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A Mixed Method Approach To Explore Lecturers' Pedagogical And Content Knowledge, Students' Perspectives And Learning Experiences In Mathematics For Social Science Course

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A MIXED METHOD APPROACH TO EXPLORE LECTURERS' PEDAGOGICAL AND CONTENT KNOWLEDGE, STUDENTS' PERSPECTIVES AND LEARNING EXPERIENCES IN MATHEMATICS FOR SOCIAL SCIENCE COURSE

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Dedication

Para mi familia, mis padres Julio Solis e Irma Arratia, mis hermanas Lizeth, Ivonne, Janeth y Karen, mis sobrinos y cuñados. Gracias a ustedes he logrado todo lo que soy. Estoy inmensamente agradecido con Dios por darme la oportunidad de contar con su apoyo. Gracias infinitamente, les amo.

I dedicate my dissertation and work to those who supported me in this journey. Thanks for being with me in this adventure through to the end. A special feeling of gratitude for my family. To my parents, Julio Solis, and Irma Arratia, thank you for your love and inspiration, ¡los amo! To my sisters Lizeth, Irma, Janeth, and Karen, who have never left me alone. My brothers-in-law, nephews and nieces, I love you all.

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Abstract

Research indicates that institutions and teachers' pedagogical and content knowledge are essential to student learning and achievement. Nevertheless, teachers' knowledge and practices have been researched primarily in K-12th grades. Therefore, there is a gap in understanding teaching practices in higher education. Moreover, research has forgotten about exploring math classes for non-STEM majors. With this in mind, this research investigates how students perceive math professors' pedagogical and content knowledge and their effect on students' learning experiences in the context of mathematics for social science classes. To achieve this, this study takes a mixed-methods approach in its use of an observation protocol, a survey, interviews, and the framework of mathematics teacher specialized knowledge (MTSK) by Carillo-Yañez et al. (2018) as a guide to understanding mathematics lecturers/professors' knowledge.

Keywords*: Pedagogical content knowledge, Mathematics teacher specialized knowledge, higher education, mathematics for social sciences.*

Table of Contents

List of Tables

Table of Figures

Chapter 1 Introduction

The present study emerged from the need to close the gap in understanding teaching practices in higher education, particularly in mathematics for social science. It is intended to explore professors' and lecturers' pedagogical content knowledge and how students perceive and benefit from this knowledge. The results of this examination will inform universities' understanding of their faculty teaching practices and, consequently, increase student learning, satisfaction, and graduation rates.

1.1 Introduction

Currently, there is an increase in researching how to improve and understand students' experiences in higher education. In particular, research has focused on exploring the quality of teaching, pedagogical practices, and their impact on students' success and retention (e.g., Hativa et al., 2001; McCoy & Byrne, 2017; Thomas, 2002). However, there is a gap in understanding this in college mathematics, particularly in mathematics for social sciences. This is reflected in the lack of existing research exploring non-STEM college classes (classes different from science, technology, engineering, and mathematics).

Indeed, most existing research on college mathematics has mainly focused on understanding student success in classes like college algebra or calculus (Ichinose & Clinkenbeard, 2016). Among the findings, research has demonstrated that women and minorities, as explained by Bressoud (2011), are more prone to not continue with their mathematics classes beyond Calculus I, even when that represents a requirement for their degree.

In general, many factors contribute to student achievement which show that teachers or professors are a crucial ingredient in college students' success. As Smith and Esch (2002) argue, teachers' knowledge directly and positively affects classroom practice and student achievement.

Nonetheless, in college, having a background in education is not a requirement to teach. Therefore, there are some instructors that ignore what good pedagogical practices are. For example, Jang et al. (2009) concluded that students recognize their mathematical content knowledge, but also, they consider that their pedagogical content knowledge needed improvement.

Then, research should find ways to understand students' needs and teaching practices in mathematics for non-STEM majors. This is important because as described by Hartmann and Sprenger (2011), the importance of mathematics and statistics in social sciences increased with the introduction and application of quantitative and mixed research methods in social contexts. In fact, "almost all social sciences rely on statistical methods to analyze data and to form hypotheses, and almost all of them use (to a greater or lesser extend) a range of mathematical methods to help us understand the social world" (Hartmann & Sprenger, 2010, p. 1).

As a consequence, Hartmann and Sprenger (2011) argue that different sub-disciplines like mathematical psychology and mathematical sociology emerged. Moreover, the development of supercomputers and modeling boosted the importance of numerical simulations of complex social phenomena. Therefore, different mathematical models and methods applied to social problems like game theory (developed by John von Neuman and Oskar Morgenstern in 1944), decision theory (introduced by Leonard Savage in 1954), and social choice theory (by Gaertner in 2006), among many others, were developed.

Then, understanding social phenomena involves much more than just using qualitative methods. It includes a depth comprehension of other procedures that might consist of mathematical processes and definitions. And the only way to properly use them is to be exposed to this mathematics that provides the necessary tools.

Therefore, there is a need for social science students to properly learn and understand mathematics. And for this reason, most of the universities' social science programs around the world require students to take at least one mathematics class (Brezavšček et al., 2020). Unfortunately, statistics show a high failure rate in mathematics classes in most non-STEM programs (Awaludin, 2015). Furthermore, mathematics has been identified as one of the first barriers encountered by students in college and, consequently, a reason for dropping out of college (Gradwohl & Eichler, 2018).

For this reason, research must be conducted in the classroom of mathematics for social sciences. However, first, it is necessary to describe what the class of mathematics for social science encompasses. According to the participating schools' websites, the topics covered in this class include linear, quadratic, exponential, and logarithmic equations, systems of equations and inequalities, matrix algebra, and the applications of mathematical finance and probabilities.

Thus, this class offers a wide spread of mathematical definitions and practices like any other STEM course. It covers the content of a regular mathematics class, but the audience of delivery is directed towards students who are not pursuing a STEM career.

1.2 Definitions

To properly understand the purpose of this research, it is needed to define the concepts of content knowledge and pedagogical content knowledge. In 1986, Lee Shulman stressed the importance of content knowledge and its components. These components are subject matter knowledge, pedagogical content knowledge, and curricular knowledge. Shulman defined content knowledge as "the amount and organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p. 9). Similarly, pedagogical content knowledge or PCK can be defined as integrating the content knowledge (CK) and the knowledge of pedagogical practices. This knowledge enables teachers to choose proper ways of presenting the material and examples. Also, this knowledge helps teachers comprehend which topics are difficult to learn and why this happens.

1.3 Purpose and Research Questions

The present study focuses on exploring mathematics for social sciences lecturers' content and pedagogical content knowledge and their effect on students' experiences and perceptions of their learning process. As mentioned before, research traditionally explores mathematics classes for STEM majors, forgetting about other mathematics courses that are an important milestone in the academic life of many non-STEM major students. Moreover, since teacher knowledge research has been conducted primarily in K-12th grades (Speer et al., 2010)., the field of lecturers' pedagogical content knowledge in college settings needs improvement.

With this in mind, this research uses the framework of Mathematics Teacher Specialized Knowledge (MTSK) (Carrillo-Yanez et al., 2018) as a guide to understand lecturers' knowledge. This framework, as will be further described in chapter 2, offers the unique feature and opportunity of exploring pedagogical content knowledge in terms of mathematics itself. This allows researchers to explore teachers' knowledge independently of the grade level, eliminating the specialized knowledge barrier described in other models (e.g., mathematical knowledge for teaching (MKT) (Ball et al., 2005)).

Then, this research is guided by the following research questions:

- What type of mathematics teacher specialized knowledge and teaching practices of the participating lecturers are observed in a mathematics for social science class?
- How do students perceive the participating lecturers' pedagogical content knowledge, and how does it affect their learning experiences?

1.4 Need for the Present Study

Benken et al. (2015) argue that “initial success in mathematics has the potential to provide students with ‘early momentum,’ that would contribute to their overall success in college” (p. 14). However, courses related to STEM, particularly to mathematics, are perceived by the non-STEM students as one of the most difficult courses (Whalen & Shelley, 2010). Thus, for many non-STEM students, mathematics for social sciences is one of the first challenges they face in higher education. For this reason, it is important that research explores how this class is affecting non-STEM students’ learning experiences.

In addition, there is a constant perception that college students are entirely responsible for their success (Thomas, 2002). For this reason, this research emerged to understand students’ needs, to improve student learning and passing rate in college mathematics. But, fixing this is not an easy task since multiple variables and factors contribute to students’ success. As Thomas (2002) described, various factors affect student continuation and persistence in post-secondary education, where the aspect of students’ academic experiences stands out.

Thus, the present study's primary objective is to explore how students perceive professors’ pedagogical knowledge practices in the mathematics class for social sciences. To do this, the present research is developed by using a mixed-method approach, and tools like a protocol to observe college classes, a survey to explore college students’ perspectives on pedagogical knowledge, and interviews were used as data sources. To further understand the data, findings across data sources were triangulated to connect the findings.

1.5 Overview of Subsequent Chapters

Chapter 2 will provide further information on how research has approached understanding college teaching practices. This chapter describes the different teachers’ knowledge models, particularly the mathematics teacher specialized knowledge (MTSK) (Carillo-Yanez et al., 2018). When exploring this, the existing gap is identified, justifying the need for this research.

Chapter 3 presents the description of the methodology used to explore this topic. It further describes the location, participants, methodology, and characteristics of the tools implemented in the data collection process. Furthermore, it depicts the different phases that constituted the development of this research and the analysis used to explore the data.

Next, Chapter 4 discusses the overall findings of each of the data methods. It describes how the instructors spent their time during the lecture. Moreover, it explores the averages and standard deviations obtained by each question and section for each of the lectures. Finally, it describes the codes that emerged from both students and instructors' interviews. It also connects the findings of each of the data sources to respond to the two research questions. First the data obtained from the observations is primarily used to answer the first research question. Thereafter, the data obtained from the interviews and the survey are mainly used to answer the second research question.

Finally, Chapter 5 discusses how the obtained data is related to the existing literature. Also, it describes the suggestions as well as the limitations of this research. Lastly, it provides an overall conclusion.

Chapter 2 Theoretical Framework and Literature Review

2.1 Students' Experiences in College Mathematics

Mccoy and Byrne (2017) explain that, as mentioned in the National Strategy for Higher Education to 2030 policy, a higher education institution is successful when students have a favorable progression and performance. However, this is a continuous challenge to higher education institutions. For instance, in some states, about 54% of first-year college students do not complete their classes. What is worse, there is only a 34% persistence rate of degree completion (Fike, 2008), where minorities have a higher risk of withdrawing (Quinn, 2013).

This phenomenon has been widely studied, particularly in mathematics for STEM majors. For example, classes for STEM majors like college algebra or calculus have a high percentage of failing students (Ichinose & Clinkenbeard, 2016; Worthley et al., 2016). Nevertheless, research has not focused on investigating what is happening in those mathematics classes for students who do not intend to pursue a career in STEM.

Even though mathematics might not be the basis of many non-STEM careers, as explained by the National Science Board (2006), students "even those students who do not pursue professional occupations in technological fields will also require solid foundations in science and mathematics in order to be productive and capable members of our Nation's society" (p. 2). For this reason, different national agencies, policymakers, and corporate leaders are interested in finding ways to help students to have a solid foundation in the principles of science, technology, and mathematics. This is a necessity for a constantly technologically evolving society (Jin & Bierma, 2013).

Particularly in mathematics for social sciences classes, Brezavšček, et al. (2020) discovered that students' achievement is closely connected to (1) attitude towards mathematics and math anxiety; (2) engagement in learning activities; (3) attitude towards involving technology in learning mathematics. Then, to help students taking mathematics for social sciences courses, we need to understand their attitudes towards mathematics, since according to research, they play a crucial role in their academic success (Popham, 2005). Understanding why students reject learning mathematics or taking mathematics courses could improve mathematics literacy. For this reason, research has explored the factors that influence college mathematics students' attitudes. For example, Goodykoontz (2008) explains that four main factors contribute to success in college mathematics classes: the teacher, teaching style, classroom environment, and assessment and achievement.

Similarly, different researchers have focused on exploring students' needs, and consequently their completion and retention rates in higher education (e.g., Longden 2004; Thomas, 2002). Among the findings, research suggests that various personal, social, and institutional reasons influence students' performance (Thomas, 2002).

Particularly, in the Interactionalist Theory developed by Tintos' (1987), he considers that students progress in their education in stages as they mature. However, these stages are influenced by integrating, formally and informally, their academic and social lives. That is, students' academic performance refers to how they interact with faculty and staff, extracurricular activities, and peer interactions, among others.

In other words, in this theory, Tinto explains that the key to helping students is the integration of students' interests, and goals, satisfaction of the course, and social experiences. To do this, he highlights five conditions: (1) high expectations of students; (2) academic support like counseling; (3) social and academic support; (4) making students feel that they are important and valuable to the school; and (5) fostering and active learning environment.

These conclusions are similar to other models. For example, Thomas (2002) concluded that among a range of students varying in age, gender, classification, and major, there are seven predictors of students' retention rate. These are (1) academic preparedness; (2) students' academic experiences; (3) students' expectations and their commitment from the institution; (4) social experiences; (5) financial and employment situations; (6) family support; and (7) the different services offered by the university.

First, the factor of (1) academic preparedness for college is related to students' readiness for a higher education institution. Then, the (2) students' academic experiences include curricula, issues about teaching and learning, accessibility and relationship with staff and faculty, and flexibility (related to deadlines and timetables). Next, (3) students' expectations and their commitment from the institution refer to what students expect from the institution and its commitment to it. Moreover, the next factor (4) is social integration or "academic and social match," which describes the social aspect and its effect on students' academic life. It also explains that "the fit between the individual's and the institution's characteristics strongly influence the student's goal commitment" (Thomas, 2002, p. 427).

Another significant aspect is (5) the students' financial and employment situation. Universities, in particular, have grown their interest in this factor in recent years. One of the main concerns is that many students with economic problems are withdrawing from universities (Thomas, 2002) since many students need at least a part-time job to survive. Similarly, (6) the support of the family plays a transcendental role in supporting students' decisions and motivation. This factor is particularly influential in those institutions with students from first-generation college families, where students usually have less support in their career. And finally,

(7) the different services provided by the university to support students' success have a crucial role in students' success.

Nevertheless, higher education institutions can only provide support to some extent to the previous factors. That is, universities have no control over variables like family support or students' financial situation. However, Webb and Cotton (2018) argued that colleges have more opportunities to control other factors that are key to students' success, such as students' academic experiences.

In fact, in a study similar to Goodykoontz (2008), Charlton, Barro, and Hornby-Atkinson (2006) concluded that teaching styles, quality, and learning culture could be related to withdrawal rate. Similarly, Thomas (2002) also explained that some "institutional habitus" encourage students to persist and succeed in their education. Indeed, she explains that students need professors who are open and willing to generate a relationship that minimizes the "social and academic distance between them" (p. 439). This can make students feel more appreciated, boost their confidence, and feel more confident to seek help when needed. Furthermore, students expect the following skills from the professors: to have a variety of teaching techniques different from the traditional methods, to consider their previous knowledge and experiences; to implement different types of activities that promote social interaction, and to not assume that all of them have the same access, time, and resources.

Thus, teaching techniques, classroom environment, teaching styles, and consideration of the students' knowledge, needs, and resources, are some of the elements that might boost and modify students' attitudes towards mathematics. In other words, students require their professors to have and demonstrate solid pedagogical content knowledge. This knowledge is recognized as the knowledge that allows teachers to know "how best to represent and formulate the subject to make it comprehensible to others, as well as knowledge on students' subject-specific conceptions and misconceptions" (Krauss et al., 2008). And for this reason, exploring this type of teacher knowledge is crucial.

For example, in a study conducted by Jang et al. (2009), they analyzed student perceptions of the pedagogical knowledge of their professors. According to students' perceptions, they concluded that most teachers did not consider their students' learning difficulties and prior knowledge, which became the weak point of college professors' teaching. Furthermore, concerning the teaching strategy, college teachers adopted an inefficient teaching approach, which couldn't stimulate students' interest in learning certain subject matter. This was happening, as perceived by the students' comments, because of teachers' subjective attitude, their explanatory lecture in class, and the speedy procession of the curriculum, which couldn't match up with students' learning status (Jang et al., 2009).

In a similar manner, Hativa (2000) found in a study that there is a difference between professors' own perceptions compared to students' experiences. In other words, even though professors considered themselves successful teachers, and that they usually accomplish their teaching goals, students felt that their teaching style was not effective. Similarly, professors thought themselves to be effective teachers with strong pedagogical knowledge that is properly implemented during the lecture; however, according to students, the way the content was delivered was not straightforward, dull, and generally ineffective.

Thus, there is incongruence between what students and professors perceive. This incongruence indicates that research must be done to explore this critical aspect of higher education. However, despite the robust literature on teaching practices in K-12th education, there is little research conducted in higher education regarding professors' teaching habits and content knowledge (Speer et al., 2010).

With this in mind, the scope of this research is to explore mathematics lecturers' pedagogical content knowledge, teaching practices, and their impact on student learning experiences. This was investigated in the context of mathematics for social science, a class that has not been much analyzed by research. And to further understand teacher knowledge, the framework of Mathematics Teachers' Specialised Knowledge, MTSK (Carrillo-Yañez et al., 2018), served as a guide.

2.2 Models Used to Understand Pedagogical Content knowledge

In 1986, Lee Shulman focused on understanding teacher knowledge. In his research he proposed that teacher knowledge is composed of subject matter knowledge, pedagogical content knowledge, and curricular knowledge. Then, in this article, two concepts stand out: content knowledge and pedagogical content knowledge.

Content knowledge can be described as "the amount and organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p. 9). Similarly, pedagogical content knowledge (PCK) is the knowledge that allows teachers to choose proper pedagogical practices to teach the content. Also, this knowledge provides the instructor with the ability to use proper examples and representations that could boost student learning. Moreover, PCK also helps teachers to comprehend the difficulties of learning mathematics.

Different authors have analyzed teachers' knowledge in different but similar ways. For example, Grossman in 1990 defined pedagogical content knowledge as the combination of content knowledge, pedagogical knowledge, and knowledge of context. Similarly, Hill, Ball, and Schilling (2008) proposed that there exists mathematical knowledge for teaching. This knowledge should not be limited to the mathematical knowledge that a person might possess or use in different professions, but also as "the subject matter knowledge that supports that teaching, for example, why and how specific mathematical procedures work, how best to define a mathematical term for a particular grade level, and the types of errors students are likely to make with particular content" (Hill et al., 2008).

Nevertheless, all the previous frameworks have been used mainly in understanding teachers' knowledge from K-12th grades. The reason for this could be because there is a perception that the K-12th teachers' specialized knowledge is different from professors' specialized knowledge (Speer, Smith & Horvath, 2010). For this reason, only a few articles have focused on analyzing professors' knowledge using Shulman's framework (e.g., Fernandez-Balboa & Stiehl, 1995; Khakbaz, 2014).

2.2.1 PCK and MKT in Mathematics Higher Education

As mentioned before, both PCK and MKT have been used mainly to explore K-12th teachers' pedagogical and content knowledge, their impact on students learning, teachers' perceptions, students' perceptions (e.g., Krauss et al., 2008; Hill et al., 2008). In addition, research has also been used in exploring pre-service teachers to develop tools to measure PCK, among others. However, only a few researchers have used the PCK or MKT frameworks to explore professors' knowledge (e.g., Major and Palmer, 2006; Pinsky, 2013).

For example, some researchers ventured in exploring professors' knowledge by using either PCK (e.g., Major and Palmer, 2006), or MKT (e.g., Pinsky, 2013), and even a combination of both PCK and MK (e.g., Khakbaz, 2014). Interestingly, in all three (Khakbaz, 2014; Major and Palmer, 2006; Pinsky, 2013), they highlighted the importance of considering the university's culture in teaching and professors' teaching perceptions. As Pinsky explains, "regardless of the content knowledge, [MKT] is shaped in large strokes by the goals of the teacher, department, school system, and nation in which it is located, and these goals, and teaching is influenced by the values of the ambient society” (p 41).

Similarly, the knowledge about students is highlighted by different authors. For example, Major and Palmer (2006) suggest that PCK comprises “knowledge of learners, knowledge of subject matter, previous experiences, ideas about pedagogical practice, and contextual cues in a dynamic, iterative process and that can be supported and encouraged through institutional intervention” (p 645). This is supported by Khakbaz (2014), who explained that one of the themes found during the interviews was knowledge about students.

As we can see, there is an emerging effort to understand teaching practices in higher education, and as Biza et al. (2016) argue, there is an increase in exploring university teachers’ knowledge, understanding what knowledge is, and how it develops. For this reason, a model that contributes to comprehending such paradigms is needed. Fortunately, the model proposed by Carrillo-Yanez et al. (2018) to explore mathematics professors’ knowledge offers a reliable framework that could be used to research such an important topic. This framework proposes that the mathematics teachers’ knowledge can be divided into two main areas: mathematical knowledge (MK) and pedagogical content knowledge (PCK).

2.2.2 The Mathematics Teacher’s Specialized Knowledge Model

As described by Carrillo-Yañez et al. (2018), MTSK is mainly influenced by two models. First, Shulman’s (1986) conceptualization of teachers’ knowledge including subject matter knowledge, Pedagogical Content Knowledge, and Curricular Knowledge. Second, the model proposed by Ball et al. (2008), Mathematical Knowledge for Teaching (MKT) defined as the “mathematical knowledge needed to carry out the work of teaching mathematics” (Bell et al., 2008, p. 395).

According to MKT, teachers’ knowledge can be divided into Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK). In this model, one particular subdomain stands out: Specialized Content Knowledge (SCK). This subdomain indicates that there is a type of knowledge that belongs to teachers. In other words, SCK states that teaching necessitates particular expertise that other professions do not need. Here, it is important to remark that the unit of analysis “is not the mathematical knowledge used by teachers to carry out their work, but rather the assessment of the mathematical knowledge needed to do so.” (Carrillo-Yañez et al., 2018, p. 238).

Then, as explained by Carrillo-Yañez et al., MTSK is a model with a supposition that teaching practices require a unique type of knowledge, including preparing for classes, planning the lesson's structure and the strategies, providing the lesson, and reflecting on the outcome of them. In particular, MTSK focuses on mathematics teaching necessary knowledge, including “meanings, the properties and definitions of particular topics, the means of building an understanding of the subject, connections between content items, knowledge of teaching mathematics, and characteristics associated with learning mathematics, among others” (Carrillo-Yañez, 2018, p. 239).

Moreover, the Mathematics Teachers’ Specialised Knowledge offers the opportunity to explore professor knowledge because, compared to other schemes, this framework analyzes teachers’ knowledge in terms of mathematics, independent of the grade level. In other methods, there is a constant perception that teachers differ in specialized knowledge from a professor;

however, MTSK perceives this specialization knowledge as a process of “becoming” instead of “being” (Zakaryan & Ribeiro, 2018).

Furthermore, since mathematics for social sciences provided the context of this research, MTSK frameworks fit since it tends to compare mathematics teachers’ knowledge to other careers where mathematics is used and applied (e.g., economics or statistics) and to other subjects different from mathematics. Then, as explained by Zakaryan and Ribeiro (2018):

> From this perspective, the analysis of the teacher’s knowledge entails better understanding of the epistemological roots of mathematics, the phenomena from which they emerge, their relations with other mathematical objects, the structure that these relationships allow to build, or the way in which these mathematical entities are generated (p. 26).

