of the most efficient methods. Metaheuristics are becoming more popular thanks to their quick convergence and low computational burden; they are able to achieve good, near-optimal solutions by performing a quick, effective, and intelligent search of the computational space (Yu & Gen, 2012).

In this research, a Multi-Objective Evolutionary Algorithm (MOEA) is presented and used to efficiently find possible solutions to the D-FACTS allocation problem and identify which meet optimality conditions. As with any other multi-objective optimization problem, it is both possible and expected that at least some objectives are in opposition to each other, and that a single optimal solution cannot be determined without the assistance of a decision maker. In such situation, a type of optimality called Pareto Optimality is considered. A Pareto-Optimal solution is one for which no other solution exists that is objectively better than it in every objective. At the end of an optimization algorithm which considers Pareto Optimality, the output will be a set of Pareto-Optimal solutions called the Pareto Front or Pareto-Optimal Set. This set of solutions can be further analyzed in a procedure called post-Pareto optimality; a set of tools used to prune very large Pareto-optimal sets in order to reduce the burden on the decision maker.

In terms of the Pareto front, the proposed MOEA has two main goals: proximity and diversity. Proximity refers to finding solutions that are as close as possible to the Pareto frontier, while Diversity means finding solutions that are spread over it, meaning that it attempts to find solutions that differ in their objective values as much as possible. The details of the algorithm are described below in section 5.2.

4.1. PARETO DOMINANCE

Before detailing the algorithm, it is important to explain the concept of Pareto Dominance. This concept is crucial in this type of optimization process, as it is used to define which solutions are good and should be stored for further analysis by a decision maker.

A Pareto non-dominated solution is one for which:

- No solution exists that is objectively better than it in all objectives being considered, and
- Is better than all other solutions in at least one objective

Conversely, a dominated solution is one for which at least one other solution exists which is strictly better than it in every objective being considered. In practice, it is easier to define a non-dominated solution as one that is not a dominated solution. The resulting set of non-dominated solutions is considered to be asymptotically closest to the utopia solution point. This proves valuable to the decision-maker as dominated solutions are objectively worse than non-dominated ones in evert single aspect (or objective) under consideration. Delivering only non-dominated solutions means delivering options which will have the best objective values and reduce the number of options being considered.

For this algorithm, the following equations are used to determine dominance:

$$If\ OF_{obj,a} \leq OF_{obj,b}\ \forall obj;\ \ D_{a,b} = 1;\ else, D_{a,b} = 0 \tag{5.1}$$

$$\sum_{a=1}^{N_{pop}} D_{a,b} = 0 \tag{5.2}$$

Eq. (5.1) states that solution a dominates solution b if all its objective function values are lower than the corresponding ones in b. Additionally, a solution b is non-dominated if it satisfies equation (5.2).

4.2. THE MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM

The proposed MOEA begins by generating sets of random values within the search space. Each of these sets is called a chromosome, and each chromosome represents a possible solution to the problem.

The algorithm currently follows the steps described below:

0. START
1. Using the reduced model consisting of Eqns. (4.3), (4.7-4.10, (4.16-4.19), and (4.26), solve the linear problem to obtain the values of $F_{k,s}$.
2. Obtain the values of $f_{k,s}$ via equation (4.27).
3. Generate an initial population using the parameters i_k^{max} and l_{max}^{alloc} defined in equations (4.20) and (4.21).
4. For each chromosome, solve the linear problem consisting of equations (4.3), (4.7-4.10), and (4.16-4.19).
5. Using the outputs from each linear problem, use a greedy algorithm to allocate the up and down spinning reserves as defined in Eqns. (4.11-4.15).
6. From this information, calculate the values for the objective functions in (4.4-4.6).
7. Use the Pareto Dominance criteria in eqns. (5.1, 5.2) on all solutions and store the non-dominated ones.
8. IF the stopping criteria have been met, go to step (13). Otherwise, go to step 9.
9. Obtain the Fitness Metrics as defined in section (5.3.4), and calculate the Aggregated Fitness Metric.
10. Rank solutions according to the Aggregated Fitness Metric obtained in step 9.
11. Select Parents from the current population as described in section (5.3.5).
12. Generate a new population as described in section (5.3.5) and return to step 4.
13. Retrieve the stored solutions and re-check for dominance to obtain the Pareto-Optimal set

14. END

The process is summarized below in the flowchart of figure 5.1. Each step of the process is also described in more detail in the following sub-sections.

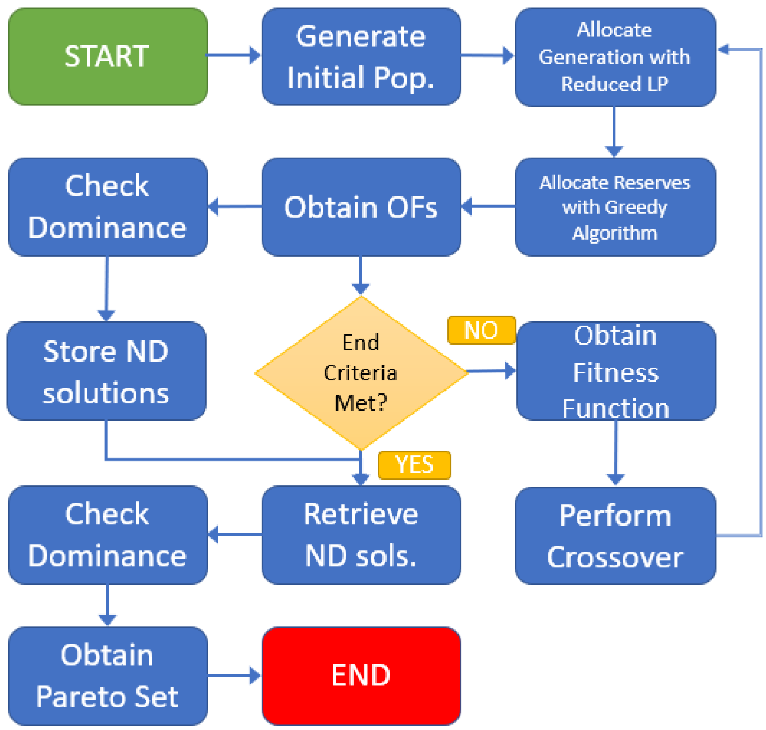

Figure 4.1 Evolutionary Algorithm Flowchart

4.2.1. The Chromosome

The chromosome is part of the foundations of a Genetic Algorithm. Each chromosome represents a solution inside the search area, and are used to generate and guide new solutions towards an optimal response. In a multi-objective algorithm, however, rather than being guided to an optimal point, they are guided towards a non-dominated optimality curve.

The chromosome used in the algorithm is a 1-dimensional vector array, representing the number of devices that can be installed in each line, based on the limits allowed by the maximum adjustment allowed and the adjustment range for each device, with the constraints defined previously in (4.20) and (4.21). The population and chromosome encoding are represented below in Fig. 5.2, where each of the N_{pop} lines represents a candidate solution or chromosome and each of the N_k columns represents a different line in the system where D-FACTS devices would be installed.

Number of devices allocated in each line

Candidate Solutions

Ind\k	1	2	3	4	5	6	7	...	N_k
1	0	45	0	150	0	30	0	...	83
2	0	0	0	43	0	0	185	...	0
3	20	0	75	0	12	0	0	...	0
4	0	30	0	51	0	0	0	...	0
5	0	0	30	0	0	12	0	...	0
6	0	0	12	0	40	0	0	...	67
7	15	0	98	0	0	0	96	...	50
8	82	0	14	0	0	69	0	...	0
9	0	0	0	6	75	30	12	...	0
...	...	...	...	...	...	...	...	...	...
N_{pop}	42	0	83	0	0	0	70	...	0

Figure 4.2: Population and Chromosome Representation

4.2.2. The Initial Generation

The first step in the algorithm is to peudo-randomly generate a pre-defined number N_{pop} of individuals as an initial ser of solutions to explore. This initial population serves to initiate the exploration of the solution space, and the size of this population will influence how quickly the algorithm can converge around an optimum point in the case of single objective optimization, or how quickly it can create a Pareto-optimal set.

For the generation of our initial population, the algorithm creates a zero-vector of length N_k, and randomly generates two sets of random values: (1) random integers between 1 and N_k to determine the locations where D-FACTS will be placed, and (2) random decimals in the [0,1] range to be multiplied by the i_k^{max} associated to the values in part 1 and rounded to the nearest integer to obtain how many devices are allocated to each line. These values are then stored in their corresponding locations on the vector. Finally, equations (4.23-4.25) are applied to the chromosome, and if the constraint in (4.24) is not met, the whole vector is multiplied by $C_{inv}^{max}/C_{inv}^{D}$ and rounded down to ensure the constraints are met.

4.2.3. The Objective Functions

Cost Function

This objective function aims to minimize not the cost of investment in D-FACTS devices but the entire system's hourly operational costs. In this function, the total cost of the devices is converted to an hourly value in order for it to be expressed in the same units and thus included in the objective function. The function consists of various expressions that will be detailed below:

$$\sum_{seg=1}^{N_{seg}} C_{g,seg}^{linear} P_{g,s}^{seg}$$

Which is the summation over the generation segments of the linearized cost and the segment's power generation, in order to obtain the total cost of a generator's operating settings

$$\sum_{g=1}^{N_g} \left(\sum_{seg=1}^{N_{seg}} C_{g,seg}^{linear} P_{g,s}^{seg} + C_g^U R_{g,s}^U + C_g^D R_{g,s}^D + C_g^{NL} \right)$$

Which is now the summation over all generators of the previously-described generation function plus the up and down reserves times their costs and the no-load costs for each generator being considered

$$\sum_{r=1}^{N_r} c_r P_{r,s}^C$$

Which is the curtailed energy from each renewable source times the curtailment cost

$$\sum_{s=1}^{N_s} P_s \left(\sum_{g=1}^{N_g} \left(\sum_{seg=1}^{N_{seg}} C_{g,seg}^{linear} P_{g,s}^{seg} + C_g^U R_{g,s}^U + C_g^D R_{g,s}^D + C_g^{NL} \right) + \sum_{r=1}^{N_r} c_r P_{r,s}^C \right)$$

Which takes both previous expressions and multiplies it by the probability of each scenario to occur, turning the problem into a stochastic one

$$\sum_{s=1}^{N_s} P_s\left(\sum_{g=1}^{N_g}\left(\sum_{seg=1}^{N_{seg}} C_{g,seg}^{linear} P_{g,s}^{seg} + C_g^U R_{g,s}^U + C_g^D R_{g,s}^D + C_g^{NL}\right) + \sum_{r=1}^{N_r} c_r P_{r,s}^C\right) + C_{inv}^D$$

Which finally adds the D-FACTS investment costs, constant through all scenarios.

As mentioned previously, the linear program used as part of the metaheuristic algorithm will optimize the system accounting mainly for the cost function and later using the full output to calculate the other objective functions. This is considered to be a realistic choice because of the capital-driven nature of most industries and of utility services.

Environmental Impact Function

The second objective function aims to minimize environmental impacts stemming from the system operations, and considering solely emissions released to the environment from the non-renewable generators, as it is assumed that both these and the renewable generators are already installed in the system and there are no external emissions considered at this time. The metric used for this study is the Global Warming Potential (GWP), which is an index of how much heat gases released from a process can trap into the atmosphere over a period of time, with a base on the heat trapped by carbon dioxide. The unit used in this study is 1kg CO_2 equivalent over a timeframe of 100 years. To calculate it, the formula below is used. Essentially, this formula only adds up the calculated GWP for each known contaminant released by every generator over every scenario and multiplies this added value by the probability of the scenario.

$$\sum_{s=1}^{N_s} P_s\left(\sum_{g=1}^{N_g}\sum_{c=1}^{N_c} GWP_{g,c,s}\right)$$

The GWP is calculated using the following formula. Essentially, each generator will release a different amount of gases depending on the amount of heat being generated. This happens to align with the linearized cost/generation functions. Thus, by multiplying each power generation segment by the corresponding heat production value we can obtain the total amount of each gaseous contaminant G being released and obtain its equivalent by using the factor W to convert it into 1kgCO_2 equivalent units.

$$GWP_{g,c,s} = \sum_{seg}^{N_{seg}} H_{g,seg}^{linear} P_{g,s}^{seg} G_{g,s} W_c$$

Line Utilization Function

The third objective function aims to improve overall health of the transmission system by minimizing the line utilization factor, as previously described, in order to prevent line degradation.

The function divides a line's utilization by its thermal limit and uses an exponent to normalize the values, and averaged using both the scenario probabilities and the number of lines, as shown in the formula below:

$$\frac{1}{N_k} \sum_{s=1}^{N_s} \sum_{k=1}^{N_k} P_s \left(\frac{F_{k,s}}{F_k^{\max}} \right)^{100}$$

Renewable Energy Integration Function

The last objective function aims to minimize the curtailment of renewable energy. This is used also as a measure of renewable energy integration into the system, which is becoming a more necessary component to modern electric grids. As so, taking the curtailed renewable energy as a percentage is one of the simplest metrics that can be used for renewable energy integration.

The function used is shown below. Essentially, it consists of adding up the total curtailed renewable energy and dividing it by the total amount of generated renewable energy, and obtaining a weighted average by the use of the scenario probabilities as the weights.

$$\sum_{s=1}^{N_s} P_s \frac{\sum_{r=1}^{N_r} P_{r,s}^C}{\sum_{r=1}^{N_r} P_{r,s}}$$

4.2.4. The Fitness Functions

The developed MOEA has two main goals. Proximity which represents the closeness to the Pareto front, and diversity that has the objective of maintaining population diversity as designed by Taboada & Coit, (2012). These two goals are evaluated with two fitness metrics which are distance-based (diversity), and dominance count-based (proximity).

The first fitness metric, which is distance-based, assigns a higher fitness value to those solutions that are farther away from other solutions in the Pareto front. The aim of giving further solutions a higher fitness value is to give these a better chance of reproducing, thus creating a wider spread of solutions. This way, it is possible to use a fitness metric to increase diversity of the Pareto-optimal solutions. The following steps are followed to assess this fitness metric:

1. **Normalization:** In this step, the objective function values of the solutions are normalized in order to ensure that all values are within the same order of magnitude to prevent unit discrepancy. The normalization of the solutions' objective value uses the following equation:

$$OF_{i,a}^{n} = \frac{OF_{i,a} - \max_{a} OF_{i,a}}{\max_{a} OF_{i,a} - \min_{a} OF_{i,a}} \tag{5.3}$$

2. **Distance Calculation**: In this second step, the distance metric for each solution is calculated. Euclidean distance is used in this step, and the following focmula is used to determine the total distances between one solution and all the others:

$$FM_{1,a} = \sum_{b=1}^{N_{pop}} \sqrt{\sum_{i=1}^{N_{obj}} \left(OF_{i,a} - OF_{i,b}\right)^2} \tag{5.4}$$

The second distance metric, which is based on dominance count, is based on the previously described concept of Pareto-dominance. The premise for this metric is that solutions which dominate others are closer to the Pareto frontier than those that are dominated, again based on the concept of Pareto dominance, which implies that a non-dominated solution is closer to the Pareto-frontier and the ideal solution than a dominated one, which is to say, it has more proximity. In this way, all solutions are compared against each other, and when a solution a dominates a solution a, the dominance count metric for this solution is increased by one. This process is based on the following formula:

$$FM_{2,a} = \sum_{b=1}^{N_{pop}} D_{a,b} \tag{5.5}$$

This is based on the dominance formula in equation (5.1). At the end of this process, the distance and dominance counts for all solutions are normalized in preparation for the final step, using a formula essentially equal to (5.3):

$$FM_{i,a}^{n} = \frac{FM_{i,a} - \min_{a} FM_{i,a}}{\max_{a} FM_{i,a} - \min_{a} FM_{i,a}} \tag{5.6}$$

The resulting normalized fitness metrics 1 and 2 are then aggregated into the final Fitness Metric used for solution ranking in the algorithm, designated $FM_{0,a}$ using equation (5.7):

$$FM_{0,a} = \sum_{i=1}^{N_{fit}} \omega_i FM_{i,a}^{n}$$

Where ω_i is used as a weight to give more importance to one metric over the others. Since the values used for the aggregation are already normalized, there is no need to transform this value any further. As there is currently no preference, all weights are set as equal and to the value of 1. The resulting aggregated fitness metric is used in step 9 of the algorithm as described above in section 4.2.

4.2.5. The Crossover Function

Crossover is the process by which a new generation, or set of solutions, is created. The process can have a single crossover point or multiple crossover points. Due to the simplicity of the chromosome, it was decided that a single crossover point is enough for the algorithm. The crossover point represents the locations in which the information from both selected parents mixes into the new solution. This is exemplified below in figure 4.3. The crossover inserts genes for the first parent into the new individual up to the crossover point $1 < p < N_k$, while the rest of the chromosome is made up of the second parent. Afterwards, if equation 4.21 is not satisfied, non-zero values in the chromosome vector are transformed to 0 until it can be satisfied.

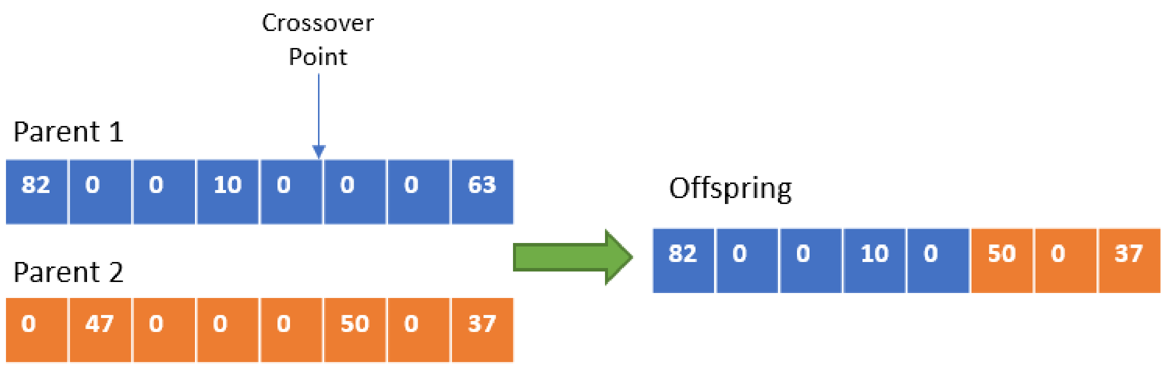

Figure 4.3: One-Point Crossover Representation

The selection of parents for the crossover also has different alternatives. There is a purely random selection process, in which two individuals are chosen from the parent population and used in crossover. This method has the drawback of not choosing better parents, which can cause the algorithm to take longer to find the optimal solutions. Another selection method is the roulette method. In this method, each solution has a probability of being chosen relative to their fitness values. This method can, in this way, have a better chance of selecting more "fit" solutions as the parents, which may create better offspring. Lastly, the method that we are going to use is called the tournament selection. This method randomly chooses two solutions to compete to be the first parent, and the one with the better fitness function gets the right to be the first parent. The same process is used to select the second parent. These two winning individuals are then used to perform the crossover process

Mutation can also occur during the crossover process. The mutation function helps the GA by inserting some random variation in the evolutionary process, to avoid it falling into local optima. It works by slightly changing the offspring from two parents with a small probability. In this sense, if mutation occurs, the origin of a value in the new chromosome will change from parent 1 to parent 2 or vice versa. This is exemplified in figure 5.4 below.

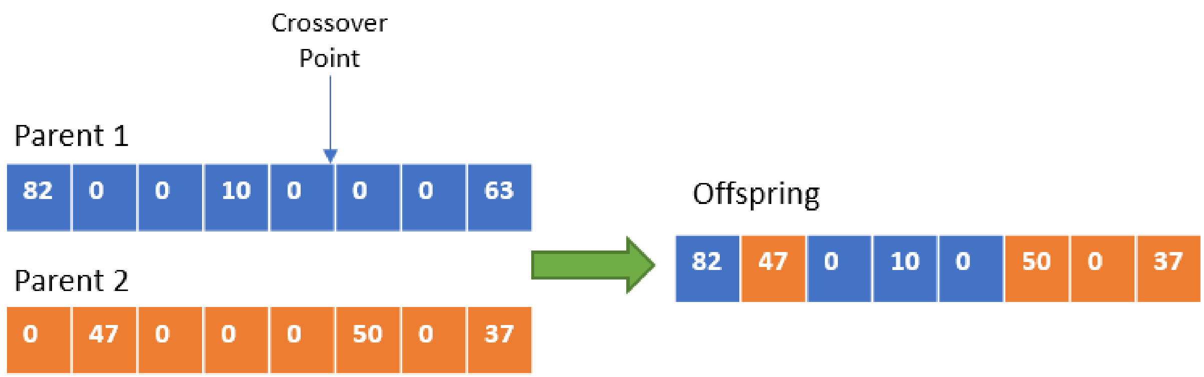

Figure 4.4: Mutation Function Example

In addition to the aforementioned process, elitism is also used to fill the new population with individuals. Elitism is a function within the GA to help keep the optimization process moving towards what it may assume at the moment to be the optimal solution. It selects the top percentage of the population in a generation, based on the fitness function, and adds it directly to the next generation of the population.

4.2.5. Obtaining the Pareto-Optimal Front

At the end of each generation and before creating the new one, all non-dominated solutions of a generation are stored separate from the rest of the algorithm. After the stopping criteria (described in the next section) have been met, these solutions are once again checked for dominance, and the dominated solutions are deleted from the set. This final set of solutions is the closest the algorithm has been able to get to the true Pareto frontier. As so, this solution set represents the Pareto-optimal set of solutions. This final set of solutions is what is then analyzed in Post-Pareto optimality procedures or presented directly to the decision-maker.

It is important to reiterate that the basic principle for a solution to be non-dominated is that there is no other single solution which is better than it in all objectives that are being considered. Generally speaking, this becomes more likely as the number of objectives increases. Brockhoff and Zitzler (2006) argue that a high number (≥3) of objectives can cause difficulties not only in approximating the Pareto-optimal set, but also in data visualization, processing time, and decision-making, as the multi-dimensionality of the solutions can present a challenge to both the researchers and the decision-makers. Dimensionality reduction methods are proposed by Brockhoff and Zitzler. However, these methods are not always suitable to use and are not very commonplace.

4.2.6. Algorithm Termination

The algorithm finishes its optimization process and goes into the Pareto-determination phase when certain criteria has been met. These criteria vary between different problems and the researcher's preference. One such criterion is based on convergence. Here, if the best solution does not change after a set number of generations, the algorithm decides that the best solution has been found and there is no need to continue calculating more iterations. This criterion is mostly applied to single-objective optimization problems but can be slightly adapted to fit multi-objective problems. A suitable change would be to stop if no new non-dominated solutions has been saved after a set number of iterations. Another, simpler stopping criterion is simply a pre-defined number of iterations. With this criterion, there is no need to check the solutions over many iterations, but simply decide how many times the iteration process will be repeated. Lastly, it is possible to use a combination of the two in order to attempt to minimize the total computational time.

In this study, a simple pre-defined number of generations is used as a stopping criterion, as determining convergence can be difficult in multi-objective optimization with a high number of objectives.

Chapter 5: Numerical Example (1)

This chapter serves to present a numerical example of the previously described algorithm. For the example, a modified version of the IEEE 1996 Reliability Test System (RTS-96) was used. This example shows a transmission system with 24 buses, connected by 38 lines where 32 generators are connected at various points in the system and where 2 buses have renewable energy systems attached to them. The objective of the example is to optimally allocate DSI-type D-FACTS devices along transmission lines, within specific parameters, with the objective of minimizing the previously described objective functions.

5.1. Numerical Data

This section will present the information on the system that will be used to perform the optimization. The changes made to the original IEEE RTS-96 system were described by Sang and Sahraei-Ardakani (2018) and will be presented below.

5.1.1. Bus Data

This sub-section presents the data for the buses in the system.

Table 5.1: Bus Data

BUS ID	LOAD (MW)	$P_{r,s}$ (MW)	Wind Curtailment Cost /MW
1	113.4	0	0
2	101.85	0	0
3	189	0	0
4	77.7	0	0
5	74.55	0	0
6	142.8	0	0

7	131.25	0	0
8	179.55	0	0
9	183.75	0	0
10	204.75	0	0
11	0	0	0
12	0	0	0
13	782.25	0	0
14	84.525	0	0
15	138.075	0	0
16	105	0	0
17	0	0	0
18	349.65	0	0
19	78.75	400	0
20	55.65	400	0
21	0	0	0
22	0	0	0
23	0	0	0
24	0	0	0

In this system, only two buses have renewable systems attached to them. Additionally, seven out of twenty-four buses have no loads on them and thus only serve as either generation points or intermediate points.

5.1.2. Line Data

This Subsection presents the data on transmission lines:

Table 5.2: Line Data

LINE ID	FROM BUS	TO BUS	Reactance	Susceptance	Thermal Limit (MW)
1	1	2	0.014	-71.4286	175
2	1	3	0.211	-4.73934	175
3	1	5	0.085	-11.7647	175
4	2	4	0.127	-7.87402	175
5	2	6	0.192	-5.20833	175
6	3	9	0.119	-8.40336	175
7	3	24	0.084	-11.9048	400
8	4	9	0.104	-9.61538	175
9	5	10	0.088	-11.3636	175
10	6	10	0.061	-16.3934	175
11	7	8	0.061	-16.3934	175
12	8	9	0.165	-6.06061	175
13	8	10	0.165	-6.06061	175
14	9	11	0.084	-11.9048	400
15	9	12	0.084	-11.9048	400
16	10	11	0.084	-11.9048	400
17	10	12	0.084	-11.9048	400
18	11	13	0.048	-20.8333	500
19	11	14	0.042	-23.8095	500
20	12	13	0.048	-20.8333	500
21	12	23	0.097	-10.3093	220
22	13	23	0.087	-11.4943	220

23	14	16	0.059	-16.9492	500
24	15	16	0.017	-58.8235	500
25	15	21	0.049	-20.4082	175
26	15	21	0.049	-20.4082	175
27	15	24	0.052	-19.2308	500
29	16	17	0.026	-38.4615	175
29	16	19	0.023	-43.4783	500
30	17	18	0.014	-71.4286	500
31	17	22	0.105	-9.52381	500
32	18	21	0.026	-38.4615	500
33	18	21	0.026	-38.4615	500
34	19	20	0.04	-25	500
35	19	20	0.04	-25	500
36	20	23	0.022	-45.4545	500
37	20	23	0.022	-45.4545	500
38	21	22	0.068	-14.7059	500

There are 38 lines in the system. The buses with the most lines connected to them are 9, 10, and 21, with 5 lines each, while the least connected is bus 24, with only one line attached.

