2019-01-01

From Mistakes, We Learn: Variations In Teacher Dis/position Toward Errors In Mathematics Classrooms

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Dedication

This dissertation is dedicated to Koke and Cris the two most important reasons to thrive on challenge, I am so fortunate to have you both as my sons, I love you both. Recuerden siempre que, equivocarse es estar vivo, saber de uno mismo y ser un mejor yo al reflexionar en el error.

“Fallor ergo sum–I err, therefore I am”

St. Augustine

FROM MISTAKES, WE LEARN: VARIATIONS IN TEACHER DIS/POSITION TOWARD ERRORS IN MATHEMATICS CLASSROOMS

by

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DISSERTATION

Presented to the Faculty of the Graduate School of

The University of Texas at El Paso

in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

Department of Teacher Education

THE UNIVERSITY OF TEXAS AT EL PASO

December 2019

Acknowledgements

En primer lugar, doy gracias a Dios por permitirme terminar mi doctorado, por darme la vida y los dones para salir adelante. Jorge, mi amado esposo, gracias por tu amor y tu paciencia. Gracias por demostrarme cuanto me amas a cada momento y creer en mi sin importar si acierto o me equivoco. Koke y Cris, lo más amado, gracias por ser pacientes con mamá, por ser mi motor para seguir adelante, para arriesgarme, para ser mejor. A ti, querida mamá, mi agradecimiento, admiración y amor por tu apoyo durante toda mi vida y por tu ejemplo de esfuerzo y tenacidad. Papá gracias por enseñarme que para lograr cualquier meta tiene que haber una sola persona que crea en mí, yo. Gracias Guimara, Char y Emilia por su amor. Nanis, amiga, gracias por tu apoyo y tu escucha. Gracias a toda la familia por sus palabras de ánimo y por los buenos momentos para sobrellevar el estrés.

To Dr. T, my admiration and appreciation are given to you for your sustained guidance during this endeavor, for listening to me and being a great mentor. Thank you for your wisdom and kindness, without your direction, patience, and expertise, this dissertation would not have been completed. Thank you to my committee members, especially to Dr. Louie, for providing encouragement while challenging me to become a better researcher. Nicole, I would like to thank you for the best gift you have given to me, which is your friendship, I admire you as a researcher, but even more as a human being. I am really blessed to know a wonderful person like you. Dr. An thank you for your kindness toward me during all my time as a doctoral student. I appreciate and admire your passion for teaching and learning.

Lidia, Diana y Laura quiero agradecerles su apoyo incondicional. Gracias por escucharme siempre y animarme a seguir. Gracias por compartir conmigo un pedazo de su vida y de su corazón, las quiero. Dios las bendiga siempre.

To my participants Bianca, Damian, and Ana thank you for your trust and for giving me the opportunity to learn from you.

Abstract

Mistakes are commonly perceived by students and teachers as an evidence of a lack of knowledge and ability (Brown, & Quinn, 2006). Recently, U.S. and Mexico mathematics education reforms has been calling to promote a positive status of errors in mathematics teaching and learning.

The purpose of this mixed-methods study is to examine secondary mathematics teachers' dis/positions toward mistakes in two contexts: their own mistakes and their students' mistakes. The study employs the frame of teacher dispositional functions (Beyers, 2011) which includes cognitive, affective, and conative characteristics. The frame provides a better understanding of teachers' dis/positions toward errors based on the type of frames they enact during classroom episodes. This study also seeks to contribute to the literature with the aim of emphasizing a critical role that teachers' disposition and framing toward mistakes play in student learning and understanding.

The study was conducted using an explanatory sequential mixed methods design. During the quantitative phase, the Error Orientation Questionnaire (EOQ) (Rybowiak, Garst, Frese, & Batinic, 1999) was used to measure mathematics teachers' disposition toward mistakes. The participants for this phase of the study (N=106) were selected using convenient sampling from the US-Mexico border region.

The qualitative phase was conducted using classroom observation protocol and semi-structured interview with a purpose of explaining the quantitative results. In this follow-up, the purposefully selected subset of teachers' (n=3) framing were closely examined at the level of a moment-to-moment classroom interaction in the context of errors (Greeno, 2009).
As integration of quantitative and qualitative phases, narratives of the selected teachers' disposition were unfolded and analyzed using meaning coding technique (Kvale and Brinkmann, 2009).

The study's main finding confirms an alignment between teachers' disposition and their invoked positional frames in mathematics classroom. Teachers' practices reflected their disposition toward mistakes from multiple perspectives including cognitive, conative, and affective characteristics. However, tensions were identified between teachers' understanding of mathematical reform that proposes a productive role of errors in mathematics learning and teachers' attempts to apply teaching strategies that incorporate error analysis. Furthermore, in some cases teaching practices had the unintentional and inadvertent effect of perpetuating correctness as paramount (Louie, 2017). Those cases provided an example of challenges that teachers face when productively using errors in the classroom as suggested by the reform movement. In this study, two opposite error frames were identified: 1) productive framing that provides student autonomy and support for using errors as tools for their learning; 2) non-productive framing that reinforces an idea of student incapacity to cope with their own mistakes and, subsequently, positioning errors as learning deficiencies. Furthermore, the study findings suggest that having a productive disposition toward mistakes does not guarantee teachers' positioning to frame errors productively in mathematics classroom.

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Chapter 1: Introduction

There are just a few things that can be stated categorically and one of those is the fact that we all make mistakes. Thus, "from mistakes, we learn" seems to be an overworked phrase, a contradiction, especially in the education context. Paradoxically, it is in school where mistakes are frowned upon by teachers and commonly perceived by them and by their students as sources of anxiety, shame, and stress. Fortunately, that situation is not experienced in all the mathematics classrooms; it is fair to say that it has been gradually changing.

Using errors as learning opportunities have been pointed out as a way of improving students' reasoning abilities. Research addressing this new approach shows an important trend in the role of errors in teaching and learning processes as instruments that promote a deeper understanding and analysis of mathematical concepts (Booth, Lange, Koedinger, & Newton, 2013; Borasi 1987,1994; Bray & Santagata 2014; Heemsoth & Heinze, 2014; Isotani et al., 2011; Kramarski & Zoldan, 2008; McLaren et al., 2012; Melis, 2005; Schleppenbach, Flevares, Sims, & Perry, 2007; Tsovaltzi et al., 2010; Zimmerman, Moylan, Hudesman, White, & Flugman, 2011).

Moreover, according to the NCTM (National Council of Teachers of Mathematics) Principles and Standards (2000) and to Planes y Programas de la Secretaria de Educación Pública (SEP) [Secretary of Public Education, Programs and Principals] (SEP, 2011/2017) students' learning from mistakes is considered an essential part of the principles of school mathematics since they provide potential learning opportunities. However, in most cases, teachers from the U.S. and Mexico were taught that errors need to be fixed since they are evidence of failure. Thus, sometimes, those beliefs and personal stories play an essential role in teachers' instructional practices (Ball, 1996).