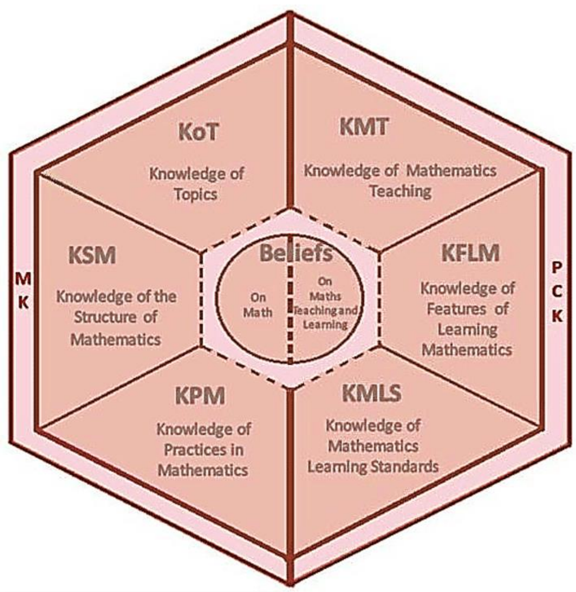

Figure 1. The Mathematics Teacher's Specialized Knowledge Model (Carrillo-Yañez et al., 2018).

Then, Carrillo-Yañez et al. proposed that MTSK can be divided into two main categories. First, Mathematical Knowledge (MK), with three subdomains, Knowledge of topics (KoT), Knowledge of the Structure of Mathematics (KSM), and Knowledge of Practices in Mathematics (KPM). And second, Pedagogical Content Knowledge with subdomains Knowledge of Mathematics teaching (KMT), Knowledge of Features of Learning Mathematics (KFLM), and Knowledge of Practices in Mathematics (KMLS). Finally, at the center of both MK and PCK is the beliefs on mathematics and mathematics teaching and learning (see Figure 1).

Mathematical Knowledge (MK). Mathematical Knowledge is one of the two main elements of the MTSK model. For Carrillo-Yañez et al. (2018), mathematics can be seen as intertwined connections of knowledge with specific rules and properties. The understanding of these connections allows teachers to teach mathematical content logically and coherently. This is supported by Loewenberg Ball, Thames, and Phelps (2008), who claim that subject content knowledge is a necessary condition to teach.

However, some models like the MKT propose a distinction between common knowledge and specialized knowledge in the subject content knowledge domain. Differentiating these domains becomes more complex in higher education because there is no clear distinction between what can be called common or specialized knowledge. For this reason, using frameworks like MKT to explore professors' knowledge becomes challenging in higher-level classes. Fortunately, MTSK overcomes this situation by "defining MK sub-domains in terms of mathematics itself (topics, connections, ways of proceeding), such that inclusion of items is independent of the level the teacher is working at" (Carrillo-Yañez et al., 2018).

Knowledge of Topics (KoT). To understand this subcategory, it is necessary first to explain that the term *topic* refers to the material covered within the content class. As explained in

Carrillo-Yañez et al. (2018), KoT comprises the *"what," "in what why,"* teachers know the content they teach. This knowledge includes concepts, procedures, facts, rules, theorems, properties, principles, definitions, methods, connections, and meanings. Additionally, this type of knowledge enables teachers to select appropriate examples with appropriate contexts and purposes. Also, KoT helps them to be aware of the use and applications of mathematics and how they can be represented. Finally, this subcategory includes what students expect to learn.

Knowledge of the Structure of Mathematics (KSM). In this subdomain, the understanding of the relation among different mathematical concepts and definitions is included. It involves the comprehension of increasing the complexity to delineate the definition of a similar mathematical concept. For example, as described by Carrillo et al., a student in kindergarten might compare two objects in size. This idea will later develop concepts of magnitude, which later can be transformed to analyze proportionality (Carrillo et al. 2018).

Similarly, this knowledge enables teachers to understand the connections of simplification (e.g., using natural numbers to explain the simplification of algebraic expressions). In addition, KSM includes the knowledge of auxiliary connections, in other words, the use of auxiliary elements to explain other topics. For example, the use of trigonometric functions to find the derivatives of certain functions. Finally, KSM includes transverse connections, which is when the teachers understand the connection between different content items. Like the connection among the concepts of root, factor, and solution. It is important to remark that auxiliary connections and connections associated with simplification are an explicit contribution of the MTSK model (Carrillo et al. 2018).

Knowledge of Practices in Mathematics (KPM). In this knowledge, the role of counterexamples stands out. To put it differently, teachers with proper KPM know when and how to use counterexamples, their definition, making deductions and inductions. Also, KPM refers to the way mathematics is produced; therefore, it also implies understanding how new mathematical knowledge is developed, allowing teachers to accept or refute students' reasoning. It is important to remark that Carrillo et al. mention that this knowledge is still under study.

Pedagogical Content Knowledge (PCK). In contrast to Shulman's PCK, MTSK considers that PCK is only related to mathematics itself. In other words, PCK is "that knowledge in which the mathematical content determines the teaching and learning which takes place" (Carrillo-Yañez et al., 2018). In this domain, the Knowledge of Mathematics Teaching (KMT), Knowledge of Features of Learning Mathematics (KFLM), and Knowledge of Mathematics Learning Standards (KMLS) are included.

Knowledge of Mathematics Teaching (KMT). This knowledge is intertwined with MK and mathematics education theories. KMT allows teachers to choose appropriate activities, examples, representations of the material, and techniques for teaching mathematics, including the knowledge of proper teaching instruments like textbooks, different types of resources, and technology. It also enables teachers to understand the limitations and obstacles.

Knowledge of Features of Learning Mathematics (KFLM). KFLM relates to the knowledge of learning mathematics where the emphasis is the mathematical content and not the learner. To develop this knowledge, Carrillo-Yañez et al. claim that the main influence is the experience. With this in mind, KFLM relates to the teachers' consciousness of how:

> Students think and construct knowledge when tackling mathematical activities and tasks. It includes understanding the process pupils must go through to get to grips with different content items, and the features peculiar to each item which might offer learning advantages or, conversely, present difficulties…takes account of the teacher's knowledge

about their student's manner of reasoning and proceeding in mathematics (in particular their error areas of difficulty and misconceptions), which informs his or her interpretation of their output (Carrillo-Yañez et al, 2018, p. 246).

In addition, teachers' KFLM allows them to understand students' learning styles and foresight topics that will be difficult for them to understand and anticipate students' strengths. Therefore, it incorporates the knowledge of students' techniques and approaches used by students to learn mathematics. For this reason, it also includes emotional aspects like students' anxiety, motivations, interests, and expectations (Carrillo-Yañez et al, 2018).

Knowledge of Mathematics Learning Standards (KMLS). KMLS refers to the knowledge about the content needed to be covered at any particular level. Therefore, teachers with KMLS know how to choose the topics that allow students to learn and understand that specific topic, which is also related to the knowledge of the structure and order of mathematical topics. Finally, by understanding the sequence of topics and their content, professors with KMLS will know how to design proper instruments that measure students' comprehension, application, and construction of mathematical knowledge.

The Mathematics Teachers' Specialised Knowledge offers the opportunity to explore professor knowledge because, compared to other schemes, this framework analyzes teachers' knowledge in terms of mathematics, independent of the grade level. In other methods, there is a constant perception that teachers differ in specialized knowledge from a professor; however, MTSK perceives this specialization knowledge as a process of "becoming" instead of "being" (Zakaryan & Ribeiro, 2018).

Furthermore, since mathematics for social sciences provided the context of this research, MTSK frameworks fit since it tends to compare mathematics teachers' knowledge to other careers where mathematics is used and applied (e.g., economics or statistics) and to other subjects different from mathematics. Then, as explained by Zakaryan and Ribeiro (2018):

> From this perspective, the analysis of the teacher's knowledge entails better understanding of the epistemological roots of mathematics, the phenomena from which they emerge, their relations with other mathematical objects, the structure that these relationships allow to build, or the way in which these mathematical entities are generated (p. 26).

2.3 Using MTSK to Explore Teachers' Knowledge

Because MTSK is a relatively new framework, only a few researchers have used MTSK to explore teachers' knowledge. For example, MTSK has been used in different grade levels from K-12th (e.g., Montes & Carrillo, 2017; Munoz Catalan et al., 2018) and in higher education (Delgado-Rebolledo & Zakaryan, 2018, 2020; Vasco & Cliement, 2018). Particularly in higher education, MTSK has been used by researchers to understand and explore professors' knowledge.

2.3.1 MTSK in K-12th Grades

Even though the use of MTSK in K-12th grades is not the scope of this analysis, it could be important to explain how it has been used in different contexts. For example, in Montes and Carrillo (2017) and Munoz Catalan et al. (2018), there is an agreement in the necessity of understanding how teachers get to know what enables teacher knowledge and what they need to do to teach mathematics properly. Also, both documents highlight the importance of helping teachers to reflect on their work. By doing this, teachers can develop a consciousness of models, concepts, students' learning styles, students' difficulties in learning different subjects and concepts, and relationships between different mathematical concepts. Lastly, in both analyses, the authors explain the importance of having a solid mathematical knowledge that is clearly reflected during the lecture.

2.3.2 MTSK in Higher Education

As described before, MTSK offers the possibility of analyzing mathematics professors' knowledge by exploring each subdomain in terms of mathematics itself. This eliminates the specialized knowledge barrier that is described in different teachers' knowledge models. However, since MTSK is considered a new model, few researchers have used it in higher education. In all of them, the authors used a qualitative approach in analyzing the interactions between the professor and students, focusing on advanced mathematics classes like real analysis and matrix algebra.

Among the findings of these analyses, the connections found between mathematical knowledge MK and pedagogical content knowledge PCK stand out. For example, Delgado-Rebolledo and Zakaryan (2018) describe how some MK subdomains supported the emergence of one or more PCK subdomains. On the other hand, they also mentioned that in some observed episodes, certain MK subdomains conditioned the occurrence of one or more PCK subdomains. In fact, this perception of seeing content knowledge CK or MK as a prerequisite for PCK is also supported by other models like MTK. As described by Loewenberg Ball Thames, and Phelps (2008), content knowledge is considered a necessary condition to teach:

> Teachers must know the subject they teach. Indeed, there may be nothing more foundational to teacher competency. The reason is simple: Teachers who do not themselves know a subject well are not likely to have the knowledge they need to help students learn this content. At the same time, however, just knowing a subject well may not be sufficient for teaching (p. 404).

Therefore, content knowledge is a necessary condition needed to develop deep pedagogical content knowledge, even though content knowledge can be conceived as a prerequisite, and not a sufficient condition for adequate learning (Abell, 2007). This was shown in Matthews (2013), whose results showed that pedagogical content knowledge "had a substantial effect on student growth in comparable classes" (p. 35). Similar research proved that content knowledge did not strongly correlate with student achievement as pedagogical content knowledge (Baumert et al., 2010). In other words, pedagogical content knowledge has a stronger relationship with student achievement.

In addition to the findings of the connections between MK and PCK, other authors discussed which MTSK subdomains were distinguishable during the observed lecture. Authors described, for example, that professors could have a deep KoT and how this affected their KFLM (Vasco & Climent, 2018). Similarly, Delgado-Rebolledo and Zakaryan (2020) described how some MK subdomains supported the emergence of one or more PCK subdomains. Also, they described the emergence of KPM and how this knowledge allowed the construction of knowledge.

2.3 Why MTSK?

It was explained before that MTSK distinguishes itself from the other frameworks because, in the mathematical knowledge domain (MK), each of the sub-domains are based on mathematics itself. Moreover, compared to other frameworks, the instructors' mathematical knowledge is developed in a process of "becoming" instead of "being" (Zakaryan & Ribeiro, 2018). That is, according to MTSK, teacher knowledge is constantly evolving and is not considered a knowledge inherent to a particular grade. This evolution implies that teacher knowledge is constantly changing depending on the situation.

In other words, MTSK eliminates the barrier of the grade level since this framework implies that the instructors' mathematical knowledge adapts to the circumstances. For instance, Scheiner et al. (2017) explained that this adaptivity is reflected in MTSK:

> The term context acquires a very different and deeper meaning than the ways it has been previously construed. This perspective assumes that context consists of situations and activities embedded in the learning-teaching complex in the immediate moment. In consequence, what signifies mathematics teacher knowledge might be better described (or can be better approached) from within the discipline. In this regard, mathematics teacher knowledge is treated not as static traits (that differ from other professions) but as interpretations of performances that are situated in the immediate context (p 64).

Thus, for these two reasons MTSK is the best fit to understand teaching practices in a mathematics for social sciences class. First, MTSK allows the researcher to explore instructor knowledge regardless of the teaching level. Moreover, this framework reflects the instructors' ability to adapt to the circumstances. In this particular case, this capability is really important since the students taking this class have diverse opinions and attitudes about mathematics, and also, none of the students are pursuing a career in STEM.

2.4 Conclusion

There is a constant perception that college students are entirely responsible for their success. As Thomas (2002) argues, there exists a "victim-blaming" situation, where students are constantly judged for not being prepared and lacking commitment to a more "rigorous" environment. Nevertheless, there is an institutional responsibility that must be noted.

It is important to remark that this research does not intend to blame professors or students, but to understand what is happening inside college classrooms. As Biza et al. (2016)

argue, there is an increase in understanding of teaching practices in higher education. And for this reason. some authors have used previous teachers' knowledge models like Shulman's PCK (1986) or Ball et al. MTK (2008). However, there are some limitations because these models distinguish between specialized and common knowledge. For this reason, most of the research about teachers' knowledge has been conducted in K-12th grade.

Furthermore, there is a conception that teaching practices in higher education are different from K-12th grades. Thus, having a framework that overcomes this barrier is crucial. Fortunately, the model proposed by Carrillo-Yañez et al. (2018), MTSK, offers a feasible option to explore mathematics professor's knowledge; but, since this is a relatively new model, only a few researchers have used this framework.

For this reason, there is still much to do in exploring teaching practices in higher education. Contributing to this vital topic could have an impact on students' learning experiences. It might also impact retaining more students in college and consequently increase the rate of students graduating from higher education. Similarly, professors might benefit from it since it will provide them with new strategies that could be used for their lectures. Finally, universities might also benefit because both students and professors will be better prepared.

Chapter 3 Methods

3.1 Introduction

This section describes the methodology employed in this research, the settings, the participants, and the instruments used to collect data. It also explains how the data analysis was performed to respond to the research questions: What type of knowledge and practices of the participating lecturers are observed in a mathematics for social science class? How do students perceive the participating lecturers' pedagogical content knowledge, and how does it affect their learning experiences?

As explained before, the present study investigates instructors' content and pedagogical knowledge in higher education in mathematics for social science and how students perceive and benefit from this knowledge. Although robust research exists to understand teacher knowledge in K-12th grade, there is a gap in understanding such a phenomenon in college settings, especially in the context of mathematics for social sciences. Then, the framework of mathematics teachers' specialized knowledge (Carrillo-Yanez et al., 2018) was used as a guide to explore this situation.

To do this, this research was guided by a mixed-method design. The data collection phase was divided into three main phases. In phase one, professors were observed by the researcher when providing their lectures using the classroom observation protocol for undergraduate STEM (COPUS) (Smith et al., 2013). Next, in phase two, the objective was to identify students' perceptions of their professors/lecturers' pedagogical content knowledge. Thus, all participating students were asked to complete the college students' perceptions of teachers' pedagogical content knowledge survey (Jang et al., 2009). Finally, phase three consisted of interviews with all professors and some participating students. This final stage enlightened the understanding of professors' backgrounds, beliefs, and perspectives, but also students' perceptions and learning experiences.

3.2 Study Design

3.2.1 Mixed-Method Design

As described before, there is an increased interest in understanding teaching practices in higher education settings. This effort to further understand this topic demands that the researcher has a comprehensive and holistic understanding of the topic and, thus, different sources of information that contribute to its comprehension. In particular, this research uses three different sources of information to explore teaching practices and knowledge and their impact on students' learning. These sources are a classroom observation protocol, a survey, and interviews.

Then, as further described in the coming sections, the researcher implemented different strategies and techniques to analyze and draw conclusions correctly. Moreover, to completely understand this research topic, the researcher integrated the results obtained from each resource. Thus, these multiple data perspectives helped the researcher to validate data by triangulating. Moreover, the results obtained from the quantitative data were further explained by using qualitative analysis.

For these reasons, this research was developed by using a mixed-method design. In a mixed method design the researcher:

> collects and analyzes both qualitative and quantitative data rigorously in response to research questions and hypotheses, integrates (or mixes or combines) the two forms of data and their results, organizes these procedures into specific research designs that provide the logic and procedures for conducting the study, and frames these procedures within theory and philosophy (Creswell & Plano Clark, 2017).

With this in mind, a mixed method approach fitted this investigation since it was the goal to have a broader understanding of pedagogical practices as well as students' experiences and perspectives. This required having a variety of collected data that helped to understand this situation. In other words, a combination of qualitative and quantitative data was used to respond to the research questions.

For instance, to respond to the first question, what type of mathematics teacher specialized knowledge and teaching practices of the participating lecturers are observed in a mathematics for social science class? The data obtained from the observations using the COPUS protocol was mainly used since the objective of it is to provide a quantitative and qualitative summary of the activities developed during a lecture. However, the data obtained from the survey and interviews were compared to the data obtained from the observations to support the information from the observations.

Next, to respond to the second research question: How do students perceive the participating lecturers' pedagogical content knowledge, and how does it affect their learning experiences? The researcher used a survey and interviews. A survey is an excellent approach to respond to this question since a "quantitative survey approach best fits the need to understand the views of participants in an entire population" (Creswell & Plano Clark, 2017, p. 7). Furthermore, as will be further explained in the coming section, interviews were used to better understand students' and professors' perspectives. Then, quantitative data and analysis were implemented to understand this data.

Thus, the researcher had a combination of both quantitative and qualitative data that was used to answer the research questions. For this reason, a mixed method approach best fitted this research since one source of data was insufficient and may not tell the whole story of the problem. With this in mind, the data collection phase was divided into three phases: Phase one consisted of observing the classes using the COPUS protocol. Next, phase two consisted of college students' perceptions of teachers' pedagogical content knowledge survey. Finally, phase three consisted of interviewing students and the instructors.

3.2.2 Researcher Perceptions and Ethical Considerations

The researcher has worked in higher education for more than five years. Particularly, he has been assigned to different activities like tutoring, research assistant, and teacher assistant in the participating institutions. Thus, the researcher has personally observed both sides of this research problem.

In other words, he has witnessed students' efforts to learn and understand mathematics for social science class, but also, he has observed how instructors prepare to be ready to teach this class. Therefore, the researcher intends to serve the scientific community by exploring what students perceive from their instructors' pedagogical and content knowledge and how this affects their learning.

With this in mind, the researcher was conscious of the different ethical concerns of developing this research neutrally. Then, the researcher was concerned with minimizing any risk (physical, mental, or social) to the participants. To do this, their names were never used in any document different from the consent form, and only pseudonyms for instructors and universities were used to describe them. This protected their privacy and ensured that students and instructors were not subject to reprimand.

3.2.3 Philosophical Assumptions

Philosophical assumptions are important when researching since they guide the direction of the research and, consequently, its' goals (Gonnerman et al., 2015). In this research, a constructivist approach was implemented since the goal of this worldview is "the understanding or meaning of phenomena, formed through participants and their subjective views" (Creswell & Plano Clark, 2017, p. 36).

3.3 Setting and Participants

3.3.1 College Setting

Two higher education institutions were considered: The University of the South and South Community College (pseudonyms). Both institutions are located in the southwestern part of the United States. The University of the South serves about 25,000 students; about 56% are female, and 44% are male. It is considered a Hispanic-Serving Research University with an 84% Hispanic student body. Also, this is considered a first-generation student college and one of the lowest-cost universities across the country in doctoral programs. Moreover, this university is also classified as an R1 Doctoral University because of the high volume of research it produces, ranked in the top 5% of research institutions.

The University of the South offers 75 undergraduate and 94 graduate degree programs. Particularly, the College of mathematical sciences has about 18 associate professors of instruction/lecturer, six professor emeritus, and 30 tenured professors. Additionally, in the College of mathematics, the ratio of professors to students is, on average, 1:20. In the context of mathematics for social science, about 14 different professors/instructors taught this class face-to-face in the Fall of 2022.

Similarly, South Community College serves over 26,000 students (about 58% are female and 42% are male) in five different campuses distributed across the city. It is also ranked as a Hispanic-serving college, where 83% of the student body is Hispanic. It is recognized as an Achieving the Dream Leader College, that is, this school is part of a network of schools that use data to develop programs to improve students' academic success. This college is also classified as one of the lowest-cost colleges in the country.

Moreover, South Community College offers 138 academic programs, with a general teacher/instructor ratio of 1:22. Particularly in the math department, South Community College has 74 part-time and full-time instructors across all the campuses, which of this 9 taught mathematics for social sciences during the Fall 2022 semester. However, the researcher observed only one campus where two instructors had this class face-to-face.

3.3.2 Mathematics Class

Both institutions offer a wide variety of mathematics classes, from remedial to doctoral courses. These classes are offered in different formats, in person, online, or hybrid. In this research, the in-person mathematics class for social sciences was selected for three reasons:

(1) This class was selected because it was noted the lack of existing literature exploring pedagogical practices or students' experiences in mathematics classes for non-STEM majors.

(2) This research will inform teaching practices and students' perspectives in non-STEM settings.

(3) This class is offered to those students who are not interested in a STEM career. Then, the professors in charge of this class should have a wide variety of pedagogical techniques that, together with their content knowledge, could promote student learning and understanding of mathematics, which means that rich data could be observed and analyzed.

(4) In-person because, it is the intention to observe the interactions between the student and the instructor in a mathematics context.

Participants.

Selection of Participants: Instructors. First, potential participating lecturers were identified from the database offered by the institutions' websites. Then, all lecturers teaching this class were invited via email to participate. Finally, those interested in the project were invited to a Zoom meeting to describe the project. Four instructors were interested in the project, three from The University of South and one from the South Community College. However, there was a time conflict with one of the classes being offered at The University of South, so only two instructors from The University of South and one from the South Community College continued in the project.

Description of Participants: Instructors. This research included three different lecturers teaching mathematics for social sciences. They differ in age, sex, ethnicity, educational background, and professional experiences.

- Mr. Morales (pseudonym). He works for South Community College and has two master's degrees, one in teaching mathematics and the second in teaching and technology. Mr. Morales is the most experienced instructor of the participating lecturers with over 15 years of teaching experience. He has also worked in K-12th grades in different positions including teacher and tutor.
- Mr. Keller (pseudonym). He works for The University of South. He has a master's degree in mathematics, and about 7 years of experience teaching different classes including statistics, math for social sciences and probability. Out of the three instructors, he is the only one that has taught senior level mathematics classes. This lecturer was the only instructor with no formal training in education.

- Mrs. Dominguez (pseudonym). She works for The University of South. She has a master's degree in teaching mathematics with 7 years of experience teaching college and high school. She has taught different classes including mathematics for social science, number concepts, and geometry. She was the only participant with a bachelor's degree in education.