5.1.3. Generator Data

This subsection will present the generator data for the system:

Table 5.3: Generator Data

ID	BUS	max	min	.S.1	.S.2	.S.3	.S.4	L.C.1	L.C.2	L.C.3	L.C.4	N.L. Cost

1	1	20	15.8	15.8	0.2	3.8	0.2	142.36	146.41	206.09	208.33	1138.68
2	1	20	15.8	15.8	0.2	3.8	0.2	142.36	146.41	206.09	208.33	1138.68
3	1	76	15.2	15.2	22.8	22.8	15.2	21.29	22.22	25.81	29.68	130.63
4	1	76	15.2	15.2	22.8	22.8	15.2	21.29	22.22	25.81	29.68	130.63
5	2	20	15.8	15.8	0.2	3.8	0.2	142.36	146.41	206.09	208.33	1138.68
6	2	20	15.8	15.8	0.2	3.8	0.2	142.36	146.41	206.09	208.33	1138.68
7	2	76	15.2	15.2	22.8	22.8	15.2	21.29	22.22	25.81	29.68	130.63
8	2	76	15.2	15.2	22.8	22.8	15.2	21.29	22.22	25.81	29.68	130.63
9	7	100	25	25	25	30	20	83.8	90.21	97.59	102.33	839.45
10	7	100	25	25	25	30	20	83.8	90.21	97.59	102.33	839.45
11	7	100	25	25	25	30	20	83.8	90.21	97.59	102.33	839.45
12	13	197	68.95	68.95	49.25	39.4	39.4	86.49	91.51	95.57	99.66	1159.93
13	13	197	68.95	68.95	49.25	39.4	39.4	86.49	91.51	95.57	99.66	1159.93

14	13	197	68.95	68.95	49.25	39.4	39.4	86.49	91.51	95.57	99.66	1159.93
15	15	12	2.4	2.4	3.6	3.6	2.4	105.45	107.02	120.88	136.95	72.68
16	15	12	2.4	2.4	3.6	3.6	2.4	105.45	107.02	120.88	136.95	72.68
17	15	12	2.4	2.4	3.6	3.6	2.4	105.45	107.02	120.88	136.95	72.68
18	15	12	2.4	2.4	3.6	3.6	2.4	105.45	107.02	120.88	136.95	72.68
19	15	12	2.4	2.4	3.6	3.6	2.4	105.45	107.02	120.88	136.95	72.68
20	15	155	54.25	54.25	38.75	31	31	18.43	19.05	19.85	20.92	252.67
21	16	155	54.25	54.25	38.75	31	31	18.43	19.05	19.85	20.92	252.67
22	18	400	100	100	100	120	80	2.21	2.24	2.3	2.36	215.08
23	21	400	100	100	100	120	80	2.21	2.24	2.3	2.36	215.08
24	22	50	50	50	0	0	0	0	0	0	0	0
25	22	50	50	50	0	0	0	0	0	0	0	0
26	22	50	50	50	0	0	0	0	0	0	0	0
27	22	50	50	50	0	0	0	0	0	0	0	0
28	22	50	50	50	0	0	0	0	0	0	0	0
29	22	50	50	50	0	0	0	0	0	0	0	0
30	23	155	54.25	54.25	38.75	31	31	18.43	19.05	19.85	20.92	252.67

31	23	155	54.25	54.25	38.75	31	31	18.43	19.05	19.85	20.92	252.67
32	23	350	140	140	87.5	52.5	70	18.74	19.84	20.61	21.78	358.23

There are a total of 32 generators in the system, attached at 10 different points in the system. Overall, the generation produced is more than enough to supply load even if all buses are at their peak load. In the table, there are four Linearized generation Segments (L.S.) with a corresponding four Linearized Cost segments (L.C.) and a column for the constant No-Load (N.L.) costs.

5.1.4. Scenario Data

This subsection presents the data for each scenario:

Table 5.4: Scenario Data

ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
R.F	0	0	0	0	0.2	0.2	0.2	0.2	0.6	0.6	0.6	0.6	1	1	1	1
Load Factor at BUS [1-24]	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.31	0.36	0.40	0.45	0.31	0.36	0.40	0.45	0.31	0.36	0.40	0.45	0.31	0.36	0.40	0.45
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.30	0.35	0.40	0.45	0.30	0.35	0.40	0.45	0.30	0.35	0.40	0.45	0.30	0.35	0.40	0.45
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95

	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95	0.65	0.75	0.85	0.95
Ps	0.04	0.05	0.04	0.01	0.20	0.19	0.15	0.03	0.09	0.06	0.04	0.01	0.06	0.03	0.01	0.00

Overall, there are 16 scenarios in the simulation: four scenarios correspond to different possible levels of load factors and four correspond to different factors indicating what percentage of the rated capacity for the renewable generators is being produced in the corresponding scenarios.

5.1.5. Heat Data

This subsection presents the data for heat output for each type of generator, differentiable by their capacity Pmax in table 5.3:

Table 5.5: Generator Heat Data

Size (MW)	Type	Corresponding Units	Fuel	Output %	MW	Net Plant Heat Rate (Btw/kWh)	Incremental Heat Rate Calculated

							by Continuous Function (Btu/kWh)
12	Fossil Steam	15, 16 ,17 ,18 ,19	#6 oil	20	2.4	16017	10179
				50	6	12500	10330
				80	9.6	11900	11668
				100	12	12000	13219
20	Combustion Turbine	1, 2, 5, 6	#2 oil	79	16	15063	9859
				80	16	15000	10139
				99	20	14500	14272
				100	20	14499	14427
50	Hydro	24, 25 ,26 ,27 ,28 ,29		100	50	Not applicable	
76	Fossil Steam	3, 4, 7, 8	Coal	20	15	17107	9548
				50	38	12637	9966
				80	61	11900	11576
				100	76	12000	13311
100	Fossil Steam	9, 10, 11	#6 oil	25	25	12999	8089
				50	50	10700	8708
				80	80	10087	9420
				100	100	10000	9877
155	Fossil Steam	20, 21, 30, 31	Coal	35	54	11244	8265
				60	93	10053	8541
				80	124	9718	8900
				100	155	9600	9381

197	Fossil Steam	12, 13, 14	#6 oil	35	69	10750	8348
				60	118	9850	8833
				80	158	9644	9225
				100	197	9600	9620
350	Fossil Steam	32	Coal	40	140	10200	8402
				65	228	9600	8896
				80	280	9500	9244
				100	350	9500	9768
400	Nuclear Steam	22, 23	LWR	25	100	12751	8848
				50	200	10825	8965
				80	320	10170	9210
				100	400	10000	9438

There are nine total different types of generators, each with different capacities and fuel types, and thus different corresponding emissions, as will be shown in section 5.1.6. The segments for the heat data actually correspond with the linearized generation segments in table 5.3.

5.1.6. Generator Emission Data

This subsection presents the emissions data for the generators based on their heat output:

Table 5.6: Emissions Data

IEEE-RTS Unit Group	**U20**	**U12,U100,U197**	**U76,U155,U350**
Unit type	GT	ST	ST
Fuel type	FO2	FO6	Bituminous Coal

Fuel sulfur content (%)	0.2	Unit-Specific	Unit-specific
Emissions Rate			
SO2 (Lbs/MMBTU)	0.2	Unit-specific	Unit-specific
NOX (Lbs/MMBTU)	0.5	0.5	Unit-specific
Part (Lbs/MMBTU)	0.036	0.1	Unit-specific
CO2 (Lbs/MMBTU)	160	170	210
CH4 (Lbs/MMBTU)	0.002	0.002	0.001
N2O (Lbs/MMBTU)	0.004	0.004	0.004
CO (Lbs/MMBTU)	0.11	0.04	0.02
VOCs (Lbs/MMBTU)	0.04	0.007	0.003

The data for the emissions was given in three groups for the fossil fuel types of generators. Note that the hydroelectric generators and the nuclear generators do not throw emissions into the atmosphere and are thus not in this table. Additionally, three of the contaminants have "Unit-specific" as the values. For simplicity, those values were set to the average of the existing values in other columns of the table.

5.1.7. Algorithm Parameters

As previously mentioned, an evolutionary algorithm needs certain parameters to be run. This subsection will briefly describe the parameters used in the algorithm before continuing on to the simulation results. These parameters are presented below in table 5.7:

Table 5.7: Algorithm Parameters

Parameter	Value
Number of Generations	100
Number of Individuals (N_{pop})	100
Mutation Factor	5%
Elitism	10%

Additionally, the following parameters are used as a base for some necessary parameters in the D-FACTS optimization problem. The values given in table 5.8 will later be modified in a sensitivity analysis to show how the solutions change based on changes in the parameters.

Table 5.8: D-FACTS Parameters

Parameter	Value
η_c, η_L	2.5%
C_{single}^{D}	\$3,000
N	10 years
Max. line reactance adjustment	±20%
I	6%
C_{inv}^{max}	25\$/hr

5.2. SIMULATION RESULTS

The algorithm was coded on MATLAB® 2019a and run on a Dell® desktop computer with an Intel® Xeon® W-2195 Processor and 256 GB of RAM. The total runtime for the algorithm was 19.59 seconds, and the results are summarized in figure 5.1 below. The first column has the cost objective in the x-axis, the second has GWP in the x-axis and the third has curtailment. The row y-axes are GWP, Curtailment, and LUF, respectively.

The Pareto fronts excluding the one outlier in the GWP objective, appear to show inverse relations between cost and LUF, curtailment and LUF, and possibly GWP and LUF, while seemingly showing direct relations between curtailment and GWP, a weak relation between cost and curtailment, and, interestingly, a positive relation between cost and GWP. The full detail of the allocation per line for each solution is given below in table 5.9. Note that some lines have been omitted from table 5.9. This is due to the fact that no D-FACTS were allocated to these lines either because the algorithm found no significant improvement to them or because they could not be installed due to the nature of the line (some lines exist as transformers with length=0 and D-FACTS cannot be assigned to them). For comparative purposes, the Objective functions for the base case with no D-FACTS installed are Cost = 73,114, GWP= 88.06, Curtailment = 64.5% and LUF= 0.053. These are given in table 5.10.

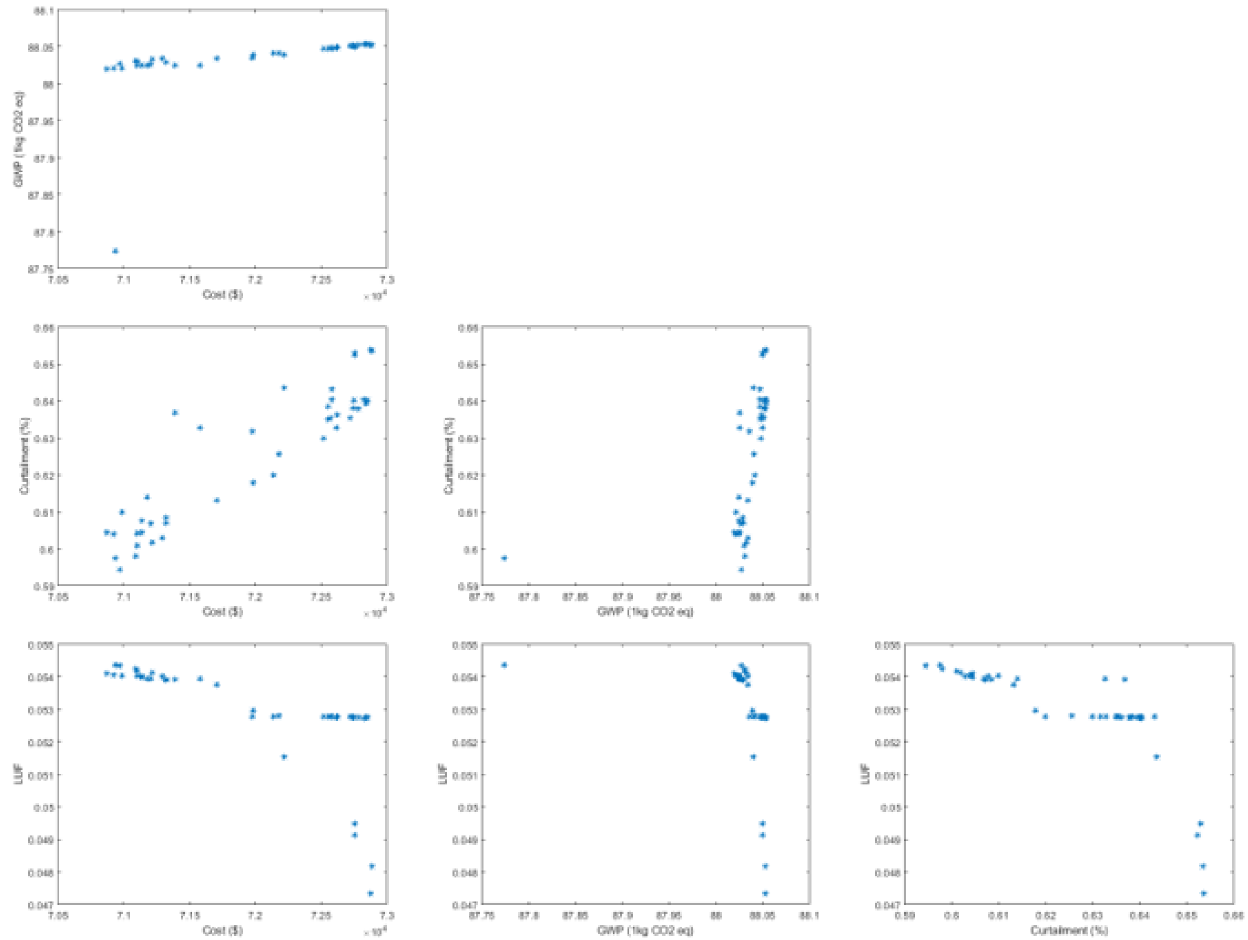

Figure 5.1: Pareto-Optimal Solutions Plot

Additionally, the average number of devices installed per line is given at the bottom of table 5.9, and it shows line 22 as the most crucial line for installing devices into, with line 23 as the second. It should be noted, however, that there are still many Pareto-optimal solutions which do not allocate D-FACTS devices in one or both of these lines. The total number of devices installed in each solution is given on the last column. As expected based on literature, the algorithm will usually attempt to allocate as many devices as possible within its parameters. However, solution 41 allocated only 675 devices, which is an interesting result, although it only remains in the Pareto set because of a slightly good LUF value and not because particularly good values in any of the other objectives.

The details of the objective function values are shown in table 5.10. The objective functions are also compared with the no-D-FACTS case (where the allocation vector is $\vec{\mathbf{0}}$) and the difference is given as a percentage for simplicity. Additionally, an average is given at the bottom to quickly compare the results from incorporating D-FACTS devices into the transmission network. Based simply off the averages, it can be seen that incorporating D-FACTS brings the average cost down to \$71,961, a 1.6% improvement from the \$73,114 from the base case; the average GWP down to 88, curtailment to 62.47% and LUF to 0.05. While on average not all these are a particularly noticeable improvement, there are solutions in which the improvement is much greater, as well as solutions in which these objectives actually worsen. This is part of the nature of a Pareto set and the process of improving the Pareto set falls under the field of Post-Pareto Optimality.

Table 5.9: Non-dominated solutions detail

	LINE																														
Sol.	**1**	**2**	**3**	**4**	**5**	**6**	**8**	**9**	**10**	**11**	**13**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**SUM**
1	6	0	0	0	0	0	0	201	0	0	0	0	0	0	720	0	0	0	0	0	72	0	0	0	0	0	0	0	0	0	0	999
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	690	0	0	0	0	0	24	0	0	0	0	108	180	0	0	0	0	1002
3	0	0	0	0	0	0	0	63	0	24	0	0	0	0	810	0	0	102	0	0	0	0	0	0	0	0	0	0	0	0	0	999
4	0	0	0	114	0	0	0	0	0	0	0	0	0	0	804	0	0	0	0	0	0	54	0	0	0	27	0	0	0	0	0	999
5	0	0	0	0	0	0	0	0	30	0	0	0	0	0	666	0	0	201	102	0	0	0	0	0	0	0	0	0	0	0	0	999
6	0	54	0	0	0	0	0	0	3	0	0	0	0	0	771	0	0	0	0	0	0	0	24	0	0	0	0	0	0	0	0	852
7	0	0	0	0	18	0	0	0	0	0	0	0	0	0	750	0	0	0	0	0	0	0	0	0	189	0	0	45	0	0	0	1002
8	12	102	0	0	135	0	0	0	0	0	0	0	0	0	663	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	912
9	24	0	0	0	0	0	0	0	0	0	216	156	0	0	603	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	999
10	0	51	0	0	0	0	0	0	0	0	153	0	0	0	633	0	0	0	99	63	0	0	0	0	0	0	0	0	0	0	0	999
11	0	0	0	0	0	0	0	0	0	0	0	234	0	0	522	0	0	0	0	0	96	0	0	0	147	0	0	0	0	0	0	999
12	0	0	0	0	0	0	0	0	156	0	0	252	0	0	543	0	51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1002
13	6	0	0	0	0	0	0	0	0	0	12	0	0	0	726	0	0	0	0	0	0	0	0	255	0	0	0	0	0	0	0	999
14	0	0	6	0	0	0	0	0	0	0	0	177	0	0	639	0	0	0	0	0	0	0	0	0	114	0	0	0	0	60	0	996
15	0	0	0	0	0	21	0	0	0	0	0	0	0	0	336	573	0	0	0	0	0	0	0	0	0	0	0	66	0	0	0	996
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	405	405	0	0	0	0	0	0	72	0	0	0	0	0	102	0	15	999
17	0	0	0	0	0	0	0	0	0	0	0	81	0	0	414	0	0	192	0	0	315	0	0	0	0	0	0	0	0	0	0	1002
18	0	0	0	0	0	0	0	0	0	0	0	0	93	0	417	0	0	0	0	0	267	0	0	0	222	0	0	0	0	0	0	999
19	0	0	0	0	0	60	0	0	0	0	0	0	0	0	246	429	0	0	0	0	0	225	0	0	0	0	39	0	0	0	0	999
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	714	0	0	0	0	222	0	0	0	0	0	6	57	0	0	0	999
21	0	0	87	0	0	0	0	0	0	0	0	0	0	0	99	594	0	0	0	0	0	207	0	0	0	0	9	0	0	0	0	996
22	0	0	0	0	0	0	0	0	0	0	0	612	0	0	0	285	0	0	0	0	0	0	9	0	0	0	0	0	0	93	0	999
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	687	132	0	0	0	0	0	87	0	0	0	0	0	0	93	0	999
24	6	0	0	0	0	0	0	0	0	0	0	0	0	408	0	0	0	0	0	0	288	168	0	0	0	0	0	0	0	0	0	870
25	21	0	0	0	0	0	255	0	0	0	0	0	0	0	0	408	0	0	0	192	0	123	0	0	0	0	0	0	0	0	0	999
26	0	0	0	0	0	84	0	237	0	0	0	0	0	0	0	309	0	0	0	0	60	0	0	0	0	0	0	309	0	0	0	999
27	0	0	0	0	168	0	0	0	0	0	12	0	0	0	0	303	0	282	0	0	0	0	0	0	0	0	0	231	0	0	0	996
28	0	0	0	60	0	405	0	0	0	0	0	0	0	0	0	321	0	0	0	0	0	0	0	0	0	213	0	0	0	0	0	999
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	264	90	324	0	0	0	0	0	0	0	0	174	0	0	0	0	852
30	21	0	0	0	0	0	0	0	0	3	0	327	0	0	0	0	0	318	0	0	0	0	0	0	0	0	330	0	0	0	0	999
31	0	231	0	0	0	0	0	0	66	0	0	0	0	0	0	300	0	0	0	279	0	0	0	0	0	0	0	120	0	0	0	996
32	0	0	0	0	0	0	0	0	0	0	0	291	0	0	0	0	0	99	0	390	0	0	0	0	0	0	201	0	0	0	15	996
33	39	0	0	333	0	0	0	0	0	0	0	327	0	0	0	0	0	0	0	300	0	0	0	0	0	0	0	0	0	0	0	999
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	99	0	0	0	393	0	0	0	156	0	0	351	0	0	0	0	999
35	0	438	0	0	0	0	0	0	0	0	0	0	0	0	0	162	0	0	0	345	51	0	0	0	0	0	0	0	0	0	0	996
36	0	0	57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	372	177	153	0	0	0	237	0	0	0	0	0	996
37	0	0	0	0	0	282	0	0	0	0	0	0	243	0	0	0	0	0	0	0	213	156	102	0	0	0	0	0	0	0	0	996
38	0	0	0	0	309	222	0	0	0	0	0	198	0	0	0	0	0	0	0	267	0	0	0	0	0	0	0	0	0	0	0	996
39	0	0	0	0	0	339	9	0	0	0	0	0	0	0	0	0	0	0	0	459	0	78	0	0	0	0	111	0	0	0	0	996
40	0	0	0	0	0	363	87	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	189	360	0	0	0	999
41	0	0	0	0	0	0	0	0	0	0	0	87	0	0	0	0	0	0	0	588	0	0	0	0	0	0	0	0	0	0	0	675
42	0	168	0	0	0	0	0	0	0	210	0	0	0	0	0	0	0	0	0	0	168	207	0	0	0	243	0	0	0	0	0	996
43	0	0	0	0	0	0	0	0	0	0	357	0	0	0	0	0	0	0	0	0	138	171	0	0	333	0	0	0	0	0	0	999
avg	3.1	24	3.5	12	15	41	8.2	12	5.9	5.5	17	64	7.8	9.5	266	136	6.3	35	4.7	85	49	36	6.8	9.6	23	19	37	28	2.4	5.7	0.7	

Table 5.10: Pareto-optimal solutions Objective Function detail

	OBJ				IMPROVEMENT (RESPECTIVE)			
Sol	COST	GWP	CURT	LUF	COST	GWP	CURT	LUF
1	$70,870	88.02	60.44%	0.054	3.07%	0.05%	6.29%	-2.10%
2	$70,921	88.02	60.40%	0.054	3.00%	0.04%	6.36%	-2.00%
3	$70,936	87.77	59.75%	0.054	2.98%	0.33%	7.37%	-2.56%
4	$70,967	88.03	59.44%	0.054	2.94%	0.04%	7.84%	-2.52%
5	$70,986	88.02	60.99%	0.054	2.91%	0.04%	5.44%	-1.94%
6	$71,092	88.03	59.80%	0.054	2.77%	0.03%	7.28%	-2.34%
7	$71,103	88.03	60.10%	0.054	2.75%	0.03%	6.82%	-2.21%
8	$71,104	88.02	60.41%	0.054	2.75%	0.04%	6.34%	-1.96%
9	$71,138	88.03	60.44%	0.054	2.70%	0.04%	6.30%	-1.84%
10	$71,139	88.02	60.76%	0.054	2.70%	0.04%	5.79%	-1.89%
11	$71,174	88.02	61.40%	0.054	2.65%	0.04%	4.80%	-1.76%
12	$71,208	88.03	60.69%	0.054	2.61%	0.04%	5.91%	-1.78%
13	$71,217	88.03	60.18%	0.054	2.59%	0.03%	6.70%	-2.14%
14	$71,290	88.03	60.29%	0.054	2.49%	0.03%	6.53%	-1.89%
15	$71,318	88.03	60.71%	0.054	2.46%	0.03%	5.87%	-1.70%
16	$71,319	88.03	60.84%	0.054	2.46%	0.04%	5.67%	-1.71%
17	$71,392	88.03	63.68%	0.054	2.36%	0.04%	1.27%	-1.73%
18	$71,578	88.03	63.27%	0.054	2.10%	0.04%	1.90%	-1.75%
19	$71,705	88.03	61.31%	0.054	1.93%	0.03%	4.95%	-1.42%
20	$71,977	88.04	63.17%	0.053	1.56%	0.03%	2.06%	0.42%
21	$71,984	88.04	61.78%	0.053	1.55%	0.02%	4.22%	0.07%
22	$72,140	88.04	61.99%	0.053	1.33%	0.02%	3.89%	0.42%
23	$72,178	88.04	62.56%	0.053	1.28%	0.02%	3.01%	0.39%
24	$72,217	88.04	64.36%	0.052	1.23%	0.02%	0.22%	2.76%
25	$72,521	88.05	62.99%	0.053	0.81%	0.01%	2.34%	0.42%
26	$72,553	88.05	63.50%	0.053	0.77%	0.01%	1.56%	0.42%
27	$72,553	88.05	63.84%	0.053	0.77%	0.01%	1.03%	0.42%
28	$72,571	88.05	63.53%	0.053	0.74%	0.01%	1.50%	0.44%
29	$72,580	88.05	64.32%	0.053	0.73%	0.01%	0.28%	0.42%
30	$72,581	88.05	64.02%	0.053	0.73%	0.01%	0.74%	0.43%
31	$72,616	88.05	63.28%	0.053	0.68%	0.01%	1.90%	0.44%
32	$72,621	88.05	63.63%	0.053	0.67%	0.01%	1.35%	0.46%
33	$72,720	88.05	63.53%	0.053	0.54%	0.01%	1.50%	0.45%
34	$72,746	88.05	63.81%	0.053	0.50%	0.01%	1.08%	0.47%
35	$72,750	88.05	64.02%	0.053	0.50%	0.01%	0.75%	0.46%
36	$72,753	88.05	65.31%	0.049	0.49%	0.01%	-1.25%	6.63%
37	$72,755	88.05	65.23%	0.049	0.49%	0.01%	-1.13%	7.31%
38	$72,786	88.05	63.78%	0.053	0.45%	0.01%	1.11%	0.47%
39	$72,826	88.05	64.03%	0.053	0.39%	0.01%	0.72%	0.50%
40	$72,839	88.05	63.93%	0.053	0.38%	0.01%	0.89%	0.47%
41	$72,851	88.05	64.00%	0.053	0.36%	0.01%	0.77%	0.48%
42	$72,879	88.05	65.37%	0.047	0.32%	0.01%	-1.35%	10.69%
43	$72,890	88.05	65.35%	0.048	0.31%	0.01%	-1.31%	9.10%
AVG	$71,961	88	62.47%	0.05	1.58%	0.03%	3.15%	0.17%

5.3. DISCUSSION

The minimum values for each objective were {\$70,879, 87.77, 59.44%, and 0.047}, respectively, for expected cost, expected GWP, expected renewable energy curtailment, and expected LUF, and the maximum values were {\$72,890, 88.05, 65.37%, and 0.054}. Of course, none of these values are present in the same solution, as these are Pareto-optimal solutions.

Up until now, the solutions were solved and analyzed under the assumption that a utility company would adjust their transmission system based on economic gain rather than ecological sustainability or long-term sustainability. These results reflect that assumption. The improvements to the system, however, do indicate that the installation of D-FACTS devices, with at least a 0.3% reduction in cost, and up to a 3.1%, which equates to up to \$2,244 an hour, or \$53,856 a day.

In terms of allocation, most solutions allocated the devices in either 4 or 5 lines, except for solution 41 which only allocated them in 2: 87 devices in line 19 and 588 in line 27. This is also the only solution to allocate only about 2/3 of the allowed devices, with most other lines allocating 996-1002 devices, although a few other solutions ranged in the 800s or around 80-90% of the allowance. This solution is worse than most others in terms of cost, GWP, and curtailment, but slightly better in terms of LUF. Further analysis would be required to determine whether this solution is worth maintaining at all.