STATEMENT OF THE PROBLEM

According to the pedagogical principles of the last Mexican education reform in 2017, a good teacher must have tools for helping students to identify the error and its origins and, in so doing it, he/she must transform students' errors into learning opportunities (SEP, 2017). In the U.S. the Principles and Standards for School Mathematics of The National Council of Teachers of Mathematics (2000) state that "[students] will be able to recognize the importance of reflecting on their thinking and learning from their mistakes." Clearly, a positive status of errors in mathematics teaching and learning can be noticed in both reforms.

Although, there is wide evidence about how students' errors promote a deeper understanding of mathematical concepts, certainly, teachers are still perceiving those from different perspectives. Willingham, Strayer, Barlow, & Lischka, (2018) stated that some teachers conceive addressing a student's error as a learning opportunity for all the students in the classroom. Whereas, some others consider addressing a student's mistake as an activity to correct that student only. Even when both teachers' positioning considered student's learning, addressing mistakes in a personal manner often make students to "see their mistakes as flaws for which their teachers will judge them" (p. 326). Definitively, it can be assumed that teachers' disposition toward error in the classroom is likely to influence students' attitudes towards learning from mistakes and, therefore, their ability to do so (Steuer & Dresel, 2011; Tulis, 2013).

Addressing errors using teachers' dispositions as an overarching perspective to understand errors is an emerging topic in mathematics education research. Math teaching standards have increased focus on disposition (Stooksberry, Schussler, & Bercaw, 2009). In this regard, some studies (Lannin, Barker, & Tonwsend, 2007; Schleppenbach et al., 2007) stated that students' and teachers' disposition are still a major concern amid educational reforms, based on a constructivist

approach, are implemented. Learning from errors it is not just about the mere act of addressing students‘ errors, but how students' errors are addressed by their teachers what might support students' learning from their mistakes. In other words, teacher's positioning when a student makes a mistake becomes decisive for taking advantage of errors. Students' learning from mistakes depends on teachers' reactions toward them (Gojak, 2013). Hence, learning from errors might represent a major challenge, since the positive status of errors claims in the U.S. and Mexico mathematics reforms and the teachers' disposition and positioning toward mistakes may or may not converge.

Reflecting on what teachers' dispositions are, and at the same time on how their disposition influence their positioning during the teaching and learning process become crucial (Stooksberry et al., 2009). Parrott (2003) stated that teachers' positioning during the specific episodes when mistakes come out are the public expressions of their disposition toward mistakes. Moreover, characterization of teachers' disposition toward mistakes become essential for examining those and then having the basis for analyzing the difference between productive and non-productive disposition once those become active by the type of frames that teachers enact during the specific moment that a student errs.

PURPOSE OF THE STUDY

This study addressed secondary mathematics teachers' disposition toward their own mistakes and their students' mistakes in the context of the U. S. and Mexico border. An exploratory sequential mixed methods design was used, which involved collecting quantitative data first and then explaining the quantitative results with in-depth qualitative data. In the first quantitative phase of the study, the Error Orientation Questionnaire (EOQ) (Rybowiak, Garst, Frese, & Batinic, 1999) was used with the aim of measuring both sides of the border mathematics teachers' disposition

toward mistakes in general terms. Also, a demographic survey was applied to assess whether years of experience, gender, and country, relate to their disposition toward mistakes.

In a second quantitative stage, a second format of the EOQ was applied to collect data from mathematics teachers to measure their dispositions toward mistakes in the context of their own mistakes and their students' mistakes to characterize and operationalize teachers' disposition in relation to cognitive, affective, and conative dispositional functional types (Beyers, 2011).

The qualitative phase was conducted as a follow-up to the quantitative results to explain them. In this exploratory follow-up, the aim was examining teachers' positionings and framings at the level of moment-to-moment interaction in the context of errors. As integration of both quantitative and qualitative phases, a narrative of three teachers' disposition was unfolded considering how teacher framing expressed their positioning during the moment that their own and/or their students' errors appeared. Consequently, constructing a greater understanding of teachers' productive and non-productive dispositions toward mistakes on three different domains---cognitive, affective, and conative (Beyers, 2011) based on the type of frames that teachers enact during class error episodes is the main purpose of this study. Also, supporting teachers to be more purposeful and consistent in their thinking and actions by understanding the convergences or divergences between their dispositions and positioning when errors emerge.

THE SIGNIFICANCE OF THE STUDY

Research on teacher disposition has focused on providing teacher education programs tools for indoctrinating, assessing, and rating candidates' dispositions which has contributed to seeing teachers' disposition as a process that concludes once candidate teachers finish their preparation (Stooksberry et al., 2009). Teacher disposition also has been addressed from a perspective that dictates the kind of disposition that a candidate teacher, or an in-service teacher, must possess with

the aim of classifying him or her as an effective teacher or in absence of those, as a non-effective one. There is limited research on addressing and examining teacher's dispositions toward mathematics and there is a gap in studying teachers' disposition toward mistakes. It was not possible to find any study addressing teachers' disposition toward mistakes in relation to their positioning and how they frame errors in their classrooms.

This study examined secondary mathematics teachers' disposition toward their own mistakes and their students' mistakes and how those were actively expressed during their teaching and learning mathematics in the context of the U.S. and Mexico border region. Unlike existing literature, this study did not aim to instill or providing a tool for inculcating certain dispositions but conversely, it aimed to provide a mean of teachers' framing during their mathematics class error episodes to illustrate the critical role that their disposition toward mistakes play.

This unique perspective allowed for the possibility of disposition toward mistakes by identifying them and the powerful effect on the way that teachers position themselves and their students in the specific context of mistakes. This mixed-methods study allowed gaining a greater understanding of teachers' productive and non-productive dispositions toward mistakes on three different domains---cognitive, affective, and conative (Beyers, 2011). Subsequently, elucidating the teachers in this study employed frames when errors come out that position their students as authors, constructors, or merely receivers of mathematics ideas and concepts.

Instead of finding a way to instill teachers dispositions toward mistakes, I advocate for drawing attention to the narratives at play in the context of errors and how these narratives intertwine with teachers' productive and non-productive dispositions toward mistakes. Since, perhaps, the recognition of familiar realities present in the classroom can help educators find a way through the understanding (of something); afterwards, this can provide a way to building tools for

the development of open dispositions toward mistakes, and consequently, being able to develop the ability to see errors like a learning tool. In the end, recognizing and seeing as valid a larger diversity of teachers' dispositions is a genuine way of bringing meaning into the connection between teachers' dispositions and their positioning which are not other than their active dispositions toward mistakes.