Selection of Participants: Students. Next, after having identified the participating professors, at the beginning of the 2022 fall semester, the researcher invited their students to participate in this study. In other words, all students 18 years or older taking classes with the participating lecturers were invited to take part in this project. Those who refused were not subject to any reprimand from the instructor.

***Description of Participants: Students*.** The second type of participant is the students. Only those students taking classes with the participating lecturers were included in this research. It is not an easy task to define the type of students that participated in the study; however, students differed in age, sex, ethnicity, educational background, and educational experiences. Regarding their major, most of the students were studying nursing, criminal justice and business. Also, most of them were classified as freshman.

3.4 Data Collection

This section provides details of the data collection process for each of the phases. First, the process for the classroom observation protocol is described. Next, the student survey process is detailed. And finally, the data collection for the interview phase is described. It is important to remark that the obtained digital data was saved in an encrypted folder in a secure shell provided by The University of South. Similarly, the collected physical data was stored in a locked filing cabinet with access only by the researcher.

3.4.1 Phase One: Classroom Observations

After identifying the participating lecturers, the researcher introduced himself to the students and collected their consent forms. Then, the first phase consisted of classroom observations. The researcher observed all three lecturers during their regular classes at least six times per instructor in the semester (from August to November 2022). In these observations, the researcher examined classroom practices, teaching strategies, professor/lecturer and students' interactions, students' attitudes, and responses; to properly identify this, the classroom observation protocol for undergraduate STEM was utilized COPUS (Smith et al., 2013).

Classroom Observation Protocol for Undergraduate STEM COPUS. The classroom observation protocol for undergraduate stem (COPUS) emerged from the need to understand what STEM professors/lecturers and students were doing during a lecture (see Appendix A). Education specialists originally designed this protocol for the University of British Columbia (UBC) and the University of Maine (UMaine). COPUS was developed primarily to characterize the activities developed in a class/session by describing what students and lecturers/professors are doing, informing how they spend their time. Thus, this protocol allows observers to provide feedback to professors/lecturers and identify their needs and strengths (Smith et al., 2013).

As described by the authors, this protocol has been successfully used to observe STEM instructors in different contexts, including different mathematics classrooms. Moreover, recall that, as described in Chapter 1, the class of mathematics for social sciences is only based on the analysis and applications of different mathematical concepts that are covered in a regular STEM class. Among these topics, the instructors covered topics like linear equations, systems of inequations, inequalities, quadratic functions, and exponential functions, among others.

Furthermore, this protocol allows the researchers to annotate comments and observations, allowing the possibility of describing the type of content and or pedagogical content knowledge that is being demonstrated at the moment. This future was particularly important to respond to the first research question, what type of mathematics teacher specialized knowledge and teaching practices of the participating lecturers are observed in a mathematics for social science class?. Then, for these reasons, the COPUS protocol was a good fit for this research.

Another benefit of using this protocol is that the authors provide videos that could help researchers unfamiliar with observation protocols. These training videos contain real-life examples lasting from 3 to 15 minutes. These recordings show various recorded classes, providing examples of how to label both what students and lecturers are doing. This feature is fundamental to the understanding of this protocol.

Regarding validity and reliability, COPUS established its validity during its development process. COPUS was created with constant feedback from experts, including science education specialists, K-12th teachers, and authors with experience in observation protocols. Similarly, when discussing the reliability of this protocol, the authors showed that the inter-rater reliability (IRR) between codes has an average Jaccard score of 0.80-0.90. To compare the reliability across all codes, the authors proved to have a Cohen's kappa IRR average score between 0.79 and 0.87, demonstrating good reliability when observers are exposed to 2 hours or fewer of training (Smith et al., 2013).

COPUS contains two main categories to code: what students and professors are doing. Each category has different labels (e.g., L listening to the instructor or Lec for lecturing/presenting a problem). The students' section contains 13 other codes, while the instructor comprises 12, and an additional section called student engagement is used to describe the percentage of student engagement. This section has three different labels, like L, describing a small fraction (10-20%) obviously engaged.

The professors' codes include: lecturing, writing on board, following up or feedback on a clicker question or activity, posing non-clicker questions, asking a clicker question, listening to, and answering questions, guiding ongoing work during learning task, one-on-one extended discussion with one or a few individuals, showing or conducting a demo, experiment or simulation, administration activities like assigning work, returning tests, etc., waiting, observing or listening to students during an activity, and other.

On the other hand, the student codes include: listening to the instructor or taking notes, individual thinking, discussing questions in groups of two or more (clicker), working in groups or worksheet activity, other assigned activity, like responding to instructor questions, answering questions for the entire class, asking questions, engaged in whole class discussion, making predictions about an experiment or demo, presentations by students, test, or quiz, waiting, and other. Moreover, a third category allows the researcher to classify students' level of engagement as low (less than 20%), medium (20% - 80%), and high percentage (80% or more).

COPUS demands to code what is happening during the lecture every two minutes. Then, this protocol requires the researcher to be an active listener and observer all class long. Therefore, about 40 times per lecture/class (80 minutes lectures), a code for each lecturer, students' activities, and student level of engagement was registered. This allowed the researcher to have rich data describing what was happening in the classroom during the lecture.

Similarly to the codes for lecturers/professors, the COPUS protocol allowed the researcher to describe what students were doing. Again, this contributes to understanding the impact of pedagogical practices on student engagement. In addition, this information is a valuable resource for understanding the impact of teacher knowledge on student engagement.

Finally, besides recording with codes what the students and instructors were doing during the observations, this protocol also provided the space for the researcher to annotate comments about the observations. This was vital to answering the first research question since it allowed the researcher to record the knowledge and teaching practices observed during the lecture. In other words, by using the definitions provided by the MTSK framework, the researcher was able to identify the knowledge and teaching practice applied at the teaching moment.

3.4.2 Phase Two: Student Survey

The second source of information consisted of students' surveys. The main objective of this survey was to explore students' perceptions about professors/lecturers' pedagogical content knowledge. To gather this data, the researcher implemented the college students' perception of teachers' pedagogical content knowledge survey developed by Jang et al. (2009).

College Students' Perceptions of Teachers' Pedagogical Content Knowledge Survey. As described by the authors, this survey emerged from the need to understand students' perceptions of their teachers' pedagogical knowledge, particularly college students. This instrument helps in "evaluating college students' perceptions of teachers' PCK to help college teachers understand better how they teach" (Jang et al., 2009, p. 597).

This survey offers the opportunity of being specific to college teachers' knowledge, mainly focusing on learning and teaching contexts. It contains 28 questions that are divided into four main categories, where each of them has 7 Likert scale questions with five options: (1) Never; (2) Seldom; (3) Sometimes; (4) Often; and (5) Always.

- Subject matter knowledge (SMK)

- Instructional representation and strategies (IRS)
- Instructional objective and context (IOC)
- Knowledge of student understanding (KSU)

Similar to the COPUS protocol, its validity was confirmed through constant feedback from experienced faculty in its development process. On the other hand, to test for inter-rater reliability, the authors proved to have an internal consistency by having an average Cronbach's value between 0.871 and 0.918.

This survey was administered to all participating students present in the last week of classes (all instructors finished their lectures by the week before the Thanksgiving break); in other words, this survey was administered in the week of November 14 to November 18, 2022, and it took the students about 10 minutes or less to complete it.

3.4.3 Phase Three: Interviews

Finally, the last source of information comes from semi-structured interviews conducted with professors and students. This type of interview allows the researcher to ask participants a series of pre-elaborated open-ended questions enabling more flexibility and control than unstructured or closed questions (Wishkoski, 2020). These interviews were conducted via Zoom and recorded using a digital voice recording.

This research conducted two types of interviews: instructors' and students' interviews (see Appendix C).

Instructors' Interviews. All participating professors were invited to participate in a semi-structured interview that lasted 45-60 minutes. This interview allowed the researcher to understand their professional and educational background, their beliefs about teaching mathematics in college settings, and their role in student's learning and academic experiences.

The interview lasted 30 to 45 minutes, and they were held during the last week of the semester (December 5 to December 9, 2022). Instructor Morales and Dominguez participated, but Instructor Keller decided not to take part in this phase.

Students' Interviews. In the present study, 11 students participated in a semi-structured interview that lasted 20 to 30 minutes during the Thanksgiving break (November 21 to November 25, 2022). The primary objective of these interviews was to explore students' perspectives about instructors' mathematics teachers' specialized knowledge (MTSK) and how this knowledge affected their learning experiences. The questions included a range of topics that covered their perceptions and viewpoints of professors' mathematical content knowledge and pedagogical content knowledge, their teaching techniques, their experiences, and their current academic status (see Appendix C).

3.5 Data Analysis

After collecting all data, the researcher analyzed it by using different approaches. For instance, the COPUS data was divided into two types: a quantitative part that included the percentage of time spent by students and instructors doing different activities. Moreover, the COPUS provided a qualitative section that served the researcher to annotate the kind of observed mathematics teacher specialized knowledge. Next, the survey was analyzed using descriptive statistics, where the interviews and observations were used to support its results. Finally, the interviews were analyzed using an open-coding approach to identify patterns.

3.5.1 Analyzing COPUS Data

As mentioned previously, the COPUS data was divided into a quantitative and a qualitative part. The quantitative data is represented by the time spent by the instructors and students performing different activities. As will be discussed in the next chapter, this data will be presented using tables (see Table 1). This allowed the researcher to have a physical representation of the gathered data. For example, if during an observation, the code of answering questions is recorded 20 times out of the 25 codes in a 50 minute-class, then 40% (20/50) will be representing answering questions. This helped explore, divide, and analyze the data by time frames (daily, weekly, monthly, and semester). Moreover, it was possible to establish how professors and students spent their time during a lecture.

Table 1

Example of the Representation of the COPUS Data

Activity	Time
Other discussions	40%
Lecturing	15%
Lecturing and writing	22%
Answering questions	13%
Demo	5%
Posing question	5%

Next, the qualitative data was analyzed by using open coding. Open coding can be defined as:

> the interpretive process by which data are broken down analytically. The purpose of open coding is to help the analyst gain new insights into the data by breaking through standard ways of thinking about (interpreting) phenomena reflected in the data… In this way, conceptually similar ones are grouped together to form categories and their subcategories. (Corbin & Strauss, 1990)

To apply open coding in the COPUS protocol, the researcher utilized the notes he collected during the observations. In these notes, the researcher included what he was observing, quotes from the instructors or students, or the activity performed by any of the participants. For instance, quotes like “the definition of function is for every value in x there exists a value in y” were registered. Also, the researcher annotated notes like “the instructor solved the system of equations by using Desmos, showing students how to use technology and how to interpret the answer.”

This feature provided the opportunity of analyzing the data in two different ways. First, by open coding, that is, by finding patterns across the teaching strategies implemented by each instructor. When the researcher did this, he found different patterns like writing on the board, reading the questions, and non-mathematics discussion, among others. Then, by using focused coding, which is the process of “identifying the most frequent or significant codes in order to develop the prominent categories” (Theron, 2015, p. 5), the researcher grouped the codes per instructor into three different teaching strategies (see section 4.4).

Second, the notes section allowed the researcher to connect instructors’ teaching strategies to one of the MTSK’s definitions. For instance, one of the categories attributed to Mr. Morales was real-life examples. This code was obtained by coding the times the instructor used real-life examples to teach mathematics. Among these examples, he explained how the slope was connected to different situations like speed. Similarly, he used examples related to personal finance to help students understand simple or compound interest. These examples were connected to the definitions of knowledge of topics (KoT) (the knowledge that allows teachers to use definitions to make connections, see section 2.2.2) and knowledge of mathematics teaching

(KMT) (the knowledge that enables teachers to choose appropriate examples to help in the understanding of the topic).

This process was applied to each of the categories that emerged, letting the researcher to respond to the first research question: what type of mathematics teacher specialized knowledge and teaching practices of the participating lecturers are observed in a mathematics for social science class?

3.5.2 Analyzing the Students' Perceptions of Professors' Pedagogical Content Knowledge Survey Data

The data obtained from the survey was analyzed using descriptive statistics, in particular, the mean and standard deviation. These values were determined because the mean is a measure of central tendency that describes the most common value in a set. By using the mean, the researcher could identify, on average, what students perceive of their instructors' pedagogical content knowledge. Also, the standard deviation was calculated. Recall that the standard deviation describes how dispersed the data is in a set relative to the mean. In this particular case, the standard deviation helped the research by describing if students have similar perceptions or opinions about their instructors.

For these reasons, the mean and standard deviation were calculated for each question and each of the survey domains (subject matter knowledge, instructional representation & strategies, instructional objective & context, and knowledge of students' understanding). First, they were computed for each instructor (using Microsoft Excel). Then, all the scores were combined in one group. This helped the researcher understand what students perceived of their instructors' pedagogical content knowledge by instructor and as a portion of the mathematics for social sciences' student body.

Furthermore, these values were used to compare and contrast students' perspectives with different instructors and pedagogical practices. This data contributed to exploring how the different teaching techniques and pedagogical knowledge yield different scenarios and different perceptions and learning experiences.

3.5.3 Analyzing Interviews

As mentioned, only professors Morales and Dominguez agreed to participate in the interview phase. Additionally, five students from Mr. Morales, three from Mr. Keller, and three from Mrs. Dominguez participated in this phase. All 13 interviews (instructors and students) were semi-structured and lasted 45 to 60 minutes for instructors and 20 to 30 minutes for students.

These interviews were transcribed verbatim, and just like with the COPUS, data analysis of the collected interviews consisted of open coding to identify initial codes (Corbin & Strauss, 1990). However, there was a significant difference. To analyze the interviews, the researcher utilized a peer review approach.

First, it is important to remark that the interviews analyzed by the other researcher did not contain the name of any of the participants or the institution. Second, the transcriptions' documents were adequately encrypted to ensure their protection. Then, after the interviews were transcribed, the researcher identified a codebook for each set of interviews. For example, Mr. Morale's group included: the pace of the class, substitute teacher, in-person tests, examples used by the instructor, in-class discussions, assignments, websites, students afraid of asking questions, and a lack of different activities. In Mr. Keller's, the initial codes were strong content knowledge, no interactions with students, lack of activities, attendance, students being afraid of asking questions, teacher support, teacher assistant, language, and the pace of the class. Finally, for Mrs. Dominguez, the initial codes included asking questions, the pace of the class, the lack of student understanding, office hours, teacher availability, student participation, activities, and break times.

Then the researcher shared the encrypted files with his peer to identify themes independently. Both researchers analyzed these interviews in groups. For instance, the interviews conducted with the five students taking classes with Mr. Morales were analyzed apart from the other instructors. The exact process was conducted for instructors Keller's and Dominguez's students.

Once the second rater completed his analysis and shared his report, the researcher quantified the number of times the codes were observed. This was done to check for the interrater agreement reliability of the codes by using Cohen's Kappa statistic (Cohen, 1960). Cohen's Kappa is commonly used in research to check for agreement between two raters (Warrens, 2015).

The obtained Cohen's Kappa for each instructor was the following: Mr. Morales: 0.7975, Mr. Keller: 0.8148, Mrs. Dominguez: 0.7857. These values, as described by Landis and Koch (1977), prove substantial to almost perfect agreement in the analysis made by the two researchers, validating the coding process (Warrens, 2015).

3.5.4 Triangulation of Data

In research, when analyzing data, triangulation refers to:

> the use of more than one approach to researching a question. The objective is to increase confidence in the findings through the confirmation of a proposition using two or more independent measures. The combination of findings from two or more rigorous

approaches provides a more comprehensive picture of the results than either approach could do alone (Heale & Forbes; 2013)

Thus, the present study aims to use triangulation to improve the trustworthiness of the results. With the use of multiple sources of information, by triangulating data, it will be possible to connect findings in a clearer, more orderly, and reliable way than by using just one source of information. In particular, this research used three types of data:

- Observation (using the COPUS protocol).
- Surveys (students' perceptions of professors' pedagogical content knowledge survey).
- Interviews (semi-structure interviews).

With this in mind, the results gathered in the observations, and supported by the interviews and surveys, helped not only in understanding how participating lecturers are spending their time during the lecture, but also it provided a clearer image of the pedagogical practices and content knowledge that are being implemented during the lecture and their impact on students learning. Then, the second question, how do students perceive the participating lecturers' pedagogical content knowledge, and how does it affect their learning experiences? was responded to by using the survey and the interviews, supported by the observed teaching practices and knowledge.

Chapter 4: Results

This chapter summarizes the results obtained from the analysis. It reviews the information gathered from observations, surveys, and interviews. First, the data obtained from observations will be described in detail. Then, the information collected from the survey will be explored and explained. Finally, the codes obtained from the interviews will be described and explained.

4.1 Observations

After obtaining the consent form, the three instructors were observed during the Fall semester of 2022. Mr. Morales, from South Community College institution, was observed seven times in the semester. Unfortunately, he was sick for a couple of weeks during the semester, limiting the number of observations. Thus, he was the instructor with the least number of observations. This instructor was observed for about 336 minutes. During this time, the observer could describe the settings and interactions between the instructor and students.

This class was held in a computer room with space for 30 students. The lecture was conducted in the afternoon. The total number of students was 18, but only 15 agreed to participate in the project. The other three students were never seen during the observations or exams in class. On average, about 13 students attended the lecture. Each student had access to a working computer with the internet and calculators provided by the institution. Also, there was a whiteboard, but the instructor never used it in any observations.

The instructor primarily used a tablet where he wrote all his notes. These notes were projected to the students for them to copy. The environment was casual and relaxed for the students and the instructor. During each observation, the instructor remained seated, writing and explaining the topic to the students, while the students were sitting on their chairs in front of a computer or moving their chairs to face the instructor.

Mainly, Mr. Morales interacted with his students by describing different real-life contexts; some applied explicitly to the topic, but also, they had some non-related mathematical discussions. It is important to remark that even though the level of interaction was notorious, these interactions decreased when the instructor discussed mathematical concepts or topics. On the other hand, when the discussion was related to real-life situations or non-mathematical conversations (e.g., discussing everyday topics like marriage), the level of interaction increased notably.

In general, the students were engaged in various conversations during the lecture. For example, Table 2, it can be observed how the time was spent during a class. On this particular day, the instructor spent most of the time discussing several topics, some non-mathematical and some mathematical-related real-life examples.

Table 2

Mr. Morales: what did the Instructor do During a Lecture?

Activity	**Time**
Group discussions	30.56%
Lecturing	22.22%
Lecturing and writing	22.22%
Answering questions	13.89%
Demo	5.56%
Posing question	5.56%

Moreover, the instructor posed questions as a way of triggering conversations among students. Similarly, this instructor tended to give his lecture by discussing the topics orally with his students, or when needed, he used his tablet to write down the formulas, examples, mathematical operations, or mathematical processes to teach the lesson. Additionally, he used different websites like Desmos to explain solutions to problems.

During this time, the students mainly listened to either the lesson or the stories and examples provided by the instructor. Moreover, thanks to the questions asked by the instructor, the students spent considerable time discussing the topics and asking questions (see Table 3). Nevertheless, the time spent responding to mathematical questions (Answering questions) during this session was minimum.

Table 3

Example: Mr. Morales: What did the Students do During a Lecture?

Activity	**Percentage of Time**
Listening	50.00%
Whole class discussion	25.00%
Student question	16.67%
Waiting	5.56%
Answering questions	2.78%

These interactions are reflected in the following table that describes the students' level of engagement. In Table 4, one can see that most of the time, the students were engaged, this happened thanks to the conversations and discussions with the instructor.

Table 4

Mr. Morales, Level of Engagement During a Lecture

Level of Engagement	Percentage of Time
Low engagement	0.00%
Medium engagement	13.89%
High engagement	86.11%

Overall, the instructor continued this trend during all the observations. This is displayed in the following table that summarizes the activities examined during all the observations. Here, it is possible to see that the instructor spent most of the time having mathematics and non-mathematics discussions with his students, lecturing, asking questions, and waiting. Here it is important to remark that the waiting also includes one day when the students were taking a test, and the instructor was waiting for them to complete it. Also, it is necessary to emphasize that even though much discussion happened during the lectures, this happened primarily when discussing a non-mathematics topic. In other words, when the instructor was discussing a mathematical concept or lecture, the level of discussion was low compared to when they had another type of discussion (e.g., last will).

Table 5

Mr. Morales, What did the Instructor do during the Observations?

Activity	Percentage of Time
Waiting	26.47%
Lecturing and writing	23.53%
Other discussions	20.59%

Lecturing	17.65%
Answering questions	5.15%
Posing question	3.68%
Demo	1.47%

Similarly, the following table describes what the students did during the observations. This display shows that the students spent most of their time listening to the lecture. However, they also spent significant time having whole class discussions and taking tests (there were four tests during the semester, including the final). In contrast, students spent less time answering and responding to mathematics-related questions.

Table 6

Mr. Morales, What did the Students do during the Observations?

Activity	**Percentage of Time**
Listening	45.52%
Taking test	23.88%
Whole class discussion	17.16%
Student questions	5.97%
Answering questions	2.24%

Finally, the overall student level of engagement reflects the high level of interaction between the instructor and the students. Table 7 reflects how most of the students were highly engaged during the lectures.

Table 7

Mr. Morales, Level of Engagement during the Observations

Level of Engagement	Percentage of Time
Low	0.00%
Medium	47.79%
High	50.74%

The observations made to Mr. Keller, who works for the University of South institution, were also during the fall semester. When discussing the observations made to Mr. Keller, it can be highlighted that the environment, classroom settings, teaching practices, and students' behavior observed were very different. The classroom had a traditional setting with space for at least 50 people, arranged in typical rows facing a chalkboard. The room has four big chalkboards, a computer in the front for the instructor, a projector, and a movable projector screen. In this class, there were a total of 45 registered students. However, the maximum number of students seen during the observations was 34, and only 29 decided to participate.

The instructor was a young mathematician that used strong mathematical language and jargon during his lectures (e.g., providing complete definitions of mathematical vocabulary like functions). Compared to the other two observed instructors, his interactions with the students were limited to solving examples and the lecture. As a result, the students were mostly quiet and taking notes. As described later, this way of conducting the class seemed to impact attendance since the number of students present during the lectures declined as time passed. For instance, during the last observation, only 17 students attended. This impacted the average of students present in the semester to 24. In other words, on average, only about 53.3% of the registered students attended the class.

Also, was observed a traditional teaching style or a teacher-centered teaching style (that is, when “students become passive learners, or rather just recipients of teachers’ knowledge and wisdom” (Ahmed, 2013, p. 22)). It was commonly observed that the instructor wrote and solved examples on the chalkboard while the student took notes. For instance, during an observation, the instructor spent 86% of the time lecturing, explaining problems, and writing on the board. During this time, the instructor asked some questions to trigger curiosity, but also, he often asked, “any questions?” without giving much time for students to respond. This can be followed in Table 8, where the ‘other section’ represents the time spent cleaning the board, taking attendance, or explaining future assignments.

Table 8

Example: Mr. Keller, What did the Instructor do During a Lecture?

Activity	**Percentage of Time**
Lecturing and writing	72.22%
Lecturing	11.11%
Other	8.33%
Posing questions	5.56%
Writing	2.78%

On the other hand, as depicted by Table 9, the students spent most of their time listening to the instructor and taking notes. The remaining time was used to answer the instructor’s questions, waiting for an answer from their classmates or the instructor, and waiting for instructions.