Of the lines where D-FACTS are most commonly allocated, line 22 stands at the top, with 20 out of 43 solutions allocating devices into it, while line 23 is second with 15 solutions. Interestingly, only four solutions allocated devices to both of these lines at the same time. Other lines to which D-FACTS were allocated to frequently are line 19, with 11 counts, line 27, with 11 counts, line 28 with 13 counts, line 29 with 10 counts, and line 34 with 10 counts. Figure 5.2 below shows the distribution of the average number of D-FACTS devices installed per line as both the total average and the average of non-zero elements from table 5.9:

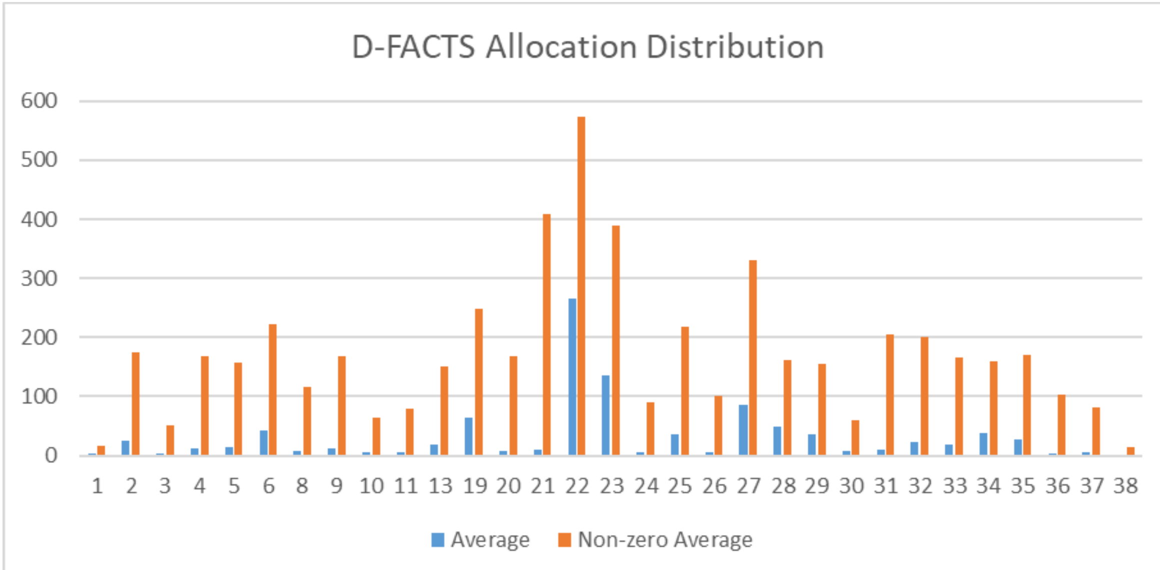

Figure 5.2: D-FACTS allocation distribution per line

As for the computational time, the algorithm benchmarked at 19.6 seconds for the given parameters and yielded a total of 43 solutions. The most similar implementation of the problem that could be found in literature was that of Sang and Sahraei-Ardakani (2019), who developed their experiment as a single-objective pure linear program, coded in Java and using the IBM CPLEX® optimizer. For comparative purposes, their algorithm was run on the same computer and, although it yielded better results on the objective of total costs, it did take a total time of 804.2 seconds to run. This means our algorithm has a runtime 97.6% smaller than a pure linear programming approach, which is a large improvement, especially if applied to larger networks.

5.4. Post-Pareto Analysis

After having performed the initial analysis based on the results of the Pareto set, further analysis will be performed to further refine the final non-dominated set and to improve parameters in subsequent experiments. As such, a series of tests will be performed to further improve the efficacy and efficiency of the algorithm.

Firstly, a correlation analysis will be performed on the objectives. The reasoning for this is that if there are two objectives that are very closely related, it becomes redundant to analyze all solutions for both of them, resulting in increased computational time over an objective that does not add much new information to out Pareto set.

Continuing on, a sensitivity analysis will be performed to test how some parameters may impact the Pareto set and how the model can be improved by tweaking these parameters within realistic limits. This can also help the decision maker by giving them various sets of solutions under different parameters so that they may make a more holistic choice by not only choosing which objectives are more beneficial, but under which parameters those exist and how those parameters affect the bottom line.

Finally, some Post-Pareto pruning techniques will be applied to discard solutions which may not be as desirable and reduce the burden on the decision maker.

5.4.1. Correlation Analysis

Correlation analysis, also known as bivariate analysis, is primarily concerned with finding out whether a relationship exists between variables and then determining the magnitude and action of that relationship. In essence, it is used to determine whether two variable follow similar trends and how similar these trends are.

The correlation coefficient between two random variables can be defined as:

$$\rho_{X,Y} = corr(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

Modern statistical software can quickly calculate this coefficient for a large set of variables and organize the result into a simple, easy to read table. Figure 6.3 below was created using Minitab® statistical software:

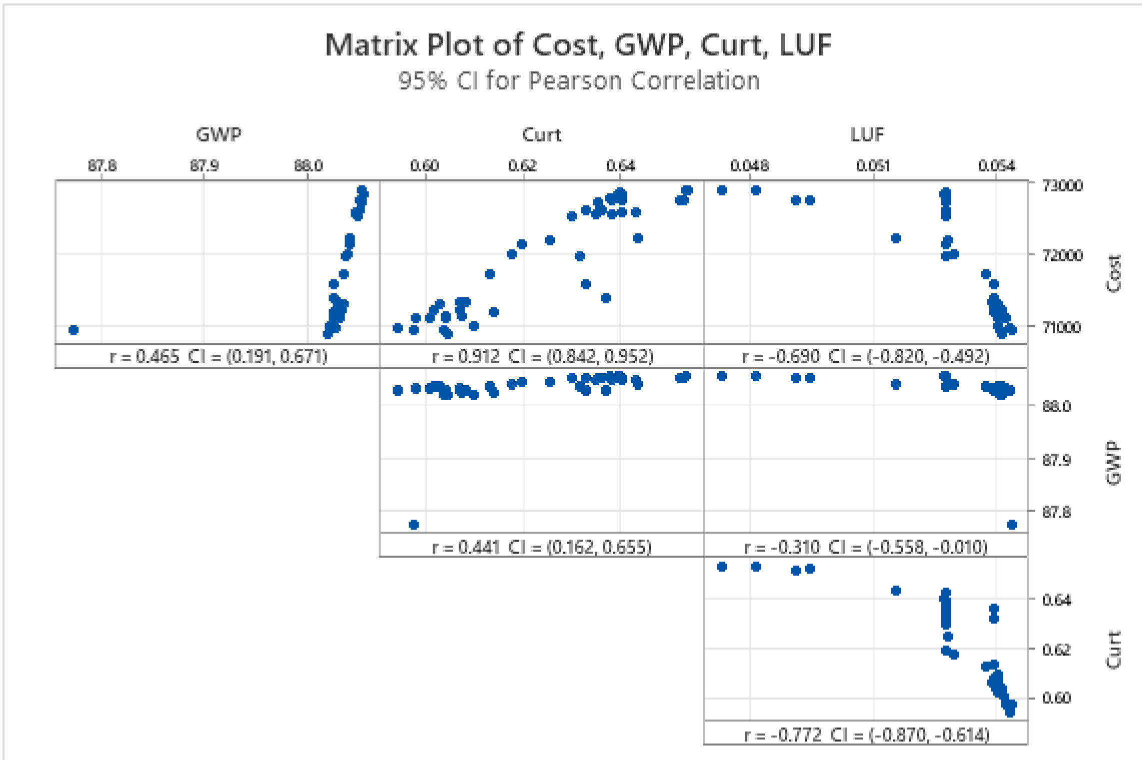

Figure 5.3: Correlation Matrix Plot

Based on the information we can obtain from the above graph, there is a very strong correlation between the objectives of total expected cost and curtailed renewable energy, with a correlation of 0.912 between them. In addition, the correlation of these two objectives with other objectives are also very similar: Cost/GWP has a correlation of 0.465 and Curtailment/GWP has a correlation of 0.441, while Cost/LUF has a correlation of -0.690 and Curtailment/LUF a correlation of -0.772. A regression analysis for the two variables is summarized below, with curtailment as the response variable and cost as the predictor.

Regression Equation: -0.971 + 0.000022*Cost

Model Summary:

S=0.0075273, R-squared = 83.16%, R-squared (adjusted) =82.75%

Unusual Observations were noted in solutions 31 and 40, with unusually large standard residuals of 3.35 and 2.23, respectively.

Since these two objectives follow a trend so similar to each other, and their R-squared value is fairly high, despite the apparent noise in the plot, it is safe to remove one of these variables to simplify the Pareto Analysis. Since Cost is a more important variable than renewable energy curtailment, it will be kept, while renewable energy curtailment will be deleted. In addition, reducing the number of objectives can allow for an improvement on the size and quality of the Pareto set.

A second case study with a reduced number of objectives is presented below in section 5.5.

5.5. Case Study 1.2: Reduced Objective Set

Upon re-running the algorithm without accounting for the objective of Renewable energy curtailment, 22 new solutions were found. These solutions are summarized in figures 5.4 and 5.5 below:

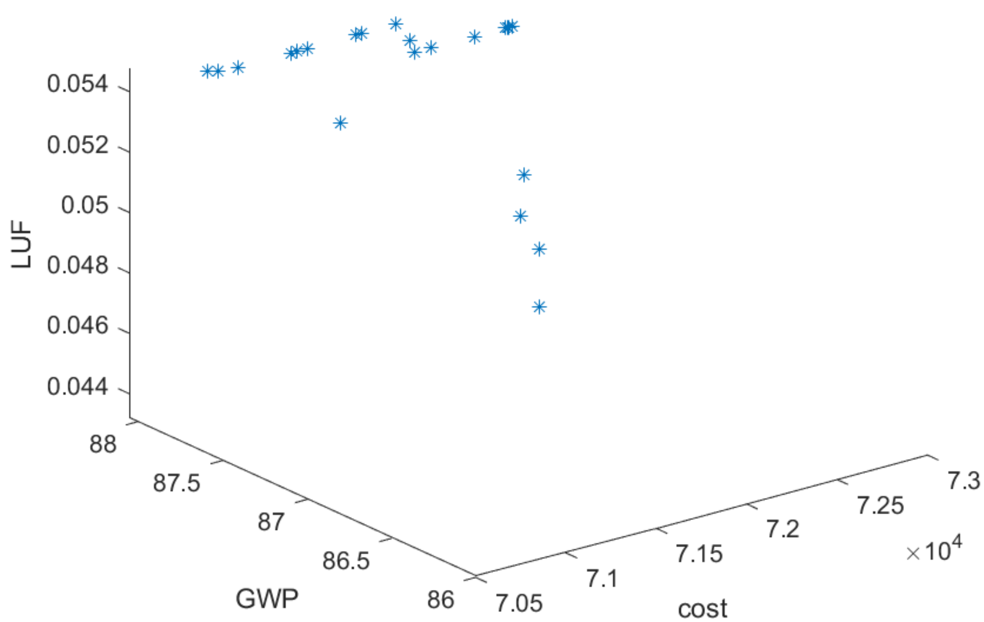

Figure 5.4: 3-D Pareto set

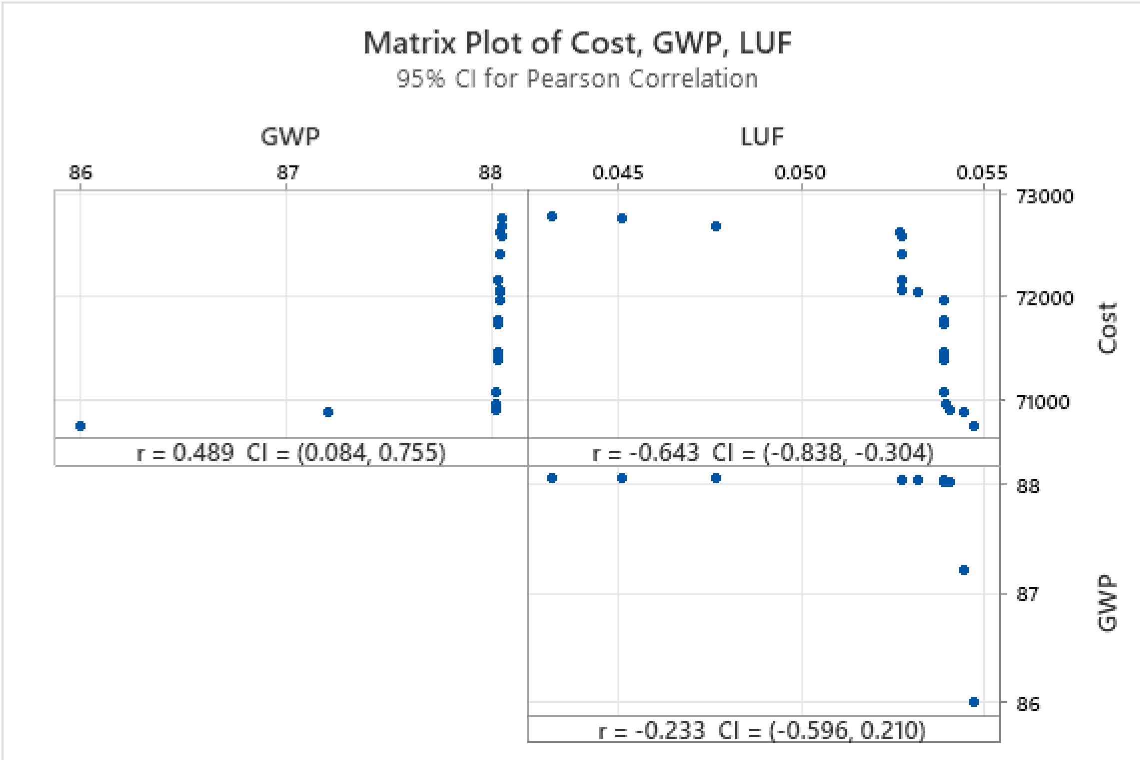

Figure 5.5: 2-D Pareto Set and Correlation Matrix Plot

Based on these values, the new objectives can be considered independent enough from each other to not reduce them further. However, no significant changes were found on the objective values to merit further discussion.

Chapter 6: Published Studies

In this chapter, further studies into the topic of D-FACTS allocation optimization, which were presented and published in various journals and/or conferences are to be included. The formatting has been altered to align with the rest of this document. For reference, the full citations will be given. The first paper included here is titled "A Computationally Efficient Evolutionary Algorithm for Stochastic D-FACTS Optimization" (Castillo Fatule et al 2021). It was published in the 2020 North American Power Symposium held online, and was the first step towards applying metaheuristic algorithms for the solving of D-FACTS allocation optimization. The second is "Co-Optimizing Operating Cost and Renewable Energy Curtailment in D-FACTS Allocation," where a second objective, namely renewable energy curtailment, was added to the problem, turning it into a multi-objective one (Castillo Fatule et al, 2021). This was published in the 2021 North American Power Symposium held in College Station, TX, USA. The third paper is "Fine-Tuning the Parameters for Solving the Multi-Objective D-FACTS Optimal Allocation Problem," which consisted mainly of a sensitivity analysis in order to improve the optimization process and study the effects of some parameters. This was published in the 2022 IEOM Society conference held in Orlando, FL, USA (Castillo Fatule et al, 2022). Next is "Analyzing the Effects of Line Switching Protocols on Multi-Objective D-FACTS Allocation Optimization," which added another congestion-relief protocol to the problem and finding that, at least in the test system being used, the combination of multiple methods can have stronger effects on the system (Castillo Fatule et al, 2022). It was pointed out for this study that there is no comparison point between the two methods individually, nor between both in conjunction and only the line-switching protocols. The main reason for the lack of this analysis is the lack of space in the conference papers. Chapter 7 will address this. In addition, other articles have been submitted for review and publication, but have not yet been accepted.

6.1. A Computationally Efficient Evolutionary Algorithm for Stochastic D-FACTS Optimization

Abstract—Flexible AC transmission systems (FACTS) are important contributors to smart transmission systems. They can offer a level of power flow control and improve transfer capability of an existing network which can be used to mitigate congestion and integrate renewable energies into a grid. Distributed FACTS is a lightweight version of FACTS which can be redeployed conveniently. It has become a more attractive power flow control technology due to its lower cost and ease of installation. This paper proposes a novel evolutionary algorithm to solve a stochastic model for D-FACTS allocation, studying their impacts on operating costs and the computational efficiency of the model. The results are presented are compared against a previously developed linear programming model and show a positive economic impact from the use of D-FACTS, as well as a significant reduction in computational time for this type of model.
Index Terms--*Distributed flexible AC transmission systems (D-FACTS), evolutionary algorithm, metaheuristics, optimal allocation, stochastic optimization*

I. Nomenclature

Indices	
k	Transmission line
g	Generator
i	The number of D-FACTS installed per phase per a certain distance for a transmission line
n	Node
s	Scenario
seg	Segment of linearized generator cost function
Sets	
$\sigma^{+}(n)$	Transmission lines with their "to" bus connected to node n
$\sigma^{-}(n)$	Transmission lines with their "from" bus connected to node n
Variables	
C_{inv}^{D}	Total investment in D-FACTS ($)
$F_{k,s}$	Real power flow through transmission line *k* in scenario *s*
$P_{g,s}$	Real power generation of generator *g* in scenario *s*
$P_{g,s}^{seg}$	Real power generation of generator g in scenario s in segment seg
$R_{g,s}^{D}$	Spinning down reserve available through generator g in scenario s
$R_{g,s}^{D}$	Spinning up reserve available through generator g in scenario s
$x_{k,i}^{D}$	Binary integer indicating D-FACTS installed in transmission line k or not; when its value is 1, it means i D-FACTS are installed on line k
$\theta_{b,s}$	Voltage angle at bus b in scenario s
$\theta_{fr,k,s}$	Voltage angle at the "from" node of line k in scenario s

$\theta_{to,k,s}$	Voltage angle at the "to" node of line k in scenario s
Parameters	
C_g^{NL}	No load cost of generator g
$C_{g,seg}^{linear}$	Linear cost of generator g in segment seg
C_g^D	Down reserve of generator g in segment seg
C_g^U	Up reserve of generator g in segment seg
C_{single}^D	Cost of a single D-FACTS unit
C_{sh}^D	Cost of a single D-FACTS unit converted to an hourly figure ($/hr)
C_{inv}^{max}	Maximum investment allowed for D-FACTS
$f_{k,s}$	Binary integer indicating direction of power flow through line k in scenario s
F_k^{max}	Thermal capacity/voltage drop limit of transmission line k
i_{max}	Maximum number of D-FACTS that can be allocated per a certain distance per phase
I	Interest rate/discount rate
$L_{n,s}$	Load at bus n in scenario s
N	Lifespan of D-FACTS
N_g	Total number of generators
N_s	Number of scenarios
N_{seg}	Number of segments for the linearized generator cost function
p_s	Probability of scenario s
P_g^{max}	Upper generation limit of generator g
P_g^{min}	Lower generation limit of generator g
S^D	Spinning down reserve requirement
S^U	Spinning up reserve requirement
X_k	Reactance of transmission line k
η_C	The maximum adjustment percentage of the line's reactance in the capacitive mode that a single D-FACTS module (1 device/phase/mile) can achieve
η_L	The maximum adjustment percentage of the line's reactance in the inductive mode that a single D-FACTS module (1 device/phase/mile) can achieve.
$\Delta\theta_k^{max}$	Maximum value of bus voltage angle difference to maintain stability for line k.
$\Delta\theta_k^{min}$	Minimum value of bus voltage angle difference to maintain stability for line k.

II. Introduction

There are a number of issues currently affecting electric grid systems in the US. One of the most significant issues is the transmission congestion, caused both by overloaded lines and the addition of new generation resources without adequate upgrading of aging systems. Some solutions to congestion include transmission expansion, energy storage implementation, demand response, and power flow control technologies [1]. In the area of power flow technologies, variable-impedance series flexible AC transmission systems (FACTS) can provide effective flow control and help implement smart transmission systems [2]. As an alternative to conventional FACTS, a relatively new technology called Distributed FACTS (D-FACTS) can be used. These D-FACTS are light-weight versions of the traditional D- FACTS, with the advantage of a lower cost and a possibility of being re-deployed after its original installation. These devices have thus become a popular subject of study in recent years [3]. D-FACTS were originally introduced by [4] with the proposition of a cost-effective alternative to FACTS. Ref. [4] proposes the different types of D-FACTS, as well as the underlying equations to model the behavior of these. Overall, there are three main types of D-FACTS, namely Distributed Series Static Compensator (DSSC), Distributes Series Reactor (DSR), and Distributed Series Impedance (DSI). DSR and DSI can adjust the impedance of a transmission line, while DSSC functions as a phase shifter [5]-[11] . Models for the efficient allocation of D-FACTS devices are still fairly scarce in literature. This can be attributed largely to the relative novelty of such devices as well as the high computational burden that such models can have. Unlike conventional FACTS devices, where just one device is installed at critical nodes in the network, D-FACTS bring the necessity to allocate a much larger number of devices along every line in the system, resulting in an exponentially larger number of variables. Some work on allocation of D-FACTS using optimization algorithms is in [12], where a Steepest Descent algorithm is used to allocate DSSC-type devices. The study used a non-linear DC-power-flow-based model which considered D- FACTS as a static change in impedance over the line where it is installed, and it does not take advantage of the adjustability of the devices. Ref. [13] proposed a Particle Swarm Optimization heuristic to reduce overloading in lines, but considering a single static scenario. Ref. [14] used a sensitivity- based technique, basing their allocation of DSSC-type devices on the sensitivity of transmission losses with respect to line impedance, and testing the findings over a simulated 118-bus system. This study also considers a single scenario and does not accommodate any uncertainties. Similarly, [15] attempted to minimize loss by using a Graph Theory approach, testing their results over different generation and load conditions. However, the model is designed to provide voltage support rather than controlling active power flow. Ref. [16] used a heuristic algorithm called backtracking search algorithm to allocate D-FACTS modules on microgrids to enhance their performance, considering various renewable energies, but only on a single scenario. Other studies proposed different optimization methods to allocate D-FACTS devices on a network. For example, [17] used a biogeography-based optimization technique to minimize cost as a function of energy price and a line utilization factor on a static scenario. [18] also used a two-stage Tabu-search metaheuristic for allocation of D- FACTS in radial distribution systems. [18] also considers the impact of renewable energies, considering the uncertainty in a Monte Carlo simulation. The two stages of their Tabu-search consist on first the allocation of the devices, and then the optimization of the output with cost as the objective function. More recently, a Linear Programming model for D-FACTS allocation considering renewable energies and uncertainties in demand was formulated and solved by [1]. This model, however, due to its formulation as a Mixed-Integer Linear Program (MILP), is forced to include a large number of binary variables, which makes it computationally intensive, and must linearize some constraints to reduce the computational burden. The most recent work on D-FACTS allocation comes in [19], who proposes a Line Utilization Factor and Particle Swarm-based algorithmic rule to allocate thrystor controlled series capacitor devices. The results show promising computational efficiency compared to other algorithms, but only considers a single scenario. In sum, existing optimal allocation model either do not fully consider the flexibility that D-FACTS has to offer, or do not consider future uncertainties, or are computationally challenging. Thus, this study aims to propose a computationally efficient evolutionary algorithm (EA) to solve a stochastic optimal allocation model for variable-impedance D-FACTS (such as DSR and DSI types) considering future uncertainty. The objective is to minimize the operating costs over the lifetime of the devices, considering uncertainties in power systems caused fluctuating demand levels. The algorithm was implemented on a modified RTS-96 test system, and results show that the proposed algorithm can give a near-optimal solution in just a fraction of the time that the previously proposed LP took to solve the problem, significantly improving the computational efficiency. The rest of the paper is organized as follows. Section III describes the formulation of the optimal allocation model as well as the EA, Section IV discusses case study results, and conclusions are drawn and future work is discusses in Section V

III. Model Formulation

A. D-FACTS Allocation Model

This paper proposes a DCOPF-based D-FACTS allocation model which allocates D-FACTS modules per phase. This model is a stochastic optimization model, which considers different scenarios to address the future uncertainty. The model differs from the mixed-integer linear program (MILP) model proposed in [1] in two aspects: (1) the proposed model is nonlinear, but the model in [1] is linear; (2) the proposed model is able to allocate D-FACTS modules per phase, but the model in [1] has to allocate D-FACTS modules per phase per a certain distance, thus, the proposed model offers more flexibility in terms of the number of D-FACTS allocated on each line. Although the model is nonlinear and allocates D-FACTS per phase, it can still be solved in a computationally efficient manner, since it will be solved using an EA

In the proposed model, series variable-impedance D- FACTS modules are allocated. Since the reactances of the lines need to be adjusted, applicable power flow constraints depend on power flow directions, as discussed by [20]:

If $\theta_{fr,k,k} - \theta_{to,k,s} \geq 0,\ \theta_{fr,k,s} - \theta_{to,k,s} / X_k^{max} \leq F_{k,s} \leq \theta_{fr,k,s} - \theta_{to,k,s} / X_k^{min}$ (1)

If $\theta_{fr,k,k} - \theta_{to,k,s} \leq 0,\ \theta_{fr,k,s} - \theta_{to,k,s} / X_k^{min} \leq F_{k,s} \leq \theta_{fr,k,s} - \theta_{to,k,s} / X_k^{max}$ (2)

In the proposed model, a base-case DCOPF model with no D-FACTS is first solved to obtain the power flow direction $f_{k,s}$ for each transmission line in every scenario, and the value for this variable is then used in the next step of the allocation problem, in which the number of devices is assigned to each line. When $f_{k,s} = 1$, the power flow direction is the same as the reference direction (from the 'from bus' to the 'to bus'), and when $f_{k,s} = 0$, the power flow direction is the opposite to the reference direction (from the 'to bus' to the 'from bus'). The formulation for optimal D-FACTS allocation is described by Equations (3) – (25).