I approached my research with the intent to learn about mathematics teacher dispositions toward mistakes and how teachers' positioning and dispositions might be intertwined during teacher-student interaction in the context of mistakes. And, in turn, unfolding teachers' disposition toward mistakes nature. The following research questions guided the study:

1. To what extent do secondary mathematics teachers' dispositions toward errors differ and/or coincide in the context of their own errors and their students' errors?
2. What teacher positional frames were unfolded at the moment when errors emerged during class?
3. How are teachers' dispositions toward mistakes aligned with teachers' positioning and framing during class?

SUMMARY

The positive status of errors is discussed in the U.S. and Mexico mathematics reforms. However, teachers' dispositions toward mistakes have been not addressed to understand how those converge or not with this positive status. There is a gap between this new educational approach that considers errors as learning opportunities and teachers' dispositions toward error. This chapter introduces a unique perspective that allows the researcher to address mathematics teachers' dispositions toward mistakes that become concrete through teachers' positioning at the moment that a mistake emerges in their classrooms. The present mixed-methods study design, supported

by the Dispositional Functions model (Beyers, 2011), and Framing (Goffman, 1974; Greeno, 2009; Hand, Penuel, & Gutiérrez, 2013) theoretical and conceptual frameworks structured and guided this study.

Chapter II: Literature Review

Mathematics Education and Error Conceptions

This literature review has two parts: The first part includes papers that provide an outlook on the role of errors in mathematics teaching and learning processes; the second part includes articles that provide a clear idea of the different types of errors that can be observed in a mathematics class, teacher mathematical dispositions, and how positioning theory has been used by mathematics educators. Thus, there are two different taxonomies that are complementary; this chapter presents two different parts which together contribute to a full understanding of the topic.

At the same time, it is indispensable to clarify that in this literature review there is not a conceptual differentiation between errors and mistakes. The differentiation was not made due to the following reasons:

1. There is no strong distinction between errors and mistakes made in the research papers that were reviewed. In these articles, the term ‘error’ is used interchangeably with the term ‘mistake’ (e.g. Borasi, 1987, 1994; Melis, 2004, 2005; Schleppenbach, Flevares, Sims, & Perry, 2007; Tulis, 2013).

2. There is an overlap between these terms and even when for certain domains these concepts have different use and connotation (e.g. linguistics), for mathematical research, authors use both concepts indistinctly.

3. Analyzing two terms’ etymologies and prefixes, there is a clear overlapping in their meaning, however, they are different in their origins. The origins of the prefix err are c.1300, from Old French error "go astray, lose one's way; make a mistake; transgress". Mistake’s origins go back to the early c. 1400, "to commit an offense;" late c. 1400, "to misunderstand, misinterpret," from a Scandinavian source such as Old Norse mistaka "take in error, miscarry".

Consequently, the taxonomy contains articles that manage these two concepts indistinctly. I discarded some of the sources that addressed topics related to neurocognitive perspective and error detection. Those articles approached mistakes based on brain responses which are associated with error detection (e.g., Ansari et al., 2011; Dehaene, 2009; Herrmann et at., 2004; Schillinger, De Smedt, Grabner, 2016), therefore, they were all excluded from the review. However, some other articles that addressed errors based on diagnosis pattern analysis (e.g., Ayres, 2001; Ketterlin-Geller & Yovanoff, 2009; Koriakin et al., 2017; Livy & Vale, 2011; Marshall, 1983; Peng & Luo, 2009; Radatz, 1979; Tariq, 2008) were added to the literature review with the aim of classifying mathematical types of errors. In this way, it was possible to have a full understanding of the different types of mistakes that may be observed in a mathematics classroom.

GENERAL OVERVIEW OF ERRORS AS LEARNING TOOLS IN TEACHING AND LEARNING MATHEMATICS

It was indispensable to examine the ways in which theoretical research about mistakes has been addressed due to several reasons. One of those reasons was that having an overview of the different approaches that have been used for studying mistakes in relation to mathematics' teaching and learning process. Studying the theoretical and philosophical paradigms that have influenced error status on mathematics education was a prerequisite for having a clear picture, because this way, the research gaps might be clearly detected.

These studies were classified as theoretical research since they "[…] focus on ideas rather than phenomena" (Arthur, Waring, Coe & Hedges, 2012, p. 10). The first ideas used around using errors as learning tools paradigm have been attributed to Rafaella Borasi (1987) who based her tenets and coined her main concepts on "the work of philosophers and historians of science such

as Kuhn [1970], Lakatos [1976] and McKline [1980] that help us realize that errors have a much more fundamental role in the growth of a discipline." (p.2)

The idea of using errors as learning tools appears in the mathematics teaching scene in the middle of the 1980 and it has become a solid part of mathematics education research. Studies conducted at all different levels of education, from kindergarten (Donaldson, 2017) to college-level (Cherepinsky, 2011; Melis, 2005; Son 2013, 2016; Zimmerman, 2011), and even in teachers' professional development (PD) (Brodie, 2014) provide proof of it.

Seeing mistakes as learning tools is a topic that has been widely addressed in some countries, while in other countries, it remains an emerging topic. For example, only one study from Latin America addressing errors from a learning perspective was found (González, Gómez, & Restrepo, 2015). Instead, errors are still being studied from the remediation perspective (Del Puerto, Minnaard, & Seminara, 2006; Pochulu, 2009; Rico, 1995; Rodríguez-Domingo, Cañadas, Molina, & Castro, 2012; Socas, 2007).

In other words, studies about the error from a remedial perspective represent the research trend in this part of the world. Paradoxically, countries as Mexico introduced an educative reform which states that "it is understood that errors are part of any building knowledge process, thus those must be used as continuous improvement sources" (SEP, 2017, p.85). However, as I mentioned, there is only one study that addresses the role of errors in the mathematical learning process.

Germany and the U.S. are countries where using errors as tools for learning is an emerging topic. This topic has continued growing in importance, particularly, in the last five years and it has been addressed from a cultural perspective by analyzing the idea of using errors as learning

mediators. In other words, it is a clear trend for conceiving mistakes as teaching and learning instruments, which is in a way Borasi's (1987) idea of using errors as a springboard for learning.

Borasi proposed a different way of seeing and using errors in mathematics education by going beyond the idea of using errors for diagnosis or remediation purposes; the author suggested exploring the opportunities the errors offer for the teaching and learning processes instead. In fact, Borasi's studies represented a watershed for addressing mathematics from a humanistic view. Indeed, using errors as springboards converges with the U.S.' and Mexico's mathematics education reforms.