Table 9

Mr. Keller, What did the Students do During a Lecture?

Activity	Percentage of Time
Listening	83.33%
Answering questions	8.33%
Waiting	8.33%

This form of conducting the class impacts students' engagement. For instance, during the same observation, most of the time, students were mediumly engaged in the lesson, while the others were distracted either by the use of a cell phone or a laptop (see Table 10).

Table 10

Mr. Keller, Level of Engagement During a Lecture

Level of Engagement	Percentage of Time
Low	2.78%
Medium	97.22%
High	0.00%

Overall, this trend for students and instructors was constant throughout the semester. As observed in Table 11, of the 500 minutes observed, the instructor spent more than 70% of the time lecturing or writing on the board. And even though he tried to incite students' curiosity by asking questions, he only spent 4% of the time responding to questions. Equally important, among all three lecturers, he was the instructor that invested more time in using or showing how to use technology or online resources to solve problems. For instance, he used different websites like Wolfram or Desmos to solve applied systems of equations.

Table 11

Mr. Keller, What did the Instructor do during the Observations?

Activity	Percentage of Time
Lecturing and writing	62.40%
Posing questions	13.20%
Lecturing	7.60%
Demo	7.20%
Answering questions	4.00%
Other	3.60%
Waiting	1.20%
Writing	0.80%

Similarly, as described before and as shown in Table 12, students spent most of the time listening and taking notes. In contrast, the time used by students to ask questions or to answer questions from the instructor was very low compared to the listening.

Table 12

Mr. Keller, What did the Students do during the Observations?

Activity	Percentage of Time
Listening	78.00%
Answering questions	6.00%
Waiting	6.00%
Student question	4.00%
Other group activity	3.20%

Other	2.80%

Regarding the overall level of engagement, there were days where the students were more committed than others; however, in general, students were mediumly engaged in the lessons (see Table 13).

Table 13

Mr. Keller, Level of Engagement during the Observations

Level of Engagement	Percentage of Time
Low	3.60%
Medium	57.60%
High	38.80%

Finally, compared to instructors 1 and 2, the practices and environment observed in the third instructor's classroom were also different. Her class was not as quiet as Mr. Keller, but it was not as relaxed as with Mr. Morales. Among all three lectures, Mrs. Dominguez was the only one with a background in education. She was a high school teacher that decided to move on to college settings. However, she only has a few years of teaching experience.

In this class, the total number of registered students was 32. However, only 28 agreed to participate. Like Mr. Keller, towards the end of the semester, the number of students attending the class decreased from 31 (maximum number of students present during an observation) to 21 (minimum number of students present during an observation). On average, 25 students showed up to class, that is, about 78% of the registered students.

This class was held in the same room as Mr. Keller. The room had a capacity of approximately 50 students, and it had chalkboards, a computer, a projector, and a screen for the projector. As explained before, it was noticed during the observations that the instructor did not use strong mathematical language and examples like Mr. Keller. Still, she did not discuss non-mathematics topics like Mr. Morales. Moreover, she used plenty of questions to lecture as a way of helping students to construct knowledge. In contrast to instructors 1 and 2, she gave breaks to students when she felt that the students were too distracted or had a low engagement level.

As an example of the previous description, Table 14 describes a regular lecture day. Here, the instructor spent most of the time lecturing and asking questions to the students by using a tablet and projecting her notes. Compared to previous instructors, lecturer-student interactions regarding mathematics examples were noticeably greater. Moreover, she did follow up on the questions asked by students and provided more information or examples related to them. Another important difference is that “waiting” also included when the professor was waiting for the students to solve a question or answer one of her questions.

Table 14

Mrs. Dominguez, What did the Instructor do During a Lecture?

Activity	Percentage of Time
Lecturing/posing question	50.00%
Lecturing	20.83%
Answering questions	16.67%
Follow up	8.33%
Waiting	4.17%

This impacted how the students reacted. For instance, students spent less time listening and spent more time answering questions. Another difference was that this instructor allowed some students to work independently on some questions. Doing this encouraged students to ask more questions than in the other observed class. However, because the instructor asked many questions, there were times when the students did not know the answer, and they were waiting for the instructor to give them the answers (see Table 15).

Table 15

Mrs. Dominguez, What did the Students do During a Lecture?

Activity	Percentage of Time
Listening	45.83%
Answering questions	29.17%
Student question	12.50%
Waiting	8.33%
Individual work	4.17%

This teaching style helped those who were interested in the class. However, some students were distracted mainly using their laptops or cell phones. This was reflected in the engagement level (see Table 16).

Table 16

Mrs. Dominguez, Level of Engagement During a Lecture?

Level of Engagement	Percentage of Time
Low	0.00%
Medium	33.33%
High	66.67%

Then, like with the other instructors, the teaching style presented by Mrs. Dominguez was constant throughout the semester. However, it was observed that she implemented different techniques, dividing her time into several activities and strategies compared to the other instructors. This can be observed in the following table (see Table 17). Here, it is illustrated how the instructor used different lecturing methods: writing on her tablet, posing questions, or simply discussing orally. Moreover, she spent some time responding to students' questions and using technology (demo) to solve questions.

Table 17

Mrs. Dominguez, What did the Instructor do During the Observations?

Activity	Percentage of Time
Lecturing and writing	30.38%
Lecturing	21.54%
Answering questions	12.69%
Lecturing and posing questions	12.69%
Posing questions	7.31%
Waiting	6.54%
Other	3.08%
Demo	1.92%
Follow up	1.92%
Writing	1.92%

By the same token, students were exposed to different teaching strategies. Among these, the instructor used questions the most as a way to engage them. In the same manner, she also asked students to work individually or to read the questions to themselves or the group. She also encouraged students to ask her as much as possible (see Table 18).

Table 18

Mrs. Dominguez, What did the Students do During the Observations?

Activity	Percentage of Time
Listening	56.54%
Answering questions	17.31%
Student question	12.31%
Waiting	6.92%
Other group activity	3.46%
Other	2.69%
Individual work	0.77%

Nevertheless, the engagement level was lower than expected. Although there was participation, there were some students who were constantly distracted by the use of their cell phones or personal computer. This is reflected in table 19, were on average, the students showed a medium level of engagement.

Table 19

Mrs. Dominguez, Level of Engagement During the Observations

Level of Engagement	Percentage of Time
Low	0.00%
Medium	70.00%
High	30.00%

Overall, each lecturer demonstrated different teaching strategies. In their unique way of teaching, each instructor showed interest in ensuring their students were learning. They did this by asking questions, telling students their availability to meet with them after class, and providing a review before each test. Also, all instructors welcomed students to ask questions and responded to their questions to the best of their abilities. Nevertheless, there was a significant difference in the environment the instructors and students created. Some of them were very relaxed, while others were more serious.

For instance, instructor's 1 students had diverse non-mathematics discussions, including topics like marriage or the health system. In instructor's 2 classes, the level of discussion was minimum, the interactions between student and instructor were limited to asking questions about the assignments or the topics, creating a quiet environment even before the class started. Finally, in instructor's 3 class, more student-instructor interactions were observed. It was observed that students felt more relaxed than with Mr. Keller. However, these interactions were limited to asking and commenting about the class and the assignments.

The main observed difference in teaching style was how the instructor taught the class. As mentioned before, Mr. Morales used examples and discussions that engaged in several discussions that were sometimes non-math related topics. Compared to Mr. Keller, he used a traditional teaching style, using the chalkboard, writing on the board, spending most of the time lecturing, and functioning as a gatekeeper of knowledge. Contrasted to instructor three, she used questioning to engage students and to guide them to construct knowledge.

On the other hand, all instructors also shared some similar teaching styles that were reflected mainly in the use of technology. In other words, all instructors used technology to help students understand or guide the lesson at some point in the semester. Particularly instructors 1 and 3 used technology all semester to write notes. Both used a tablet that was used while explaining the concepts or examples. These notes were projected for the students so they could take notes. More discreetly, Mr. Keller used technology to show students how to solve complex problems (during the lesson on systems of equations).

Another similarity in the teaching style was found when the instructors connected mathematics to real-life situations. For all instructors, the applied situations given to students were related to money and personal finance. Most of the time, instructors connected students' personal experiences by connecting mathematics to money-related situations. It was not observed that instructors used applications connected to different scenarios or majors. As will be further described in the discussion chapter, this situation impacted students' mathematics experiences.

In general, all three had a teacher-centered teaching style. This is described in the following table that summarizes the observations of the three instructors. Here, one can notice how lecturers spend most of their time lecturing. Also, the use of websites or tools used to solve problems or following up questions is minimum (see Table 20).

Table 20

What did the Instructors do During the Observations?

Activity	**Percentage of Time**
Lecturing and writing	41.46%
Lecturing	15.37%
Waiting	8.70%
Posing questions	8.85%
Answering questions	7.76%
Other	6.99%
Lecturing and posing questions	5.12%
Demo	3.88%

Writing	1.09%
Follow up	0.78%

In the same way, Table 21 reinforces the teacher-centered orientation of the classes (see Table 21). This table provides a stronger argument than the previous table. Here, one can observe that students spent most of their time listening and taking notes. This implies that the time they had to practice during class was minimum. In fact, only Mrs. Dominguez allowed students to work a couple of times individually during a lecture. Similarly, it was never observed the use of different strategies like group work. The only group work observed during the semester was when Mrs. Dominguez asked the class to solve a problem during a short period of time.

Table 21

What did the Students do During the Observations?

Activity	**Percentage of Time**
Listening	62.58%
Answering questions	9.78%
Student questions	7.76%
Waiting	6.21%
Taking test	4.97%
Other	2.17%
Other group activity	2.64%
Whole class discussion	3.57%
Individual work	0.31%

Finally, the overall engagement level was constant for all instructors. Most of the students responded to the instructors' requirements by staying alert, writing notes, and responding to the instructors' questions (see Table 22).

Table 22

Overall Level of Engagement During the Observations?

Level of Engagement	Percentage of Time
Low	1.40%
Medium	60.71%
High	37.89%

4.2 Survey

The survey used in this research was developed by Jang et al. (2009), and it is called assessing students' perceptions of college teachers' PCK. This survey was used because, as described by the authors, "the uniqueness of the survey was that it was specifically related to college teachers' knowledge within the particular teaching and learning context. This was important because research had shown college teachers' knowledge influenced students' perceptions of the learning environment" (Yang et al., 2009, p. 603).

This survey contains 28 Likert questions with five different option choices: (1) Never; (2) Seldom; (3) Sometimes; (4) Often; and (5) Always. The questions were divided into four different domains: (1) SMK, subject matter knowledge; (2) , instructional representation and strategies; (3) IOC, instructional objective and context; and (4) KSU, knowledge of students' understanding.

Subject matter knowledge (SMK) helps understand what students think of their instructor's content knowledge. In other words, it describes if the students believe their instructors know the content they are teaching. Then instructional representation and strategies (IRS) explore what students think of how instructors use examples, analogies, or metaphors to help them learn mathematics.

Furthermore, the instructional objective and context (IOC) section helps in exploring what students perceive of their instructors' pedagogical practices, like attitudes, classroom management, and knowledge of the curriculum. And last, knowledge of students' understanding (KSU) evaluates students' perspectives on teachers' student understanding during instruction and learning. Moreover, at the end of the survey, the students had the opportunity to express themselves by having the space to write any comment about the class (Comments: In this course, if you have any learning difficulty or opinion, please describe it as follows).

The survey was conducted in the last week of classes, and a total of 51 students participated. For each domain, descriptive statistics were calculated. These values helped in the analysis of what students perceive about their instructors' teaching practices and knowledge.

4.2.1 Students' Surveys, Mr. Morales

In this class, of the 15 students that signed the consent form, only 13 students were present in the final week of classes. For each of the questions in the first category, SMK (Subject Matter Knowledge), the mean and standard deviation were calculated.

Table 23

Mr. Morales: Mean and standard deviation for Domain A

Mr. Morales	Mean	Standard deviation
Q1: My teacher knows the content he/she is teaching	4.77	0.44
Q2: My teacher clearly explains the content of the subject.	4.00	1.22

Question 3: My teacher knows how theories or principles of the subject have been developed.	4.92	0.28
Question 4: My teacher selects the appropriate content for students.	4.54	0.97
Question 5: My teacher knows the answers to questions that we ask about the subject	4.62	0.65
Question 6: My teacher explains the impact of subject matter on society.	4.46	1.13
Question 7: My teacher knows the whole structure and direction of this class.	4.46	1.20
Average	4.54	0.84

By looking at the data, students gave high values in this domain to the instructor. Recall that this dimension explores what students think of the content knowledge of their instructor. In other words, students believe that Mr. Morales has solid content mathematical knowledge. Nevertheless, the lowest score was for question two, “my teacher clearly explains the content of the subject.” In contrast, the highest score was given to question three, “my teacher knows how theories or principles of the subject have been developed.”

Moreover, the values for the standard deviation are high. This could indicate that students differ in their perceptions, especially in question 2, which according to students, has the lowest score. Indeed, question 2 discusses what students think of how the instructor explains the concept. Similarly, the question with the slightest standard deviation is question 3, which is the question with the highest average score given by students. This shows that most students have the same perception.

Additionally, the average of each option choice is depicted in the following Table (see Table 24). This table reflects that most of the students gave a score of 5 to most of the questions, and only a few believed that the instructor had a weak content knowledge.

Table 24

Mr. Morales: Most Popular Answer Choice Given by Students in Domain A

Score	Percentage
1	1.10%
2	5.49%
3	6.59%
4	13.19%
5	73.63%

The values obtained in the second domain, IRS (Instructional Representation and Strategies) are shown in Table 25. In this table, one can observe that students gave lower scores than in the previous section (average score of 4.44). As mentioned before, this section explores how students perceive the use of examples and metaphors to help them learn mathematics. Moreover, in contrast to the previous section, the standard deviation is lower (standard deviation of 0.69) than in the last domain. This indicates that there is more consensus in the score among students.

Table 25

Mr. Morales: Mean and standard deviation for Domain B

Mr. Morales	Mean	Standard deviation
Question 1: My teacher uses appropriate examples to explain concepts related to the subject matter.	4.54	0.97
Question 2: My teacher uses familiar analogies to explain concepts of subject matter.	4.62	0.77
Question 3: My teacher's teaching methods keep me interested in this subject.	4.08	1.44
Question 4: My teacher provides opportunities for me to express my views during class.	4.69	0.85
Question 5: My teacher uses demonstrations to help explaining the main concept.	4.38	1.19
Question 6: My teacher uses a variety of teaching approaches to transform subject matter into comprehensible knowledge.	4.15	1.63
Question 7: My teacher uses multimedia or technology (e.g., PowerPoint) to	4.85	0.38

express the concept of subject.		
Average	4.47	1.03

In this section, the lowest mean average was given to question 6 (my teacher uses a variety of teaching approaches to transform subject matter into understandable knowledge). Also, this question has the highest standard deviation. On the other hand, the highest score was given to question 1 (My teacher uses appropriate examples to explain concepts related to the subject matter), which also has the lowest standard deviation.

The summary of the scores for all questions within domain B is represented in the following table. Here it can be observed that students gave lower scores than before, reducing the number of times when students gave a perfect score (see Table 26).

Table 26

Mr. Morales: Most Popular Answer Choice Given by Students in Domain B

Score	Percentage
1	3.30%
2	7.69%
3	4.40%
4	7.69%
5	76.92%

Next, section C is IOC (Instructional Objective and Context). Here the students had a more divided opinion about their instructor's practices. This domain describes what students perceive about their instructor's pedagogical methods. It can be observed that the scores are lower than in the previous two sections. Remarkably, two questions have a score lower than 4.

Additionally, the standard deviation for all questions is greater than 1. This indicates that students had different opinions about these questions (see Table 27).

Table 27

Mr. Morales: Mean and standard deviation for Domain C

Mr. Morales	Mean	Standard deviation
Question 1: My teacher makes me clearly understand objectives of this course.	4.18	1.25
Question 2: My teacher provides an appropriate interaction or good atmosphere.	4.45	1.04
Question 3: My teacher pays attention to students' reaction during class and adjusts his/her teaching attitude.	4.36	1.43
Question 4: My teacher creates a classroom circumstance to promote my interest in learning.	4.18	1.40
Question 5: My teacher prepares some additional teaching materials.	3.73	1.62
Question 6: My teacher copes with our classroom context appropriately.	4.45	1.21
Question 7: My teacher's belief or value in teaching is active and aggressive.	3.80	1.55
Average	4.16	1.35

Mainly questions 5, 2, and 7 stand out. Question 5 (my teacher prepares some additional teaching materials) has the lowest mean value, with a score of 3.73. And same as with the other domains, this question has the highest standard deviation (1.62). On the other hand, questions 2 (my teacher provides an appropriate interaction or good atmosphere) and 7 (my teacher's belief or value in teaching is active and aggressive) have the highest mean value (4.45). However, the question where students had more consensus, in other words, with the lowest standard deviation, was question 2, with a value of 1.04.

The difference in opinion can be observed in the following table. In this table, students were granted more diverse scores than in previous questions, especially scores 1 and 2.

Table 28

Mr.Morales: Most Popular Answer Choice Given by Students in Domain C

Score	Percentage
1	6.62%
2	13.25%
3	2.60%
4	11.95%
5	65.58%

Finally, domain 4, KSU (Knowledge of Students' Understanding), or the domain that explores how students perceive instructors' methods to check if they are learning during and after the lesson. Among all four categories, this domain has the students' lowest scores and the highest standard deviations. Particularly question 6 (my teacher's assignments facilitate my understanding of the subject) has the lowest average with a score of 3.64. And consistent with the previous domains, it is also the question with the highest standard deviation.

By the same token, three questions have a higher mean value of 4; these questions are the first one (my teacher realizes students' prior knowledge before class), question 4 (my teachers' assessment methods evaluate my understanding of the subject), and question 5 (my teacher uses different approaches (questions, discussions, etc.) to find out whether I understand). Nevertheless, the question with the lowest standard deviation was question 7 (my teacher's test helps me realize the learning situation).

Table 29

Mr. Morales: Mean and standard deviation for Domain D

Mr. Morales	Mean	Standard deviation
Question 1: My teacher realizes students' prior knowledge before class.	4.00	1.26
Question 2: My teacher knows students' learning difficulties of subject before class.	3.82	1.60
Question 3: My teacher's questions evaluate my understanding of a topic.	3.73	1.56
Question 4: My teacher's assessment methods evaluate my understanding of the subject.	4.00	1.18
Question 5: My teacher uses different approaches (questions, discussion, etc.) to find out whether I understand.	4.00	1.55
Question 6: My teacher's assignments facilitate my understanding of the subject.	3.64	1.57
Question 7: My teacher's tests help me	3.91	0.94

realize the learning situation.		
Average	3.87	1.38

Lastly, the following table reflects the increase in the standard deviation. This represents how students have a more diverse opinion about their instructor's practices to measure and understand their learning. It is observed that all answer choices were chosen more often than before.

Table 30

Mr. Morales: Most Popular Answer Choice Given by Students in Domain D

Score	Percentage
1	11.69%
2	7.79%
3	10.39%
4	22.08%
5	48.05%

4.2.2 Students' Surveys, Mr. Keller

Similarly, to Mr. Morales, the mean and standard deviation were calculated for each of the domains in the survey. In this class, 16 students participated in the survey. First, the following values were obtained for section A SMK (Subject Matter Knowledge) (see Table 31). The average score obtained for Mr. Keller was lower than the previous instructor. Additionally, the standard deviations were greater.

Table 31

Mr. Keller: Mean and standard deviation for Domain A

Mr. Keller	Mean	Standard deviation
Q1: My teacher knows the content he/she is teaching	4.81	0.54
Q2: My teacher clearly explains the content of the subject.	4.13	1.02
Question 3: My teacher knows how theories or principles of the subject have been developed.	4.44	0.89
Question 4: My teacher selects the appropriate content for students.	4.44	0.96
Question 5: My teacher knows the answers to questions that we ask about the subject	4.56	0.73
Question 6: My teacher explains the impact of subject matter on society.	3.64	1.01
Question 7: My teacher knows the whole structure and direction of this class.	4.56	0.73
Average	4.37	0.84

Particularly, students gave the lowest score of 3.64 to Mr. Keller in question 6 (my teacher explains the impact of subject matter on society). Although the values for the standard deviation were high, this question did not have the highest value. For Mr. Keller, the highest standard deviation was found in question 2 (1.02), my teacher clearly explains the content of the subject. This indicates more discrepancy in students' perceptions than in question 6.

By comparison, the question with the highest average was given to question 1 (4.81), or my teacher knows the content he/she is teaching, which also has the lowest standard deviation (0.54). This implies that students have similar perceptions about their lecturer's content knowledge.

Then, as with instructor one, the average of each answer choice for the first domain was calculated. Here, it can be observed that the higher values obtained for the standard deviation for this instructor are reflected in how students used lower score values. Nevertheless, most of the students used five as the most used score.

Table 32

Mr. Keller: Most Popular Answer Choice Given by Students in Domain A

Score	**Percentage**
1	0.0%
2	4.72%
3	14.80%
4	19.39%
5	61.10%

For the second domain, students used lower values to respond to the questions (see table 33). This is reflected in the overall average score (4.13), almost 0.24 less than in the previous section. Similarly, when can observe an increase in the standard deviation, from 0.84 to 1.41, describing more discrepancy in students' perceptions. In particular, question 3 (my teacher's teaching methods keep me interested in this subject) had the lowest score, with a mean of 3.38 and the highest standard deviation (1.41).

In the same way, the highest mean was given to question 1 (my teacher uses appropriate examples to explain concepts related to the subject matter), which is also the question with the lowest standard deviation (0.58).

Table 33

Mr. Keller: Mean and standard deviation for Domain B

Mr. Keller	Mean	Standard deviation
Question 1: My teacher uses appropriate examples to explain concepts related to the subject matter.	4.75	0.58
Question 2: My teacher uses familiar analogies to explain concepts of subject matter.	4.20	1.21
Question 3: My teacher's teaching methods keep me interested in this subject.	3.38	1.41
Question 4: My teacher provides opportunities for me to express my views during class.	4.63	0.62
Question 5: My teacher uses demonstrations to help explaining the main concept.	4.44	0.73
Question 6: My teacher uses a variety of teaching approaches to transform subject matter into comprehensible knowledge.	3.63	1.31
Question 7: My teacher uses multimedia or technology (e.g.,	3.94	1.29

PowerPoint) to express the concept of subject.		
Average	4.13	1.02

The previous values are reflected in the following table (see table 34), where it can be observed a significant reduction in the average use of answer choice four and a substantial increase in the use of answer choices three and four. This explains the large values obtained for the standard deviation.

Table 34

Mr. Keller: Most Popular Answer Choice Given by Students in Domain B

Score	**Percentage**
1	3.57%
2	7.26%
3	16.25%
4	17.86%
5	55.06%

Next, section C or IOC (instructional objective and context) had lower values than the previous two domains; compared to domain one, section C reduced in average 0.45 average points (from 4.37 to 3.92), and compared to domain two, a reduction of 0.21 (from 4.13 to 3.92) see Table 35. Similarly, the average standard deviations of domains 1 and 2 increased from 0.84 and 1.02 to 1.07.