$$\min\left(\sum_{s=1}^{N_s} p_s\left(\sum_{g=1}^{N_g}\left(\begin{array}{c}\sum_{seg=1}^{N_{seg}} C_{g,seg}^{linear} P_{g,s}^{seg} + C_g^U R_{g,s}^U \\ +C_g^D R_{g,s}^D + C_g^{NL}\end{array}\right)\right) + C_{inv}^D\right) \quad (3)$$

$$P_{g,s} = \sum_{seg=1}^{N_{seg}} P_{g,s}^{seg} \quad (4)$$

$$P_g^{min} \le P_{g,s} \le P_g^{max} \quad (5)$$

$$-F_k^{max} \le F_{k,s} \le F_k^{max} \quad (6)$$

$$\sum_{k\in\sigma^+(n)} F_{k,s} - \sum_{k\in\sigma^-(n)} F_{k,s} + \sum_{g\in g(n)} P_{g,s} = L_{n,s} \quad (7)$$

$$\sum_{g=1}^{N_g} R_{g,s}^U \ge S^U \quad (8)$$

$$\sum_{g=1}^{N_g} R_{g,s}^D \ge S^D \quad (9)$$

$$R_{g,s}^U \le P_g^{max} - P_{g,s} \quad (10)$$

$$R_{g,s}^D \le P_{g,s} - P_g^{min} \quad (11)$$

$$R_{g,s}^U \ge 0 \quad (12)$$

$$R_{g,s}^D \ge 0 \quad (13)$$

$$\Delta\theta_k^{min} \le \theta_{fr,k,s} - \theta_{to,k,s} \le \Delta\theta_k^{max} \quad (14)$$

$$\theta_{1,s} = 0 \quad (15)$$

$$x_{k,i}^D f_{k,s}(1 + i\eta_L)X_k F_{k,s} \ge x_{k,i}^D f_{k,i}(\theta_{fr,k,s} - \theta_{to,k,s}) \quad (16)$$

$$x_{k,i}^D f_{k,s}(1 + i\eta_C)X_k F_{k,s} \le x_{k,i}^D f_{k,i}(\theta_{fr,k,s} - \theta_{to,k,s}) \quad (17)$$

$$x_{k,i}^D (1 - f_{k,s})(1 + i\eta_L)X_k F_{k,s} \le x_{k,i}^D (1 - f_{k,i})(\theta_{fr,k,s} - \theta_{to,k,s}) \quad (18)$$

$$x_{k,i}^D (1 - f_{k,s})(1 + i\eta_L)X_k F_{k,s} \ge x_{k,i}^D (1 - f_{k,i})(\theta_{fr,k,s} - \theta_{to,k,s}) \quad (19)$$

$$\left(1 - \sum_{i=1}^{i_{max}} x_{k,i}^D\right)X_k F_{k,s} \ge \left(1 - \sum_{i=1}^{i_{max}} x_{k,i}^D\right)(\theta_{fr,k,s} - \theta_{to,k,s}) \quad (20)$$

$$\left(1 - \sum_{i=1}^{i_{max}} x_{k,i}^D\right)X_k F_{k,s} \le \left(1 - \sum_{i=1}^{i_{max}} x_{k,i}^D\right)(\theta_{fr,k,s} - \theta_{to,k,s}) \quad (21)$$

$$\sum_{i=1}^{i_{max}} x_{k,i}^D \le 1 \quad (22)$$

$$C_{inv}^D = \sum_{k=1}^{N_{br}} \sum_{i=1}^{i_{max}} 3iC_{sh}^D x_{k,i}^D \quad (23)$$

$$C_{inv}^D \le C_{inv}^{max} \quad (24)$$

$$C_{sh}^D = C_{single}^D \frac{I(1+I)^N}{8760((1+I)^N - 1)} \quad (25)$$

In this formulation, the objective is to minimize the total operation cost, including generation dispatch cost, reserve cost, and no-load cost and also D-FACTS investment, considering all the scenarios and their probabilities. (4) and (5) are the generation constraints, where (4) segments the production in order to linearize the cost curve, and (5) denotes the lower and upper limits for each generator producing a load. Eq. (6) indicates the transmission limits for each line. Based on previous models, the capacity of short lines (0–50 miles) is set to their thermal limit; for medium lines (50–156 miles), the capacity is determined by the voltage drop limit, and long lines (more than 156 miles), are limited by the angular stability limit. Eq. (7) defines the power balance at each bus in each scenario. Eqs. (8)–(13) are the reserve requirements and (14) and (15) are the bus voltage angle constraints. In order to indicate the quantities and locations for D-FACTS installation, a binary integer variable, $x_{k,i}^{D}$, is introduced in Eqs. (16) – (23). For each line, $x_{k,i}^{D}$ is an array with i_{max} elements. When $x_{k,i}^{D} = 1$, the number of D-FACTS modules that are allocated to line k is i; if no $x_{k,i}^{D}$ is 1 for all i for a line k, no D-FACTS is allocated to line k. Eqs. (16) and (17) shows the DC power flow equations when D-FACTS modules are installed on the line and the power flow direction is the same as the reference direction, Eqs. (18) and (19) shows the DC power flow equations when D-FACTS modules are installed on the line and the power flow direction is the opposite to the reference direction, and Eqs. (20) and (21) shows the DC power flow equations when D-FACTS modules are not installed on the line. Eq. (22) makes sure that $x_{k,i}^{D}$ can only set a single value for the number of devices for the line. Eq. (23) defines the total investment cost, and this cost is limited in Eq. (24). Eq. (25) calculates the cost of each D-FACTS modules as an hourly figure considering the discount rate and lifespan of the modules.

B. The Evolutionary Algorithm

EAs are stochastic search algorithms inspired by natural evolution processes. They have been used in several complex problems arising in real-world applications which need effective algorithms that are able to achieve good (but not necessarily optimal) solutions by performing an effective and intelligent search of the space of possible solutions [21]-[23].

In the present research, we develop a new EA to solve the D-FACTS allocation problem to improve the computational efficiency and reduce the number of variables that need to be calculated. This allows us to remove linearization in some constraints and simplify the formulation. It is thus possible to make the solution as accurate while reducing the computational time.

The algorithm starts by generating random values within possible solution ranges, before testing if these values violate any constraints. If the constraints are not violated, the algorithm then calculates and stores the objective function value for the solution. Otherwise, it assigns a value of infinity. This is repeated for both steps of the solution process, obtaining the power flow directions $f_{k,s}$ before the second step.

Two main search operators in an EA are crossover and mutation. Crossover is the process of exchanging chromosome material to create a new offspring, and mutation helps to diversify the population. The chromosome developed in our EA is a 3-dimensional array where each row represents a different set of variables (Power generation, reserves, D-FACTS allocation for each generator or line), stored in an orderly fashion within each row, with the values for each scenario stored in the 3rd dimension. In the first three rows, generation and reserve levels for each available generator is encoded as a percentage of the possible power generation for each generator. e.g., if a generator's minimum possible generation is 30 MW, and the maximum is 40MW, a value of 0.5 in the corresponding gene means that the generator is set to 30 + (40 - 30) × 0.5= 35MW. A negative value, on the other hand, indicates that the generator will handle no load.

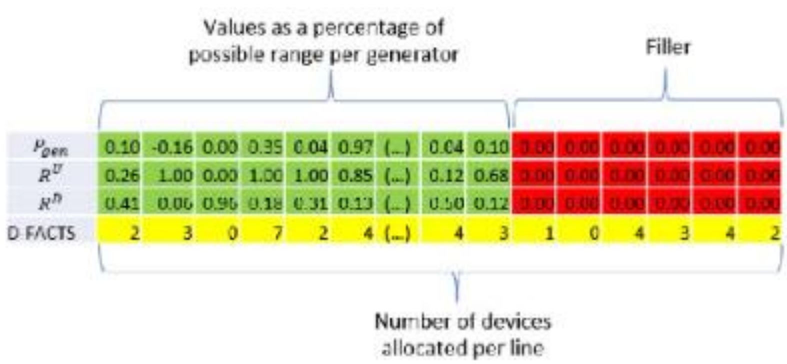

Figure 6.1.1. Chromosome Example

An example of one layer of the chromosome is presented in Fig. 1. The full chromosome consists of N_s layers of this type of encoding, for each possible scenario. The section denoted in yellow (the allocation of D-FACTS devices), however, is constant for all scenarios, as it is not possible to change the installed number of devices on such short demand. The standard procedure for an EA is summarized in Fig. 2.

In evaluating a possible solution, the following steps are taken, for each scenario:

1) Verify that the assigned generation meets demand. If not, use a simple linear transformation to assure demand is met.

2) Obtain power flows and voltage angles for each transmission line and bus based on the load and generation.

3) Verify that these flows and angles meet the constraints.

If the constraints are not satisfied for any single scenario, the solution is marked as unfeasible and its objective function value set to infinity, otherwise, the objective function value is obtained via (25) and assigned to the chromosome. After each iteration, the solutions are ranked based on their objective functions.

In a new iteration, solutions are created via a triple single-point crossover. A cut-point is generated in each dimension of the array, and two randomly selected solutions are combined in a checkerboard pattern to generate a new one. This process repeats until a set number of iterations has been run or there is no improvement for the optimal objective function value after a set number of iterations. Mutation can occur on newly generated solutions by switching around values in the generator configurations (green section of Chromosome shown in Fig. 1).

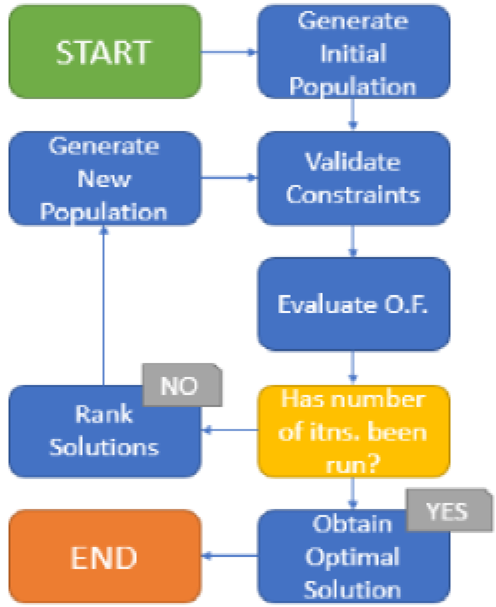

Figure 6.1.2. Evolutionary Algorithm Flowchart

IV. Case Studies

A. Simulation Setup

In this study, the model in Section II-A is adopted to study the cost-effectiveness of D-FACTS using the new algorithm proposed in section II-B, and the computational efficiency of the solution method is discussed. The case studies use the same RTS-96 test system as Ref. [1] except that this study uses the original generator settings and fuel prices provided by the RTS-96 test system. Uncertainties in the model is represented using four different load scenarios, and the load factors are 0.65, 0.75, 0.85, and 0.95, respectively.

It is assumed that each D-FACTS module is designed to be able to adjust the line's reactance by ±2.5% per phase per mile, and the maximum reactance adjustment range for a 3-phase line is ±20% [4]. D-FACTS results were obtained with the D- FACTS allocated evenly per line per phase.

D-FACTS costs were determined based on industry data and previous academic studies [1]. It is assumed the cost for D- FACTS to be \$100/kVA; where the compensation level in kVA depends on the parameters of the transmission line in which the D-FACTS device is installed. For simplicity, the compensation level for the most demanding line was adopted. In the RTS-96 system, the largest compensation level is 30kVA/module. Thus, a cost of $3000/module is used in this study. The hourly cost of the devices is based on Eq. (25), with an assumed lifespan of 30 years and a discount rate of 6%. Eq. (24) puts an investment limit on the D-FACTS modules. For the case studied, a D- FACTS allowance cost of $25/hour was assumed.

B. Comparison of Cost Savings

To demonstrate out algorithm's performance against previously developed linear programs, the results of [1] are used to compare the effectiveness of the proposed algorithm. In order to do this, simulations were carried out under similar conditions as the specific scenario we are evaluating, described previously in Section IV-A. D-FACTS can reduce congestion in the network as well as the expected dispatch cost. The expected dispatch cost in each simulation case was obtained from the objective function of the base case and D-FACTS optimization models, and compared with the results of the linear programming problem solution in [1]. The resulting expected costs from both cases and both algorithms are presented in Table I and illustrated in Fig. 3. It can be seen that, similar to the results obtained from the linear programming solver, there are significant savings between the base case and the D-FACTS case. The cost obtained through the EA is slightly higher than those obtained through the linear programing solver, and this is due to the nature of such types of algorithms, which cannot guarantee the finding of an optimal solution. However, the differences between the operating costs solved by the linear programming solver and the EA are less than 2%, which is not significant. According to the solutions, the linear programming formulation's solution requires the installation of about 20% more D-FACTS modules compared to the EA. The difference in devices can be partly attributed to the 'per phase, per mile' formulation used in the linear model in Ref. [1], which can make the problem less computationally intensive by reducing the range of some variables, but may restrain the flexibility in D-FACTS allocation. The results show that the proposed method has merit in terms of solution quality and reliability

Table 6.1.1. Comparisons Between Results Of Different Solution Algorithms

	Linear Program	Evol. Algorithm	Difference (%)
Base Case	$79,500	$80,269	0.96%
D-FACTS	$76,175	$77,705	1.96%
# of modules	936	753	19.5%
Savings %	4.18%	3.19%	-

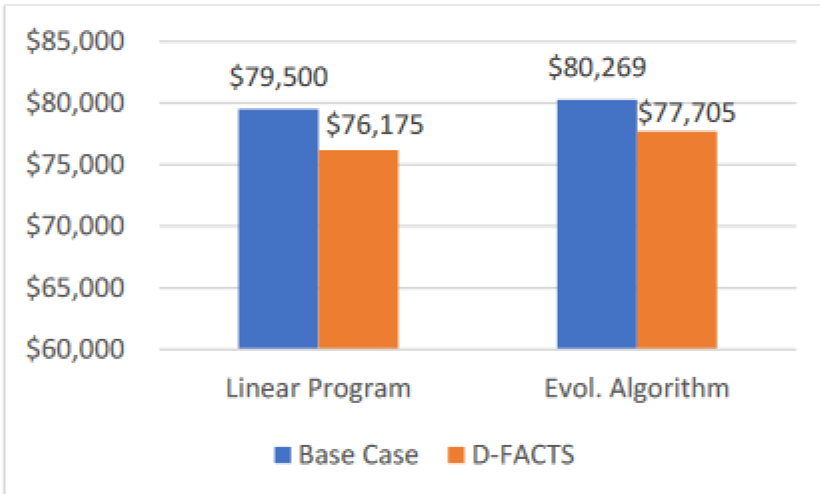

Figure 6.1.3. Cost comparison between algorithms for base and D-FACTS cases

C. Comparison of D-FACTS Allocation

For a closer comparison, Table II below shows the allocation for the D-FACTS modules in each algorithm. Lines where no devices were allocated in either algorithm are not included in the table. The comparison shows that while the linear programming method concentrated the devices into two critical lines in the network, the EA instead distributed them throughout the entire system, and the number of D-FACTS modules allocated on each line does not have to follow a 'per phase per mile' resolution when solved using the EA. This shows that the EA is able to offer more flexibility in D-FACTS allocation in a computationally efficient manner.

Table 6.1.2. D-FACTS Allocation per algorithm

	Devices	
Line No.	L. P.	E. A.
1	0	12
2	0	63
3	0	18
4	0	66
6	0	6
8	0	45
9	0	27
10	0	15
11	0	18
16	0	27
19	0	54
21	0	6
22	720	87
23	0	3
24	0	54
25	0	21
26	0	21
27	0	51
28	216	6
29	0	27
30	0	18
31	0	18
32	0	3
33	0	36
37	0	24
38	0	27
Total	**936**	**753**

D. Computational Efficiency

In order to show the computational efficiency of this type of model, the EA was coded on Matlab®, and run with a population size of 200 and 100 iterations. Both programs were run on a Dell Inspiron 17-7779 with 16 GB of RAM and an i7-7500U processor. The runtimes of both algorithms were obtained and are illustrated in Fig. 4. There is a noticeable difference in runtime between the two methods, with the EA being roughly a third of the linear program. This shows that the EA-based method is significantly more computationally efficient than methods such as branch and bound for this type of model.

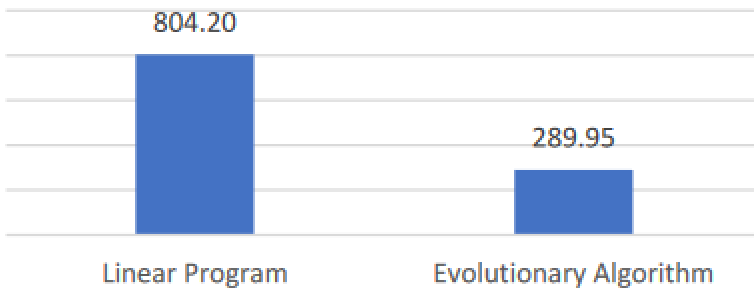

Fig. 6.1.4. Runtime Comparisons for both algorithms (seconds)

V. Conclusions and Future Work

This study presented an EA-based approach to solve a stochastic D-FACTS allocation model, which optimally assigns D-FACTS devices per phase to transmission lines. It mitigates transmission congestion and reduces generation dispatch costs, resulting in better social welfare in the electricity market. The model considers uncertainties caused by fluctuating loads in the system. The algorithm was used to allocate D-FACTS under static investment limits and conditions, and its effectiveness was compared to a previously developed linear model. Results show that the proposed algorithm can solve the D-FACTS optimally allocation problem much more computationally efficient than the Linear Programming model for D-FACTS allocation without significantly increasing the objective function value.

This approach to solving a D-FACTS allocation problem is novel, and it is still necessary to further test its applicability over different scenarios. In a future work, the algorithm will be tested against other solution methods with varying investment levels and different operating conditions including with the implementation of renewable energy sources

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6.2. Co- Optimizing Operating Cost and Renewable Energy Curtailment in D-FACTS Allocation

Abstract— Modern energy grids have become extremely complex systems, requiring more variable and active flow control. As a remedy to this, Distributed Flexible AC Transmission Systems (D-FACTS) are cost-efficient devices used to mitigate power flow congestion and integrate renewable energies. The objective of this study is then to propose an efficient multiple objective evolutionary algorithm to solve a stochastic model for D-FACTS allocation which aims to optimize both the utilization of renewable resources and the total operating costs of the grid. The model was implemented on a modified RTS-96 test system, and the results show that optimally allocating D-FACTS modules using the proposed model can significantly reduce power system operating costs and improve the integration of renewable energy sources. Additionally, the case study shows that the model is very computationally efficient.

Index Terms--Distributed flexible AC transmission systems (D-FACTS), evolutionary algorithm, metaheuristics, multi-objective optimization, optimal allocation, stochastic optimization

I. Nomenclature

Indices	
a, b	Solutions
c	Contaminant
k	Transmission line.
g	Generator.
n	Node.
s	Scenario.
seg	Segment of linearized generator cost function.
i	Objective or Fitness Function
Sets	

$\sigma^{+}(n)$		Transmission lines with their "to" bus connected to node n.
$\sigma^{-}(n)$		Transmission lines with their "from" bus connected to node n.
$g(n)$		Generators connected to node n.
$r(n)$		Renewable generators connected to node n.
Variables		
C_{inv}^{D}		Total investment in D-FACTS ($).
$D_{a,b}$		Dominance of solution a over solution b
$F_{k,s}$		Real power flow through transmission line k in scenarios s.
FMi_{a}		Value of fitness function i for solution a
$OF_{i,a}$		Value of objective function i for solution a
$P_{g,s}$		Real power generation of generator g in scenarios s.
$P_{r,s}^{C}$		Curtailed renewable generation from renewable generator r in scenario s
$P_{g,s}^{seg}$		Real power generation of generator g in scenarios s in segment seg.
$R_{g,s}^{D}$		Spinning down reserve available through generator g in scenario s.
$R_{g,s}^{U}$		Spinning up reserve available through generator g in scenario s.
x_{k}^{D}	k	Integer indicating the number of D-FACTS installed on transmission line
$\theta_{b,s}$		Voltage angle at bus b in scenarios s.
$\theta_{fr,k,s}$		Voltage angle at the "from" node of line k in scenarios s.
$\theta_{to,k,s}$		Voltage angle at the "to" node of line k in scenarios s.
Parameters		
C_{g}^{NL}		No load cost of generator g.
$C_{g,seg}^{linear}$		Linear cost of generator g in segment seg.
C_{g}^{D}		Down reserve cost of generator g.
C_{g}^{U}		Up reserve cost of generator g.
C_{single}^{D}		Cost a of single D-FACTS unit ($).

C_{sh}^{D}	Cost a of single D-FACTS unit converted to an hourly figure ($/h).
C_{inv}^{max}	Maximum investment allowed for D-FACTS.
$f_{k,s}$	Flow direction for line *k* in scenario *s*.
F_{k}^{max}	Thermal capacity/voltage drop limit of transmission line k.
i_{k}^{max}	Maximum number of D-FACTS that can be allocated per line.
I	Interest rate/discount rate.
l_{max}^{alloc}	Maximum number of lines in which D-FACTS devices may be allocated
l_{k}	Length of line *k*
$L_{n,s}$	Load at bus n in scenario s.
N	Lifespan of D-FACTS.
N_{g}	Total number of generators.
N_{k}	Total number of lines.
N_{s}	Number of scenarios.
N_{seg}	Number of segments for the linearized generator cost function.
N_{pop}	Population size for the algorithm.
N_{r}	Number of renewable generators.
p_{s}	Probability of scenario s.
P_{g}^{max}	Upper generation limit of generator g.
P_{g}^{min}	Lower generation limit of generator g.
$P_{r,s}$	Renewable generation produced by renewable generator r in scenario s
$P_{r,s}^{C}$	Renewable energy curtailed from renewable generator *r* in scenario *s*.
S^{D}	Spinning down reserve requirement g.
S^{U}	Spinning up reserve requirement g.
X_{k}	The reactance of transmission line k.
X_{k}^{max}	The maximum reactance of line *k* if D-FACTS are installed on this line.
X_{k}^{min}	The minimum reactance of line *k* if D-FACTS are installed on this line.

η_C, η_L	The maximum adjustment percentage of the line's reactance in the capacitive or inductive mode that a single D-FACTS module (1 device/phase/mile) can achieve.
$\Delta\theta_k^{max}$	Maximum value of bus voltage angle difference to maintain stability for line k.
$\Delta\theta_k^{min}$	Minimum value of bus voltage angle difference to maintain stability for line k.

II. Introduction

The American electric grid system is constantly facing problems with their transmission systems. While some issues such as technical failures or natural disasters are hard to plan for, others, such as capacity and congestion, can be solved in the design of the system. Transmission congestion occurs when a line is overloaded to the point that it starts to deteriorate, which can then cause a myriad of issues from short-circuiting to line breakage. As a way of mitigating this problem, variable-impedance series flexible AC transmission systems (FACTS) can be used to provide effective power flow control as part of smart transmission systems [1]. FACTS can help improve the utilization of an existing network and provides a more sustainable and reliable power delivery network.

Distributed FACTS (D-FACTS) are considered a light-weight version of FACTS, in which many smaller devices are used in place of a single large one, and installed along the transmission line rather than at a substation. In recent years, D-FACTS technology has advanced considerably, and is now being implemented in various projects [2]. However, it is important to also consider environmental impacts when looking at possible large infrastructure projects, even more so now as climate changes are becoming more extreme. For this reason, this study will consider not only the cost-effectiveness of installing DFACTS devices on an existing power grid, which has been previously shown by [3] to improve considerably by the installation of the devices, but also will consider the integration of renewable energy sources as a rudimentary measure of environmental impacts. Since D-FACTS devices can improve the transmission capacity of a network, it is expected then that by improving flow capacity around the renewable energy sources the power generated by traditional generators would be reduced and the integration of renewable energies would improve.

The topic of D-FACTS allocation is still a relatively new one, with a still limited number of studies and models published. The models have varying levels of complexity according to their objective. The earliest work on D-FACTS allocation is by Li *et al* [4], who in 2009 proposed a non-linear DC-based optimization model for allocating D-FACTS devices with the objective of minimizing congestion in the lines. Said study used a static value for the reactance adjustment of each device. Ref. [5] proposed a Particle Swarm Optimization (PSO) algorithm for D-FACTS device allocation with the objective to reduce loads in overloaded lines. This model used a static generation and load. Other proposed methods include the use of graph theory [6], linear programming, and mixed-integer programming [3], [7]. However, these types of optimization methods can be very computationally intensive.

Studies such as [8]-[9] recommend the use of D-FACTS devices for effectively controlling energy flow in systems with distributed generation, including less predictable sources such as renewable energies. Ref. [10] also remarks that FACTS devices have been successfully used for smart power flow control. By extension, it can be said that the modular D-FACTS can be at least as effective for the same purpose. Among the advantages of D-FACTS devices are enhanced grid utilization, increased flexibility and power flow control, and increased security and reliability [11]. Most importantly, D-FACTS improve power quality by stabilizing nonlinear loads, thus working to maintain sinusoidal waves in the current [12].

Multiple Objective Optimization of D-FACTS allocation problems have previously been performed with deep focuses on specific issues of transmission lines. Ref. [13], for example, attempts to minimize voltage deviation and power losses, maximize voltage stability, and optimize load balancing using an enhanced bacterial foraging optimization (EBFO) method. Particle Swarm Optimization methods have also been used by [14] with the purpose of simultaneously optimizing the VA rating, power loss, and undervoltage problems by allocating unified power quality conditioners, which are devices with similar applications as D-FACTS. Newer algorithms, such as the lightning search algorithm, have also been successfully used by [15] to improve power loss, voltage deviation, and voltage stability by allocating D-FACTS. However, despite the optimization of multiple objectives in these studies, they are considered as separate objectives and are optimized separately as a show of the improvements offered by the devices rather than by simultaneous optimization. In addition, renewable energy curtailment has yet to be considered as an objective to optimize in existing literature.

In order to fill these gaps, this paper proposes a Multiple Objective Evolutionary Algorithm (MOEA) to solve a D-FACTS optimal allocation problem. The innovation of this algorithm is that it considers not only cost savings resulted from using D-FACTS, but also the renewable energy curtailment, and the two objectives are considered simultaneously in the optimization problem. The algorithm can generate a pareto front with feasible solutions, and system planner could choose the best solution based on their needs. Additionally, the proposed algorithm is very computationally efficient.

The remainder of this paper is organized in the following manner: Sec. **Error! Reference source not found.** will describe the formulation of the optimal allocation model as well as the MOEA used to solve it; Sec. **Error! Reference source not found.** will discuss the simulation setup and the results from the case study, and finally conclusions and future work directions are presented in Sec. **Error! Reference source not found.**.