TWO DIFFERENT PARADIGMS: ERRORS AS LEARNING DEFICIENCIES TO ERRORS AS SPRINGBOARDS OF LEARNING

In mathematics instruction there are different, sometimes contradictory, interpretations and uses of errors. A common way of error analysis is the one that involves error patterns—a recurrent error that is commonly committed by students all over the world. That phenomenon was named "epistemological obstacle" by Gaston Bachelard in 1948, and it was subsequently adopted by Guy Brousseau to the mathematics education field (as cited in Radford, 1997). In this regard, Radford (1997, p. 29) explained how "the concept of epistemological obstacle gives to Brousseau a way to interpret some of the recurrent and non-aleatorial mistakes that students make when they learn a specific topic." Brousseau (2006) applied that same perspective in his *Theory of Didactic Situations*, in which errors are stressed.

Error diagnosis has helped mathematics educators to understand students' difficulties or common error patterns. It is easy to find research studies that focused on common errors that students make when specific topics are being taught. For example, studies that provided a clear scenario about decimal fractions, students' misconceptions, and error analysis (Brown & Quinn,

2006; Resnick et al., 1989; Tirosh, 2000) provide valuable information about teachers' and students' knowledge about the topic. However, these studies only analyzed and documented common students' errors, leaving aside teachers' and students' humanistic side because those are focused on errors from a perspective of diagnosing a learning problem.

That error approach has been used by researchers and teachers as a tool to identify learning difficulties and curriculum planning (Borasi, 1996). However, studies about error patterns did not take into account students' participation as agents that can use their own mistakes or their mistakes as learning tools, nor, address errors from their potential as a tool for facilitating teaching and learning.

Borasi (1987) established a different interpretation of errors. Her perspective about errors enables students to participate in the process not only of detection, but also, in the process that involves error's explanation, analysis, correction, and discussion. This transformation is rooted in a constructivist framework where the students play an active role in their own learning process. In constructivism, errors assume a new role since the student is the creator/builder of his/her knowledge and reality through a trial and error process; in this sense, students are capable of learning from their/others' errors (Kilpatrick, 1987).

Borasi provided a new perspective in which error is a helpful tool that students can use to improve their mathematics skills. From her view, errors are used as vehicles to construct a deeper mathematics conceptual understanding. The error according to Borasi:

> can be used as a motivational device and as a starting point for creative mathematical explorations, involving valuable problem solving, and problem posing activities […] Errors can foster a deeper and more complete understanding of mathematical content, as well as of the nature of mathematics itself. (1987, p. 7)

Addressing errors from this perspective allows students not only to identify and adjust their mathematical procedures but importantly, it allows them to change attitudes toward errors and toward mathematics. Thus, students can move forward to analyze conceptual, reasoning, and procedural mistakes instead of calculation mistakes only. Consequently, mathematical errors become instruments that promote a deeper understanding which provided a reason to name them as springboards (Borasi, 1987/1994; Booth, Lange, Koedinger, & Newton, 2013; Bray & Santagata 2014; Heemsoth & Heinze, 2014; Isotani et al., 2011; Kramarski & Zoldan, 2008; McLaren et al., 2012; Melis, 2005; Santagata 2005; Schleppenbach, Flevares, Sims, & Perry, 2007; Tsovaltzi et al., 2010; Zimmerman, Moylan, Hudesman, White, & Flugman, 2011). From this point of view, errors are perceived as the starting point to explore mathematic concepts by supporting students to gain a deeper understanding, improve their critical thinking, redefine their problem-solving skills, and contribute to acquiring metacognitive skills (Kramarski & Zoldan, 2008; Mathan & Koedinger, 2005).

Using errors as springboards has given rise not only to different educational and instructional approaches but also to different types of research studies that are closely connected with this idea. For example, some research studies have focused on the cultural aspects of error-handling practices (Santagata, 2005; Stingler et al., 2006). Cultural issues are influential with students' and teachers' disposition toward errors; furthermore, the ways that they respond to failure may vary significantly depending on their culture.

There are some cultures, especially Eastern cultures (Japan and China), which have been considering errors as learning mediators for a long time with very good outcomes (Stingler et al., 2006). For example, Stingler et al. (2006) found that the U.S. students produce the same number and similar type of mathematical mistakes as Chinese students, but teachers respond in a very

different way. The U.S. teachers were more likely to follow errors with statements or immediate corrections, whereas Chinese teachers asked follow-up questions to prompt student discussion (Schleppenbach et al., 2007). Santagata (2004/2005) provided another important example of these types of studies by addressing differences between teachers from Italy and the U.S. handling of errors. Santagata (2004/2005) found that the U.S. teachers showed to be commonly worried about students' self-esteem, so their strategy was to try not to pay too much attention to errors. On the contrary, in Italy, teachers focus and make a strong and direct emphasis on mistakes to make students responsible for their errors.

ERRORS AS TOOLS FOR CREATING LEARNING OPPORTUNITIES

Teachers of mathematics are familiar with the idea of avoiding mistakes. For instance, once a problem has been posed it must be carefully solved and explained to or by the students, a process where there is no room for mistakes. Thinking about the use of correct examples or procedures is very easy because many of us have had that experience. However, teaching and learning from errors as an instructional tool provide a completely different perspective.

Another important issue that researchers stress is how teachers apply the idea of the use of errors as opportunities to promote the development of students' critical thinking. Teachers need to have and show good attitudes toward errors, in order to be able to set the basis to generate a positive error climate. For example, teachers should avoid expressing too much concern or pay too much attention to a specific student's error, which could make matters worse by making the student more self-conscious about its failure causing anxiety or shame. The opposite can also be the case when a teacher ignores mistakes or corrects them by her or himself with the aim of decentralizing the attention. Borasi (1994) suggested that teachers should correct an error only after giving students an opportunity to notice and correct the error themselves. For example, Schleppenbach (2007)

promoted the idea that "Only by asking students' questions about errors can teachers create the kind of inquiry environment envisioned by researchers like Borasi (1994) and Kazemi (1998)." Teachers' questions have to be versatile, questions that open discussions, even class debates. A type of inquiry that promotes and encourages analysis and discussion, more than a mere explanation.

The use of errors as a tool to enhance students' mathematics skills and conceptual knowledge is relatively a new research topic. According to this approach, students play a leading role in detecting, analyzing, and correcting their own and others' errors. Consequently, the instruction is student-centered, since the teacher's role is to design and facilitate learning activities that provoke and cause conflict with the purpose of inviting students to reconsider their thinking structures (Engler et al., 2004).

However, posing a problem that engages students' critical thinking is not an easy job, nor it is to develop exercises that have one or more mistakes to analyze. Melis (2005, p.3) stated that "it is an art to design examples that include an obvious inconsistency and provoke conflicts." When students are working with correct examples, they do not need to explain why a strategy was misused or incorrect. Nevertheless, when students are working with erroneous examples they have to analyze and explain the problem. According to Tsolvalti et al. (2010) "erroneous examples are worked solutions that include one or more errors that the student is asked to detect and/or correct" (p. 356). Working with erroneous examples require considerable effort by the teachers to design challenging problems with errors that invite students to reflect.