In particular, question 7 (my teacher's belief or value in teaching is active and aggressive) had the lowest average score with 3.29 points and the highest standard deviation of

1.49. On the contrary, the highest mean score was given to question 6 (my teacher copes with our classroom context appropriately) with a mean of 4.29 and the lowest standard deviation of 0.83.

Table 35

Mr. Keller: Mean and standard deviation for Domain C

Mr. Keller	Mean	Standard deviation
Question 1: My teacher makes me clearly understand objectives of this course.	3.81	1.05
Question 2: My teacher provides an appropriate interaction or good atmosphere.	4.25	0.93
Question 3: My teacher pays attention to students' reaction during class and adjusts his/her teaching attitude.	4.25	1.13
Question 4: My teacher creates a classroom circumstance to promote my interest in learning.	3.88	0.96
Question 5: My teacher prepares some additional teaching materials.	3.73	1.10
Question 6: My teacher copes with our classroom context appropriately.	4.29	0.83
Question 7: My teacher's belief or value in teaching is active and aggressive.	3.29	1.49
Average	3.92	1.07

This increase in the standard deviation and reduced average scores can be observed in the following image. It can be noticed that the average of times that a five was given as an answer was decreased significantly from the last two sections (from 61.10 and 55.06% to 41.88%), while the number of times a student scored a question with a three increased considerably (from 14.08 and 16.25% to 27.21%).

Table 36

Mr. Keller: Most Popular Answer Choice Given by Students in Domain C

Score	Percentage
1	3.06%
2	6.37%
3	27.21%
4	21.48%
5	41.88%

Last, the average scores in domain D or KSU (knowledge of students' understanding) were the lowest scores for Mr. Keller. For this section, the mean score was 3.69, the lowest score of all instructors in all the domains. At the same time, the standard deviation is, on average, 1.14, the highest value for this instructor.

The lowest value obtained was for question 2 (my teacher knows students' learning difficulties of the subject before class), with a mean value of 3.13 and a high standard deviation of 1.26. On the other hand, the highest average value (4.06) were questions 4 (my teacher's assessment method evaluates my understanding of the subject) and 6 (my teacher's assignment facilitates my understanding of the subject).

Table 37

Mr. Keller: Mean and standard deviation for Domain D

Mr. Keller	Mean	Standard deviation
Question 1: My teacher realizes students' prior knowledge before class.	3.25	1.24
Question 2: My teacher knows students' learning difficulties of subject before class.	3.13	1.26
Question 3: My teacher's questions evaluate my understanding of a topic.	3.69	1.08
Question 4: My teacher's assessment methods evaluate my understanding of the subject.	4.06	1.06
Question 5: My teacher uses different approaches (questions, discussion, etc.) to find out whether I understand.	3.69	1.35
Question 6: My teacher's assignments facilitate my understanding of the subject.	4.06	1.00
Question 7: My teacher's tests help me realize the learning situation.	4.00	1.03
Average	3.69	1.14

Thus, the high standard deviation values are represented in the following table. One can see how the distribution of answer choices was more evenly distributed among options, especially in options three and four, while the number of times that students chose five was significantly reduced.

Table 38

Mr. Keller: Most Popular Answer Choice Given by Students in Domain D

Score	Percentage
1	5.36%
2	9.82%
3	26.79%
4	25.89%
5	32.14%

4.2.3 Students' Surveys, Mrs. Dominguez

For Mrs. Dominguez, a total of 26 students signed the consent form. However, only 22 students participated in the survey. The scores obtained for this instructor were the most stable. For instance, only one of the domains had an average standard deviation value greater than 1. Moreover, none of the means of students' scores were below 4.0.

In particular, domain 1 or SMK (subject matter knowledge), the average score was 4.44 with an average standard deviation of 0.69 (see Table 39). The lower value was 3.59 for question 6 (my teacher explains the impact of subject matter on society). This, as consistent with most of the other described tables, also has the highest standard deviation (1.22). At the same time, the highest mean value was 4.82 in question 1 (my teacher knows the content he/she is teaching). Moreover, it has the lowest standard deviation.

Table 39

Mrs. Dominguez: Mean and standard deviation for Domain A

Mrs. Dominguez	Mean	Standard deviation
Q1: My teacher knows the content he/she is teaching	4.82	0.39
Q2: My teacher clearly explains the content of the subject.	4.05	0.72
Question 3: My teacher knows how theories or principles of the subject have been developed.	4.55	0.60
Question 4: My teacher selects the appropriate content for students.	4.68	0.65
Question 5: My teacher knows the answers to questions that we ask about the subject	4.73	0.46
Question 6: My teacher explains the impact of subject matter on society.	3.59	1.22
Question 7: My teacher knows the whole structure and direction of this class.	4.68	0.84
Average	4.44	0.69

Once again, the previous values are reflected in the next table (see Table 40). Here one can be observed that most of the value scores granted by the students were 4 and 5 (88% of all scores). And for this reason, the average standard deviation was low.

Table 40

Mrs. Dominguez: Most Popular Answer Choice Given by Students in Domain A

Score	Percentage
1	0.65%
2	3.90%
3	7.14%
4	27.27%
5	61.04%

Overall, section B or instructional representation and strategies (IRS), behaves like the previous domain. The mean is 4.29, with an average standard deviation of 0.90. Here, the lower mean value is 3.68 for question 3 (my teacher's teaching methods keep me interested in this subject), with the highest standard deviation of 1.17. By the same token, the highest mean value was 4.68 for question 1 (my teacher uses appropriate examples to explain concepts to the subject matter). However, the lowest standard deviation was for question 5 (my teacher uses demonstration to help explain the main concept) with a value of 0.67.

Table 41

Mrs. Dominguez: Mean and standard deviation for Domain B

Mrs. Dominguez	Mean	Standard deviation
Question 1: My teacher uses appropriate examples to explain concepts related to the subject matter.	4.68	0.72
Question 2: My teacher uses familiar analogies to explain concepts of subject matter.	4.18	0.96

Question 3: My teacher's teaching methods keep me interested in this subject.	3.68	1.17
Question 4: My teacher provides opportunities for me to express my views during class.	4.32	1.04
Question 5: My teacher uses demonstrations to help explaining the main concept.	4.62	0.67
Question 6: My teacher uses a variety of teaching approaches to transform subject matter into comprehensible knowledge.	3.95	1.05
Question 7: My teacher uses multimedia or technology (e.g., PowerPoint) to express the concept of subject.	4.64	0.73
Average	4.29	0.90

Then, the average answer choice used by students remained more or less similar to the previous domain. Option five was reduced by only a few points. However, option 4 decreased significantly (about 8%), and option three increased by about (9%). This explains why the average decreased and the standard deviation increased.

Table 42

Mrs. Dominguez: Most Popular Answer Choice Given by Students in Domain B

Score	**Percentage**
1	1.95%
2	3.90%
3	16.30%

4	19.60%
5	58.26%

As mentioned before, the overall averages for Mrs. Dominguez are similar. Indeed, the scores obtained for section C (instructional objective and context, IOC) are very similar to the previous domain. For example, the average for section C is 4.30, while the average in the prior domain was 4.29. Similarly, the average standard deviation is 0.89, while the previous section was 0.90.

Particularly, the lowest average score belongs to question seven (my teacher's belief or value in teaching is active and aggressive), with a value of 3.91 and the highest standard deviation of 1.11. Additionally, the highest average score awarded by the students was questions two (my teacher provides an appropriate interaction or good atmosphere) and three (my teacher pays attention to students' reactions during class and adjusts his/her teaching attitude) with a value of 4.68.

Table 43

Mrs. Dominguez: Mean and standard deviation for Domain C

Mrs. Dominguez	Mean	Standard deviation
Question 1: My teacher makes me clearly understand objectives of this course.	4.09	1.11
Question 2: My teacher provides an appropriate interaction or good atmosphere.	4.68	0.72
Question 3: My teacher pays attention to students' reaction during class and adjusts	4.68	0.78

his/her teaching attitude.		
Question 4: My teacher creates a classroom circumstance to promote my interest in learning.	4.05	1.00
Question 5: My teacher prepares some additional teaching materials.	4.23	0.87
Question 6: My teacher copes with our classroom context appropriately.	4.50	0.74
Question 7: My teacher's belief or value in teaching is active and aggressive.	3.91	1.06
Average	4.30	0.89

Moreover, the averages of the answer choices are more similar to domain one than domain 2. Here, the percentage of students choosing three decreased compared to domain A, while the rate of option 4 increased. The number of students choosing option 5 decreased (see Table 44).

Table 44

Mrs. Dominguez: Most Popular Answer Choice Given by Students in Domain C

Score	**Percentage**
1	0.65%
2	5.84%
3	11.69%
4	25.97%
5	55.84%

Next, section 4 had a reduction of the mean and, consequently, an increase in the standard deviation. However, this change is minimal compared to the changes observed in the other two instructors. For instance, the mean obtained in the KSU (knowledge of students' understanding) domain is 4.04, with an average standard deviation. The difference between the highest average (domain A) score and this domain is only 0.26, while the difference between the highest and the lowest average scores for the other instructors was 0.67 for both instructors 1 and 2 (domains one and four for both instructors).

For this instructor, students gave the lowest score to question 2 (my teacher knows students' learning difficulties of the subject before class) with an average score of 3.41 and a standard deviation of 1.40, the highest value for this instructor. While the lowest value was granted for question 5 (my teacher uses different approaches (questions, discussion, etc.) to find out whether I understand) with a value of 4.36 and a standard deviation of 0.79.

Table 45

Mrs. Dominguez: Mean and standard deviation for Domain B

Mrs. Dominguez	Mean	Standard deviation
Question 1	4.00	1.15
Question 2	3.41	1.40
Question 3	4.05	1.13
Question 4	4.09	1.19
Question 5	4.36	0.79
Question 6	4.14	0.99
Question 7	4.27	0.94
Average	4.04	1.08

Table 46

Mrs. Dominguez: Most Popular Answer Choice Given by Students in Domain D

Score	Percentage
1	3.90%
2	7.79%
3	12.99%
4	30.52%
5	44.81%

4.2.4 All Students' Survey

Much information can be obtained when analyzing all the data together. First, the highest score given by students was to Mr. Morales, question 3(my teacher knows how theories or principles of the subject have been developed). Similarly, the scores obtained by Mr. Morales in domain A were the highest among all domains for all instructors. On the other hand, the lowest score obtained was question 2, domain D for Mr. Keller (3.13, my teacher's questions evaluate my understanding of a topic).

When discussing, section A (subject matter knowledge) (see Table 47) has the highest average values and the lower standard deviation of all domains. As mentioned before, the score given to Mr. Morales in question 3 has the highest score of all questions, but also, Mr. Morales had the highest domain A average score. Likewise, this question also has the lowest standard deviation of all the questions. This indicates that students perceived their instructor to have solid content knowledge. Moreover, Mr. Keller has the lowest average score among the three instructors, and the average standard deviation found in domain A for Mrs. Dominguez was the lowest among all lecturers.

On the other hand, question 6 (my teacher explains the impact of subject matter on society) was the question with the lowest score and the highest standard deviation of section A. This is noticeable in Mr. Keller, who obtained an average score of 3.59. Then this might suggest that students cannot see the impact of mathematics in their everyday life.

Table 47

All Instructors: Mean and standard deviation for Domain A

Section A	Mean	Standard deviation
Q1: My teacher knows the content he/she is teaching	4.80	0.46
Q2: My teacher clearly explains the content of the subject.	4.06	0.99
Question 3: My teacher knows how theories or principles of the subject have been developed.	4.64	0.59
Question 4: My teacher selects the appropriate content for students.	4.55	0.86
Question 5: My teacher knows the answers to questions that we ask about the subject	4.64	0.61
Question 6: My teacher explains the impact of subject matter on society.	3.90	1.12
Question 7: My teacher knows the whole structure and direction of this class.	4.57	0.92
Average	4.45	0.79

In the following domains, all the scores decrease, and the standard deviation increases. For instance, domain B (instructional representation and strategies) (see Table 48) fell 0.15 on the average score and an increase of 0.20 in the standard deviation. Here, the highest score and the lowest standard deviation correspond to question 1 (my teacher uses appropriate examples to explain concepts related to the subject matter). Indicating that students feel that the examples used during class are good for them to grasp the concept. By the same token, the lowest average score and the highest standard deviation were given to question 3 (my teacher's teaching methods keep me interested in this subject). This could be translated to students feeling the need to have different or more varied strategies during the lecture.

In this domain, the highest average score was also given to Mr. Morales, while the lowest score was also given to Mr. Keller. However, lecturer 2 obtained the highest score (4.75) among all three in question 1, but at the same time, the score obtained in question 3 was the lowest (3.38) of all lecturers.

Table 48

All Instructors: Mean and standard deviation for Domain B

All instructors	Mean	Standard deviation
Question 1: My teacher uses appropriate examples to explain concepts related to the subject matter.	4.66	0.75
Question 2: My teacher uses familiar analogies to explain concepts of subject matter.	4.33	0.98
Question 3: My teacher's teaching methods keep me interested in this subject.	3.71	1.34
Question 4: My teacher provides opportunities for me to express my views during class.	4.55	0.84

Question 5: My teacher uses demonstrations to help explaining the main concept.	4.48	0.86
Question 6: My teacher uses a variety of teaching approaches to transform subject matter into comprehensible knowledge.	3.91	1.33
Question 7: My teacher uses multimedia or technology (e.g., PowerPoint) to express the concept of subject.	4.47	0.80
Average	4.30	0.99

In domain C (instructional objective and context), the mean went down to 4.13, a difference of 0.32, and an increase of 0.32 in the standard deviation compared to section A. Here the highest average mean was identified in question 2 (my teacher provides an appropriate interaction or good atmosphere). This explains why students feel comfortable with the environment created by the teacher and students. On the contrary, the lowest average score and the highest standard deviation were given to question 7 (my teacher's belief or value in teaching is active and aggressive). Thus, students might perceive instructors' teaching style as more passive than aggressive.

For this domain, the highest score was given to Mrs. Dominguez (4.31) and the lowest to Mr. Keller (3.93). The highest average for question 2 was also given to Mr. Keller. Notably, question 7 was one of the few questions where all instructors gave an average score value below 4, especially to Mr. Keller with the lowest score of 3.29.

Table 49

All Instructors: Mean and standard deviation for Domain C

All instructors	Mean	Standard deviation
Question 1: My teacher makes me clearly understand objectives of this course.	4.03	1.14
Question 2: My teacher provides an appropriate interaction or good atmosphere.	4.46	0.89
Question 3: My teacher pays attention to students' reaction during class and adjusts his/her teaching attitude.	4.43	1.11
Question 4: My teacher creates a classroom circumstance to promote my interest in learning.	4.03	1.12
Question 5: My teacher prepares some additional teaching materials.	3.90	1.20
Question 6: My teacher copes with our classroom context appropriately.	4.41	0.93
Question 7: My teacher's belief or value in teaching is active and aggressive.	3.66	1.37
Average	4.13	1.11

Finally, section D (Knowledge of students' understanding) had a reduction in the mean of 0.58 points and a reduction in the standard deviation of 0.43 points compared to the first domain.

This section contains the question with the lowest score and the highest standard deviation given by the students in all the surveys, which is question 2 (my teacher knows students' learning difficulties of the subject before class). This might suggest that students perceive that instructors do not reflect on the challenges of learning new material and concepts. In contrast, the highest average score in this domain was question 7 (my teacher's tests help me realize the learning situation). Therefore, this suggests that students feel comfortable with the tests.

When discussing the scores of domain D, Mrs. Dominguez was the only one that maintained an average above 4 points (4.05). And once more, Mr. Keller received the lowest average given by students (3.70), in particular, in question 2 students gave only an average score of 3.13: being this question another example of a question with an average score below 4 points for all instructors.

Table 50

All Instructors: Mean and standard deviation for Domain D

All instructors	Mean	Standard deviation
Question 1: My teacher realizes students' prior knowledge before class.	3.75	1.22
Question 2: My teacher knows students' learning difficulties of subject before class.	3.45	1.42
Question 3: My teacher's questions evaluate my understanding of a topic.	3.82	1.26
Question 4: My teacher's assessment methods evaluate my	4.05	1.15

understanding of the subject.		
Question 5: My teacher uses different approaches (questions, discussion, etc.) to find out whether I understand.	4.02	1.23
Question 6: My teacher's assignments facilitate my understanding of the subject.	3.95	1.18
Question 7: My teacher's tests help me realize the learning situation.	4.06	1.11
Average	3.87	1.22

Then, we can observe that the scores given to Mr. Morales in domains A and B were the highest among all domains. Similarly, Mr. Keller obtained the highest scores among all three in domains C and D. It is also important to remark that Mrs. Dominguez was the only instructor that maintained an average score above 4 points in all domains. Moreover, Mr. Keller obtained the lowest scores in all domains.

4.2.5 Comments Section

A total of 14 students added a comment at the end of the survey (see table 51). These comments were varied and described what they thought of the class, mathematics, or their instructor.

Table 51

All Instructors: Students' Comments

Mr. Morales
Good course, easy to navigate mostly on your own

This course taught me a lot reality wise not math because I still don't understand
Mr. Keller
Good class just a lot of content.
I really don't like math in my opinion. So it's usually boring unless I understand the concept really well.
Professor 2 [name omitted] really knows his stuff, just wish he would help his students grasp the content better. But the reviews he gives are great! Definitely need to apply yourself though.
I like how he teaches, explains and that he is always willing and able to help if you need it.
I get distracted really easy but that's because of me, it's not because of the teacher.
Just about every class period did the same which got monotonous.
Only opinion is that I love this course and the teacher he takes time to teach and make sure we understand would recommend to others.
I do have a hard time understanding Mr. Keller (name omitted) at times.
We all learn differently, therefore is hard to satisfy everyone.
Mrs. Dominguez
Good use of technology, engaging, lack in clarity when explaining new concepts.
A little fast to learn but good explanation.
I really enjoyed the course.

As can be seen, most of the comments were positive. They describe instructors' good practices, like being available for the students. On the other hand, a few of them explained the difficulties they faced, like having to deal with the pace of the class or lack of engagement.

4.3 Interviews

The final phase of this research consisted of semi-structured interviews. A total of 11 students and two instructors participated in the interviews. Five students from Mr. Morales and three from Mr. Keller and Mrs. Dominguez agreed to participate in this phase. Moreover, Mr. Morales and Mrs. Dominguez were interested in contributing to this phase. Unfortunately, Mr. Keller decided not to participate.

For the students, the interview lasted between 20 to 30 minutes long. It is important to remark that the students who participated in this phase were those who participated during the class. On the other hand, instructors' interviews lasted longer, about 30 to 40 minutes. Each interview was recorded digitally with the participant's approval. After performing the analysis of interviews, the following themes describe the results for each instructor.

4.3.1 Students' Interviews

Mr. Morales. As explained before, five students from instructor one participated in this phase. These students shared some generalities. Among these, four out of five are considered a freshman, and only one sophomore. This last student was the only participating student among all that has previously taken the mathematics class for social science. The rest of the students are new to the institution and are coming straight from high school. As described by them, they all learned the content covered in the class. Moreover, all participants mentioned they felt comfortable having the assignments and exams through a website.

Then, after analyzing the transcripts, three codes were identified. The codes are the instructor's teaching practices, real-life examples, and classroom environment.

Students' Needs. During the interviews, students had the opportunity to explain and describe their needs. In particular, they explained how they felt the class pace was too fast for them. For instance, students expressed that they wished the instructor could "slowdown [*sic*] in the course." Some felt the class was moving too quickly, and they did not have the chance to review the content again. Even some explained that if they were to teach the lesson, they would go "a little bit slower."

Moreover, students were satisfied with how the instructor used real-life situations and related them to the topics being covered during the lesson. As explained by one student, he liked how "[the instructor] put it into real-life examples, and that's what I really like." Another student described how "he really showed us how the world is set… [the instructor] uses real-world problems and he compares them with how we solve the problems" or "he compares things that are currently happening and relate them to the question or to the example that is given to us."

However, students feel that even though the instructor used good examples to relate the topics to real-life situations, they needed more in-depth explanations of the mathematical processes. This is described by students when they were exposed that they needed more "details as far as like questions" or "if he could do better in explaining himself more when it comes to math." Another participant explained that he felt that the instructor "isn't clear when he explains his stuff, so there are things I understand and there are things I don't understand at all."

Then, they exposed that maybe a solution to this situation was using different activities during the lesson. Primarily, they explained that they wanted to have group activities where they could help each other. In fact, students explained that "group activities would be a good benefit to some people." For example, another student explained that one solution is to have "some group work, maybe more talking with my classmates, having the chance to talk to them." Similarly, another supported the idea of different activities since "I learned with examples, so maybe if we do more group activities, or maybe if people who do get it if they could explain the way they get it."

No Connections to Math. Besides expressing their needs, students expose what they thought about the class and mathematics and their connections to their life and their major. For example, most of the students indicated that they dislike mathematics. By the same token, most students do not perceive any connections between the concepts covered in the class and their life. Indeed, one student indicated, "I feel like there is no purpose… but math itself, I feel, is not relevant to any major." Another student responded when asked if mathematics was related to his major, "to my career? I don't think so!" and others answered no.

Then, students only perceive the usefulness of mathematics when it comes to their finances. For instance, a student expressed that math only relates to him "as far as figuring out how much taxes will be, or how much money." Another student mentioned that mathematics is connected to their life if they want to be "financially stable," or if they want to know "the proper way of doing math when paying bills."

Asking Questions. The last code obtained after analyzing the interviews is how students feel when asking for help during the lecture. As expressed by the participants, asking questions directly to the instructor is less comfortable than one might think. In other words, students said that they are afraid and uncomfortable when it comes to asking questions. Some believe that "you do get nervous when you ask questions…because nobody wants to look stupid…you don't want to be pointed out, so students, like myself, are afraid of asking questions."

Another student thinks that "some students in there that don't understand it and they just are too afraid to speak up… [and] they are not enough confident in themselves." He even explains that when they do ask questions, they feel "like we are a burden even though he does tell us that it is ok to ask questions, he just makes us feel we're asking too much."

Mr. Keller. Three students from Mr. Keller agreed to participate in this phase. Same as with the previous students, these students shared some characteristics. In this case, all three students are freshmen. Moreover, they all feel they learned enough from this class, and all three participants mentioned feeling comfortable using the class's website to have the activities and tests.

In this case, three different codes were obtained: instructor's content and pedagogical knowledge, instructors' knowledge, asking questions, and adaptation and students' responsibilities.

Instructor's Content and Pedagogical Knowledge. During the interviews, students explained what they thought of the class, Mr. Keller, and his teaching practices. Among this, they described what they perceived of the instructor's content knowledge. For instance, one of them mentioned, "he was really good at math that is for sure." Similarly, another participant expressed that he believes the instructor "is a very very very good mathematician, he is really good at what he does, he is really good at it."

However, students perceive that they need more than having strong content knowledge. Most of them felt the instructor needed to improve his teaching style when dealing with students' needs. For example, one student expressed, "just because he knows what he is talking about he thinks we know?" Another classmate mentioned that the instructor "sometimes seems like he is talking to himself when he is teaching."