III. Model Formulation

A. D-FACTS Allocation Mathematical Model

The proposed model is a DC Optimal Power Flow (DCOPF) -based D-FACTS allocation model, which has the purpose of allocating D-FACTS modules in each phase with the objective of optimizing cost and renewable energy utilization while maintaining load satisfaction. It is a stochastic optimization model, where different scenarios with associated probabilities are considered to assess some level of future uncertainties. It differs from the Mixed-Integer Linear Program model presented in [3] in the following key aspects: (1) The proposed model in this study is nonlinear, while the model in [3] is linear; (2) the proposed model allocates the total number of devices per phase for a transmission line, while the model in [2] allocated them per phase per mile in each line, which allows our model more accuracy in finding the required number of devices to optimally allocate along the system; and (3) the proposed model simultaneously optimizes both total costs and renewable energy usage. Despite introducing a second objective function and removing linearity constraints, it is still possible to solve the problem in a computationally-efficient manner thanks to the use of the Multiple Objective Evolutionary Algorithm discussed in Sec. III-B. In the proposed model, series variable-impedance D-FACTS modules are allocated. Since the reactances of the lines are to be adjusted, the applicable power flow constraints will depend on power flow directions, as discussed by **Error! Reference source not found.** :

If $\theta_{fr,k,s} - \theta_{to,k,s} \geq 0$,

$$\theta_{fr,k,s} - \theta_{to,k,s}/X_k^{max} \leq F_{k,s} \leq \theta_{fr,k,s} - \theta_{to,k,s}/X_k^{min} \quad (1)$$

If $\theta_{fr,k,s} - \theta_{to,k,s} \leq 0$,

$$\theta_{fr,k,s} - \theta_{to,k,s}/X_k^{min} \leq F_{k,s} \leq \theta_{fr,k,s} - \theta_{to,k,s}/X_k^{max} \quad (2)$$

The model for optimal D-FACTS allocation considering energy reserve requirements and multiple scenarios is described by Equations (3) – (26):

$$\min OF_1 = \sum_{s=1}^{N_s} P_s \left(\sum_{g=1}^{N_g} \left(\sum_{seg=1}^{N_{seg}} C_{g,seg}^{linear} P_{g,s}^{seg} + C_g^U R_{g,s}^U + C_g^D R_{g,s}^D + C_g^{NL} \right) + \sum_{r=1}^{N_r} c_r P_{r,s}^C \right) + C_{inv}^D \quad (3)$$

$$\min \mathrm{OF}_2 = \left(\sum_{s=1}^{N_s} \left(P_s \sum_{r=1}^{N_r} \frac{P_{r,s}^C}{P_{r,s}} \right) \right) \quad (4)$$

$$P_{g,s} = \sum_{seg=1}^{N_{seg}} P_{g,s}^{seg} \quad (5)$$

$$P_g^{min} \le P_{g,s} \le P_g^{max} \quad (6)$$

$$-F_k^{max} \le F_{k,s} \le F_k^{max} \quad (7)$$

$$\sum_{k\in\sigma^+(n)} F_{k,s} - \sum_{k\in\sigma^-(n)} F_{k,s} + \sum_{g\in g(n)} P_{g,s} + \sum_{r\in r(n)} \left(P_{r,s} - P_{r,s}^C\right) = L_{n,s} \quad (8)$$

$$\sum_{g=1}^{N_g} R_{g,s}^U \ge S^U \quad (9)$$

$$\sum_{g=1}^{N_g} R_{g,s}^D \ge S^D \quad (10)$$

$$R_{g,s}^U \le P_g^{max} - P_{g,s} \quad (11)$$

$$R_{g,s}^D \le P_{g,s} - P_g^{min} \quad (12)$$

$$R_{g,s}^U \ge 0 \quad (13)$$

$$R_{g,s}^D \ge 0 \quad (14)$$

$$\Delta\theta_k^{min} \le \theta_{fr,k,s} - \theta_{to,k,s} \le \Delta\theta_k^{max} \quad (15)$$

$$\theta_{1,s} = 0 \quad (16)$$

$$f_{k,s}\left(1 + \frac{x_k^D}{l_k}\eta_L\right) X_k F_{k,s} \ge f_{k,s}\left(\theta_{fr,k,s} - \theta_{to,k,s}\right) \quad (17)$$

$$f_{k,s}\left(1 + \frac{x_k^D}{l_k}\eta_C\right) X_k F_{k,s} \le f_{k,s}\left(\theta_{fr,k,s} - \theta_{to,k,s}\right) \quad (18)$$

$$0 \le x_k^D \le i_k^{max} \quad (19)$$

$$\sum_{k=1}^{N_k} \frac{x_k^D}{\max(x_k^D, 1)} \le l_{max}^{alloc} \quad (20)$$

$$C_{inv}^D = \sum_{k=1}^{N_{br}} \sum_{i=1}^{i_{max}} 3iC_{sh}^D x_{k,i}^D \quad (21)$$

$$C_{inv}^D \le C_{inv}^{max} \quad (22)$$

$$C_{sh}^D = C_{single}^D \frac{I(1+I)^N}{8760((1+I)^N - 1)} \quad (23)$$

$$0 \le P_{r,s}^C \le P_{r,s} \quad (24)$$

In this formulation, the objectives are first to minimize the total expected operational costs, including generation and reserve costs as well as D-FACTS investment costs (3), and the curtailed renewable energy (as a percentage) to use as a rudimentary environmental impact metric (4). Equation (5) segments the generation to work with the linearized form of the generation cost curve, while (6) serves to establish the upper and lower generation limits for each generator that serves the system. Equation (7) indicates the transmission limits for each line, considering that the flow can be described as positive or negative based on the flow direction. Based on previous models, the capacity of short lines (0-50 miles) is set to their thermal limits, the capacity of medium lines (50-156 miles) is determined by their voltage drop limit, and the for long lines (over 156 miles) the capacity is set by voltage angle stability limits. The power balance at each bus in the system is defined by (8) indicating the load at each bus to be equal to all the power generated at each bus plus the incoming line transfers minus the outgoing line transfers. Eqns. (9)-(14) define the reserve requirements, with (11) and (12) specifically defining reserve capacities. Equations (15) and (16) define the voltage angle constraints, setting the angle at bus 1 to be 0 by definition. Furthermore, (17) and (18) show the DC power flow equations for each line, regardless of whether D-FACTS are installed in the line or not, considering the flow direction. Equation (19) serves to define the limit of how many D-FACTS may be installed at each line based on the adjustment limits for the simulation and the length of the line. Eq. (20) limits the number of lines at which the devices may be installed, in order to assist with feasibility of implementation and prevent too many lines from having devices allocated to them, which could make installation unfeasible under realistic circumstances. Eq. (21) defines the total investment cost, which is limited in (22) to account for possible budget limits, and calculated into an hourly figure in (23) considering the discount rate and expected lifespan of the modules. Additionally, (24) defines the upper and lower bounds for the renewable energy curtailment.

B. The Evolutionary Algorithm

A metaheuristic algorithm is a type of stochastic search algorithm inspired by natural evolution processes. In its base form, an EA seeks to encode a computational problem into a set of strings which can represent a solution to a problem, and then combine these using a heuristic process in order to approximate an optimal solution. Since its proposal in 1975, this type of algorithm has been used for solving various computationally burdensome problems arising in real-world applications which then require computationally efficient algorithms to achieve near-optimal solutions within a reasonable timeframe by performing an intelligent search of the solution space [17]. In the present research, we develop a new MOEA to efficiently find possible solutions to the problem and identify which meet optimality conditions.

As this is a multiple-objective optimization problem, it is possible and expected that the objective functions are in opposition to each other, and thus a single solution cannot be obtained without receiving input from a decision-maker who either has experience in the field or who works in the industry and has some priorities already in mind. In this case, a type of optimality called Pareto Optimality is used. Pareto-optimal solutions are those for which no other solution is equal or better in all objectives. At the end of a multi-objective optimization problem, the result is then a set of Pareto-optimal solutions or a Pareto front to be presented to a decision maker.

Line	1	2	3	4	5	6	7	8	9	…	N_k
# DFACTS	125	0	0	63	0	0	0	81	0	…	15

Figure 6.2.1. Chromosome Example

First, the problem is solved without considering D-FACTS in order to obtain the flow directions which will be used in (17) and (18). The algorithm then starts by generating random values within the solution space in order to pre-allocate the D-FACTS devices within the network, following constraints (19) and (20). These are used in a reduced LP problem with constraints (3-8, 15-24) in order to minimize the generation costs and renewable energy curtailment throughout the system, using cost as the objective function and then obtaining the energy curtailment from the LP output. After, the reserves defined in (9-14) are allocated by the use of a greedy algorithm, to further reduce computational burden from the LP. The greedy algorithm works by allocating reserves from the available space described in (11-14) based on the lowest possible cost, until the up and down spinning reserve requirements formulated in (9-10) are satisfied. Figure 1 shows how a solution or chromosome may be interpreted within the algorithm, each value within the vector representing the number of D-FACTS allocated at each line, with 0 representing that no devices are installed in that line. A set of solutions (or population) is thus stored as a 2-D matrix in which each row is a chromosome vector that represents a solution.

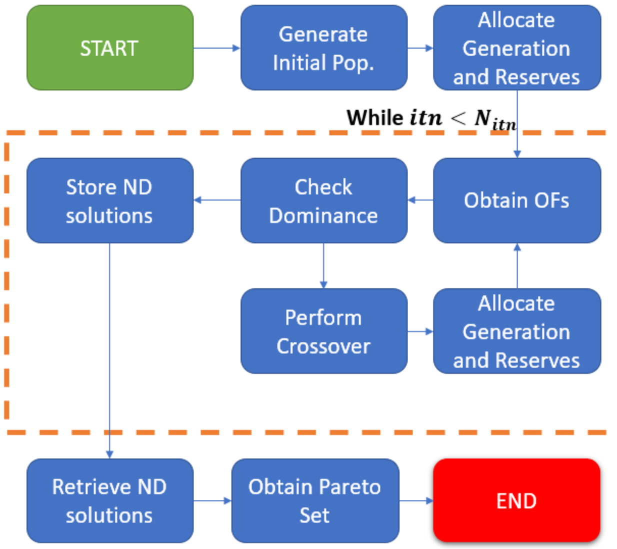

Figure 6.2.2. Evolutionary Algorithm Flowchart

Fig. 2 contains a flowchart detailing the steps taken by the algorithm. At first, a random first set of solutions is generated and the LP and greedy algorithm mentioned before are used to obtain the objective function values described in (3,4). After each iteration, all current solutions are cross-checked for dominance, with the non-dominated solutions being stored separately. In order to rank the solutions for the crossover step, two new fitness metrics are incorporated and later combined to obtain a single combined fitness metric with which to rank solutions prior to crossover. The first metric is based on dominance count, to ensure proximity to the true Pareto frontier, while the second metric is based on inter-solution distance, to ensure a widespread range of solutions, as described in the method developed by [18]. Both metrics are then normalized and added to create a combined fitness metric in order to rank the solutions for elitism and perform a crossover. The metrics are described below in (25)-(27). In Eq. (25), $D_{i,j} = 1$ if $OF1_i \leq OF1_j$ & $OF2_i \leq OF2_j$ or 0 if otherwise. In these equations, i and j refer to a solution number, while Npop refers to the number of solutions at each iteration. OF1 and OF2 refer to the objective functions described in (1) and (2), normalized to the [0,1] range.

$$FM1_i = \sum_{j \neq i}^{N_{pop}} D_{i,j} \qquad (25)$$

$$FM2_i = \sum_{j=1}^{N_{pop}} \left(\left|OF1_i - OF1_j\right|^2 + \left|OF2_i - OF2_j\right|^2\right)^{\frac{1}{2}} \qquad (26)$$

$$FM_i = \frac{FM1_i}{\max FM1} + \frac{FM2_i}{\max FM2} \quad (27)$$

At the end of each iteration, a part of the solutions continues into the next iteration by a process called elitism, which serves to ensure that good solutions are not lost as the algorithm progresses. Additionally, a new set of solutions is created via a single-point crossover. A cut-point is generated, and two randomly selected solutions are combined by taking parts of the parent solutions before and after the cut-point, and enforcing the limits in Eqs. (19-23) afterwards to ensure validity of the solutions. This process repeats until a set number of iterations has been run or there is no improvement for the optimal objective function value after a set number of iterations and no new solutions are being added to the non-dominated storage. Another aspect of generating a new population is mutation, which can occur on newly generated solutions by randomly switching some values within the new solutions. Mutation is used in an EA to add variety into the solution space and preventing the algorithm from converging around local optima.

IV. Case Study

A. Simulation Setup

In this study, the model in Section III-A is adopted to study the cost savings associated to the installation of D-FACTS devices using the algorithm proposed in section III-B. The computational efficiency of the solution method is also briefly discussed. The case study uses the same modifications over the IEEE RTS-96 test system as Ref. [3]. Uncertainties in the model are represented by four different load scenarios, with load factors of 0.65, 0.75, 0.85, and 0.95, as well as by renewable energy generation factors of 0, 0.2, 0.6, and 1, resulting in a total of 16 scenarios. It is assumed that each D-FACTS module is capable of adjusting the reactance of the line in which it is installed by $\pm 2.5\%$ per phase per mile, and that the maximum reactance adjustment range allowed for a 3-phase line is $\pm 20\%$. This would result in a line limit of $i_k^{max} = \frac{20}{2.5} l_k = 8 l_k$ devices per line. D-FACTS allocation results were obtained with the D-FACTS allocated evenly per line per phase.

Device costs were determined based on industry data and previous academic studies**Error! Reference source not found.**. Based on studies which account for it, the cost for a D-FACTS device is assumed to be \$100/kVA; where the compensation level in kVA is dependent on the parameters of the transmission line in which the D-FACTS device is installed. For simplicity, the compensation level for the most demanding line was adopted. In the modified RTS-96 system, the largest compensation level is 30kVA/module. Thus, a total cost of \$3000/module is used in this study. Since the cost functions are on hourly units, it is necessary to use Eq. (23) to convert this to an hourly value, considering an expected lifespan of 30 years and a discount rate of 6%. Additionally, industry practices would impose a limit on investment for the installation of modules, denoted by (22). An allowance of \$25/hour is assumed in this study.

B. *Trade-off between Cost and Renewable Energy Curtailment*

The MOEA was run with the following parameters: 500 individuals in the population iterated over 100 generations, with 5% elitism and 5% chance of mutation. The algorithm was run on a Dell computer with 256GB of RAM and an Intel ® Xeon ® W-2195 CPU, and had a total runtime of 52.62 seconds. With two objectives, namely, minimizing both power system operating costs and renewable energy curtailment, a Pareto front consisting of nondominated feasible solutions was generated from the optimization problem and shown in Fig. 3. From the results, it can be seen that a solution with a low cost can have a high percentage of renewable energy curtailment, and vice versa. The solutions obtained have a cost ranging between \$71,075 and \$71,307 and a renewable energy curtailment ranging between 59.48% and 60.45%, while the base case solution has an expected cost of \$78,716 and a curtailment of 64.45%. It can be seen that all solutions provide some level of improvement to both objective functions, with some solutions providing a more reduced cost function while others provide a lower level of curtailment. Based on such a pareto front, a system planner can choose a solution according to their budget and renewable energy integration goals.

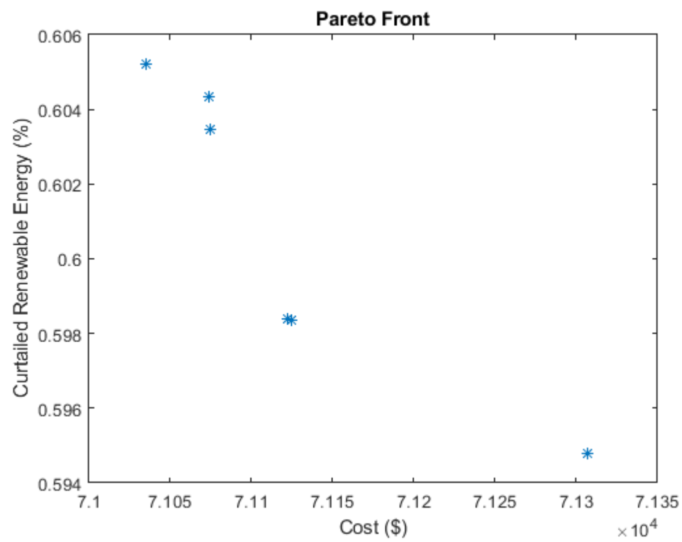

Figure 6.2.3. Pareto-Front representation of the objective functions of non-dominated solutions

C. D-FACTS Allocation

To compare the results, we listed the cost savings, renewable energy curtailment reduction and optimal D-FACTS allocation from a few representative solutions and compared them with the base case, in which no D-FACTS was allocated, in Table I. The base case solution with no D-FACTS is also presented and highlighted with light blue color in the table. In terms of the number of D-FACTS allocated, the number tends to vary over all the solutions between 900-1000 devices. In terms of locations, all the obtained solutions placed a large number of D-FACTS modules in line 22, which leads to the conclusion that this is a crucial line in the transmission of the renewable energy, and so installing the devices along this line helps distribute more renewable energy and thus reduce the total curtailment and costs.

Table 6.2.1. Selected Solutions

Cost ($)	Cost Reduct.	Curt.	Curt. Reduct.	Line	Alloc.
78,716	-	64.45%	-	-	-
71,075	9.71%	60.35%	10.52%	4	255
				13	72
				22	675
71,122	9.65%	59.84%	11.28%	22	738

				36	90
				37	174
				11	12
				20	177
71,307	9.41%	59.48%	11.82%	22	711
				23	75
				36	21

D. Computational Efficiency

In terms of computational time, a similar problem which does not consider renewable energy and only the objective of minimizing costs was run on a full mixed-integer linear program with a total computational time of 804.50 seconds. This new approach, with a solution time of 52.62 seconds, has at least a 93% improvement in computational time over a conventional MILP solver.

V. Conclusions and Future Work

This study presented a metaheuristic multi-objective approach for solving a scenario-based stochastic D-FACTS allocation model. The model optimally allocates D-FACTS devices along transmission lines per phase. It mitigates transmission congestion, reduces power system operating costs, and facilitates the integration of renewable energies, which can result in better social welfare in the electricity market as well as lower environmental impacts. The model considers uncertainties introduced by fluctuating loads in the system and volatile renewable energy generation. The algorithm allocates D-FACTS modules with static investment limits, optimizing the objective of minimizing both total costs and the percentage of curtailed renewable energy. Additionally, computational times show that this type of algorithm is very efficient in solving the problem. In future work, a more in-depth study of environmental impacts, using more thorough methods such as Global Warming Potential of the whole system including non-renewable generators. Additionally, for a more realistic allocation of devices, constraints will be introduced to limit the number of lines in which devices may be installed, as it is not necessarily realistic to install devices throughout most of the system. Other work includes refining of the algorithm to further reduce computational time as well as making it feasible to apply over larger networks.

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6.3. Fine-Tuning the Parameters for Solving the Multi-Objective D-FACTS Optimal Allocation Problem

Abstract

Distributed Flexible AC Transmission Systems (D-FACTS) and D-FACTS allocation are new topics that are gaining traction in the field of power systems. The reason for this is that they are a simple yet effective tool for improving power flow control, power system flexibility, and reducing overall power systems cost by manipulating some properties of the transmission lines on which they are installed. So far, most research has focused on improving the algorithms used to optimally allocate the D-FACTS along existing networks in order to maximize or minimize a certain objective. However, much of this research has been based on the assumption that all the parameters are pre-defined and immutable. The key objective of this study is thus to study how the changing of different parameters may affect the final solutions found by the optimization algorithm. The key parameters studied are the line reactance adjustment limit, the number of lines on which D-FACTS are allowed, and the investment cost limit. Results show that all these parameters have an effect on the final solution set, and decisions need to be made by carefully weighing the available resources, convenience for deployment, and the potential benefits that could be brought by utilizing D-FACTS devices.

Keywords

Power Systems, D-FACTS, Optimization, Multi-Objective Optimization and Sensitivity Analysis.

1. Introduction

American electric grids are facing an increasing number of issues, some of which are related to an ever-increasing changing climate; from the now yearly phenomenon of wildfires caused by fallen PG&E lines in California to the weeks-long outages in Texas in early 2021 due to freezing temperatures. While some of the issues may be weather-related and harder to control or prepare for, many issues stem from increased demand, such as congestion and line overloading.

Congestion is such a major issue in U.S. power grids that $40 billion were spent on congestion reduction projects in 2018 alone, and yet congestion costs are still measured in the billions of dollars per year (US DOE, 2020). In order to help mitigate this problem, we propose the use of variable-impedance series flexible AC transmission systems (FACTS), which help provide effective power flow control as part of smart transmission systems (Li, *et al*, 2010). FACTS, as well as Distributed FACTS (D-FACTS), can help improve the utilization of an existing network and provide a more reliable and sustainable power delivery network (Gotham and Heydt, 1998).

As an extension of FACTS technology, D-FACTS are a lightweight version of traditional FACTS. These have the associated benefits of reduced cost and spatial footprint, as well as a capacity for being re-deployed based on shifting power needs. Traditional FACTS require large spaces for installation next to buses in the system, but D-FACTS can be installed along existing transmission lines or towers in a modular fashion (Sang and Sahraei-Ardakani, 2019). That is why D-FACTS devices have become more popular and are being implemented in various electric grid improvement projects throughout the country, and their benefits in the integration of renewable energies into new grids has been previously demonstrated by Gandoman et al (2018). However, D-FACTS allocation is still a new field and has not been studied in full detail. The expectations and some preliminary studies indicate that they will be similarly useful in the integration of renewable energy into existing and established grids, but more research is needed to see to what extent this will be possible and under what conditions and parameters they would be able to do so.

Although arguably more versatile and effective, the allocation of D-FACTS modules rather than traditional FACTS devices introduces nonlinearities to the optimal allocation model which can be computationally exhausting to solve (Sang & Sahraei-Ardakani, 2018). For some time, the challenge was then to improve on the computational complexity of the algorithms in order to solve the allocation problem. Castillo Fatule (2021) proposed a new formulation and algorithm to eliminate the nonlinearities by the use of metaheuristic optimization and greatly reducing the computational time for this problem.

Since the benefits of incorporating D-FACTS devices into a network have been thoroughly studied and demonstrated, and progress has been made into improving the optimization process, the main objective in this research is now to estimate under which parameters the D-FACTS devices will be able to perform best to improve upon the objectives being studied.

The main objectives considered for the optimization process are the total expected system costs, the total expected Global Warming Potential (GWP), and Line Utilization Factor (LUF). These three objectives are to be minimized simultaneously, but they are conflicting with each other, thus a multi-objective optimization process is utilized to obtain a set of non-dominated Pareto-optimal solutions. The optimization algorithm will be then executed repeatedly while varying the parameters being studied in order to perform a sensitivity analysis of the parameters.

The key objective of this study is thus to use Design of Experiments tools to study the change of the objective function values in the Pareto sets and analyze the impact of each of the parameters being modified.

2. Literature Review

FACTS and D-FACTS are thyristor-based controllers designed to manage series impedance, shunt impedance, phase angle, or some other parameter in electric transmission systems (Hingorani, 1993). Some of the most common types of FACTS are Static Series Var Compensator (SSVC), used to control voltage, Thyristor Controlled Series Capacitor (TCSC) used to increase transfer capability and stability, Static synchronous series Compensator (SSSC) used for power transmission series compensation with synchronous voltage, and Unified Power Flow Controller (UPFC) for enhancing steady state, dynamic, and transient stability (Murali *et al*, 2010). Similarly, there is a distributed version for most of these types of devices.

Previous research such as Habur and O'Leary's (2004) has shown that the installation of FACTS devices can not only improve the stability of transmission networks, but also reduce operational costs and open the possibility for increased sales by utility companies. Others such as Wibowo et al (2011) and Torino et al (2003) have used FACTS to reduce congestion, stabilize voltage, integrate new energy sources into the grid and improve network security. In addition, it is by design that all the applications for which traditional FACTS can be used are also applicable for D-FACTS devices.

The allocation of FACTS devices has been a relatively established research field for some time now. Jordehi (2015) shows various uses Particle Swarm Optimization (PSO) algorithms for determining optimal location and size of STATCOM-type FACTS devices in power systems, optimal type and location of multi-type FACTS devices including TCSC, SVC, and UPFC to maximize voltage stability, optimal FACTS settings to optimize system loadability and installation costs, minimization of copper losses, etc. In fact, various authors have implemented different methods for optimizing the allocation of FACTS and D-FACTS devices around a specific parameter in the power system, including optimizing voltage stability during outages as a Line Stability Index (LSI) using a modified PSO algorithm (Srivastava et al, 2014); minimizing total operation and installation costs (Mohamed et al, 2010); optimization of system security and installation costs using Genetic Algorithms (GA) (Radu and Besanger, 2006); as well as various other parameters by the use of less commonly used optimization algorithms.

Similarly, the allocation of D-FACTS devices has also been a popular field in recent years since D-FACTS were first proposed by Divan and Johal in 2005, emphasizing their added benefits of a reduced investment cost, space requirements, system stress, and reliability requirements. In addition to these benefits, it has been studied that the potential economic benefits of D-FACTS devices outperform those of FACTS devices (Sang and Sahraei-Ardakani, 2018). This is not considering the long-term benefit of re-deployability that D-FACTS have, which have yet to be studied given the additional complexity for the problem, but that promises further reduced costs if a situation arises in which the network configuration changes and re-allocation becomes necessary.

D-FACTS also help integrate new renewable energy sources into the grid. An analysis performed by Suresh and Sreejith (2017) showed that the use of D-FACTS can improve the voltage profile and reduce line power loss when integrating new power sources to the grid. It is estimated that by 2050 at least 20% of energy in global grids will come from renewable sources (Jha et al, 2017). As it stands, power flow control devices will become more relevant in managing distribution networks and grid congestion. Also, more than FACTS, D-FACTS are more attractive control devices for the dynamic management of voltage, reactive power, and power quality, since they can provide more precise and dynamic management of microgrids to mitigate the uncertainty associated with the incorporation of renewable energies (Gupta and Kumar, 2016; Gaigowal and Renge, 2016).

The problem of allocating D-FACTS has been studied through various algorithms and mathematical models. One very common optimization method is Linear Programming (LP), formulated in 1947. However, the allocation of D-FACTS created non-linearities in the equations used to model the problem, not to mention that for problems that are NP-hard or NP-complete, a pure LP approach is computationally expensive. A better approach to avoid excessive computational burdens is the use of a metaheuristic algorithm. Metaheuristic algorithms are advanced search algorithms designed to search for an optimal solution by testing possible solutions around the search space, testing their objective functions, and adjusting the solutions further until convergence around an optimum or some other condition occurs. For the purpose of this study, a genetic algorithm will be used to search the solution space by testing possible combination of D-FACTS allocated along the power network and using these combinations to eliminate the nonlinearities in the mathematical model to solve the reduced problem using a LP approach. GAs were first proposed by Holland in 1975 around the idea of mimicking natural evolution processes. Their basic procedure is to initialize a population or set of possible solutions, evaluate them, and, while the termination criteria is not reached, select solutions for the next population and perform crossover and mutation before evaluating this new population (Srinivas and Patnaik, 1994). Although not as much as PSO methods, GAs have been also used to allocate FACTS and D-FACTS devices such as the allocation of multi-type FACTS for improving voltage stability by Baghaee et al (2008), among others.