Some authors argued that using errors as tools for creating learning opportunities can be directly observed and recorded by the student that made the errors or from an anonymous student (Tsovaltzi et al., 2010). In this regard, Ingram, Baldry, & Pitt (2014) argued that sometimes it

could be difficult to work on your own mistakes, especially in the case of teamwork, due to the factors that come into play. For example, students' beliefs, attitudes, and feelings need to be open to critique. Working on unidentified third-party mistakes can help as well, without any emotional burden. Thus, fictitious students' examples could serve due to "the student reviewing the errors is freed from embarrassment – and possible demotivation – of having their own errors exposed" (McLaren et al., 2012, p.223).

Zimmerman et al. (2011) and Booth et al. (2013) argued that using errors as learning tools not only give the possibility to the students of analyzing the problem-solving process, but errors also contribute to the development of different mathematical skills. Students not only became aware of evaluating their mathematical processes, but they develop a deeper understanding when they have the opportunity of providing arguments about their mistakes. This reasoning level is achieved by the continuous contact with exercises that promote self-reflective opportunities and self-explanation of the underlying concepts inherent in it, as part of daily assessments, with the aim of comparing, contrasting, evaluating and establishing strategies to solve future problems (Adams et al. 2014).

Some authors suggested that working on erroneous and correct examples together would generate better results due to the use of the correct one as a pattern or contrasting tool (Booth et al., 2013; Cherepinsky, 2011; Tsolvasti et al., 2010; Zimmerman et al., 2011). Booth et al. (2013, p.25) argued that "asking children to explain a combination of correct and incorrect examples can be even more effective." Working with both types of examples (correct and incorrect) have a double purpose, one of which is providing self-feedback and the other one is self-explanation. Self-explanation of a combination of correct and incorrect examples improves conceptual understanding and concept validation (Booth et al., 2013; Zimmerman et al., 2011). In fact,

working with errors is a key competence, especially in the context of informal learning or in a self-learning environment (Lannin, Baker & Townsend, 2007; Mathan & Koedinger, 2005; Schleppenbach, Flevares, Sims & Perry, 2007; Tsolvasti et al., 2010). Thus, using errors as instructional tools is also a way to improve self-questioning.

Moreover, error analysis could be an instrument that facilitates a better understanding of the multiple representations of a mathematical object (Kramarski & Zoldan, 2008). For this reason, in order to have a deeper understanding of a mathematical concept, it is necessary to have the ability to make conversions between the multiple representations (e.g. graphic, table, equation, etc.) (Duval, 1999). Making conversions between different representations might support students' error analysis by guiding students' mathematical abilities in different domains.

On the other hand, Isotani et al. (2011) stated that using errors for learning is not as effective for learning as researchers have hypothesized, since their effectiveness depends on the learning techniques, context (e.g., e-learning, lab setting, etc.), and students' level of knowledge since the advanced students learn more from errors than low-level ones. In other words, students with a higher level of knowledge would benefit more from erroneous examples or error analysis more than those that are struggling to understand a concept. The profit level of using errors as springboards for instructional technique depends proportionally on the student's level of understanding (Heemsoth & Heinze, 2014; Lannin, Barker, & Townsend, 2014; Leikin, Waisman, Leikin & Shaul, 2013; Tsolvasti et al., 2010).

RESEARCH ON TEACHERS' BELIEFS ABOUT ERRORS IN MATHEMATICS

As I mentioned earlier, mathematics education researchers have analyzed instructional strategies used by teachers to handle students' errors. Those studies focused on identifying error-handling practices with the aim of evaluating those practices effectively. In addition to those

studies, there is also a small number of studies that addressed mathematics teachers' beliefs about errors in relation to their teaching and learning practices. Those studies stated that students' and teachers' attitudes toward errors are a major concern due to the negative feelings that emerge when educational reforms or academic programs introduce an instructional approach indicating the use of errors from a productive perspective (Lannin, Barker, & Tonwsend, 2007; Schleppenbach et al., 2007; Steuer, Rosentritt, & Dresel, 2013; Tulis, 2013).

Education research on teachers' beliefs demonstrated that beliefs have a strong influence on teaching and learning (Cabello & Burstein, 1995; Pajares, 1996). Pajares (1992) provided an overview of a variety of ways and kind of beliefs that have been studied in education, for example, teacher beliefs about subject areas like reading, mathematics, or sciences. However, the most common approach to studying teachers' beliefs is to examine teachers' beliefs in relation to teaching and learning practices (Barkatsas and Malone, 2005; Perry, Tracey, and Howard, 1999; Stipek, Givvin, Salmon, and MacGyvers, 2001). Furthermore, teachers' beliefs possess certain peculiarities associated with the different factors that influence their practices, including teachers' beliefs about errors.

Addressing errors using teachers' beliefs as an overarching perspective to understand errors is an emerging topic in mathematics education research. Students' and teachers' beliefs and attitudes toward mistakes are influenced by a wide variety of elements and factors. It can be assumed that teachers' error management behavior in the classroom is likely to influence students' attitudes towards learning from mistakes (Steuer & Dresel, 2011; Tulis, 2013). In this regard, Tulis (2013) found that teachers' beliefs about errors will impact their error management in the classroom, which in turn is highly likely to influence students' attitudes towards learning from

mistakes. Borasi (1987) found that mathematics teachers perceive students' errors as valuable tools and resources for remediation teaching strategies.

Similarly, Santagata (2005) examined teachers' beliefs about mistakes and their error handling practices in relation to how cultural factors impact both, finding that teachers' mistakes handling activities are influenced by their beliefs and cultural practices. Bray (2011) and Tulis (2013) noticed that teachers' positive beliefs about errors and appropriate ways of error-handling practices will impact and change their students' attitudes about errors. The effects of teachers' error handling practices on their students' academic performance are very significant because the way that teachers handle mistakes will affect students' motivation to persist on more complex tasks (Santagata, 2005). Therefore, teachers who perceive and manage errors positively encourage students to move forward to analyze conceptual mistakes instead of mistakes related to the accuracy only.

From that perspective, students and teachers can change their attitudes about errors and improve their analysis skills, stimulate critical reasoning, and increase their enthusiasm by analyzing erroneous answers. However, Tsamir, Rasslan, and Dreyfus (2006) argued that mathematics teachers reject the use of error-based tasks because they believe that this type of task may cause students embarrassment and frustration. Tulis, Steuer & Dresel (2017) stated that students that believe that error is a natural part of the learning process are less likely to experience negative feelings when they are receiving error feedback and at the same time, they are more inclined to view errors as learning opportunities and correct them to overcome their knowledge gaps. Additionally, Tulis (2013) was interested in the support or inhibition that classroom environment might offer to students to make them able to learn from errors. Tulis referred to Oser and Spychiger (2005), who identified the presence of two different error cultures in classrooms: A

positive one, characterized by the encouragement of students to identify, discuss, and reflect on errors. On the opposite, a negative is identified by students perceiving errors as threatening, as leading students to think about errors as poor knowledge and ability.