Students are also concerned with how he connected or bounded with his students. Students said "I don't know, I don't want to say he was not a good teacher… but he doesn't feel as open in my opinion." Moreover, they also describe that the instructor "doesn't really talk much to students." This has affected students' perceptions of his teaching style and the classroom environment in different ways.

For instance, participants believe that there is a lack of engagement. As explained by a student who said there "wasn't a lot of engagement, you know, in the class, with the way he was teaching." And students perceived this impacted the number of students present during the class. Indeed, they mentioned, "it is like an empty classroom, is like, well none wants to learn? Why to take the class? Maybe [he] should require students to come to class."

By the same token, students recognize the instructor's effort to help them understand the material. They mentioned the professor's willingness to stay after class if they need it since, as perceived by them, he "just wants to make sure you learn." Nevertheless, the time spent per topic did not help in their learning process. Indeed, one participant explains that he felt that "he [the instructor] will just go really quickly, he would just keep on going." Similarly, another classmate felt that "he [the instructor] does a good job at teaching, but it feels rushed, it seems we went by so fast you know? And sometimes we don't touch back on that material, we just move further and further on, and sometimes I feel… I could say unmotivated."

These perceptions of the time spent in the lesson and the instructor's attitudes affected students in different ways, and one of the consequences that students highlighted the most was the sense of openness to asking questions to the instructor.

Asking Questions. Students explained that they believed the instructor tried to be open to students' questions. However, their perception was different. For instance, one of the students mentioned she would like a space where it "is ok to have wrong answers." And according to their perceptions, this was not precisely the case since, in their experience, “when I heard a student asking a question, he kind of makes a face like your dumb.”

Thus, these micro expressions affected students’ intentions to ask questions. For example, another student expressed that when he discussed this situation with his classmates, he said “it seems like some are too scared to ask or hesitant in a way.” As explained by the participants, this situation can result from two conditions. First, students don’t want to be “put in the position where they are in the spot, and they have to speak in front of everyone.” Second, because, as described before, the unintentional instructor’s attitudes and “the way of responding to questions.” This situation is critical to them since “this is a freshman course, it’s gonna be younger kids and they can be intimidated.” However, students know that the learning process is a two-way street where they must participate actively. Thus, they also describe their responsibilities in this process.

Adaptation and Students’ Responsibilities. Students know that the instructor can only partially be responsible for their learning. For instance, they mentioned the importance of attending the class concerning the low assistance rate. One student considered, "I think is just whether you want to come to class or not… it is just school, so either you go to class or not.” Similarly, another mentioned, “even if he thinks that no one even cares about the class, he shouldn’t be discouraged because no one shows up, there are kids that do show up, and school is important to them.”

In the same way, students are conscious of the importance of being capable of adapting to different teaching styles and college settings. For example, participants highlighted how important it was for them to “adapt and mold to his teaching style.” And doing this is really important because knowing how to do this could help them in knowing “what is required to success, and what I need to do to put me in the right path.”

Mrs. Dominguez. Like Mr. Keller, three students from Mrs. Dominguez’s class agreed to participate in this third phase of the research. Unlike Mr. Morales and Mr. Keller, there was one freshman, junior, and senior student. Two did not like mathematics; nevertheless, they all felt they had learned enough from the class and the instructor. Moreover, like the other students, they all felt comfortable having the assignments and the tests online. Then, after analyzing the interviews, two codes arose from the data: instructor’s teaching practices and non-connected mathematics.

Instructor’s teaching practices. During the observations, it was noticed that the instructor used various teaching approaches. This was also reflected in the interviews. For instance, one student mentioned, “she definitely kept the pace of the class in a very good way, she did not rush concepts, she makes sure to take her time to explain it to everybody, I really liked that.” In the same way, another student expressed that she felt that “I had a difficult time with the math, but it was interesting; I think the teacher has to do with this; she does give a good vibe, and she understands what she is talking about” or “they way she taught like some stuff will make it a little bit easier.”

Indeed, students recognize how she made sure that students understood by stopping, answering, and asking questions. For example, one participant mentioned, “I like how she will stop to make sure that everyone will be caught up and make sure to answer everybody’s questions even if it was repetitive.” Likewise, another student explained that “she [the instructor] was always looking around, she did notice students, sometimes you didn’t have to ask, she will ask you: what is wrong? And she saw your face, and if you have a question, she will try to answer it for everyone."

Then this teaching style helped them in being engaged with the class. For instance, one student said that "she will encourage lots of input from students, which was good in keeping us engaged in class." Moreover, when the instructor felt the students were getting distracted, she would stop the lesson and give them a break time. The students well received these practices since they expressed that "when she would see that we were getting kind of tired or when we weren't really paying attention, she would give us breaks, so I thought that was pretty good, and it is pretty good in mathematics especially."

Equally important, students explained how valuable it was to them when the instructor was available during and after class. For instance, a student mentioned "and you can see that she is available for you, inside and outside the class, that was wonderful." But even though students know that the instructor is there to solve their questions, they still perceive the need for different activities to improve their learning.

For example, students proposed using different types of activities like hands-on activities. As one participant expressed, "I need like more hands-on activities…more hands-on activities during class would have helped." Similarly, students express that "more hands-on activities because I feel like just lecturing and lecturing every day is boring for some people, so trying different things, and different activities to get the students engaged."

Non-connected mathematics. Thus, this lack of hands-on activities or real-life applications makes students believe mathematics is useless and ineffectual for their careers. This is clearly reflected in some of the students' comments like the following: "I don't think it would really benefit everybody unless you are specifically studying math or science, but for people in criminal justice or like in other major that don’t really need math I don’t think this class valuable to them.”

Moreover, just like with the students of the other instructors, students only see a real-life connection between mathematics and their finances. For instance, when asked a student if she could think of one way of relating mathematics to her career, she explained that “a little bit, not much because I’m studying psychology… maybe with the money, knowing how much to charge and stuff like that.” Similarly, when asked the same question, another participant mentioned, “I wouldn’t know, honestly, I don’t know why they [the university] would me make take this class… I don’t think math would really help with my major.”

4.3.2 Instructors’ Interviews

Then, besides allowing the students to express themselves, the instructors also had the chance to do it. That is, the observed lecturers were also invited to participate in a 30 to 40-minute interview. Two of the three instructors agreed to participate in phase three. Allowing the instructors to take part in the interview provided a different perspective on the research.

Thus, by analyzing the interviews and connecting them with the students’ interviews, the following codes emerged: the importance of pedagogical knowledge, the importance of applications, and the importance of learning.

Pedagogical Knowledge. During the interviews, both instructors reflected on pedagogical practices in college. They both explained what teaching is for them. Mr. Morales mentioned that, to him, teaching is not “to open the book, turn your back to the students, and write on the board for 60 minutes” but instead, it is to “give the best of me, teach the content, make sure no one has questions, and explain the topic and make it relevant to the student’s lives.” Similarly, Mrs. Dominguez mentioned that her “passion is to teach, to help them to think, to help them make sense… to make them understand that everything they are going to learn can be applied."

However, this responsibility cannot rely only on the instructor; students must actively participate in their learning process. As described by Mr. Morales, "it is a relation, 50%, and 50%, there has to be a commitment, if the student is not willing to learn, and the instructor is not willing to give the best of him, it can't be done." Nevertheless, instructors feel some students are not as committed as they would like. For instance, Mrs. Dominguez explained how she thinks that some "students, they want to have everything as fast as the technology works. They don't want to sit, they don't want to think, they don't want to be part of the discussion."

Then, instructors perceived that the classroom environment and caring for their students could be some ways of helping students to learn. For example, instructors highlight the importance of asking questions and making mistakes as part of their process. Indeed, Mr. Morales mentioned that it is necessary to "create in the classroom a place where you can be wrong, a place where you can ask any question and avoiding mentioning to the student: that is a good question! because all questions are good questions." This argument was also supported by Mrs. Dominguez, who explained that teachers should know how to respond to questions, she believes that teachers should "never say no you are incorrect, or no that is not the answer, what I usually do Is to guide them, help them construct the answer."

Moreover, lecturers also emphasized that caring for the student is important. In other words, as explained by Mrs. Dominguez, the lecturer should "understand that they are individuals, they are not just a number." In fact, she explained that all teachers "should be close to students, to be available for students' questions." Likewise, for Mr. Morales, caring for the students includes being aware of who is there and their needs “to make them [students] feel they are part of the class, of the team, at least for the semester.”

Then, students could be more motivated to learn mathematics by doing this. Nevertheless, the instructors believe that to do this; they should consider students’ interests and needs. In other words, to make the topics meaningful to their lives.

Importance of Mathematics. Then, both instructors agree that it is essential for them to connect mathematics with students’ real-life situations. Examples include “when they [students] open an account, money, how to invest money.” With this in mind, Mrs. Dominguez believes she tries to discuss “something that is usually happening in their lives, bringing the real life to the class.”

In the same manner, Mr. Morales tries to “give problems related to their life with topics relevant to them, we must find ways of proving them that math is everywhere.” But what is most important, Mr. Morales seeks to make them believe that “math is involved in everything, and they must realize it, that mathematics is the key to everything.”

However, helping students realize that mathematics can be applied in their life is a challenging task. Especially when students cannot see these connections.

Learning Mathematics. Teachers are aware of making mathematics relevant to students’ lives. However, students come with previous attitudes that could prevent them from learning mathematics. In this interview, the instructors described two factors preventing students from learning mathematics: being afraid of asking questions and memorizing the steps without comprehending them.

According to instructors, students could be afraid of asking questions because they don’t want to be pointed out. In addition, as explained by Mr. Morales, “they [students] are ashamed of being perceived as the student who doesn’t know or who doesn’t understand, especially when people [instructor or students] use phrases like: but it is so easy! or, you don’t know how to do it?”

Then, to overcome this situation, instructors explained the importance of constructing a welcoming environment. As described before, it is a space free of judgment where students can feel it is ok to make a mistake. For instance, Mrs. Dominguez explained that she promoted this environment just by performing actions like knowing their names, “I knew the students’ name because I wanted to be close to them… and when I saw the face of “I don’t understand” I would ask.” Similarly, Mr. Morales mentioned “it is important to create in your classroom an atmosphere where anyone can ask questions, any question, it doesn't matter.”

Also, both lecturers explained that sometimes mathematics is perceived as a recipe with instructions and steps they must follow. This perception only allows us to see beyond the questions covered in class. Mrs. Dominguez emphasizes transforming that knowledge into applicable knowledge that can be taught to others. For her, to learn is to “eat this knowledge, but also digest it and make that knowledge your knowledge… if you understand it, if you make it your own knowledge, you are going to be able to use it and teach it.”

Similarly, Mr. Morales believes that many students need help making the connection Mrs. Dominguez described. For him, in the process of learning mathematics, “what you learned in kindergarten you would use it for the rest of your life, it doesn’t matter if you learned it during third, fourth or twelfth grade, you will always use that knowledge, and it will always be related one to another.” Thus, students should not see mathematics just as a group of formulas that can be learned, but instead, students should “learn it [mathematics] and understand it, not just memorize the formula, but you should understand how the procedure works and why, you might forget the formula, but if you learn why, you will never forget the process."

4.4 Answering Research Questions

This section discusses how the obtained information previously described can be used to respond to the research questions. Recall that the research questions are the following:

- What type of knowledge and practices of the participating lecturers are observed in a mathematics for social science class?
- How do students perceive the participating lecturers' pedagogical and content knowledge and how does it affect their learning experiences?

4.4.1 Research Question 1

The data gathered from the COPUS protocol served as a guide to answering the first research question (What type of mathematics teacher specialized knowledge and teaching practices of the participating lecturers are observed in a mathematics for social science class?). Remember that this protocol allows the researcher to measure what the teacher and students are doing during a lecture and observe what is happening.

Then, the observed teaching practices will be described to answer the first research question. Next, by using MTSK, the teaching practices will be connected to the type of knowledge that was evidenced during the lecture. With this in mind, a summary of the observations made for each instructor will be presented.

Mr. Morales: Observed Teaching Practices. The teaching style of Mr. Morales was very different from the other two instructors. First, Mr. Morales used a tablet to write his notes that were projected onto a screen. The examples that he provided were not from a worksheet or a book. It was never seen during the observations for the professor to walk between the students or write on any of the available whiteboards.

During the lecture, Mr. Morales would solve a few questions, and in between the examples, he would discuss some examples that were sometimes nonrelated to mathematics. This helped students to be engaged in the class. In addition, a high volume of interaction between the students and the instructor was noticed.

For example, the table provided in section 4.1 (see Table 2) represented a typical lecture day. In this table, one can observe that the discussion took most of the lecture time. Particularly, on this day, the students and instructor discussed a non-math-related topic. Then this interaction was also reflected in the data obtained by observing students (see Table 3). It is noted that students spent much time listening to the instructor, but also, they spent a significant amount of time having a whole class discussion and asking questions. Therefore, the level of engagement in this particular lesson was high (see Table 4).

The tables mentioned above are just examples of what happened during the observations. In general, this behavior was constant throughout the semester. So then, what teaching practices were observed in Mr. Morales' class? To answer this question, three teaching practices will be described, these strategies distinguished Mr. Morales from the other instructors:

Real life examples. As mentioned before, this teacher spent many minutes discussing different topics that were and were not related to mathematics. By doing this, the instructor maintained the average engagement of the students. This can be observed in the overall average engagement level (52% high and 49% medium overall).

Moreover, students felt these discussions were helpful. For instance, one participant claimed during the interview, "he really showed us how the world is set…he uses real world problems, he compares things that are currently happening and relate them to the question or to the example." Similarly, another participant expressed that "I think everybody enjoys his stories."

In the same way, the scores obtained from the survey indicate that most students enjoy this teaching approach, particularly in domain B, which is related to how instructors represent the topics. For instance, in questions: B1 (my teacher uses appropriate examples to explain concepts related to the subject matter), and B2 (my teacher uses familiar analogies to explain concepts of subject matter), is where he got scores of 4.54 and 4.62, respectively.

Finally, Mr. Morales intentionally used this teaching strategy to engage and help students to learn mathematics. During the interview, Mr. Morales emphasized the need to make mathematics real to the students and make it meaningful. For instance, when he was asked, what is one way to motivate students to learn mathematics? He responded: "I give them questions that are related to their life, topics that are relevant to them."

Class Environment. Thus, due to having flexible topics, this class's environment was different than in the other two classes. It was perceived as a more relaxed and informal atmosphere where students talked and asked the instructor more mathematics and non-mathematical questions. Moreover, this welcoming environment could be observed in two aspects. First, the number of students present during the observations was constant throughout the semester. Second, this was the only observed class where students had whole-class discussions (see Table 5).

Then this was also reflected in students' perceptions measured by the survey. Particularly in domain C (instructional objective and context), questions 2 (my teacher provides an appropriate interaction or good atmosphere) and 6 (my teacher copes with our classroom context appropriately), where he got an average score of 4.45 for both questions.

In the same way, Mr. Morales is aware of the importance of this teaching practice. Indeed, he claims that "we must create a place, a classroom where anyone can ask questions, any type of question, it doesn't matter, like my mentor said, it is not wrong to ask questions, what is wrong is not to ask."

In Class Exams. The last teaching strategy implemented by Mr. Morales that distinguished his class was that he was the only teacher to give online, in-person tests. In other words, the students needed to be present during class to take the unit test and the final. This decision could be beneficial to students since the teacher is there to respond to any questions they may have during the test.

Observed Instructor's Knowledge. During the observations, solid mathematical content knowledge (MK) was notable. Particularly, a strong knowledge of topics (KoT). Recall that this definition involves the instructor's understanding of the concepts, processes, rules, applications, connections, and meanings. Furthermore, this knowledge allows teachers to choose appropriate examples that contribute to students' understanding of the application of mathematics.

Thus, the knowledge of topics was observed during the whole class discussions or when the teacher provided some background or history of the concepts. Moreover, KoT was also recognized by the students during the interviews when they mentioned how important it was for them to have these real examples. Additionally, students acknowledged this knowledge in the survey. For instance, the average score provided to the instructor's MK (domain A) was 4.54. This acknowledgment was strongly recognized in question 1A (my teacher knows the content he is teaching), where the students gave an average mean of 4.77.

Regarding the pedagogical content knowledge, it was primarily observed to be a solid KMT (knowledge of mathematics teaching). As defined before, KMT is the knowledge of choosing appropriate examples and representations of the content needed to teach a specific topic. Therefore, this knowledge is intertwined with KoT. Moreover, KMT allows teachers to choose appropriate teaching instruments, including technology.

Additionally, it was often observed that Mr. Morales described the origin and development of different mathematical theories, definitions, and ideas. This knowledge can also be known as the knowledge of practices in mathematics (KPM). Students were somehow aware of this when in question 3A (my teacher knows how theories or principles of the subject have been developed), students gave an average mean of 4.92, the highest score for Mr. Morales, and the question with the lowest standard deviation of all, for this instructor (0.28).

Particularly, when describing the use of technology, this instructor utilized different electronic devices like a tablet. But also, he employed other digital resources like websites and an online graphing calculator. Students appreciated these multiple uses of various digital tools. For this reason, in the survey, in section C, question 7 (my teacher uses multimedia or technology to express the concept of subject), students gave an average score of 4.85, the highest score among all instructors for this question.

Mr. Keller: Observed Teaching Practices. Mr. Keller's teaching practices were very similar to Mrs. Dominguez's; they even shared the same classroom. Particularly, the classroom and teaching practices of Mr. Keller were conducted in a traditional style. In this teaching style, as Michel et al. (2009) described, the professor is in charge of delivering the information in a monologue style to the students. In other words, the students have little to no interaction with the professor and their classmates. They only listened to the instructor and took notes.

In a regular lecture, this instructor spent most of the time lecturing, either by discussing the topic orally or by writing and solving a question on the board. This impacted the relationship he had with the students. As perceived by the observations, students were mostly quiet or just listening to the instructor. The time students had to ask questions or respond to Mr. Keller's questions was minimal. Therefore, a significant reduction in the number of students attending the class was observed. However, it is unclear if they stopped attending the class because they dropped the course or just decided not to attend.

On the other hand, Mr. Keller spent a lot of time-solving examples on the board. All these questions varied in difficulty and complexity from the easiest to the most abstract situations. He was the only instructor that used different websites to solve problems, particularly complex systems of equations and optimization problems.

As an example, the Table presented in the previous chapter (see Tables 11 and 12) depicts the traditional lecturing style of Mr. Keller. In these tables, one can observe that he spent about 83% of the time lecturing (lecturing and lecturing and writing). Whereas students spent about 84% of their time listening to the instructor and writing notes.

By the same token, this impacted students' engagement (see Table 13). For instance, during the same lecture, some students were constantly distracted by using their laptops, but mostly they were distracted by their phones.

So, what teaching practices were observed during the lectures in the semester? For Mr. Keller, the following strategies were observed: traditional teaching style and mathematical language, worksheets, and technology.

Traditional Teaching Style. As described before, Mr. Keller lectured in a traditional way. This teaching style was seen during all the observations. For instance, the following process was observed every time Mr. Keller was about to introduce a new topic. First, the instructor asked students to pick up a set of worksheets with examples that were used to teach the chapter or the section.

With this in mind, Mr. Keller usually began the lectures by explaining or defining the terminology used in the lesson. It was noticed that he was the only instructor that used proper mathematical definitions with jargon and the writing style of a mathematician. For instance, he was the only one that provided exact definitions of terms like function. In fact, this mathematical language was constantly observed during the lectures. It was common that the lecturer would ask questions and lecture using advanced mathematical jargon.

Then, once Mr. Keller introduced the concepts, he would usually read one of the examples aloud and begin writing on the board. During this time, the instructor would continue explaining the process and steps necessary to complete the problem. Following this, the interactions between Mr. Keller and students were minimal. The number of times observed students asked questions was minimal (see Table 13). Here one can notice that students asked questions less than 5% of the time.

Moreover, the time Mr. Keller used to ask the students questions was also minimal compared to the time he spent lecturing. Table 12 shows that the instructor spent about 13% of the time asking questions to the students (see Table 12). However, these questions included questions like “any question?” that were not responded to by the students.

In the previously discussed tables, the time students spent answering questions was about 6% of the time. In comparison, they spent about 6% waiting. By the same token, as mentioned before, Mr. Keller spent about 13% of the time asking questions. This pattern was continuously observed throughout the semester. Consequently, this situation was also reflected in the student survey. In question 5, section D (my teacher uses different approaches (questions, discussion, etc.) to find out whether I understand), the students gave an average score of 3.73.

Worksheets. Similar to Mrs. Dominguez, Mr. Keller used worksheets to guide the instruction. These worksheets contained the definitions and examples that were used during class. As described before, Mr. Keller provided each student with a copy. This practice made accessible to students the opportunity to follow the instructor's pace. Also, by doing this, the students did not spend time copying the problems. Furthermore, all students had access to the same definitions and notes. And, for this reason, in survey question 5C (my teacher prepares some additional reaching materials), Mr. Keller obtained an average score of 4.44.

It is important to remark that these worksheets used during this semester are a new strategy that the mathematics department of the institution is implementing. In other words, all the instructors teaching this class had the option to use them or not during the lecture.

Technology. Finally, the last distinguishable instruction practice observed during the observation was the use of technology. It was explained before that Mr. Keller was the only one that showed students how different websites could help them solve complex systems of equations. Moreover, he also compared the website’s strengths and weaknesses, and which was better to use depending on the problem they were solving.

Additionally, Mr. Keller was the only one who implemented another type of teaching strategy suggested by the mathematics department. This strategy used music videos with lyrics related to the topics covered in the class. In particular, the song played during the observed lesson was related to the topics of systems of equations.

Even though Mr. Keller used technology to improve students’ understanding, he was the lecturer that used technology the least amount of time. Students acknowledged this lack of use of technology, such as, in question 7, section B (my teacher uses multimedia or technology to express the concept of the subject), they gave their instructor a score of 3.94.

Observed Instructor's Knowledge. Without a doubt, it was observed that Mr. Keller had the most solid mathematical knowledge among all instructors—particularly a strong knowledge of topics (KoT). Mr. Keller defined each of the concepts using mathematical language and solved all questions without notes. Students also noted this by scoring section A with the highest value among all domains. More specifically, question 1A (my teacher knows the content he is teaching) was the question with the highest score in his survey.

Moreover, Mr. Keller also proved to have a strong knowledge of the structure of mathematics (SKM). As a review, SKM is the knowledge that allows the instructor to understand the relation of concepts and definitions. It also includes the understanding that different mathematical concepts can be described when increasing the complexity of the topics. It was observed during the lectures that Mr. Keller used to relate other concepts when including previous definitions and knowledge to describe new mathematical concepts.

Similarly, it was also observed that Mr. Keller has a solid knowledge of practices in mathematics (KPM). Recall that KPM is knowing how mathematical knowledge is produced, allowing the instructor to accept or refuse students' reasoning. Moreover, this knowledge is also understanding counterexamples. Then, it was observed during the lectures to the instructor using counterexamples as a teaching practice. Also, the students distinguished this knowledge in question 3C (my teacher knows how theories or principles of the subject have been developed), giving him an average score of 4.44.