Multi-Objective optimization is an optimization procedure used to optimize more than one conflicting objective function simultaneously. While some approaches attempt to use linear functions to combine the objectives into a single aggregated objective, the approach we take in this study is instead to store all the solutions that can be objectively be considered “not worse” than any others by not being worse in all objectives compared to the rest. This is usually called a non-dominated set of solutions or a Pareto set. This concept implies that there will be no other solution in the feasible region which is quantifiably better in all objectives than the Pareto-Optimal set, and allows for trade-offs to be made from the decision-maker’s point of view (Zitzler et al, 2002). Because of the nature of a Pareto-optimal set, the decision maker must have some expertise in the field for which the problem is solved, as well as knowledge on the resulting set of solutions, in order to make an educated choice as to which solution would be best, as well as to give weights to the objectives being evaluated. The Pareto-optimal set can help reduce the design alternatives from a feasible region into optimal trade-offs (Yancang et al, 2010).

In this study, the multi-objective algorithm used is one similar to the Non-Dominated Sorting Genetic Algorithm (NSGA), an approach in which all non-dominated solutions are classified into a separate category and assigned a fitness value based around population size, but a different fitness assigning metric, as proposed by Taboada *et al* (2017). This metric is based on both inter-solution distance and dominance, and is based around attempting to increase the spread of solutions over the Pareto set as well as improve proximity to the true Pareto frontier. This method has been proven useful in D-FACTS allocation with the dual objectives of minimizing total system costs and renewable energy curtailment (Castillo Fatule et al, 2021), as well as in solving various other engineering problems including some in the areas of logistics and biofuel production.

As shown in the literature, D-FACTS optimization is an active research topic in the area of power systems and transmission system optimization, but there is still much to study in the field. Very few studies consider quantified environmental impact metrics as an objective to optimize, which becomes a more significant topic as climate change effects worsen. Additionally, while some studies such as Sang and Sahraei-Ardakani (2019) have considered varying load scenarios, none have accounted for generator or line failures, opting for less computationally-burdensome deterministic approaches, considering only the most likely scenario of optimal operating conditions. Also, sensitivity analysis of the operating parameters for the D-FACTS devices has not been a focus on any research in the found literature. As such, the present research will use some previously-developed multi-objective optimization algorithms in order to analyze the effects of adjusting some of the parameters of D-FACTS allocation.

3. Methods

In order to solve the D-FACTS allocation algorithm, the following mathematical model was created based on Sang and Sahraei-Ardakani's (2019) model. This model was created not only to address the nonlinearities created by the incorporation of the D-FACTS devices but also to account for the calculations of environmental impacts, and combine some constraints that become redundant after addressing the nonlinearities in the previous models. The full model is described below. In addition, the flow direction of power lines is relevant when adjusting impedance, so the following DC power flow constraints are applicable:

$$If\ \theta_{fr,k,s} - \theta_{to,k,s} \geq 0, \left(\theta_{fr,k,s} - \theta_{to,k,s}\right)/X_k^{\max} \leq F_{k,s} \leq \left(\theta_{fr,k,s} - \theta_{to,k,s}\right)/X_k^{\min}$$

$$If\ \theta_{fr,k,s} - \theta_{to,k,s} \leq 0, \left(\theta_{fr,k,s} - \theta_{to,k,s}\right)/X_k^{\min} \leq F_{k,s} \leq \left(\theta_{fr,k,s} - \theta_{to,k,s}\right)/X_k^{max}$$

Having considered flow constraints, the objective functions for the model are as follows:

$$\min OF_1 = \sum_{s=1}^{N_s} P_s \left(\sum_{g=1}^{N_g} \left(\sum_{seg=1}^{N_{seg}} C_{g,seg}^{linear} P_{g,s}^{seg} + C_g^U R_{g,s}^U + C_g^D R_{g,s}^D + C_g^{NL} \right) + \sum_{r=1}^{N_r} c_r P_{r,s}^C \right) + C_{inv}^D \quad (1)$$

$$\min OF_2 = \sum_{s=1}^{N_s} P_s \left(\sum_{g=1}^{N_g} \sum_{c=1}^{N_c} GWP_{g,c,s} \right) \quad (2)$$

$$\min OF_3 = \frac{1}{N_k} \sum_{s=1}^{N_s} \sum_{k=1}^{N_k} P_s \left(\frac{F_{k,s}}{F_k^{\max}} \right)^{100} \quad (3)$$

These objective functions correspond with the three key objectives considered in this study which are (1) minimize total system operational costs, including a transformed investment cost,

(2) minimize the total environmental impacts expressed in terms of Global Warming Potential (100kg CO_2 equivalent), and (3) minimize the line utilization factor, a measure of congestion proposed by Das *et al* (2009). The model constraints are below:

$$P_{g,s} = \sum_{seg=1}^{N_{seg}} P_{g,s}^{seg} \tag{4}$$

$$P_g^{min} \leq P_{g,s} \leq P_g^{max} \tag{5}$$

$$-F_k^{max} \leq F_{k,s} \leq F_k^{max} \tag{6}$$

$$\sum_{k\in\sigma^+(n)} F_{k,s} - \sum_{k\in\sigma^-(n)} F_{k,s} + \sum_{g\in g(n)} P_{g,s} + \sum_{r\in r(n)} \left(P_{r,s} - P_{r,s}^C\right) = L_{n,s} \tag{7}$$

$$\sum_{g=1}^{N_g} R_{g,s}^U \geq S^U \tag{8}$$

$$\sum_{g=1}^{N_g} R_{g,s}^D \geq S^D \tag{9}$$

$$R_{g,s}^U \leq P_g^{max} - P_{g,s} \tag{10}$$

$$R_{g,s}^D \leq P_{g,s} - P_g^{min} \tag{11}$$

$$R_{g,s}^U, R_{g,s}^D \geq 0 \tag{12}$$

$$\Delta\theta_k^{min} \leq \theta_{fr,k,s} - \theta_{to,k,s} \leq \Delta\theta_k^{max} \tag{13}$$

$$\theta_{1,s} = 0 \tag{14}$$

$$f_{k,s}\left(1 + \frac{x_k^D}{l_k}\eta_L\right) X_k F_{k,s} \geq f_{k,s}\left(\theta_{fr,k,s} - \theta_{to,k,s}\right) \tag{15}$$

$$f_{k,s}\left(1 + \frac{x_k^D}{l_k}\eta_C\right) X_k F_{k,s} \leq f_{k,s}\left(\theta_{fr,k,s} - \theta_{to,k,s}\right) \tag{16}$$

$$0 \leq x_k^D \leq i_k^{max} \tag{17}$$

$$\sum_{k=1}^{N_k} \frac{x_k^D}{\max(x_k^D, 1)} \leq l_{max}^{alloc} \tag{18}$$

$$GWP_{g,c,s} = \sum_{seg}^{N_{seg}} H_{g,seg}^{linear} P_{g,s}^{seg} G_{g,s} W_c \tag{19}$$

$$C_{inv}^D = \sum_{k=1}^{N_k} 3x_k^D C_{sh}^D \tag{20}$$

$$C_{inv}^{D} \leq C_{inv}^{max} \tag{21}$$

$$C_{sh}^{D} = C_{single}^{D} \frac{I(1+I)^{N}}{8760((1+I)^{N}-1)} \tag{22}$$

$$0 \leq P_{r,s}^{C} \leq P_{r,s} \tag{23}$$

$$f_{k,s} = \frac{F_{k,s}}{|F_{k,s}|} \tag{24}$$

Equation (4) segmentizes the power generation at each generator in order to match with the segments of the linearized cost functions. Eq. (5) defines the minimum and maximum generation level for each generator. Eq. (6) defines the maximum flow capacity for each line, in either positive or negative direction. Eq (7) defines the load at each bus as the sum of all incoming energy flows minus the sum of all outgoing flows, plus the sum of all power generated at this bus via traditional generators and the sum of all non-curtailed renewable energy attached to this bus. Equations (8-9) define the up and down reserve requirements for the system, with eqs. (10-12) defining the reserves at each generator. Eq. (13) defines the voltage angle stability limits at each bus and eq. (14) defines the angle at bus 1 to be 0 as a reference point.

Eqs. (15-16) are the DC power flow equations considering the reactance adjustment effect of the D-FACTS devices in both inductive and capacitive modes, and the flow directions. Eq. (17) serves to define the number of D-FACTS that can be installed at each line, with eq. (18) limiting the number of lines in which the devices may be installed. Eq. (19) serves to define the Global Warming Potential of the generator configurations. Eq. (20) defines the D-FACTS investment cost, which is then limited by eq. (21). Eq. (22) converts the cost of the devices into an hourly value via a compound interest function in order to have the same units as the rest of the data, and finally eq. (23) defines the curtailment of renewable energy as no more than the amount produced. Eq. (24) is not part of the optimization model, but is used as an intermediate step in order to obtain the power flow directions required in other equations.

In addition, the following equations describe the Pareto dominance criteria:

$$If\ OF_{obj,a} \leq OF_{obj,b}\ \forall obj;\ \ D_{a,b} = 1;\ else, D_{a,b} = 0 \tag{25}$$

$$\sum_{a=1}^{N_{pop}} D_{a,b} = 0 \tag{26}$$

Where a solution is non-dominated if it meets equation 26

The fitness metrics used in the algorithm for parent selection are:

$$FM_{1,a} = \sum_{b=1}^{N_{pop}} \sqrt{\sum_{i=1}^{N_{obj}} \left(OF_{i,a} - OF_{i,b}\right)^2} \tag{27}$$

$$FM_{2,a} = \sum_{b=1}^{N_{pop}} D_{a,b} \tag{28}$$

And an aggregated fitness metric is obtained by normalizing both these values and adding them together.

For solving the problem associated to this model, a modified multi-objective evolutionary algorithm is proposed. This algorithm follows the following steps:

0. START
1. Using a reduced model consisting of Eqns. (1), (4-9), (13-17), and (23), with $x_k^D = 0\ and\ f_{k,s} = 1$ solve the linear problem to obtain the values of $F_{k,s}$.
2. Obtain the values of $f_{k,s}$ via equation (24).
3. Generate an initial population using the parameters i_k^{max} and l_{max}^{alloc} defined in equations (17) and (18).
4. For each chromosome, solve the linear problem consisting of equations (1), (4-7), (13-17), and (20-23).
5. Using the outputs from each linear problem, use a greedy algorithm to allocate the up and down spinning reserves as defined in Eqns. (10-14).
6. From this information, calculate the values for the objective functions in (2, 3).
7. Use the Pareto Dominance criteria in Eqns. (25-26) on all solutions and store the non-dominated ones.

8. IF the stopping criteria have been met, go to step (13). Otherwise, go to step 9.
9. Obtain the Fitness Metrics and calculate the Aggregated Fitness Metric.
10. Rank solutions according to the Aggregated Fitness Metric obtained in step 9.
11. Select Parents from the current population.
12. Generate a new population and return to step 4.
13. Retrieve the stored solutions and re-check for dominance to obtain the Pareto-Optimal set
14. END

This process is summarized below in fig. 1:

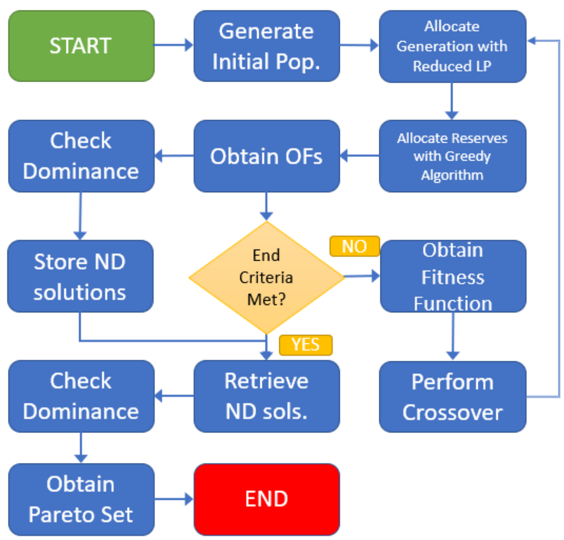

Figure 6.3.1. Evolutionary Algorithm Flowchart

Tournament selection for the parents and single-point crossover method are used in the algorithm for creating a new population.

4. Data Collection

As the base of analyzing electrical grids, a modified version of the IEEE 1996 Reliability Test System was used. The modifications are described in Sang and Sahraei-Ardakani's (2017) study. These modifications consist mainly of modifying load and generation capacities in order to produce additional congestion in the system.

In summary, the IEEE RTS-96 is a fairly simple network consisting of 24 buses, 38 transmission lines, 32 traditional generators, and 2 renewable energy sources. It was originally proposed by Cliff Grigg in the 1996 IEEE Winter Power meeting. While there are extensions to increase the system to three areas, only a single area is used for this study.

In addition, the base case used as reference for this study is the one described by Castillo Fatule (2021), with the following parameters for the D-FACTS allocation. The number of lines in which devices may be installed is limited to 5 lines for feasibility in installation (corresponding to eq. 18), the maximum change in reactance is 20% of the line's current reactance for stability purposes (reflected in eq. 17), and the hourly investment limit has been set to $25/hr (corresponding to eq. 21).

These are the three key parameters that will be modified for the sensitivity analysis. In addition, the other parameters used in the algorithm which will not be studied are the following:

- Number of generations used in the MOEA: 100
- Number of Individuals in each generation: 100
- Mutation Factor: 5%
- Elitism: 10%
- Reactance change in inductive and capacitive mode per device: 2.5% per phase per mile
- Life Expectancy of the devices: 10 years
- Interest rate on the devices: 6%

For reference, the results from the original study are summarized below in figure 2:

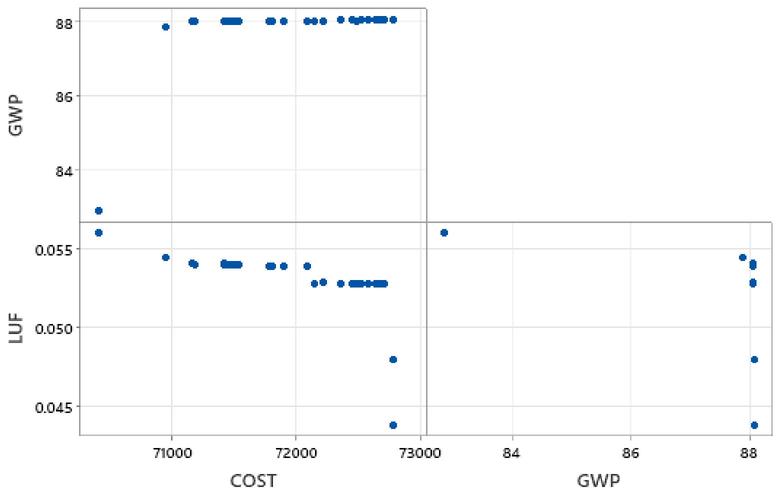

Figure 6.3.2. Matrix scatterplot for Cost, GWP, and LUF for original optimization problem without sensitivity analysis

Results and analysis from the sensitivity analysis are presented in section 5 below.

5. Results and Discussion

5.1. Numerical Results

The sensitivity analysis involved changing three factors in the D-FACTS parameters in order to study their effects on the non-dominated Pareto set. These factors and their levels are summarized below in Table 1.

Table 6.3.1. Sensitivity Analysis Factors

Factor	**Reference Level**	**Test Levels**
Ch_{lim}	20%	15%, 25%
n_{lines}	5	10
C^D_{max}	\$25	\$20, \$30, \$35

The twenty-four experiments were carried out using the algorithm described in section 3 on a Dell Computer with an Intel® Xeon ® W-2195 CPU and 256GB of RAM, with an average computing time of 19s.

The results of the tests are summarized below in Table 2. The objectives in the Pareto Front are summarized as Best value, Worst value, and Average value for each of the objectives being considered. The number of non-dominated solutions are also recorded under NOBS in the table. For the purpose of simplicity, these values will also be used in the sensitivity analysis.

In Addition, Table 3 will show the p-values for the ANOVA analysis of each of the below columns against the factors being studied, including an interaction between the maximum investment limit and the line limit, which was found to be significant in some cases. In Table 3, an asterisk will be used to indicate factors that have a small but significant effect at a 10% significance level, two asterisks for a moderately significant effect on the response variable at significance level of 5% is used for the statistical analysis, and three asterisks will be shown on highly significant effects with a significance level of 1%. Interestingly, the reactance change limit is not a very significant factor (marked with ** or *** in table 3) in any of the responses except in the worst-case LUF. Additionally, the worst-case responses for both cost and GWP are seemingly independent from all the factors with a slightly significant effect from the reactance change limit in the worst-case cost response). However, worst-case responses in one objective function usually correspond with best-case responses in a different objective function, and thus we cannot automatically assume that there is no benefit from studying these responses.

Table 6.3.2. Summarized Experiment Data

EXP	BCOST	BGWP	BLUF	WCOST	WGWP	WLUF	ACOST	AGWP	ALUF	NOBS	CHLIM	NLINES	CDMAX
1	71011.2	88.025	0.050	72925.0	88.054	0.054	72220.9	88.042	0.053	24	0.2	5	20
2	70406.0	82.874	0.044	72778.5	88.051	0.056	71947.5	87.848	0.053	28	0.2	5	25
3	70446.2	83.556	0.044	72786.2	88.050	0.055	71696.7	87.696	0.053	24	0.2	5	30

4	69651.6	75.957	0.043	72777.2	88.050	0.060	71135.9	85.530	0.054	14	0.2	5	35
5	71214.4	88.026	0.048	72899.3	88.053	0.054	72124.5	88.041	0.053	20	0.15	5	20
6	70847.7	87.041	0.048	72561.8	88.045	0.055	71721.4	87.952	0.053	12	0.15	5	25
7	70903.5	86.792	0.047	72889.6	88.053	0.058	72107.6	87.944	0.053	17	0.15	5	30
8	70342.6	78.277	0.046	72869.9	88.052	0.060	71536.2	87.005	0.054	23	0.15	5	35
9	71045.5	88.023	0.053	72779.9	88.051	0.054	71955.9	88.038	0.054	23	0.25	5	20
10	71045.5	88.023	0.053	72779.9	88.051	0.054	71955.9	88.038	0.054	23	0.25	5	25
11	70410.0	83.209	0.043	72790.4	88.051	0.055	71623.1	87.544	0.053	23	0.25	5	30
12	70402.9	83.299	0.043	72737.3	88.050	0.055	71529.9	87.628	0.052	23	0.25	5	35
13	71671.0	88.034	0.051	72847.7	88.052	0.054	72481.0	88.046	0.053	22	0.2	10	20
14	71389.1	88.028	0.046	72810.9	88.051	0.054	72207.1	88.041	0.053	20	0.2	10	25
15	71210.1	88.018	0.045	72717.3	88.049	0.055	71893.1	88.036	0.053	27	0.2	10	30
16	70701.2	86.089	0.049	72597.9	88.048	0.057	71941.9	87.939	0.053	23	0.2	10	35
17	71863.9	88.036	0.052	72880.6	88.053	0.054	72416.1	88.045	0.053	28	0.15	10	20
18	71459.9	88.028	0.049	72880.2	88.053	0.054	72321.6	88.043	0.053	20	0.15	10	25
19	70915.4	88.019	0.047	72899.6	88.053	0.056	72269.9	88.042	0.053	34	0.15	10	30
20	70673.6	86.474	0.045	72868.8	88.052	0.055	71952.9	87.988	0.053	32	0.15	10	35
21	71846.4	88.034	0.050	72882.8	88.053	0.054	72442.6	88.046	0.053	28	0.25	10	20
22	71397.3	88.026	0.053	72765.2	88.050	0.054	72301.8	88.043	0.053	29	0.25	10	25
23	71089.8	88.022	0.044	72743.2	88.050	0.054	72068.8	88.039	0.052	20	0.25	10	30
24	70745.3	83.447	0.053	71670.8	87.833	0.054	72575.2	88.047	0.060	23	0.25	10	35

Table 6.3.3. ANOVA p-values

Response Var	Ch lim p-value	N lines p-value	CDmax p-value	Nlines*CDmax
Best Cost	0.778	0.000***	0.000***	0.473
Average Cost	0.997	0.000***	0.000***	0.144

Worst Cost	0.092*	0.372	0.065*	0.107
Best GWP	0.947	0.005*	0.000*	0.010***
Average GWP	0.848	0.038**	0.019**	0.012**
Worst GWP	0.202	0.315	0.160	0.170
Best LUF	0.437	0.148	0.010***	0.255
Average LUF	0.314	0.596	0.192	0.300
Worst LUF	0.034**	0.017**	0.001***	0.045**

After removing the non-significant factors for each response column, the following regression equations were obtained. Only responses with at least one significant factor at a 5% significance level were considered:

$$BestCost = 71872 + 120.6 * N_{lines} - 66.59 * C_{max}^{D}$$

$$AvgCost = 72243 + 88.6 * N_{lines} - 32.34 * C_{max}^{D}$$

$$BestGWP = 107.84 - 1.603 * N_{lines} - 0.958 * C_{max}^{D} + 0.0796 * N_{lines} * C_{max}^{D}$$

$$AvgGWP = 91.69 + 0.357 * N_{lines} - 0.1637 * C_{max}^{D} + 0.01604 * N_{lines} * C_{max}^{D}$$

$$BestLUF = 0.05618 - 0.00031 * C_{max}^{D}$$

$$WorstLUF = 0.04734 - 0.01525 * Ch_{lim} - 0.000779 * N_{lines} + 0.000474 * C_{max}^{D} - 0.000039 * N_{lines} * C_{max}^{D}$$

Based on these equations we can perform some analyses on the effects on the significant variables on the Pareto front. First, increasing the number of lines over which the D-FACTS may be allocated actually increases both the total costs and GWP of the system, but it decreases the LUF. A reasoning for this is that it can reduce strain on more lines as the devices are allocated throughout the system, but by not focusing improving the transmission around more cost-efficient and environmentally friendly generators, it forces the system to rely on more expensive and/or more polluting generators which are more distributed throughout the system.

Another interesting observation is the effect of increasing the investment limit over the objectives. While it reduces both the cost objective and the Global Warming Potential throughout the system, as is expected for an improved network with improved transmission capacity from more efficient generators, it actually has an undesirable effect (at least linearly) on the worst-case of line utilization. This can also be attributed once again to the increased transfer capability from the generator nodes and into the rest of the network, with those specific lines being utilized more heavily, but it still has a desirable effect on the best-case LUF, possibly due to an overall network health improvement.

Further, the effect of the number of lines in which D-FACTS may be installed actually has different effects on the best-case and the average-case for the GWP. While part of this result can be explained by the interaction effect with the investment limit, it is hard to fully understand this difference, especially when considering that these results come from a metaheuristic algorithm, and so we not only not have a full representation of the Pareto frontier, but also we may be looking at a local optimal set rather than the true optimal set.

Overall, however, increasing the investment limit on the DFACTS devices in order to install more of them seems to have, overall, a positive effect on the objective functions being studied. Section 5.2 below has some illustrative graphs to help further the analysis.

5.2. Graphical Results

Below, Figure 3 is a scatterplot with trend line of the cost vs investment limit, for both 5 lines and 10 lines max. allocation and with both the best and average cases for the objective. Figure 4 is a scatterplot with trend line of the average cases for GWP. Only the average cases were used here since the range of the best cases is very wide and there are outliers in the data which impair visualization by over-expanding the axis range. Finally, Figure 5 shows the LUF values against the investment limit in all three cases.

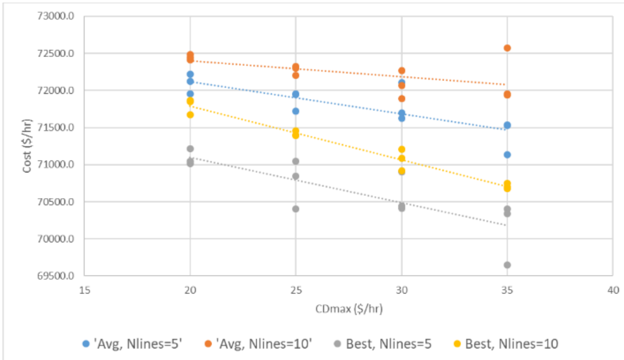

Figure 6.3.3. Expected Cost vs. Investment limit

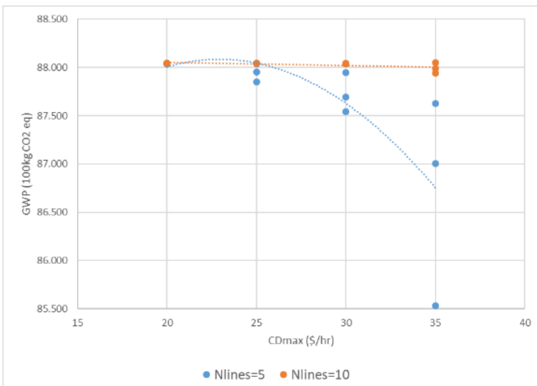

Figure 6.3.4. Average-case Global Warming Potential vs. Investment limit

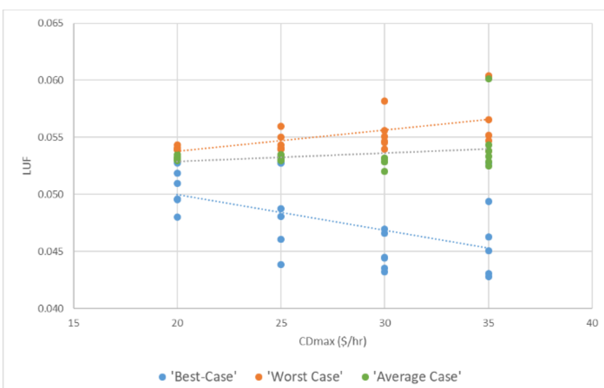

Figure 6.3.5. Line Utilization Factor vs. Investment limit

Overall, as described in the analysis in section 5.1, it's very apparent that both the Cost and Global Warming Potential appear to go down as the investment limit increases. Although there will be a limit to how much this limit can increase before the objectives stop improving, this value is larger than the scope of this sensitivity analysis.

Furthermore, in the LUF, while the worst and average case do slightly increase with the investment limit, the best case does have a downward trend, although it has very high variability which is attributed to noise, as it was found that none of the other factors have a significant enough effect on this variable.

In addition, as reflected in figure 3, an increased number of lines in which installation of D-FACTS devices does not improve the overall system costs or environmental impacts. In Figure 3, the trends of the best and average cases have very similar slopes despite the number of lines, but the intercepts are different based on whether we look at the cases with 5 lines max. or with 10 lines max. In Figure 4, the effect is even more pronounced. The trend for the case with 5 lines max appears to decrease almost quadratically as the investment limit increases (although this may be caused by to outliers in the data, since the regression equations do not have any significant quadratic terms), while the case with 10 lines decreases linearly and at a much slower rate.

6. Conclusion

This study performed a sensitivity analysis of the D-FACTS allocation problem taking into account three key parameters relevant to network properties rather than the optimization algorithm. These parameters were the amount of reactance change allowed on transmission lines, which affects voltage stability and total transfer along the line; the maximum number of lines over which D-FACTS may be installed, which affects how many lines will receive the devices and the feasibility of having the resources to perform the installation; and finally the monetary investment limit, which affects the amount of devices which will be installed throughout the system. The effects of these parameters are measured based on the resulting changes in the objective functions of the solutions remaining in the Pareto-optimal set after running the optimization algorithm, which are summarized into best-case, average-case, and worst-case for total expected operating costs, expected Global Warming Potential, and Line Utilization Factors, which correspond to economic, environmental, and sustainable effects.