It becomes evident that the main support of a friendly error environment or culture is the teacher, who in turn will show openness to implicitly or explicitly stating the rules to the classroom error management. According to Schleppenbach (et al. 2007), the first idea about students' and teachers' beliefs and attitudes toward mistakes when they are using errors as an instructional tool is that teachers have to create special conditions in their classroom. This type of classroom should be a place where errors are natural and openly discussed. A place where every student has the possibility of addressing, analyzing, and correcting a mistake, but even more specific, a place where every student has the right to be wrong and not judge or evaluate for that.

From this type of setting, many scholars develop a concept arises that encompasses different issues around students' and teachers' attitudes toward errors which is error climate or error culture (Steuer, Rosentritt, & Dresel, 2013; Tulis, 2013). Those researchers concluded that environments that have a positive error climate produce an enhancement in all learning process areas and vice versa. On one side, when students work in a positive error culture, they acquire a higher level of concept understanding, self-motivation, and academic positive self-concept. On the other side, when there is a negative error climate, students develop anxiety among other negative feelings (Tulis & Ainley, 2011). In this sense, error culture is a concept that brings together not only teachers' positive attitudes and reactions, but negative ones as humiliating students and/or express annoyance, disappointment, or hopelessness about their ability to learn mathematics (Tulis, 2013).

As may be seen, the way teachers perceive mistakes and their preponderant role in the construction of the error culture are two key aspects to use errors as instructional tools in an effective way. Although, sometimes teachers are not able to establish an error culture in their classroom since, as some researchers argued, the U.S. teachers prefer to use expressions that avoid the idea of mentioning that a student has made a mistake or error (Santagata, 2004/2005).

According to Santagata and Bray (2016, p. 549) teachers prefer giving clues to the students that have made a mistake or even hide their students' mistakes since they believe that "errors will confuse or demotivate students". Therefore, they argued about the relevance of focusing teachers' PD on learning how errors embody a useful and effective teaching tool with the aim of changing teachers' beliefs, attitudes, and practices (Santagata and Bray, 2016). It is not only necessary to provide training related to a positive management error by changing attitudes and beliefs; indeed, the main objective would be training teachers to develop the necessary skills to design and pose the type of problems that train students just to detect errors and correct them, but provide them with examples that amplify opportunities to rationalize such problems.

TEACHER'S DISPOSITIONS

According to Katz (1993, p.2) "a disposition is a tendency to exhibit frequently, consciously, and voluntarily a pattern of behavior that is directed to a broad goal." Spliter (2010) defined disposition in the education field and we can notice that a common denominator in those definitions are words like "tendencies," "attitudes," "believes," "values," "actions," "patterns," and "behaviors"; he focused his discussion on how researchers have addressed defining dispositions, origins, nature, characteristics, and scopes.

It is important to highlight that not only Spliter (2010) focused on understanding beyond dispositions per se, but for deepening on their origins and nature. For example, while talking about

disposition nature from an epistemological perspective, John Dewey stated that dispositions are not a state of possession, but the state of performance (as cited by Dottin, 2008). From Dewey's definition, I would like to make a brief analysis of (1) possession and (2) performance etymologies, in order to understand the nature of disposition.

Possession's etymological origins are from the Latin word posse which means to be able and sidēre which means to sit; consequently, possession means to be able to be sit. On the other hand, performance is a compound word formed by the root perform which means to execute, and the suffix -ance which means action or process. Contrasting those two words, we can learn that according to Dewey a disposition nature is not latent but active. Regarding disposition origins, Katz (1993) stated that dispositions are not acquired by students in the process of teaching and learning, but they assimilate dispositions by experiencing them from people who are exhibit specific dispositions around them. More explicitly, students assimilate dispositions from the dispositions that their teachers exhibit.

Experiences and environmental conditions support the manifestation of dispositions (Rogoff, Gauvain, and Ellis, 1990). In this regard, it is essential to understand that teachers are the ones who construct, provoke, and encourage all the different types of experiences and conditions that take place in a classroom. Thus, it is important that a teacher models productive dispositions to their students.

MATHEMATICAL DISPOSITIONS

There is a body of work addressing students' and teachers' dispositions toward mathematics; these studies began to arise when an evaluation standard named as "mathematical disposition" was proposed by the National Council of Teachers of Mathematics. This concept was released on The Curriculum and Evaluation Standards for School Mathematics (1989). In this

document, NCTM stated that "disposition refers not simply to attitudes but to a tendency to think and to act in positive ways," adding that "this kind of information is best collected through informal observation" (NCTM 1989, p. 233). In this regard, disposition goes beyond the idea of including attitudes, but it includes such habits of mind as interest, curiosity, perseverance, confidence in using mathematics, and interest on the role that mathematics plays in the society and culture.

In mathematics education, disposition is a concept that has been widely used by researchers trying to establish and examine mathematics desirable teachers' inclinations or tendencies that pre-service teachers should demonstrate to be considered as a professional and effective teacher (Cruz, 2017; Varol, 2011). In this vein, in teacher education, disposition is a concept that has been studied since they are conceived as predictors of future behaviors. Additionally, those type of studies is focused not only on examining how preservice teachers' mathematical dispositions influence their knowledge and teaching (Feldhaus, 2012; Siegfried, 2012) but in trying to model and instill determinate kinds of dispositions (Varol, 2011). Although, there is a research gap of in-service teachers' effective dispositions, with a different purpose than stilling them or examining their influence and relationship of those and students' outcomes. In this regard, studies assessing mathematics teachers' productive and not-productive dispositions toward mistakes is a topic that seems not being addressed, yet.

The National Research Council (in Kilpatrick, Swafford, and Findell [2001]) defined productive disposition as "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (p. 131). Then, productive dispositions toward mathematics involve tendencies of a teacher to behave in particular ways by perceiving mathematics as something valuable and in this sense, understanding learning mathematics as a

process where her/his consciousness of the context of a situation, and her/his dispositions' awareness will position her/him in a way to direct actions to what the context requires for desired outcomes to be reached.

For considering dispositions, according to Beyers (2011), there are three different types of dispositions toward mathematics as mental processes: cognitive, affective, and conative. In this way, Beyers argued that organizing dispositions toward mathematics by considering those three modes of mental functioning allows us to analyze students' (teachers') dispositions in a systematic way.

MATHEMATICS TEACHERS' DISPOSITIONS TOWARD ERRORS

For introducing and using errors as real learning experiences with students, it is essential starting as soon as possible; this because, according to Donaldson (2017), pre-K young children show no concern about mistakes; however, around the ages between five and six, children start showing fear of erring. Consequently, Donaldson (2017) stated that introducing strategies for learning from mistakes should be as early as kindergarten, but to do so, teachers' responses to their students' mathematical mistakes should be well thought and supportive. However, teachers' responses are not automatically positive based on a productive status of errors, instead, teachers' responses are directly influenced by their dispositions toward errors (Wagner & Herbel, 2009), thus examining teachers dispositions might be an essential condition to know if using errors as learning tools is an appropriate approach, in all the cases.