Regarding the observed pedagogical knowledge, a solid knowledge of mathematics learning standards (KMLS) was noted, which is reflected in the appropriate selection of instruments to measure student understanding of the topic. This was also appreciated by students when in the survey, students gave the highest scores to questions 6 (my teacher's assignments facilitate my understanding of the subject) and 7 (my teacher's test helps me realize the learning situation) with 4.06 and 4.0 respectively.

Mrs. Dominguez: Observed Teaching Practices. Mrs. Dominguez also had a particular teaching style. First, the environment was more relaxed than with Mr. Keller but less than with Mr. Morales. Second, this instructor employed a teaching style different from the other two. She used a questioning method as a way to guide the class and teach them how to solve problems. Students also identified this teaching technique during the interview. For example, one student mentioned, "she will encourage lots of input from students, which was good in keeping us engaged."

Moreover, this style of conducting the class was also identified by the COPUS protocol. As an example, the data obtained from a randomly selected observation indicates that she spent almost half of the time asking questions while lecturing. On the other hand, she propitiated student participation. It can also be observed in Table 17 how students react to this teaching strategy. Here, students spent considerable time, compared to the other instructors, responding to the questions posed by her. Consequently, this teaching strategy also impacted the student engagement level (see Table 18).

So, what teaching strategies were observed in Mrs. Dominguez's class? To answer this question, three main teaching practices that stood out from the observations will be described: questioning, break times, and use of technology.

Questioning. As mentioned before, this instructor's leading teaching practice was questioning. In a regular lecture, she would constantly ask questions as a way of guiding and helping students. This trend remained constant all the semester (see Table 17). Here, she spent about 20% of the time asking questions (posing questions and lecturing and posing questions). Moreover, because the level of interaction between the instructor and the student was high, she also spent considerable time answering questions from the students (about 13% of the time).

Because of this, students' reaction and behavior to this teaching practice was also constant during the semester. In this case, the time used by students to pose and answer questions was about 29%. Furthermore, this was also reflected in the survey in domain D question 5 (my teacher uses different approaches (questions, discussions, etc.) to find out whether I understand), where students gave an average score of 4.36), the higher value obtained among all instructors for this question.

The students also perceived this teaching strategy. For instance, during the interview, the students explained, "I like how she will stop to make sure that everyone will be caught up and make sure to answer everybody's questions even if it was repetitive." Similarly, another student replied, "she was always looking around, she did notice if students had questions, because sometimes you didn't have to ask, she will ask you…but if you have a question, she will try to answer it for everyone."

Furthermore, Mr. Keller also reflected on the importance of questioning during the interview. For instance, she explained that she will:

> usually look at my students' faces, and when I see the face of "I don't understand" I ask, Do you have a question? And they usually say no, I don't have any questions, but yes you have!... and what I usually do I will guide them by telling them, what do you think about this? About that? I guided them to the correct answer.

Break Times. Besides using questioning to engage students, she also employed a technique that was not observed with the other instructors: she would give break times. This teaching strategy was observed several times, and it was implemented only when Mrs. Dominguez saw a low engagement level. This break usually lasted for 4-6 minutes, and the students had the opportunity to walk outside the classroom.

This strategy was found helpful because, usually, after the breaks, the engagement level increased significantly. And like the other teaching technique, this was also reflected in the student survey, particularly in section C, question 3 (my teacher pays attention to students' reactions during class and adjusts her teaching attitude), where she received an average score of 4.68. While in the interview, a student explained, "when she would see that we were getting kind of tired or when we weren't really paying attention, she would give us breaks, so I thought that was pretty good."

Use of Technology. Finally, Mrs. Dominguez also used technology constantly during her lectures. Just like Mr. Morales, she would use a tablet to write her notes, which were projected onto a screen. Moreover, she also used a digital calculator that was also cast into the screen whenever she needed to solve a problem. This again was also perceived by the students in the survey in question 7 from section B (my teacher uses multimedia or technology to express the concept of subject), where she received an average score of 4.64.

Observed Instructor's Knowledge. Mrs. Dominguez demonstrated solid mathematical knowledge. Particularly a solid understanding of practices in mathematics (KPM) intertwined with knowledge of topics (KoT). This knowledge helped her guide her students, question them, and refute or accept students' reasoning and solutions. In the survey, this was reflected in section

A, question 5 (my teacher knows the answers to questions that we ask about the subject) which the students scored with an average of 4.62.

Yet, for Mrs. Dominguez, it was observed that a more solid pedagogical knowledge than content knowledge when compared to the other instructors. Remarkably, the knowledge of features of learning mathematics (KFLM, knowledge that allows instructors to recognize student learning difficulties and styles) and the knowledge of mathematics learning standards (KMLS, knowledge (knowledge needed to teach at a certain level, the structure and the required instruments to assess students' learning)—reflected in the average of 4.05 given to domain D (knowledge of students' understanding), the highest among all three instructors. Furthermore, Mrs. Dominguez echoed this knowledge's importance during the interview. She mentioned, "my passion is to teach, to help them think, to help them make sense."

Additionally, KFLM also includes understanding the emotional aspects of teaching mathematics, like students' needs, enthusiasm, and anxiety during their learning process. In some way, this knowledge demands the instructor to be close to the students' demands. With this in mind, Mrs. Dominguez demonstrated this knowledge during the lecture when she was constantly looking for ways to answer students' questions. Also, she stated many times during the class that students were welcomed during office hours if they needed extra help. As well she recognized the importance of this during the interview by stating that many other instructors:

> don't get close to the students. They just see them as a number. So I believe that we should be more like personal, to be able to understand they are individuals, they are not just a class…students are a reflection of the teacher, if the teacher doesn't care the student doesn't care.

4.4.2 Research Question 2

The second research question is: how do students perceive the participating lecturers' pedagogical and content knowledge, and how do they affect their learning experiences? To answer these questions, the data obtained from the survey and the students' interviews, combined with what during the observations, were used.

The following table summarizes the average score obtained by all instructors in all domains, where the highlighted values represent the average scores lower than 4 obtained on each domain.

Table 52
Survey's Average Scores

Domain	Average score
Domain A: Subject matter knowledge	
Q1: My teacher knows the content he/she is teaching	4.80
Q2: My teacher clearly explains the content of the subject.	4.06
Question 3: My teacher knows how theories or principles of the subject have been developed.	4.64
Question 4: My teacher selects the appropriate content for students.	4.55
Question 5: My teacher knows the answers to questions that we ask about the subject	4.64
Question 6: My teacher explains the impact of subject matter on society.	3.90
Question 7: My teacher knows the whole structure and direction of this class.	4.57
Average domain A	4.45
Domain B: Instructional representation and strategies	
Question 1: My teacher uses appropriate examples to explain concepts related to the subject matter.	4.66

Question 2: My teacher uses familiar analogies to explain concepts of subject matter.	4.33
Question 3: My teacher's teaching methods keep me interested in this subject.	3.71
Question 4: My teacher provides opportunities for me to express my views during class.	4.55
Question 5: My teacher uses demonstrations to help explaining the main concept.	4.48
Question 6: My teacher uses a variety of teaching approaches to transform subject matter into comprehensible knowledge.	3.91
Question 7: My teacher uses multimedia or technology (e.g., PowerPoint) to express the concept of subject.	4.47
Average domain B	4.30
Domain C: Instructional objective and context	
Question 1: My teacher makes me clearly understand objectives of this course.	4.03
Question 2: My teacher provides an appropriate interaction or good atmosphere.	4.46
Question 3: My teacher pays attention to students' reaction during class and adjusts his/her teaching attitude.	4.43
Question 4: My teacher creates a classroom circumstance to promote my interest in learning.	4.03
Question 5: My teacher prepares some additional teaching materials.	3.90
Question 6: My teacher copes with our classroom context appropriately.	4.41
Question 7: My teacher's belief or value in teaching is active and aggressive.	3.66
Average domain C	4.13
Domain D Knowledge of students' understanding	
Question 1: My teacher realizes students' prior knowledge before class.	3.75
Question 2: My teacher knows students' learning difficulties of subject before class.	3.45

Question 3: My teacher’s questions evaluate my understanding of a topic.	3.82
Question 4: My teacher’s assessment methods evaluate my understanding of the subject.	4.05
Question 5: My teacher uses different approaches (questions, discussion, etc.) to find out whether I understand.	4.02
Question 6: My teacher’s assignments facilitate my understanding of the subject.	3.95
Question 7: My teacher’s tests help me realize the learning situation.	4.06
Average domain D	3.87

In this table, the values below four were considered as low. We can observe that, as expected, the most substantial knowledge for these instructors is what students perceive of the lecturers' mathematical knowledge. This knowledge, as described before, is necessary to teach mathematics coherently and logically. However, even though this knowledge is necessary, it is not a sufficient condition to properly teach students. Then, within the same domain, the lowest score was given to question 6 (my teacher explains the impact of subject matter on society).

For domain B, the average score went down by 0.17 points to 4.13. Recall that this domain includes knowing how the instructors represent and present the content to the students. Here, the lowest scores were given to questions 6 (my teacher uses a variety of teaching approaches to transform subject matter into comprehensible knowledge) and particularly to question 3 (my teacher’s teaching methods keep me interested in this subject).

Next, in domain C, the domain that describes the class environment and context, the score went down again by 0.32 compared to domain A. In this case, the lowest values were given to questions 5 (my teacher prepares some additional teaching material) and 7 (my teacher’s belief or value in teaching is active and aggressive).

Finally, domain D (it describes what students perceived about their instructors' awareness of their learning) received the lowest average score among all. In this case, the difference between this domain and section A is 0.58, obtaining an average of 3.87 points. It can also be observed that in this case, four of the seven questions received a value below 4.0: question 1 (my teacher realizes students' prior knowledge before class), question 2 (my teacher knows students' learning difficulties of subject before class, which was the question that received the lowest score among all (3.45), question 3 (my teacher questions evaluate my understanding of a topic), and question 6 (my teacher's assignments facilitate my understanding of the subject).

Students' Perceptions of Instructors Pedagogical and Content Knowledge. With the information obtained from the survey and with the analysis gained from the interviews, the following trends about students' perceptions of their lecturers' pedagogical and content knowledge were obtained: (1) strong mathematical knowledge; (2) mathematics can only be related to personal finance; (3) teaching methods and practices; and (4) students' needs.

Strong Mathematical Knowledge. According to the collected data, students have the same opinion about their strong instructors' mathematical knowledge. This was consistently repeated in all sources of information. First, as mentioned before, in the survey, domain A (related to mathematical knowledge), particularly question 1, received the highest score. Moreover, the scores given to questions like 3 (my teacher knows how theories or principles of the subject have been developed) and 5 (my teacher knows the answers to questions that we ask about the subject), within the same domain, also support this.

Similarly, students' opinion about instructors' content knowledge was also recognized during the interview process. Comments like "he is a very very very good mathematician," or "he compares things that are currently happening and relates them to the question or to the example," or "I went to her office yesterday to get a little bit of help and yeah it was fine, she was able to help me…she understands what is talking about."

Moreover, their strong mathematical knowledge was also observed in every lecture for each instructor. During the observations, it was noted how the instructors would solve each of the examples without hesitation. Moreover, the instructors also provided definitions and the history or the background to most of the concepts covered during the lesson.

Mathematics can Only be Related to Personal Finances. In addition, students feel that mathematics can only be used in topics related to students' finances or money. Therefore, it can be assumed that the applications covered during the lectures only impacted students in money-related topics. This conclusion can be observed in the low score obtained in domain A question 6 (my teacher explains the impact of subject matter on society).

Again, most of the students also reflected on how they think that applying mathematics only equals finance and money. For instance, some of the comments made by the students when asked if they can see any application of the content covered during the class in their life were "with the money, knowing how much to charge and stuff like that," "in figuring out how much taxes will be, how much money," "the proper way of doing math when paying bills," "I can only see the business part of it," "business involves money and money involves numbers."

On the other hand, during the observations, it was noted that there was a whole chapter related to mathematical business, where different topics included taxes, interests, and bank accounts, among others. And when comparing this chapter to others, they had few applications as the finance chapter. Moreover, the problems with applications used were not as connected to students' reality and interests as the financial problems.

Teaching methods and practices. Among all identified codes in students' perceptions, their experiences with their instructors' teaching methods and practices stand out. In fact, many of the questions in the survey with low average scores are related to this topic. Particularly, questions 5 and 7 from domain C (my teacher prepares some additional teaching material, and my teacher's belief or value in teaching is active and aggressive) and questions 1, 2, 3, and 6 from domain D (my teacher realizes students' prior knowledge before class and my teacher knows students' learning difficulties of subject before class, my teacher questions evaluate my understanding of a topic, and my teacher's assignments facilitate my understanding of the subject).

Students gave low scores to all the previous questions, indicating, in an implicit manner, that they did not feel comfortable at all with their instructors' teaching strategies. Explicitly, during the interviews, students stated their opinions about their instructors' lecturing practices. For example, comments like "seems like he is talking to himself," "he isn't sometimes clear when he explains stuff," "in new concepts she wasn't very clear about the delivery in doing so," "I would get a little bit frustrated trying to understand his teaching methods," "they [instructors] should keep the students more engaged" were expressed during the interviews.

Moreover, some of the comments described above were recognized during the observations. For instance, it was noted that sometimes the instructor explained the topics by providing example after example without giving much time for the students to digest the material. Furthermore, there were cases where it was observed that instructors used definitions or examples that could be difficult to understand for students with a lack of mathematical knowledge. Similarly, it was common for instructors to ask "any questions" without providing much time for the students to respond. Finally, it was also noted a lack of different activities that could boost students' engagement.

So, what do students explain that they need to help them understand mathematics better?

Students' needs. During the interviews, most of the students explained that there could be three ways of improving their learning: modifying the pace of the class so it doesn't feel rushed, having different activities during the instruction, and increasing the engagement level. This, like the other codes, can be identified in the survey in questions 3 and 6 from domain B (my teacher's teaching methods keep me interested in this subject, and my teacher uses a variety of teaching approaches to transform subject matter into comprehensible knowledge), where both questions obtained low scores.

For instance, students explained that they needed "more hands-on activities during the class, that would have helped. I feel like just lecturing after lecturing every day is boring, so trying different things and different activities to get the student engaged." Other students explained "maybe doing groups or something that could allow us to participate a little bit more could have helped students, not only to engage them more but to understand the concept," or "[doing] some group work maybe being more talkative with my classmates, having the chance to talk to them." Also, others expressed, "I think group activities would be a good benefit to some people" or "so maybe if we do more group activities."

Regarding the pace of the class, students also stated that "he'll start going a little fast, at a fast pace." Other students said: "it seems we went by so fast, you know?" "he should slow down in the course," "go a little bit slower," "she needs to slow down," "everything was fast…really fast," or "we did not have enough time in class."

Then, with the previous information and as described by students, what is the impact on students' learning experiences?

Impacts of Instructors' Pedagogical and Content Knowledge on Students' Learning. Students' learning experiences and perspectives were impacted by their instructors' teaching practices and knowledge. Even though each instructor applied different teaching strategies, leading to different MTSK, the result at the end was basically the same, students were not satisfied with their learning experience.

For instance, some students were affected emotionally, impacting their self-esteem and motivation: "we just move further and further on, so sometimes I feel, I could say, unmotivated." Similarly, another student expressed, "maybe I'm just a bad student, but yeah, mathematics doesn't seem to click." Furthermore, student attendance, or student retention, was also affected. Recall that attendance for Mr. Morales was reduced from 18 students to 13 (about a 28% reduction).

Similarly, attendance for Mr. Keller changed from a maximum of 34 students present in class to 17 (50% reduction). And finally, Mrs. Dominguez's student attendance changed from 32 students to 21 (a 34% reduction). Moreover, one student expressed this during the interview: "it is like an empty classroom, is like, well none wants to learn? Why take the class?"

Additionally, students from all three instructors mentioned that instructors' attitudes and the classroom environment affect the comfort level of asking questions. All students agree to have feelings of being hesitant to ask questions. Indeed, some indicated that "asking feels like we are a burden… [the instructor] makes us feel we're asking too much." Other students mentioned that "what I get from my classmates seems like some are too scared to ask or hesitant in a way." By the same token, another student explained that some classmates "are not good at math, I guess they don’t want to be put in the position where they are in the spot, and they have to speak in front of everyone.” Then, as explained by students, much has to do with the environment created by the instructor. For example, one participant indicated that “although it is the students’ choice in their own participation, a lot involves the professor to make the students feel comfortable in the class…people are afraid to ask questions, and it was the case in this class as well.”

Finally, even though all students indicated in the interview that they understood the content of the class well, the main consequence of all the previous data is that students perceive that they cannot see the connections of the learned material to their life. For instance, when asked if they could explain why they needed to know the material, many explained: “no, I can’t think of a reason why” or “to be honest, not really,” “I don’t think so,” “a little bit not so much,” “I feel like there is no purpose,” and “I wouldn’t know honestly…I don’t think it would really benefit everybody unless you are specifically studying math or science.”

4.5 Conclusion

The two research questions were what type of knowledge and practices of participating lecturers are evidenced in a mathematics for social science class? And, how do students perceive the participating lecturers’ pedagogical and content knowledge, and how does it affect their learning experiences? Both research questions were answered by connecting all three sources of information; the observations made using the COPUS protocol, the survey for assessing the students’ perceptions of professors’ pedagogical content knowledge, and the interviews conducted with the students and the instructor.

Consequently, for the first question, it was observed that each instructor taught using different styles and approaches, primarily using real-life examples, traditional teaching style, and questioning. These styles demonstrated that all instructors possess solid mathematical knowledge, while all have varied strengths when discussing their pedagogical knowledge.

On the other hand, students' perceptions about their instructors' pedagogical and content knowledge can be summarized in four codes: (1) strong mathematical knowledge; (2) mathematics can only be related to finances; (3) teaching methods and practices; and (4) students' needs. And all this impacted students' learning experiences mainly in three ways: (1) their self-esteem and motivation; (2) their comfort level when asking questions; and (3) their perceptions of the usefulness of mathematics in their life.

With this in mind, the last chapter will discuss how these findings relate to the existing literature. Moreover, it will also describe some recommendations as well as the limitations of this research.

Chapter 5: Discussion, Conclusion, and Recommendations

Previous chapters described the methodology, analysis and results of this research. These findings can be summarized in the following table.

Table 53

Summary of findings.

Research question	**Data source**	**Main results**	**Implications**
What type of mathematics teacher specialized knowledge and teaching practices of the participating lecturers are observed in a mathematics for social science class?	COPUS protocol	• Observed strong mathematical content knowledge. • Observed varied pedagogical content knowledge. o Mr. Morales: KMT; Mr. Keller: KMLS; Mrs. Dominguez: KFLM.	• Strong mathematical content knowledge is not enough for students to learn mathematics.
How do students perceive the participating lecturers' pedagogical and content knowledge and how does it affect their learning experiences?	Student survey Interviews	• In the survey, students recognized their instructors' CK. However, they also indicated their low level of satisfaction in some pedagogical and teaching practices. • During the interviews, students reflected on what was needed to improve their learning experiences in this class. • Students' perceptions about their instructors' knowledge and teaching practices affected their: o Their self-esteem and motivation. o Their comfort level when asking questions. o Their perceptions about the usefulness of mathematics in their life.	• Students seek different activities different from traditional styles. • Students feel the need to have a space where they feel safe to ask questions.

5.1 Instructor's Pedagogical and Content Knowledge

The previous chapters discussed what the researcher and students perceived about the instructor's pedagogical and content knowledge. It was explained that during the observations, the instructors demonstrated solid mathematical content knowledge. Students also perceived this during the interviews, and it was also reflected in the survey. In a similar study, Jang (2011) found that students also observed their instructors to possess deep content knowledge.

However, when discussing instructors' pedagogical knowledge and practices, they all applied different approaches during the lecture. For instance, Mr. Morales used real-life stories and questions to engage students. Similarly, Mrs. Dominguez used questions to help students construct knowledge and guide the lecture. Nevertheless, Mr. Keller used a more traditional teaching style where he guided the lecture by solving questions on the board most of the time on his own.

Even though the instructors used different approaches to engage students, the class lecture was mainly a teacher-center teacher style. In other words, the instructors were perceived as the gatekeepers of knowledge. This traditional teaching style in college mathematics has been used for a long time. As described by Abdulwahed et al. (2012), "mathematics teaching (or instruction) in higher education has long embraced traditional methods: non-interactive ways of teaching mathematics (ways in which the student is the receiver of delivery from the teacher, but only minimally a participant)" (p. 49).

When discussing the observed pedagogical knowledge, all instructors exhibited the use, to some extent, of all three subdomains of the PCK. Nevertheless, Mr. Morales demonstrated a strong knowledge of mathematics teaching (KMT). Similarly, Mr. Keller demonstrated a solid knowledge of mathematics learning standards (KMLS), and Mrs. Dominguez showed proficiency in the knowledge of features of learning mathematics (KFLM).

Although all interviewed students mentioned they learned enough and recognized instructors' strong mathematical content knowledge, students' learning experiences were not always positive. The negative perceptions affected students' motivations to learn mathematics, students' self-esteem, and students' confidence to ask questions.

5.2 Students' Perceptions of Instructors' Knowledge

As discussed in the previous chapter, participating students' perceptions of instructors' pedagogical and content knowledge was explained in four codes: (1) strong mathematical knowledge; (2) mathematics can only be related to finances; (3) teaching methods and practices; and (4) students' needs. These perceptions have impacted students' learning experiences in different ways.

For instance, students concluded the semester by taking with them the impression that their instructors have deep mathematical knowledge. Also, students believe they learned enough from their lectures. However negatively, some students felt that the teaching style affected their motivation and self-esteem. This impacted their level of comfort when asking questions, and also, students could not recognize any significant application of mathematics in their life except in their finance.

These conclusions are similar to the conclusions of other research. For example, in a study conducted by Kasa et al. (2022), using a modified version of the survey implemented in this research, they concluded that students had low perceptions of their instructors' IRS (instructional representation and strategies). In Kasa et al., IRS refers to "the extent to which the teacher selects teaching tactics that challenge students' initial concepts and activities for instruction” (p. 3). These low scores guided Kasa et al. (2022) to conclude that students had difficulty understanding mathematics, consequently impacting their learning. Similar conclusions were found in this research. For instance, during the interviews, students mentioned that they could not perceive how mathematics can be applied. Also, low scores were given for questions where students reflected on this.

Furthermore, based on the definition of IRS, low scores in this domain indicate that students were not pleased with the activities performed during instructional time. This was also a conclusion of this research, reflected mainly in students’ perceptions of their instructors’ teaching methods and practices in the survey and during the interviews.

Similarly, the scores obtained in this research were similar to those obtained in Jang's study (2011). Just like in the analysis provided in the last chapter, in Jang’s (2011) research, students gave the highest score to the Subject Matter Knowledge (SMK) domain, following instructional Representation and Strategies (IRS), then Instructional Objects and Context (IOC), and finally, Knowledge of Students’ Understanding (KSU) received the lowest score.

Again, in both research, results imply that students recognize instructors’ strong mathematical content knowledge. However, students’ perceptions of their instructors' pedagogical and teaching practices are not as strong as the students’ perceptions of their content knowledge. Moreover, in the same study conducted by Jang (2011), he also found that students had difficulties with their assignments and the class pace. This was also perceived by the participants in this study when several students claimed that the class pace was too fast for them.