The sensitivity analysis was performed mainly by the use of statistical regression analysis, with a significance level of 5% as a standard. These calculations were assisted by the use of Minitab ® software. Statistical analysis found that the reactance change limit has little impact on most of the objectives and that it is not a very significant parameter in the optimization process. On the other hand, number of lines and investment limit have a much more significant effect, with number of lines having an apparent inverse relation with most objectives, while the investment limit has a direct relation with improving the objective function values. The main drawback, however, is that investors and decision-makers may oppose increasing their investments despite obvious long-term benefits. Since each D-FACTS device has an estimated cost of $3000, and the hourly cost is calculated to be approximately 2.5 cents, an increase of $5/hr to the investment limit translates to approximately $600,000 in initial investment, which would discourage change and risk-averse executives, even despite previous studies estimating hourly savings in the tens of thousands of dollars.

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6.4. Analyzing the Effects of Line Switching Protocols on Multi-Objective D-FACTS Allocation Optimization

Abstract—Distributed Flexible AC Transmission Systems (D-FACTS) and their allocation are emerging topics in the field of Power and Transmission Systems. They are simple yet effective tools for improving power flow control, system flexibility, and overall reliability. Line switching, on the other hand, refers to the practice of disconnecting transmission lines to change the topology of the transmission network and reroute the power flow. This study proposes a co-optimization model of D-FACTS and transmission switching and analyzes the synergy between the installation of D-FACT devices and line switching practices to reduce both the operating costs and environmental impacts of power systems. Case studies were carried out on a modified RTS-96 test system, and results show favorable results when implementing both practices in the test system.

Keywords—Distributed flexible AC transmission systems (D-FACTS), evolutionary algorithm, line switching, multi-objective optimization, optimal allocation

I. Nomenclature

Indices	
a, b	Solutions
c	Contaminant
k	Transmission line.
g	Generator.
n	Node.
s	Scenario.
seg	Segment of linearized generator cost function.
i	Objective or Fitness Function
Sets	
$\sigma^{+}(n)$	Transmission lines with their “to” bus connected to node n.
$\sigma^{-}(n)$	Transmission lines with their “from” bus connected to node n.
$g(n)$	Generators connected to node n.
$r(n)$	Renewable generators connected to node n.
Variables	
C_{inv}^{D}	Total investment in D-FACTS ($).
$D_{a,b}$	Dominance of solution a over solution b
$F_{k,s}$	Real power flow through transmission line k in scenarios s.

FMi_a	Value of fitness function *i* for solution *a*
$OF_{i,a}$	Value of objective function *i* for solution *a*
$P_{g,s}$	Real power generation of generator g in scenario s.
$P_{r,s}^{C}$	Curtailed renewable generation from renewable generator r in scenario s
$P_{g,s}^{seg}$	Real power generation of generator g in scenarios s in segment seg.
$R_{g,s}^{D}$	Spinning down reserve available through generator g in scenario s.
$R_{g,s}^{U}$	Spinning up reserve available through generator g in scenario s.
x_k^D	Integer indicating the number of D-FACTS installed on transmission line k
$\theta_{b,s}$	Voltage angle at bus b in scenarios s.
$\theta_{fr,k,s}$	Voltage angle at the "from" node of line k in scenarios s.
$\theta_{to,k,s}$	Voltage angle at the "to" node of line k in scenarios s.
z_k	Indicator of whether a line *k* is switched on or off
Parameters	
C_g^{NL}	No load cost of generator g.
$C_{g,seg}^{linear}$	Linear cost of generator g in segment seg.
C_g^D	Down reserve cost of generator g.
C_g^U	Up reserve cost of generator g.
C_{single}^{D}	Cost a of single D-FACTS unit ($).
C_{sh}^{D}	Cost a of single D-FACTS unit converted to an hourly figure ($/h).
C_{inv}^{max}	Maximum investment allowed for D-FACTS.
$f_{k,s}$	Flow direction for line *k* in scenario *s*.
F_k^{max}	Thermal capacity/voltage drop limit of transmission line k.
$H_{g,seg}^{linear}$	Linearized Heat production of generator *g* in generation segment *seg* (MMBTU/MW)
$G_{g,c}$	Gaseous contaminant *c* released by generator *g* (kg/MMBTU)
$GWP_{g,c,s}$	Global Warming Potential caused by contaminant *c* from generator *g* in scenario *s*
i_k^{max}	Maximum number of D-FACTS that can be allocated per line.
I	Interest rate/discount rate.
l_{max}^{alloc}	Maximum number of lines in which D-FACTS devices may be allocated
l_k	Length of line *k*
$L_{n,s}$	Load at bus n in scenario s.
N	Lifespan of D-FACTS.
N_g	Total number of generators.
N_k	Total number of lines.
N_s	Number of scenarios.
N_{seg}	Number of segments for the linearized generator cost function.
N_{pop}	Population size for the algorithm.
N_r	Number of renewable generators.
p_s	Probability of scenario s.
P_g^{max}	Upper generation limit of generator g.
P_g^{min}	Lower generation limit of generator g.
$P_{r,s}$	Renewable generation produced by renewable generator r in scenario s
$P_{r,s}^{C}$	Renewable energy curtailed from renewable generator *r* in scenario *s*.

S^D	Spinning down reserve requirement g.
S^U	Spinning up reserve requirement g.
W_c	GWP factor for contaminant c (1kg CO_2 eq.)
X_k	The reactance of transmission line k.
X_k^{max}	The maximum reactance of line k if D-FACTS are installed on this line.
X_k^{min}	The minimum reactance of line k if D-FACTS are installed on this line.
η_C, η_L	The maximum adjustment percentage of the line's reactance in the capacitive or inductive mode that a single D-FACTS module (1 device/phase/mile) can achieve.
$\Delta\theta_k^{max}$	Maximum value of bus voltage angle difference to maintain stability for line k.
$\Delta\theta_k^{min}$	Minimum value of bus voltage angle difference to maintain stability for line k.
z^{lim}	Maximum number of lines that can be switched off

II. Introduction

Energy infrastructure is used to generate, transmit, and distribute electricity. While investments have increased significantly to meet energy demands, the transmission and distribution sections still struggle with reliability, as over 70% the approximate 600,000 miles of transmission lines in the U.S. are nearing their expected lifespan [1]. Despite recent government initiatives to improve grid reliability and resistance to extreme conditions [2], it is expected that simple infrastructure expansion is not enough to fully remedy the problem. As an option to help in mitigating some of the issues in the transmission side of the grid, flexible AC transmission systems (FACTS) devices can be used to provide effective power flow control as part of smart transmission systems [3]. These same benefits can also be obtained by the use of D-FACTS, which are smaller, lightweight versions of traditional FACTS devices, with the added benefits of improved reliability, portability, and redeployability [4].

While D-FACTS devices have become more widespread in recent years, and their economic benefits have long been proven, it is important to consider environmental impacts when looking at large infrastructure projects. Some positive environmental effects have been previously associated with the integration of D-FACTS devices, as they can also improve renewable energy integration into power grids [5]. However, more specific measures of environmental impacts are not commonly studied for the D-FACTS allocation problem. As such, one objective of this study is to include the optimization of Global Warming Potential (GWP) alongside with the expected cost. Since D-FACTS devices can improve transmission capacity in the network, it is expected that, by improving the integration of renewable energy and reduce the generation from fossil fuel-based sources, the environmental impacts of power systems can be reduced.

To effectively utilize D-FACTS, D-FACTS modules need to be optimally allocated. D-FACTS allocation optimization is still a relatively new topic, with a limited number of studies and mathematical models published. These models have different levels of complexity depending on the main objective of the study, whether it being minimizing line congestion or system costs. One of the earliest studies on D-FACTS allocation can be attributed to Li *et al.* [6], who proposed a non-linear DC-based optimization model for allocating D-FACTS devices with the objective of minimizing congestion within the lines. Other studies such as Das *et al.* (Das, Prasaj, Harley, & Divan, 2009), who proposed a particle swarm-based approach for D-FACTS allocation in order to reduce load in overloaded lines using static generation and loads. Other proposed methods include the use of graph theory [8], linear programming [9], and mixed-integer programming[10]. However, these methods can be very computationally intensive, creating heavy burdens when analyzing larger systems. As a solution, a metaheuristic approach was created to avoid excessive computational times while maintaining optimal or near-optimal solutions.

The use of D-FACTS devices has been recommended for effective flow control in systems with distributed generation, including less predictable sources such as renewable energies [11]. The D-FACTS devices are also advantageous for improving grid utilization, increasing flexibility and power flow control, as well as increasing grid security and reliability. Multiple-objective optimization of their allocation has previously been performed with focus on specific issues. Ref. [12], for example, used an enhanced bacterial foraging optimization method in order to minimize voltage deviation and power losses, maximize security, and optimize load balancing by the use of D-FACTS. Particle swarm optimization has also been used by [13] to optimize VA rating, power loss, and undervoltage problems by allocating unified power quality conditioners (a type of D-FACTS). However, despite the optimization of multiple objectives, they are considered as separate problems and thus optimizes separately rather than simultaneously in order to present the advantages of the devices rather to optimize all objectives. In addition, environmental impacts have not been considered by other authors in the found literature.

Unlike the fairly recent development of D-FACTS, Transmission switching has been in use since the 1980s in order to relieve overload and voltage problems, reduce losses, and improve security [14]. Nowadays, it is used more commonly to reduce generation costs in congested systems, for day ahead or real time planning, by taking some lines out of service [15]. Based on the DCOPF problem, a mixed-integer program was proposed by [16] to minimize generation costs while simultaneously choosing to open some transmission-line switches, and this problem is named as optimal transmission switching (OTS). Related studies found that significant savings were possible by the use of transmission switching.

Despite all the research in both fields, there have been no previous studies which consider the effects of both transmission switching and D-FACTS allocation. In order to fill this gap, this paper proposed a multiple-objective evolutionary algorithm (MOEA) to solve a D-FACTS optimal allocation and transmission switching problem. The main innovations of this algorithm are that it (1) considers optimization of not only cost but also environmental impacts in the form of GWP, and (2) incorporates both D-FACTS allocation and transmission switching to control the power flow and reduce operating costs as well as the environmental impacts. The algorithm can generate a Pareto front with feasible solutions, and a system planner or utility company can thus choose the best solution based on their needs and requirements.

The remainder of this paper is organized as follows: Sec. III will describe the mathematical formulation of the model as well as the algorithm to solve it, Sec. IV will propose the simulation setup and results from the case study, and finally Sec. V will contain the concluding remarks.

III. Mathematical Model

A. D-FACTS Allocation and Transmission Switching Model

The proposed model is based on the DC Optimal Power Flow (DCOPF) problem. Variable-impedance type of D-FACTS modules are used in this study, and these modules mainly affect real power flow. Thus, the DCOPF model is adopted instead of ACOPF to improve computational efficiency. This allocation model has the main purpose of allocating both D-FACTS modules in each phase of the lines and choosing lines to switch off with the objective of optimizing cost and environmental impacts while meeting the load demands. It is a stochastic optimization model, where different scenarios with associated probabilities are considered in order to account for some level of uncertainty. It differs from the model in [10] in the following key aspects: (1) The proposed model in this study is nonlinear, while the model in [10] is linear, however, the algorithm described in Sec. III-B can solve the model computationally efficiently despite the nonlinearities; (2) the proposed model allocates the D-FACTS devices in a per-phase basis for each line rather than per-phase per-mile for each line; (3) the proposed model has the additional objective of minimizing the expected GWP; and (4) the proposed model co-optimizes the location of D-FACTS modules and transmission switching.

Despite the introduction of additional objectives and nonlinearities, the problem can still be solved in a computationally-efficient manner by using the algorithm described in section III-B. In the proposed model, variable-impedance series D-FACTS modules are allocated. As the reactances of the lines are to be adjusted, the applicable power flow constraints will depend on power flow directions, as discussed by [17]:

If $\theta_{fr,k,s} - \theta_{to,k,s} \geq 0$,

$$\theta_{fr,k,s} - \theta_{to,k,s}/X_k^{max} \leq F_{k,s} \leq \theta_{fr,k,s} - \theta_{to,k,s}/X_k^{min} \tag{1}$$

If $\theta_{fr,k,s} - \theta_{to,k,s} \leq 0$,

$$\theta_{fr,k,s} - \theta_{to,k,s}/X_k^{min} \leq F_{k,s} \leq \theta_{fr,k,s} - \theta_{to,k,s}/X_k^{max} \tag{2}$$

The model for optimal D-FACTS allocation considering reserve requirements, multiple scenarios, and line switching practices is described below by Equations (3)-(26):

$$\min OF_1 = \sum_{s=1}^{N_s} P_s \left(\sum_{g=1}^{N_g} \left(\sum_{seg=1}^{N_{seg}} C_{g,seg}^{linear} P_{g,s}^{seg} + C_g^U R_{g,s}^U + C_g^D R_{g,s}^D + C_g^{NL}\right) + \sum_{r=1}^{N_r} c_r P_{r,s}^C\right) + C_{inv}^D \quad (3)$$

$$\min \mathrm{OF}_2 = \sum_{s=1}^{N_s} P_s \left(\sum_{g=1}^{N_g} \sum_{c=1}^{N_c} GWP_{g,c,s}\right) \quad (4)$$

$$P_{g,s} = \sum_{seg=1}^{N_{seg}} P_{g,s}^{seg} \quad (5)$$

$$P_g^{min} \le P_{g,s} \le P_g^{max} \quad (6)$$

$$-z_k F_k^{max} \le F_{k,s} \le z_k F_k^{max} \quad (7)$$

$$\sum_{k\in\sigma^+(n)} F_{k,s} - \sum_{k\in\sigma^-(n)} F_{k,s} + \sum_{g\in g(n)} P_{g,s} + \sum_{r\in r(n)} \left(P_{r,s} - P_{r,s}^C\right) = L_{n,s} \quad (8)$$

$$\sum_{g=1}^{N_g} R_{g,s}^U \ge S^U \quad (9)$$

$$\sum_{g=1}^{N_g} R_{g,s}^D \ge S^D \quad (10)$$

$$R_{g,s}^U \le P_g^{max} - P_{g,s} \quad (11)$$

$$R_{g,s}^D \le P_{g,s} - P_g^{min} \quad (12)$$

$$R_{g,s}^U \ge 0 \quad (13)$$

$$R_{g,s}^D \ge 0 \quad (14)$$

$$\Delta\theta_k^{min} \le \theta_{fr,k,s} - \theta_{to,k,s} \le \Delta\theta_k^{max} \quad (15)$$

$$\theta_{1,s} = 0 \quad (16)$$

$$z_k f_{k,s} \left(1 + \frac{x_k^D}{l_k} \eta_L\right) X_k F_{k,s} \ge z_k f_{k,s} \left(\theta_{fr,k,s} - \theta_{to,k,s}\right) \quad (17)$$

$$z_k f_{k,s} \left(1 + \frac{x_k^D}{l_k} \eta_C\right) X_k F_{k,s} \le z_k f_{k,s} \left(\theta_{fr,k,s} - \theta_{to,k,s}\right) \quad (18)$$

$$GWP_{g,c,s} = \sum_{seg}^{N_{seg}} H_{g,seg}^{linear} P_{g,s}^{seg} G_{g,s} W_c \quad (19)$$

$$0 \le x_k^D \le i_k^{max} \quad (20)$$

$$\sum_{k=1}^{N_k} \frac{x_k^D}{\max(x_k^D, 1)} \le l_{max}^{alloc} \quad (21)$$

$$C_{inv}^{D} = \sum_{k=1}^{N_{br}} \sum_{i=1}^{i_{max}} 3iC_{sh}^{D} x_{k,i}^{D} \quad (22)$$

$$C_{inv}^{D} \leq C_{inv}^{max} \quad (23)$$

$$C_{sh}^{D} = C_{single}^{D} \frac{I(1+I)^{N}}{8760((1+I)^{N}-1)} \quad (24)$$

$$0 \leq P_{r,s}^{C} \leq P_{r,s} \quad (25)$$

$$\sum z_k + z^{lim} \geq N_k \quad (26)$$

In this formulation, the objectives are first to minimize the total expected operating costs over all scenarios considering generation and reserve costs as well as the D-FACTS investment costs (3), and the total GWP of the generators (4). Eqn. (5) segments the generation to be on the same intervals as the linearized cost curve, while (6) denotes the upper and lower limits for each generator in the system. Eqn. (7) indicates the transmission limits for each line, accounting for the switched lines by using the binary variable z_k, which takes a value of 1 for lines in service, and 0 for lines that are switched off. Based on previous models, the capacity of short lines (0-50 miles) is set to their thermal limits, the capacity of medium lines (50-156 miles) is determined by their voltage drop limit, and for long lines (over 156 miles) the capacity is set by voltage angle stability limits. The power balance at each bus is defined by (8), indicating the load at each bus to be equal to all power generated at the bus plus all incoming transfers and minus all outgoing transfers. Eqns. (9)-(14) define the reserve requirements, with (11) and (12) defining the reserve capacities. Eqns. (15) and (16) define the voltage angle constraints and set the angle at bus 1 to 0 for reference. Furthermore, (17) and (18) show the DC power flow equations at each line, regardless of the installation of D-FACTS and considering the flow direction and the line status. Eqn. (19) defines the GWP for each generator based on the Intergovernmental Panel on Climate Change 5th Assessment Report (AR5). A limit on the number of D-FACTS per line is defined in (20), while the total number of lines with D-FACTS devices is restrained in (21). The cost conversion and investment limit for D-FACTS are defined in (22)-(24). Finally, (25) defines the limits for renewable energy curtailment and (26) defines the maximum number of lines that may be switched off. In addition, as the power flow direction $f_{k,s}$ is a necessary but unknown parameter for optimizing the problem, a reduced model consisting of Eqns. (3), (5)-(8), (15)-(18) is used to calculate a simplified base scenario and the flow directions are then obtained using Eqn. (27), with all the values for $f_{k,s}$ and x_k^D initialized to 1 and 0 respectively, turning Eqns. (17) and (18) into the DC power flow equation.

$$f_{k,s} = \frac{F_{k,s}}{\max\left(\left|F_{k,s}\right|, 1\right)} \tag{27}$$

B. The Evolutionary Algorithm

A metaheuristic algorithm is a type of stochastic search algorithm that seeks to iteratively approach an optimal solution to a problem by performing a guided search over a solution space. An evolutionary algorithm, in turn, is a metaheuristic algorithm that mimics natural evolutionary processes by encoding a problem into a set of strings that can represent a solution to this problem. Since its proposal in 1975, this type of algorithm has been used for solving various computational problems that arise in real-world applications which require efficient algorithms to solve them within reasonable time frames. In the present research, we develop and present a new MOEA to efficiently find possible solutions to the problem and identify which of them meet optimality conditions. Since this model is nonlinear and nonconvex, an optimal solution cannot be guaranteed. However, previous research, as done by [18], shows that this type of algorithm can reduce the computational time by a large margin.

As this is a multi-objective optimization problem, it is both possible and expected that the objectives are in opposition to each other, and thus a single solution cannot usually be obtained without receiving input from an outside decision-maker. For this situation, a type of optimality called Pareto Optimality is used. A Pareto-optimal solution is a solution for which no other existing solution is equal or better than it in all objectives. At the end of the optimization process, the result should be a set of Pareto-optimal solutions, called a Pareto front.

k	1	2	3	4	5	6	7	8	9	...	N_k
X_k^D	60	0	0	0	-1	0	96	0	0	...	36

Fig. 6.4.1. Chromosome Example

In the proposed algorithm, the first step is to generate an initial set of possible solutions. These are encoded following the example in Fig. 1: A possible solution consists of a vector of length N_k, with positive values representing how many D-FACTS are to be installed at each line, and a -1 value representing lines that are to be shut off. Initially, these values are allocated randomly following constraints (20)-(24) and (26), with the negative values representing a z_k of 0 and all other representing a value of 1.

As line switching can affect the power flow directions, the values for $f_{k,s}$ are re-calculated for each candidate solution. Afterwards, this information is used to consecutively solve reduced LPs consisting of (3)-(8), (15)-(19), and (25) while the reserves in (9)-(14) are allocated using a greedy algorithm in order to further reduce the computational time. The objective functions are then obtained and dominance is checked in order to store the non-dominated solutions before performing a crossover and generating new solutions until the termination criteria are satisfied. Fig. 2 contains a more detailed flowchart of the steps taken by the algorithm.

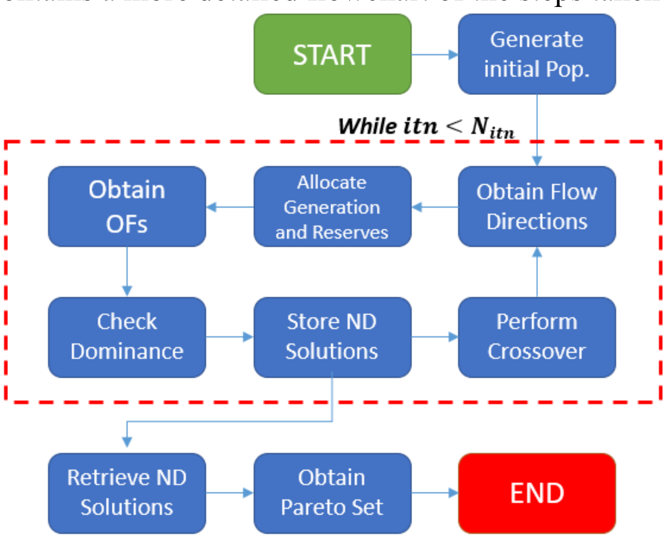

Fig. 6.4.2. MOEA Flowchart

At each iteration, after evaluating each solution, all current solutions that resulted in valid configurations are cross-checked for dominance, with the non-dominated solutions being stored separately. In order to rank the solutions for the crossover step, two new fitness metrics are calculated and aggregated to obtain a single combined fitness metric with which to rank the solutions prior to crossover. The first metric is based on dominance count to ensure proximity to the true Pareto front, while the second is based on inter-solution distance, in order to ensure a wider range of solutions, as described by the method developed by [19]. Both metrics are normalized before being added into a combined fitness metric that can be used to rank each solution. These metrics are shown below in (28)-(30). In Eq. (28), $D_{i,j} = 1$ if $OF1_i \leq OF1_j$ & $OF2_i \leq OF2_j$, or 0 if otherwise. In these equations, *i* and *j* refer to a solution number, while N_{pop} refers to the total number of solutions at each iteration. OF1 and OF2 refer to the objective functions described in (3) and (4), respectively, normalized to the [0,1] range.

$$OF1_i = \sum_{j \neq i}^{N_{pop}} D_{i,j} \qquad (28)$$

$$FM2_i = \sum_{j=1}^{N_{pop}} \left(\left| OF1_i - OF1_j \right|^2 + \left| OF2_i - OF2_j \right|^2 \right)^{\frac{1}{2}} \quad (29)$$

$$FM_i = \frac{FM1_i}{\max FM1} + \frac{FM2_i}{\max FM2} \qquad (30)$$

At the end of each iteration, a small part of the solutions is kept and inserted into the next iteration by a process called elitism to ensure good quality of solutions. Additionally, a new set of solutions is generated via single-point crossover, where randomly chosen solutions will be split along a cut-point and their parts combined. To avoid falling into local optima, there is also a small chance of mutation occurring, in which case some genes will be moved around the solution. After generating a new solution, they are adjusted if needed to ensure the enforcement of (20)-(24). This whole process repeats until a number of iterations has been run.

IV. Case Study

A. Simulation Setup

In this study, the model described in Sec. III-A is adopted to study the cost and environmental impact reductions associated with the installation of D-FACTS devices and implementation of line-switching protocols using the algorithm described in Sec. III-B. The test system used is the IEEE RTS-96, with some modifications as described in [10] so as to simulate additional congestion. To model uncertainty, various load and renewable energy production scenarios are created. For the load scenarios, load factors of 0.65, 0.75, 0.85, and 0.95 are applied to the system, while renewable generation factors of 0, 0.2, 0.6, and 1 are considered, resulting in a total of 16 scenarios after combining them. It is assumed that each D-FACTS module is capable of adjusting the reactance of the line in which it is installed by ±2.5% per phase per mile, and that the maximum reactance change allowed for a 3-phase line is ±20%. This would result in a limit of $i_k^{max} = \frac{20}{2.5} l_k = 8 l_k$ devices per phase.

The cost of the devices was determined based on industry data and previous academic studies. Based on the literature, the cost of a single D-FACTS device is assumed to be approximately \$100/kVA; where the compensation level in kVA is dependent on the parameters of the transmission line on which it is installed. For simplicity, the compensation level of the most demanding line was adopted. In the RTS-96, this value is 30kVA/module, and so a total cost of \$3000 per D-FACTS module was adopted. Since all generation-related costs are in hourly units, (24) is used to convert this cost into an hourly rate, considering an expected lifespan of 30 years and a discount rate of 6%. Additionally, industry practices would impose a limit on investment on the modules, denoted by (23). An allowance of \$25/hour is assumed. The MOEA was run with the following parameters: 200 individuals in the population iterated over 100 generations, using 5% elitism and 5% chance of mutation. The algorithm was run on a DELL® computer with 256GB of RAM and an Intel® Xeon® W-2195 CPU, with each simulation averaging around 165 seconds. The number of lines at which D-FACTS could be installed was limited to 5 lines, as the sensitivity study in [20] showed little improvement of the system above this number. The number of lines that can be switched off will vary between 0 and 10% of the existing lines in the system, rounded to the nearest number. As the RTS-96 has 38 lines, the largest number of lines that can be switched was set to 4.

B. Simulation Results

This model alleviates transmission congestion by using both transmission switching and D-FACTS, and the alleviation can be reflected in the optimal generation dispatch cost. A higher optimal generation dispatch cost reflects a higher congestion level. The results from each of the simulations are summarized below in Fig. 3 and Table I. For reference, the base-case solution in which no D-FACTS are installed, and no lines are switched off is also included. This solution is marked in Fig. 1 with a red circle. Due to space limitations, all Pareto fronts are graphed together in Fig. 1, with different color and shape markers to differentiate the solutions from each different value of z_{lim}. The results in Table I show that the optimal generation dispatch cost were reduced by using transmission switching and D-FACTS, which reflected a reduced level of transmission congestion.