The way that teachers respond to productive errors can encourage or discourage student thinking and learn (Gojak, 2013). For example, teachers who handle errors inappropriately are likely to increase students' error strain disposition which is characterized by a fear of making mistakes. In contrast, if the teacher treats an error as a natural part of the teaching and learning

process then their students would not develop a non-productive disposition toward errors. The foregoing will be understood to mean that to change students' dispositions toward errors, teachers' dispositions should be modified in the first place.

Teachers' disposition to use mistakes as learning opportunities require to develop skills for forecasting and anticipating students' errors to properly jump into them by designing the proper didactic situations in order to have planning responses to every type of conceptual or procedural mistakes as it may apply. In mathematics, there are some topics where the same errors are frequently committed. Hence, teachers might be open to noticing based on their professional experience, the most common mistakes committing by students repeatedly, in order to facilitate the anticipating task (Lannin, Barker & Townsend, 2007; Schleppenbach et al., 2007).

Teachers' disposition for anticipating, detecting, and explaining students' mistakes, rather than correcting them, only is needed for supporting students learning (Tsovaltzi et al., 2012; Melis, 2005). That idea is based on the importance that the development of mathematical thinking has on a level that the students are able to argue their responses. Once students detect a problem or a mistake to explain it should be more valuable than to correct it. Indeed, understand a mistake constituted a to correct it. In this regard, feedback is a key element for learning from errors approach. Having the disposition for learning from mistakes provides opportunities for debating among peers or/and as a part of an active learning (Tsovaltzi et al., 2012). Giving immediate feedback to students has been controversial due to the lack of opportunities for critical thinking and reflection, which at the same time, limit students' opportunities for learning from their mistakes and, in turn, develop productive dispositions toward mistakes.

To conclude, some of the studies referenced above have analyzed the evolution of research in mathematics education with regard to mathematics teachers' beliefs about errors and their

strategies to handle them by using quantitative methods. Some others have focused on cultural aspects of educational practices related to mathematics teachers' beliefs about mistakes, while others have studied errors-handling strategies from a student learning perspective. Only a few studies have explored teachers' responses to their students' errors.

The idea of using the construct of disposition and even more dispositions toward mathematics is an emerging topic. Furthermore, the idea of addressing teachers dispositions toward mistakes from a holistic approach. Then, for considering the mathematics teacher's disposition toward mistakes, I used three domains (Beyers, 2011) -- cognitive, affective, and conative.

My goal by conducting this research was not to instill specific dispositions; instead, my main goal was to assess teachers' disposition toward mistakes so that they can be more productive and consistent in their thinking and actions regarding mistakes. My goal was to explore how experienced teachers were inclined to think and act in particular ways when error emerged during their mathematics class. Thus, this research project contributes to explaining teachers' disposition toward their own mistakes and their students' mistakes using a mixed-methods approach. Teachers' error-handling practices were also examined, as well as their competence to consider errors as a natural means of the teaching and learning process. This study provides relevant information of teachers' cognitive, affective, and conative disposition toward errors and the relationships between their thinking and their instructional practices to analyze the suitability of using mistakes as part of their instructional approaches.

THEORETICAL AND CONTEXTUAL FRAMEWORKS

Without any doubt, the theoretical and/or conceptual frameworks are not only an essential part of any research study, and those are necessary since they provide a coherent structure to clearly

explain a phenomenon thru their specific lens. Grant and Osanloo referred to the theoretical framework as "the foundation from which all knowledge is constructed" (nd, p.12). In this regard, Ravitch and Riggan (2017, p. 8) defined the conceptual framework "as the overarching argument for the work—both why it is worth doing and how it should be done…an argument for importance (reason) and method (rigor)". Moreover, theoretical and conceptual frameworks influence and impact every research process stage since they shape and support research conceptualization and questions, the study design, data collection and analysis, and the way in which findings are conceived and reported (Ravitch and Riggan, 2017).

The components of the theoretical and contextual frameworks were grounded in the disposition toward mathematics framework (Beyers, 2011) and Framing (Greeno, 2009). The analysis of the operationalized constructs and their convergence with teachers positioning was supported and guided by Framing (Greeno, 2009; Hand, Penuel, & Gutiérrez, 2013; van de Sande and Greeno, 2012). Connections between those theoretical and conceptual frameworks are illustrated in Figures 2.1 and 2.2. These connections are explained in detail below.

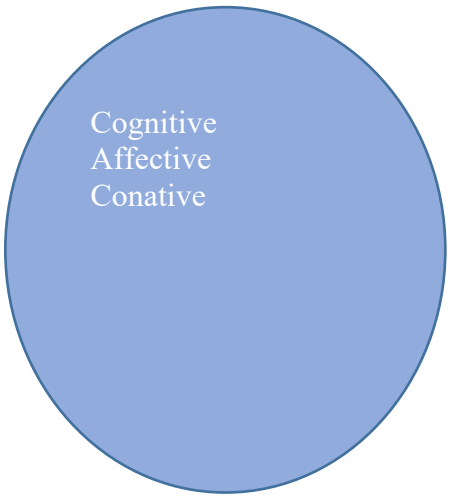

Figure 2.1 Dispositional Functions (Beyers, 2011)

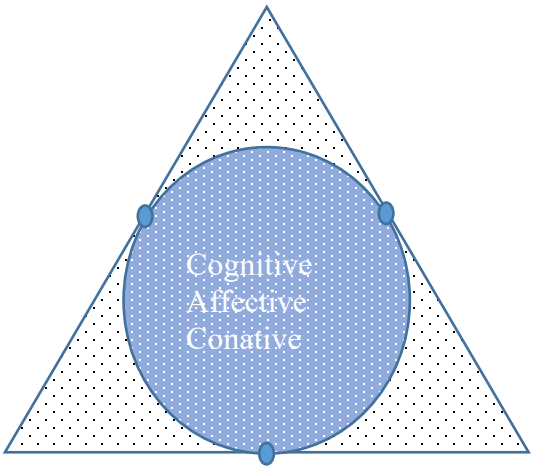

Figure 2.2 Dis/position toward errors framework

DISPOSITION CONSTRUCTS

It is common to hear that mathematics teachers need to help their students to develop good and productive mathematics disposition. Even the NCTM (1989), standards have highlighted its importance for almost three decades (Lappan, 1999). Recently, NCTM's seventy-third yearbook examined motivation and disposition (Brahier, 2011). However, even when we can easily notice that the disposition construct has been highly used since its introduction to education, inconsistencies, ambiguity, and loose ends still are presented (Schussler, 2006). These inconsistencies range from the definition of disposition to its use in empirical research.