Similarly, just like the conclusion obtained by Jang et al. (2011)., the students in this research argued the importance of having diverse activities like group work. Furthermore, as described in the previous chapter, this was reflected in the interviews and survey, but the researcher also observed it.

For these reasons, students' needs must be considered to improve their learning experiences. In this research, students mentioned the need to have a class at a slower pace, have the opportunity of different in-class activities, and increase class engagement. These findings were similar to the work developed by Chuyun (2020), who found that, according to students, effective teaching happens when there exists a teacher-student relationship when the level of engagement increases with students' participation, group work, or activities, and with real-life examples that connect the theory in the books with real-world practices.

Likewise, Bjälkebring (2019) explained that math anxiety ("commonly defined as a feeling of tension, apprehension, or fear that interferes with math performance" (Ashcraft, 2002, p.181)) could be reduced if students can help each other and with group activities. Moreover, he also explains that real-life examples boost students' interest and motivate them to learn and understand mathematics better. Similarly, in the research conducted by Brezavšček et al. (2020), they concluded that college social science students' accomplishment in mathematics is heavily influenced by mathematics anxiety, confidence, student engagement, and background knowledge from high school.

5.3. Implications

Based on the findings about students' learning experiences, the following implications are described.

5.3.1 Strong Mathematical Knowledge is not Enough

It was mentioned before that students recognized during the interviews and in the survey that their instructor possesses solid mathematical content knowledge. However, having strong content knowledge is insufficient for teaching (Milner, 2012). This was exhibited in the survey, in questions related to pedagogical knowledge and practices (especially in domain D), that instructors need to improve their pedagogical knowledge and habits.

As Mrs. Dominguez reflected during the interview, these practices include different practices that help students create bonds with their instructors. For instance, Chuyun (2020) describes that students do not want to be considered just a number. Also, they expect to develop a student-teacher relationship where the instructor gets to know their students with mutual respect and whom students feel comfortable approaching.

Thus, if instructors and higher education institutions are interested in increasing mathematics literacy, they must realize how meaningful these connections are. Instructors should reflect on how important this is to students and what they can do to create this environment.

5.3.2 Need for Different Activities

Besides having a solid student-instructor relationship, students still need something more. Among these needs, participating students explained that they needed something more than just lecture after lecture. This was reflected in the survey in questions related to various activities during the lecture.

As mentioned, students would like to be exposed to different activities promoting student participation. Doing this could also reduce the teacher-centered teaching style, becoming more student-centered. Moreover, group activities help students understand mathematics, reduce mathematics anxiety, or in the development of new ideas Chuyun (2020); Clarke and Jopling (2009); Koçaka et al. (2009).

Then, if instructors and universities are interested in improving students' learning, different activities should be implemented inside the mathematics classrooms. These approaches could help students understand concepts and reduce mathematics anxiety. As Koçaka et al. (2009) explain:

> Learning by the way of group work, which is one of the integrated approach theories encourages students to discuss, criticise and be more attentive whilst rescuing them from memorising information. The researches and our observations show that students who studies mathematics in group work comprehend problems in a better way, put forward new ideas, are in control of the objectives and learn by applying what they understand instead of memorising mathematics (p. 2365).

5.3.3 Freedom to Ask Questions

Turner and Patrick (2004) explain that participation supports students' learning process. There are different types of participation, some observable (e.g., verbal participation and interaction) and others not (e.g., watching or thinking). As explained by Turner and Patrick (2004), participation allows students to reflect on their knowledge and recognize misconceptions. Similarly, instructors benefit from it because it will enable them to explore if students comprehend and identify learning problems.

Different factors influence student participation, including motivation and classroom environment (Turner & Patrick, 2004). When discussing the students' motivation, the authors highlight the role of personal goals as an incentive to learn. At the same time, this motivation is also influenced by the classroom environment (Ames, 1992). This environment is key since it sets instructors' expectations and provides students with clear goals and a structure that could impact students' learning (Turner & Patrick, 2004).

Then, students' perceptions of the environment and their motivation are related to instructors' practices and discourse (Ames, 1992). This statement was reflected in Turner and

Patrick's (2004) research, where they discovered that teachers' expectations and students' personal goals heavily influenced students' participation.

Within the context of this research, various students mentioned being hesitant to ask questions to the instructor. Students highlighted two reasons why they were reluctant to participate. First, they did not want to be called or perceived as 'dumb.' Second, some students mentioned the environment created by the instructor.

Both of these reasons have been recognized by research. First, research indicates that the fear of looking uneducated influences students' decision to participate in class (Jacobson, 2013). Second, as mentioned before, instructors' expectations and discourse are essential in helping students to participate actively in the lesson.

Thus, teaching styles and discourses that promote a safe environment for students to ask questions and make mistakes are transcendental in promoting student participation. As explained by Abdullah et al. (2012):

> positive traits of instructors and the method or style of teaching employed are important motivating factors to stimulate verbal engagements among students in the classroom. An instructor's traits that favored by students are friendly, know each student well, do not criticize the students, always show a good mood, and approachable. With these traits, students do not feel afraid and ashamed to speak up in class (p. 520).

Then, we can see that all implications are connected. In other words, when instructors work on their pedagogical teaching practices, it is easier for them to realize how important it is to bond with students, creating an atmosphere that welcomes students' questions and mistakes. This could also help move to a student-centered teaching style where students could be more comfortable asking questions.

5.4 Conclusions, limitations, and future work

Researchers in different contexts have deeply explored teaching practices in K-12 settings (e.g., Carrillo (2017); Munoz Catalan et al., (2018)); however, more research needs to be conducted in higher education (Speer et al., 2010). Particularly, in college mathematics for social sciences, only one research describing the factors influencing mathematical achievement was found (Brezavšček et al. (2020)). Thus, studies need to be developed that help understand teaching practices and students' experiences in college.

With this in mind, this research focused on understanding how teaching practices and instructors' pedagogical and content knowledge affect students' experiences and perspectives in mathematics for social science classes. Then, this study was developed using a mixed method approach in three stages: class observations using the COPUS protocol, a student survey called students' perceptions of professors' pedagogical content knowledge, and interviews.

Then the MTSK model was used to explore instructors' content and pedagogical content knowledge. It was mentioned before that this model fitted this research because two main reasons. First, MTSK differentiates from other models because other frameworks argue that there exists specialized teacher knowledge that is different for every teaching grade. However, MTSK argues that this specialized knowledge is not static, and it is constantly changing. Then this knowledge becomes a knowledge of 'becoming' instead of 'being.'

Second, since this knowledge constantly evolves, it implies it is also adaptive knowledge. In other words, this knowledge must adjust to the teaching circumstances. Particularly in this project, this skill is important since instructors need to adapt and find ways of motivating students not interested in pursuing a career in STEM. This is a significant teacher ability since, as described in Chapter 2, students' attitudes and motivation are important ingredients that affect their learning process.

Then, using MTSK, the researcher concluded that participating instructors possess solid mathematical knowledge with various teaching practices and pedagogical content knowledge. Moreover, the results obtained from the survey, together with the interviews, revealed how teaching strategies affected students' learning experiences. It was concluded that students acknowledge their instructors' content knowledge; however, students need more than that; they require different strategies that better support their learning and engagement in the class.

For instance, students mentioned the need for different in-class activities that promote learning and student participation. Then, instructors are essential for this to happen; they should create an atmosphere where students feel welcomed, respected, and considered, but also free of judgment, where they can ask questions and make mistakes. Doing this could create a bond between the instructor and student, facilitating student learning. As explained by Chuyun (2020), "effective teaching does not happen when teachers pour information into students and make students take notes; it happens when students are allowed to share opinions and feedback and are involved throughout the process" (p. 324).

However, the conclusions of this research are limited by several factors. For instance, the number of participating lecturers and students was small to generalize the findings. Moreover, different types of institutions (two and four-years), instructors with varied teaching experience, grading tools, and course expectations (e.g., grading approach) also influenced and limited the obtained results. Also, other factors influenced the pace of this research; for example, Mr. Morales was sick for several days, limiting the number of observations. Similarly, Mr. Keller did not participate in the interview, limiting the analysis of instructors' perspectives. Also, since attendance was not mandatory for any of the classes and the assignments and tests were online, the number of students present during instruction varied significantly, affecting the observation of student-instructor interactions and the number of surveys. Also, the students who participated in the interview phase were the same students who participated in class, restricting the perspective of other students who only participated a little during the course.

Then there are multiple suggestions for future work. First, a more extensive project is recommended to allow for more observations and, thus, more surveys and interviews. Also, as explained before, the students who participated in the interviews were the same students who participated in class. Then, a deeper qualitative analysis needs to be done to consider the experiences of those students who are hesitant to participate.

Second, the number of conducted research exploring college student participation in mathematics classes is minimum. Therefore, it is suggested that future work also focuses on understanding why students resist participating and how this affects their experiences in college. Finally, there is an increase in the number of online classes. Since this class is considered a core class for many non-STEM programs, research should also focus on how this virtual environment affects student learning and progression.

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Glossary

Achieving the Dream Leader College. Community colleges are part of a national initiative that seeks student success using data to make decisions. This initiative also pursues to encourage community engagement and research.

Hispanic-Serving Research University. Universities that belong to the Alliance of Hispanic-Serving Research Universities. These universities are Hispanic serving institutions that belong to the top 5% in the country for research.

In addition, teachers' KFLM allows them to understand students' learning styles and foresight topics that will be difficult for them to understand and anticipate students' strengths. Therefore, it incorporates the knowledge of students' techniques and approaches used by students to learn mathematics. For this reason, it also includes emotional aspects like students' anxiety, motivations, interests, and expectations.

Knowledge of features of learning mathematics (KFLM). KFLM relates to the knowledge of learning mathematics where the emphasis is the mathematical content and not the learner. KFLM relates to the teachers' consciousness of how:

Knowledge of mathematics learning standards (KMLS). KMLS refers to the knowledge about the content needed to be covered at any particular level. Therefore, teachers with KMLS know how to choose the topics that allow students to learn and understand that specific topic. For this reason, KMLS is also related to the knowledge of the structure and order of mathematical topics. Finally, by understanding the sequence of topics and their content, professors with KMLS will know how to design proper instruments that measure students' comprehension, application, and construction of mathematical knowledge.

Knowledge of mathematics teaching (KMT). This knowledge is intertwined with MK and mathematics education theories. KMT allows teachers to choose appropriate activities, examples, representations of the material, and techniques for teaching mathematics. Including the knowledge of proper teaching instruments like textbooks, different types of resources, including technology. In addition, it enables teachers to understand the limitations and obstacles.

Knowledge of practices in mathematics (KPM). In this knowledge, the role of counterexamples stands out. To put it differently, teachers with proper KPM know when and how to use counterexamples, their definition, making deductions and inductions. Also, KPM refers to the way mathematics is produced; therefore, it also implies understanding how new mathematical knowledge is developed, allowing teachers to accept or refute students' reasoning.

Knowledge of the structure of mathematics (KSM). The understanding of the relation among different mathematical concepts and definitions are included. It involves the comprehension of increasing the complexity to delineate the definition of a similar mathematical concept. Similarly, this knowledge enables teachers to understand the connections of simplification, for example, using natural numbers to explain the simplification of algebraic expressions. In addition, KSM includes the knowledge of auxiliary connections, in other words, the use of auxiliary elements to explain other topics. Finally, KSM includes transverse connections. That is, when the teachers understand the connection between different content items.

Knowledge of topics (KoT). KoT comprises the *what, in what why,* teachers know the content they teach. This knowledge includes concepts, procedures, facts, rules, theorems, properties, principles, definitions, methods, connections, and meanings. Additionally, this type of knowledge enables teachers to select appropriate examples with appropriate contexts and purposes, awareness of uses, applications, and how they can be represented. Finally, in this subcategory, it is included what is expected from the students to learn.

Mathematical knowledge (MK). Mathematical Knowledge is one of the two main elements of the MTSK model. For Carrillo-Yañez et al. (2018), mathematics can be seen as intertwined connections of knowledge with specific rules and properties. The understanding of these connections allows teachers to teach mathematical content logically and coherently.

Mathematics specialized teaching knowledge (MTSK). The model is used to explore teacher knowledge. This model was developed by Carrillo-Yañez et al., (2018). This framework suggests that teacher knowledge can be divided into two subdomains: mathematical knowledge and pedagogical content knowledge.

Pedagogical content knowledge (PCK). PCK is "that knowledge in which the mathematical content determines the teaching and learning which takes place" (Carrillo-Yañez et al., 2018). In this domain, the Knowledge of Mathematics Teaching (KMT), Knowledge of Features of Learning Mathematics (KFLM), and Knowledge of Mathematics Learning Standards (KMLS) are included.

> Students think and construct knowledge when tackling mathematical activities and tasks. It includes understanding the process pupils must go through to het to grips with different content items, and the features peculiar to each item which might offer learning advantages or, conversely, present difficulties…takes account of the teacher's knowledge about their student's manner of reasoning and proceeding in mathematics (in particular their error areas of difficulty and misconceptions),

which informs his or her interpretation of their output (Carrillo-Yañez et al, 2018, p. 19).

Appendices

Appendix A: Classroom Observation Protocol for Undergraduate STEM – COPUS

This protocol allows observers, after a short 1.5 hour training period, to reliably characterize how faculty and students are spending their time in the STEM classroom.www.cwsei.ubc.ca/resources/COPUS.htm † For further information, see:

† *This protocol was adapted from: Hora MT, Oleson A, Ferrare JJ. Teaching Dimensions Observation Protocol (TDOP) User's Manual. Madison: Wisconsin Center for Education Research, University of Wisconsin–Madison; 2013.*

Smith MK, Jones FHM, Gilbert SL, and Wieman CE. 2013. The Classroom Observation Protocol for Undergraduate STEM (COPUS): a New Instrument to Characterize University STEM Classroom Practices. CBE-Life Sciences Education, Vol 12(4), pp. 618-627

Observation codes 1. Students are Doing

L Listening to instructor/taking notes, etc.

Ind Individual thinking/problem solving. Only mark when an instructor explicitly asks students to think about a clicker question or another question/problem on their own.

CG Discuss clicker question in groups of 2 or more students

WG Working in groups on worksheet activity

OG Other assigned group activity, such as responding to instructor question

AnQ Student answering a question posed by the instructor with rest of class listening

SQ Student asks question

WC Engaged in whole class discussion by offering explanations, opinion, judgment, etc. to whole class, often facilitated by instructor

Prd Making a prediction about the outcome of demo or experiment

SP Presentation by student(s)

TQ Test or quiz

W Waiting (instructor late, working on fixing AV problems, instructor otherwise occupied, etc.)

O Other – explain in comments

2. Instructor is Doing

Lec Lecturing (presenting content, deriving mathematical results, presenting a problem solution, etc.)

RtW Real-time writing on board, doc. projector, etc. (often checked off along with Lec)

FUp Follow-up/feedback on clicker question or activity to entire class

PQ Posing non-clicker question to students (non-rhetorical)

CQ Asking a clicker question (mark the entire time the instructor is using a clicker question, not just when first asked)

AnQ Listening to and answering student questions with entire class listening

MG Moving through class guiding ongoing student work during active learning task

1o1 One-on-one extended discussion with one or a few individuals, not paying attention to the rest of the class (can be along with MG or AnQ)

D/V Showing or conducting a demo, experiment, simulation, video, or animation

Adm Administration (assign homework, return tests, etc.)

W Waiting when there is an opportunity for an instructor to be interacting with or observing/listening to student or group activities and the instructor is not doing so

O Other – explain in comments

Date: ____________ **Class:** __________________ **Instructor:** ____________________ **No. students** ______ **Observer Name:** _____________________

Classroom arranged how? ____

1. **L**-Listening; **Ind**-Individual thinking; **CG**-Clicker Q discussion; **WG**-Worksheet group work; **OG**-Other group work; **AnQ**-Answer Q; **SQ**-Student Q; **WC**-Whole class discuss; **Prd**-Predicting; **SP**-Student present; **TQ**-Test/quiz; **W**-Waiting; **O**-Other

2. **Lec**-Lecturing; **RtW**-Writing; **FUp**-Follow-up; **PQ**-Pose Q; **CQ**-Clicker Q; **AnQ**-Answer Q; **MG**-Moving/Guiding; **1o1**-One-on-one; **D/V**-Demo+; **Adm**-Admin; **W**-Waiting; **O**-Other

For each 2 minute interval, check columns to show what's happening in each category (or draw vertical line to indicate continuation of activity). OK to check multiple columns.

COPUS																													
	1. Students doing													2. instructor doing												3. Engagement			Comments: EG: explain difficult coding choices, flag key points for feedback for the instructor, identify good analogies, etc.
min	L	Ind	CG	WG	OG	AnQ	SQ	WC	Prd	SP	T/Q	W	O	Lec	RtW	Fup	PQ	CQ	AnQ	MG	1o1	D/V	Adm	W	O	L	M	H	
0 - 2																													
2																													
4																													
6																													
8 - 10																													
	L	Ind	CG	WG	OG	AnQ	SQ	WC	Prd	SP	T/Q	W	O	Lec	RtW	Fup	PQ	CQ	AnQ	MG	1o1	D/V	Adm	W	O	L	M	H	
10 - 12																													
12																													
14																													
16																													
18 - 20																													
	L	Ind	CG	WG	OG	AnQ	SQ	WC	Prd	SP	T/Q	W	O	Lec	RtW	Fup	PQ	CQ	AnQ	MG	1o1	D/V	Adm	W	O	L	M	H	
20 - 22																													
22																													
24																													
26																													
28 - 30																													

1. **L**-Listening; **Ind**-Individual thinking; **CG**-Clicker Q discussion; **WG**-Worksheet group work; **OG**-Other group work; **AnQ**-Answer Q; **SQ**-Student Q; **WC**-Whole class discuss; **Prd**-Predicting; **SP**-Student present; **TQ**-Test/quiz; **W**-Waiting; **O**-Other
2. **Lec**-Lecturing; **RtW**-Writing; **FUp**-Follow-up; **PQ**-Pose Q; **CQ**-Clicker Q; **AnQ**-Answer Q; **MG**-Moving/Guiding; **1o1**-One-on-one; **D/V**-Demo+; **Adm**-Admin; **W**-Waiting; **O**-Other

For each 2 minute interval, check columns to show what's happening in each category (or draw vertical line to indicate continuation of activity). OK to check multiple columns.

page 2

	1. Students doing													**2. instructor doing**												**3. Engagement**			**Comments:** EG: explain difficult coding choices, flag key points for feedback for the instructor, identify good analogies, etc.
min	L	Ind	CG	WG	OG	AnQ	SQ	WC	Prd	SP	T/Q	W	O	Lec	RtW	Fup	PQ	CQ	AnQ	MG	1o1	D/V	Adm	W	O	L	M	H	
30 - 32																													
32																													
34																													
36																													
38 - 40																													
	L	Ind	CG	WG	OG	AnQ	SQ	WC	Prd	SP	T/Q	W	O	Lec	RtW	Fup	PQ	CQ	AnQ	MG	1o1	D/V	Adm	W	O	L	M	H	
40 - 42																													
42																													
44																													
46																													
48 - 50																													

Further comments:

Appendix B: Survey

Assessing Students' Perceptions of College Teachers' PCK

Directions for students:

This questionnaire contains five statements about teaching practices which could take place in this class. You will be asked how often each practice takes place. There are no "right" or "wrong" answers. Your opinion is what is wanted. Think about how well each statement describes what this class is like for you. You will be asked to describe freely your personal comments to the course in the end. Be sure to give an answer for all questions.

Draw a circle around

1. if teaching practice takes place Never
2. if teaching practice takes place Seldom
3. if teaching practice takes place Sometimes
4. if teaching practice takes place Often

A. SMK (Subject Matter Knowledge)		**C. IOC (Instructional Objective & Context)**	
1	My teacher knows the content he/she is teaching.	1	My teacher makes me clearly understand objectives of this course.
2	My teacher explains clearly the content of the subject.	2	My teacher provides an appropriate interaction or good atmosphere.
3	My teacher knows how theories or principles of the subject have been developed.	3	My teacher pays attention to students' reaction during class and adjusts his/her teaching attitude.
4	My teacher selects the appropriate content for students.	4	My teacher creates a classroom circumstance to promote my interest for learning.
5	My teacher knows the answers to questions that we ask about the subject.	5	My teacher prepares some additional teaching materials.
6	My teacher explains the impact of subject matter on society.	6	My teacher copes with our classroom context appropriately.
7	My teacher knows the whole structure and direction of this SMK.	7	My teacher's belief or value in teaching is active and aggressive.
B. IRS (Instructional Representation & Strategies)		**D. KSU (Knowledge of Students' Understanding)**	
1	My teacher uses appropriate examples to explain concepts related to subject matter.	1	My teacher realizes students' prior knowledge before class.
2	My teacher uses familiar analogies to explain concepts of subject matter.	2	My teacher knows students' learning difficulties of subject before class.
3	My teacher's teaching methods keep me interested in this subject.	3	My teacher's questions evaluate my understanding of a topic.
4	My teacher provides opportunities for me to express my views during class.	4	My teacher's assessment methods evaluate my understanding of the subject.
5	My teacher uses demonstrations to help explaining the main concept.	5	My teacher uses different approaches (questions, discussion, etc.) to find out whether I understand.
6	My teacher uses a variety of teaching approaches to transform subject matter into comprehensible knowledge.	6	My teacher's assignments facilitate my understanding of the subject.
7	My teacher uses multimedia or technology (e.g. PowerPoint) to express the concept of subject.	7	My teacher's tests help me realize the learning situation.

Comments:

In this course, if you have any learning difficulty or opinion, please describe it as follows.

____

____

Thanks for filling in this questionnaire

Appendix C: Interview Protocols

Interview (Professors' interview)
1. Can you please provide your academic background?
2. Can you please describe your professional experience background?
3. When did you decide to become a math professor?
4. Can you please describe the biggest challenges you have had when teaching math for non-STEM students?
5. What do you think your responsibilities are regarding teaching math for non-STEM students?
6. How do you plan your classes?
7. Can you please describe what you do to ensure students' learning?
8. How do you assess students' learning?
9. How do you think non-STEM students learn mathematics?
10. Can you please describe good pedagogical practices in teaching math for non-STEM students?
11. How important do you think is to have good pedagogical practices?
12. Can you please describe any professional training in pedagogical practices? If any
13. What do you think is the number one factor for students' success and learning?
14. How important do you think you are in students' success and learning?
15. Is there anything else you would like to add?

Interview 2 (Students' interview)
1. Can you please provide your major and classification?
2. Do you like mathematics? Why or why not?
3. What do you think about your MATH 1320?
4. What did the professor do good?
5. What do you think the instructor could have done better during instruction?
6. What will you need to enhance your learning?
7. Do you feel that the techniques used in class were enough to motivate your interest in mathematics?
8. How much do you feel that you learned in this class?
9. What was the biggest pedagogical practice or strength?
10. What was the biggest pedagogical practice or weakness?
11. Is there anything else you would like to do?

Vita

Julio Solis was born in Texas. In December 2012, he obtained a bachelor's degree in applied mathematics at the University of Texas at El Paso. Later, in May 2018, he obtained a masters in computational science from the same university. He has about 8 years of teaching experience in mathematics, and three years as a research assistant.

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