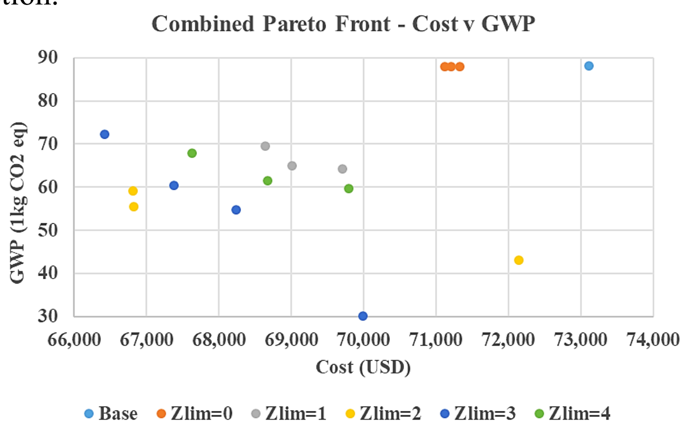

Fig. 6.4.3. Combined Pareto Fronts for all simulations

Table 6.4.1. Detailed Solutions

Sol.	$\sum x_k^D$	D-FACTS locations	z^{lim}	Lines OFF	Cost	GWP
Base	0	-	0	-	73,114	88.06
1	999	2,22,27,36	0	-	71,207	88.026
2	999	10,18,22	0	-	71,331	88.023
3	999	21,22,24,36	0	-	71,117	88.022
4	999	1,8,22,36	1	35	68,639	69.49
5	996	9,11,18,19,33	1	29	69,007	64.97
6	981	8,22,35	1	29	69,705	64.26
7	828	11,24,28,30	2	29,32	66,817	59.07
8	915	5,8,21,37	2	29,33	66,823	55.46
9	894	5,24	2	3,29	72,141	43.17
10	957	24,31,32,36	3	16,29,33	66,425	72.25
11	609	1,12,37	3	9,29,33	67,375	60.49
12	990	3,5,19,20,37	3	11,29,32	68,237	54.76
13	990	19,33,36,38	3	3,29,30	69,993	30.23
14	984	2,6,10,30	4	16,29,35,37	67,632	67.91
15	657	8,11,21,22	4	13,29,34,37	68,674	61.48

16	858	4,23,25,32	4	5,11,29,37	69,797	59.66

Based on the results, the largest cost savings can be obtained with a maximum switch-off of three lines, while the largest reduction on environmental impacts can be obtained with switching only two. The most common line to switch off seems to be line 29, appearing in 12 out of 13 solutions. Other lines that are commonly off in these solutions are lines 33 and 37, each appearing 3 and times. Another interesting observation is an apparently inverse relation between the number of lines switched off and the number of D-FACTS modules installed. On average, there are 999 modules installed when no lines are switched off, but this number reduced to 992, 879, 886, and finally 833 when 4 lines are switched off. This overall down trend is because less input from the D-FACTS is required to optimize the objective functions and obtain the desired flow when the power flow is partially controlled by the line switching protocols,

To further showcase the overall objective value improvements based on the number of lines switched off, each objective value is plotted against the number of switched lines in Figs. 4 and 5. It can be seen that there is a downward trend on the cost for the first three switched lines, while a downward trend exists in for the GWP until the second switched line. After the 3rd and 2nd line, the average cost and average GWP start increasing again, respectively. It is worth noting that the results are only for non-dominated solutions. While constraint (26) does not prevent the algorithm from switching off fewer than z_{lim} lines, no non-dominated solution had fewer than z_{lim} lines switched.

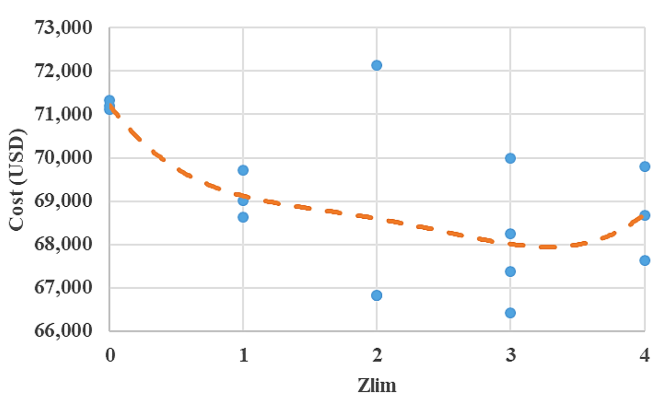

Fig. 6.4.4. Cost vs. $z^{\lim}$ with average trendline

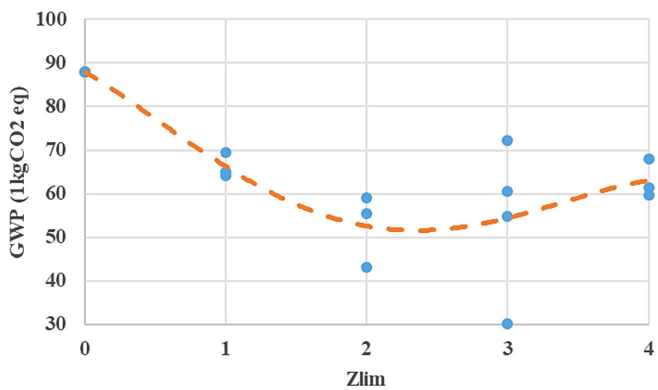

Fig. 6.4.5. GWP vs. $z^{\lim}$ with average trendline

V. Conclusions

This study presented a metaheuristic multi-objective approach for solving a scenario-based stochastic D-FACTS allocation and line switching co-optimization model which optimally allocates D-FACTS devices considering transmission switching. This model mitigates congestion and reduces both expected operating costs and environmental impacts of power systems. The algorithm allocates D-FACTS modules with static investment limits while optimizing the objectives of expected total cost and expected GWP in a computationally efficient manner. Results show that, with transmission switching, fewer D-FACTS modules are needed to achieve effective power flow control. Results also show that there is an inverse relationship between the operating costs and GWP, and a trade-off needs to be made by decision-makers.

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Chapter 7: Further Studies

7.1. Line Switching Analysis

As it was mentioned in Chapter 6, the comparison of pure line-switching approaches to congestion relief against pure D-FACTS and combined D-FACTS and line-switching approaches was not performed. This subsection aims to address this. The mathematical model is the same as the one presented in the 4th publication in Chapter 6, which is also similar to the one presented in section 3.3, with some modifications to accommodate the line switching and different objective functions. This model is also shown below in eqs. (7.1 – 7.24). For this analysis, the D-FACTS allowance, C_{max}^{inv} is set to 0. This allows us to be able to run the exact same program with no significant changes, and observe the results of switching system lines off without any D-FACTS installed.

$$\min OF_1 = \sum_{s=1}^{N_s} P_s \left(\sum_{g=1}^{N_g} \left(\sum_{seg=1}^{N_{seg}} C_{g,seg}^{linear} P_{g,s}^{seg} + C_g^U R_{g,s}^U + C_g^D R_{g,s}^D + C_g^{NL} \right) + \sum_{r=1}^{N_r} c_r P_{r,s}^C \right) + C_{inv}^D \tag{7.1}$$

$$\min \mathrm{OF}_2 = \sum_{s=1}^{N_s} P_s \left(\sum_{g=1}^{N_g} \sum_{c=1}^{N_c} GWP_{g,c,s} \right) \tag{7.2}$$

$$P_{g,s} = \sum_{seg=1}^{N_{seg}} P_{g,s}^{seg} \tag{7.3}$$

$$P_g^{min} \leq P_{g,s} \leq P_g^{max} \tag{7.4}$$

$$-z_k F_k^{max} \leq F_{k,s} \leq z_k F_k^{max} \tag{7.5}$$

$$\sum_{k \in \sigma^+(n)} F_{k,s} - \sum_{k \in \sigma^-(n)} F_{k,s} + \sum_{g \in g(n)} P_{g,s} + \sum_{r \in r(n)} \left(P_{r,s} - P_{r,s}^C \right) = L_{n,s} \tag{7.6}$$

$$\sum_{g=1}^{N_g} R_{g,s}^U \geq S^U \tag{7.7}$$

$$\sum_{g=1}^{N_g} R_{g,s}^D \geq S^D \tag{7.8}$$

$$R_{g,s}^U \leq P_g^{max} - P_{g,s} \tag{7.9}$$

$$R_{g,s}^D \leq P_{g,s} - P_g^{min} \tag{7.10}$$

$$R_{g,s}^U \geq 0 \tag{7.11}$$

$$R_{g,s}^D \geq 0 \tag{7.12}$$

$$\Delta\theta_k^{min} \leq \theta_{fr,k,s} - \theta_{to,k,s} \leq \Delta\theta_k^{max} \tag{7.13}$$

$$\theta_{1,s} = 0 \tag{7.14}$$

$$z_k f_{k,s} \left(1 + \frac{x_k^D}{l_k} \eta_L \right) X_k F_{k,s} \geq z_k f_{k,s} \left(\theta_{fr,k,s} - \theta_{to,k,s} \right) \tag{7.15}$$

$$z_k f_{k,s}\left(1+\frac{x_k^D}{l_k}\eta_C\right)X_k F_{k,s} \le z_k f_{k,s}(\theta_{fr,k,s}-\theta_{to,k,s}) \tag{7.16}$$

$$GWP_{g,c,s} = \sum_{seg}^{N_{seg}} H_{g,seg}^{linear} P_{g,s}^{seg} G_{g,s} W_c \tag{7.17}$$

$$0 \le x_k^D \le i_k^{max} \tag{7.18}$$

$$\sum_{k=1}^{N_k} \frac{x_k^D}{\max(x_k^D, 1)} \le l_{max}^{alloc} \tag{7.19}$$

$$C_{inv}^D = \sum_{k=1}^{N_{br}} \sum_{i=1}^{i_{max}} 3iC_{sh}^D x_{k,i}^D \tag{7.20}$$

$$C_{inv}^D \le C_{inv}^{max} \tag{7.21}$$

$$C_{sh}^D = C_{single}^D \frac{I(1+I)^N}{8760((1+I)^N-1)} \tag{7.22}$$

$$0 \le P_{r,s}^C \le P_{r,s} \tag{7.23}$$

$$\sum z_k + z^{lim} \ge N_k \tag{7.24}$$

In this model, the new variables incorporated are z_k, which has a value of 0 if line k is switched off and 1 if it is switched on, and z^{lim}, the maximum number of lines which can be switched off. The new equation (7.24) simply limits the number of lines that can be switched off as a number lesser or equal to the limit z^{lim}. The program was executed with z^{lim} values of 0 (base case), 1, 2, 3, and 4, the same values as in the conference article. For reference, the results from that article are below in table 7.1. Table 7.2, in contrast, has the results with $c_{inv}^{max} = 0$.

Table 7.1. Solutions from line switching model

Sol.	$\sum x_k^D$	**D-FACTS locations**	z^{lim}	**Lines OFF**	**Cost**	**GWP**
Base	0	-	0	-	73,114	88.06
1	999	2,22,27,36	0	-	71,207	88.026
2	999	10,18,22	0	-	71,331	88.023
3	999	21,22,24,36	0	-	71,117	88.022
4	999	1,8,22,36	1	35	68,639	69.49
5	996	9,11,18,19,33	1	29	69,007	64.97
6	981	8,22,35	1	29	69,705	64.26
7	828	11,24,28,30	2	29,32	66,817	59.07
8	915	5,8,21,37	2	29,33	66,823	55.46
9	894	5,24	2	3,29	72,141	43.17
10	957	24,31,32,36	3	16,29,33	66,425	72.25
11	609	1,12,37	3	9,29,33	67,375	60.49
12	990	3,5,19,20,37	3	11,29,32	68,237	54.76
13	990	19,33,36,38	3	3,29,30	69,993	30.23
14	984	2,6,10,30	4	16,29,35,37	67,632	67.91
15	657	8,11,21,22	4	13,29,34,37	68,674	61.48
16	858	4,23,25,32	4	5,11,29,37	69,797	59.66

Table 7.2. Solutions from line switching models without D-FACTS

Sol.	z^{lim}	Lines OFF	Cost	GWP
1	0	-	73,114	88.06
2	1	29	69,063	65.52
3	2	29, 35	69,032	65.52
4	2	3, 4	77,783	58.42
5	2	3, 5	81,431	25.26
6	2	29, 36	69,039	65.52
7	2	3, 12	79,648	50.02
8	3	9, 31, 34	70,027	86.28
9	3	4, 5, 29	70,173	66.19
10	3	12, 34, 35	72,384	54.10
11	3	3, 6, 32	79,054	44.46
12	3	16, 30, 34	68,701	71.52
13	3	15, 29, 35	69,321	68.37
14	4	8, 14, 28, 35	68,359	66.40
15	4	5, 34, 35	65,221	54.06
16	4	12, 13, 29, 33	77,378	46.41
17	4	3, 6, 33	79,054	44.46
18	4	4, 13, 29, 35	69,103	65.26
19	4	8, 12, 13, 36	83,102	63.22
20	4	3, 5, 25, 37	83,299	26.80

To compare the solutions from the cases with D-FACTS and the cases without, the below figures were created. The case where $z^{lim} = 0$ will not be considered for the comparisons.

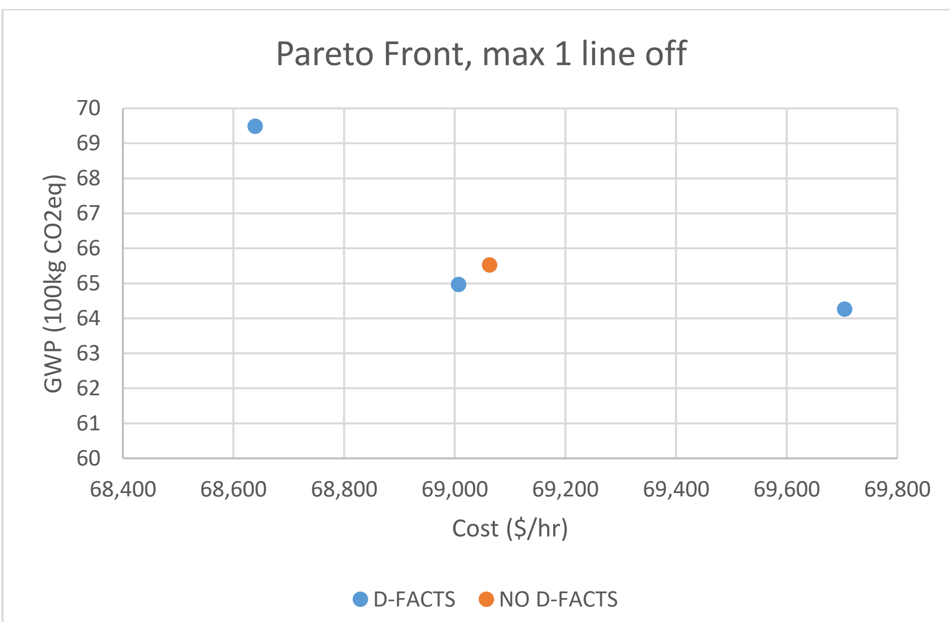

Figure 7.1. Pareto front of solutions with $z^{lim} = 1$

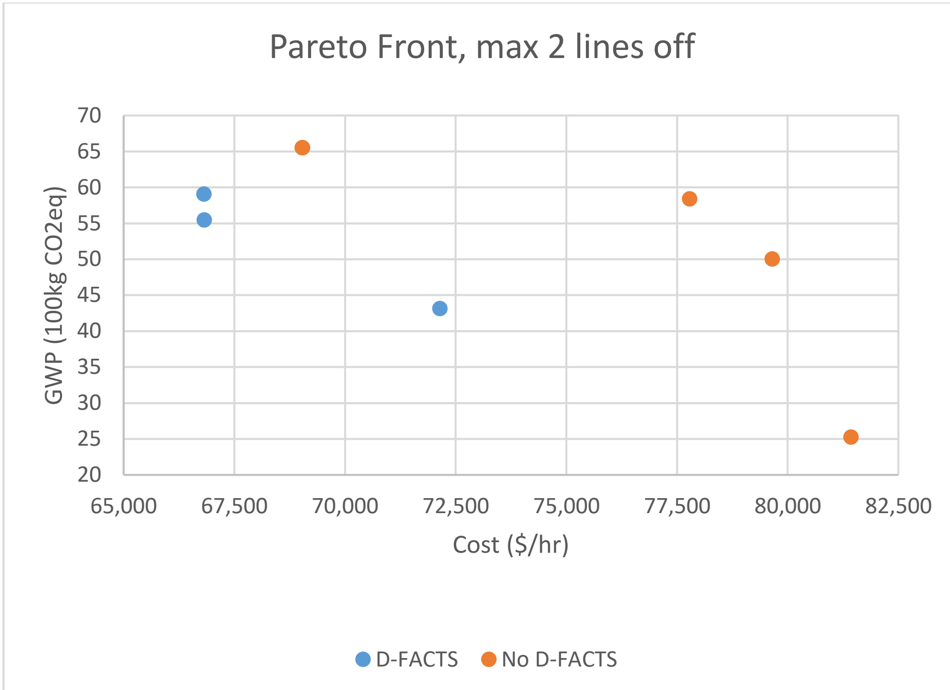

Figure 7.2. Pareto front of solutions with $z^{lim} = 2$

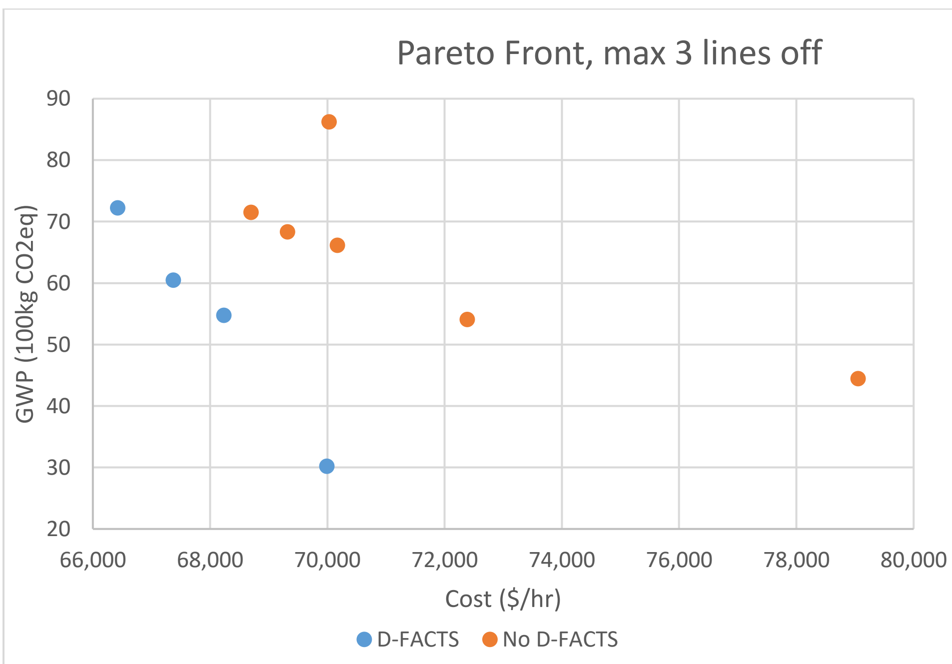

Figure 7.3. Pareto front of solutions with $z^{lim} = 3$

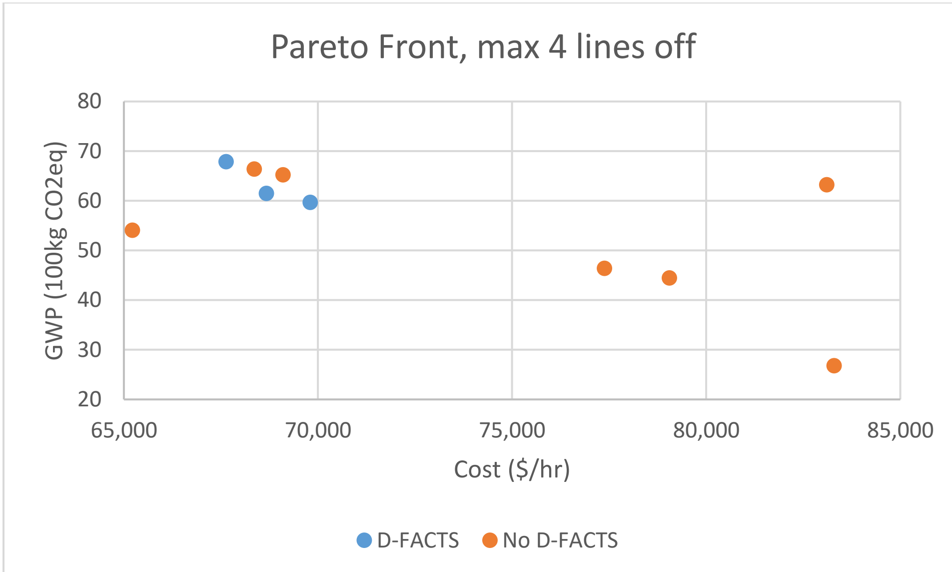

Figure 7.4. Pareto front of solutions with $z^{lim} = 4$

As can be seen in figures 7.1-7.4, the inclusion of D-FACTS generally does improve the system operating conditions even further than simply using line-switching protocols. It must be mentioned, however, that the use of D-FACTS can bring about logistical issues regarding deployment of devices, as well as installation time and costs, which are not issues that would arise with a pure line-switching approach to congestion relief. However, D-FACTS can bring about improved and more precise flow control, particularly over more congested networks where line switching is less feasible.

In effect, line switching is a very powerful tool for congestion relief, but its effects are limited precisely by how rudimentary of a technique it is. In combination with D-FACTS, however, it can become an even more useful tool for precise power flow control and together both can have the ability to improve electric transmission systems by reducing costs and emissions as well as increasing the penetration of renewable energies into the grid.

Chapter 8: Conclusions and Future Work

8.1. CONCLUSIONS

The main objective of this work is to analyze the D-FACTS allocation problem using a modified version of the IEEE RTS-96 test system in order to show the cost and environmental benefits of installing DSI-type devices throughout the transmission system in order to best serve the system loads and optimize resource utilization. For this, a new multi-objective evolutionary algorithm was developed in order to efficiently and effectively allocate D-FACTS devices. Stochasticity is considered in the algorithm in the form of various scenarios which model distinctive behaviors for both the system loads and the renewable energy generation. As the system was studied as a multiple objective optimization problem, a Pareto-optimal set of solutions was created and presented, to be further analyzed in further studies.

In order to demonstrate the effectiveness of the algorithm on transmission networks, the D-FACTS allocation problem was presented with a new formulation, with some equations combined or reduced to better adapt to the algorithm's implementation. This formulation simplifies previous attempts at optimizing D-FACTS allocation thanks to the properties of metaheuristic algorithms: by having the EA randomly generate allocations, and by pre-calculating the flow directions on a reduced LP, many of the nonlinearities that arose in previous models were converted back into linear equations, reducing the number of variables needed to be solved as well as the number of equalities and inequalities needed to be balanced.

The results showed, as expected, some level of conflict between various objectives, as well as some synergies between others. For example, there is a direct relation between the total system cost and the curtailment of renewable energy, as renewable energies have no associated generation cost once the generators have been installed and would thus reduce the total cost of the system by satisfying loads, if the transmission lines were capable of transmitting all the power. However, transmission capacity limits, storage, and reliability are usually the biggest limitations for the integration of renewable energy into existing power grids, and thus it is not possible to fully utilize these resources without larger infrastructure investments.

As shown, the results indicate that the implementation of D-FACTS devices does have significant benefits to both network sustainability and generational dispatch costs, which both would result in both short- and long-term economic benefits to utility companies, their subsidiaries, and their customers.

In addition to this, various articles were presented in which this author expands upon the research on D-FACTS devices, by performing a sensitivity analysis in which the most important factors for further reducing costs were found to be the investment limits and the number of lines in which the devices were allowed to be installed. Additionally, various secondary objectives were proposed and analyzed, as well as the incorporation of other congestion-reducing protocols such as line switching. In the studies, it was generally found that the deployment of D-FACTS devices on transmission grids has positive effects even in (and even more in) conjunction with other protocols and technologies thanks to their flexibility and versatility in power flow control applications.

8.2. FUTURE WORK

Significant work remains to be done under this research, both in the analysis of benefits on the implementation of D-FACTS devices and in the analysis of the algorithm's applicability in other networks.

For starters, other elements in the network could have associated probabilities for reliability. Some data in the RTS-96 system includes generator reliability. For further analysis of the benefits of D-FACTS devices, new scenarios can be created in which some generator or line failures may occur, to not only expand the scenario pool, but also study how and if the use of D-FACTS devices can help mitigate outages and improve power systems reliability metrics such as expected energy not supplied (EENS) or others.

To guarantee the efficacy of the algorithm, it must be tested in other case studies and compare the results with existing benchmarks. For this purpose, data has been acquired for a 2,000 bus system, on which the algorithm will be tested in future studies for the purpose of both validating the algorithm and also once again showing the benefits of D-FACTS devices on transmission networks. In addition to this system, updates have been proposed to the IEEE Reliability Test System which was used in the current case study. The publication by Barrows *et al.* (2020) delineates a new update to the 1996 system, updating some line and generator capacities and bus loads for more updated simulations of energy demand, removing outdated generator types, and including new generator types and renewable energy systems not accounted for in the older version. While this new RTS has not been yet officially adopted, it could still prove very beneficial as a tool to demonstrate the applicability of D-FACTS devices in more modern grids with more renewable generation and heavier demand loads. However, these test systems need modifications in order to simulate congestion, and are thus not suitable for D-FACTS allocation in their current state. Further work needs to be done to create congested test systems before the algorithm can be implemented and tested. At the current time, while the algorithm is capable of running with these new data sets, the obtained results are not useful for studies, as there are no benefits for the implementation of D-FACTS devices in a non-congested system.

Finally, other objectives such as social impacts or power system reliability metrics could be added as objectives to optimize, and other objectives, or a combination of them, could be used for the main objective function in the LP, in order to further diversify the solution process. More research is required to see how social impacts can be measured in this situation and whether prioritizing a different objective would be a realistic or feasible situation in industry. In addition, studies on health impacts have been submitted for review.

Other than testing the functionality of the algorithm, other studies are currently underway on testing marginal costs and emissions on congested systems before and after the implementation of D-FACTS into a system based on the El Paso, TX distribution network.

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Vita

Eduardo Jose Castillo Fatule was born in Santo Domingo, Dominican Republic. He coursed his education there and started a bachelor's in Industrial Engineering at the Instituto Tecnologico de Santo Domingo (INTEC) before transferring to the University of Texas at El Paso to complete his degree, which he did in 2016. Following this, he also completed a Master's degree also in Industrial Engineering at UTEP in 2019, and a Master's in Computational Science in 2021. He is currently working towards a Ph.D. degree in Computational Science at UTEP.

Eduardo has also been working as a Research Assistant and Teaching assistant at UTEP since 2017, and has presented research at the IISE 2015 and 2016 annual conferences, at the IEOM 2015 conference, where he got 2nd place in the graduate student paper competition. He has also presented his research at the 2019 INFORMS conference, and at the 2020, 2021, and 2022 North American Power Symposium, and the 2022 IEOM conference.

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