Some authors have referred to teachers' disposition as a pattern of acts that are displayed in a particular context, or the trend of teachers' actions (Katz & Raths,1985). Schussler (2006) provided a broader view of teachers' disposition explaining them as "the core that affects, guides, and supports teachers' external behaviors, thoughts, and the context of their teaching" (p.258). Similarly, Schussler stated that disposition is the internal schemata that dictate how cognition, beliefs, and values will be shown during an interaction; Schussler provided a clear idea about what elements need to be encompassed on the analysis of disposition. Those elements are founded on

"the tripartite classification of mental activities into cognition, affect, and conation" (Hilgard, 1980, p.4).

In that same vein, Beyers (2011) provided a framework to study students' dis/positions toward mathematics that classifies mental processes from this tripartite approach that involves cognitive, affective, and conative elements. Thus, he considered these three functions as what constitutes disposition toward mathematics. For this dissertation, these are the three constructs that were operationalized to make mathematics teachers' disposition toward errors measurable by the Error Orientation Questionnaire (EOQ) and subsequently observable.

Furthermore, Beyers' (2011) dispositional functions types, cognitive, affective, and conative, served as a base for conceptualizing and operationalizing the disposition construct in relation to mathematics teachers' disposition toward mistakes. In this sense, disposition is represented by the circle that is inscribed in the triangle, since disposition is understood as the core of the dis/position reciprocal relationship. Seeing disposition as inscribed in the positioning area provides an image of the idea that "dispositions are at the root of teachers' decisions to think and to act" (Schussler, 2006).

Considering features of mathematics teachers' disposition toward errors as elements of the cognitive, affective, or conative mode of mental functioning may afford one a more systematic way to organize, measure, and conceptualize them around the disposition toward errors of the EOQ. The reciprocal relationship between teachers' disposition toward mistakes and teachers' positioning when an error emerges is represented by three points that are tangential to the framing triangle segments. These triangle segments that are the existing or not existing framing relationship between teachers' disposition toward mistakes and their positioning during the error

episodes that take place during the classroom context, provide me an opportunity for gathering evidence of how teachers position according to their disposition on those moments of interaction.

DIS/POSITION

The public expression of the concept of disposition is shown by positioning in a particular circumstance and context given its inherent nature (Parrott, 2003). In other words, positioning becomes the active nature of dispositions. Hence, the aim of analyzing teachers' positioning that takes place in the mathematics class in episodes where errors are involved is to demonstrate the critical role that teachers' disposition play toward errors.

Theoretical frameworks addressing participants' positioning and how the other participants are positioned by them allows researchers to examine and analyze what people are doing in a situation in a specific context and dynamics nature (Harré,1995; Harre & Slocum, 2003). Since positioning focuses on moments of action and interaction, it allows scholars to examine teachers' kinds of participation according to what they say and do in their classrooms at the specific moment that mistakes emerged and are addressed or not by teachers (Wagner & Herbel-Eisenmann, 2009). Harré et al., (2009) argued that positioning is locally constituted and "happens in the course of interaction; such it is a discursive process" (p. 10).

Studying positions allowed me to interpret the moment-by-moment meanings of people speaking and acting in the context of errors. In this sense, Harré and Slocum stated that positions that are adopted by participants during moment-to-moment interactions allow unfolding interactions for understanding episodes of daily life (2003). Furthermore, narratives about participant positioning provided a context in which interactions are taking place in the episode of errors. Depending on a specific context, teacher positioning toward errors may vary.

FRAMING

"Activity always occurs in some context or framing…and framing is constructed on the interaction" (Greeno, 2009, p. 269-270). Greeno offered some theoretical assumptions that allow researchers to understand, conceptualize, and frame concepts. He distinguished two aspects of framing: epistemological and positional framing. Positional framing refers to the ways in which participants positioning themselves and the others when in the activity they are interacting in, and framing is being constructed in a particular context. In this sense, according to Goffman, this type of framing allows understanding, "What is it that is going on here?" (as is cited by Louie, 2017, p. 491). In other words, framing provides a tool for analyzing and connecting the operationalized constructs (dispositions) to which is happening in the course of a moment-to-moment teachers' interaction (positions) in the context of errors.

Framing involves the activation of different types of resources (cognitive, behavioral, cultural, affective) by the actors participating in a situation, some of these resources are constantly activated becoming an established way in which participants orient themselves (Hammer, Elby, Scherr, & Redish, 2005; Louie, 2017). This study analyzed what kind of dispositions were activated during an activity where errors emerged by teachers positioning. It examined features of classroom activities that involve errors when the students were positioned as authors or receivers of mathematics ideas and what their conceptions were about mathematics mistakes or in regard to positioning them as actively producing math knowledge from their approach to errors.

Thus, I focused on teachers' instructional practices when errors emerged and the alignment of those with frames that were directly connected to their affective, cognitive, and conative dispositions. I generated a matrix that allowed me to understand the relevance of positions

designated for teachers to themselves and their students in the context of errors and how those implicitly constituted teachers' framings.

SUMMARY

There is a relatively sizeable body of related errors as learning tools literature that focused mainly on teaching methods and strategies using this approach. Within these teaching-oriented articles, there are some others that address student-teachers', teachers', and students' dispositions toward mathematics or toward some specific aspect related to mathematics. Analyzing these articles, I noticed that in the teaching-oriented articles that use mistakes as learning tools, it is taken for granted that teachers' dispositions toward mistakes go in the same direction, regarding the positive status of errors in mathematics. The issue of assessing mathematics teachers' dispositions toward mistakes, however, does not appear to be directly explored in the related literature. Thus, the present study provides a unique perspective. The theoretical framework guiding this study was Framing (Greeno, 2009). Beyers' (2011) cognitive, affective, and conative dispositional functions types served as a base for conceptualizing and operationalizing the disposition construct in relation to mathematics teachers' disposition toward mistakes in relation to the eight domains of the EOQ.

Chapter III: Methodology

In this chapter, I address the research questions and the research methods that guide this study on secondary mathematics teachers' dispositions toward their own mistakes and their students' mistakes. I conducted this study by using a mixed methods research design which is described below along with the rationale for using it. The sampling strategies, settings, instrumentation for quantitative and qualitative data collection, and validity and reliability are discussed in this chapter. Finally, details about data analysis are also provided in this section.

RESEARCH QUESTIONS

A mixed-methods sequential explanatory design is one of the types of mixed-method designs. The starting point of this design is the quantitative phase followed by the qualitative phase which builds on the quantitative phase (Creswell, Plano Clark, Gutmann, & Hanson, 2003). Consequently, I developed the research questions in two different phases. During the first phase, data collection was guided by the quantitative question (1), subsequently, I used the quantitative results to develop the qualitative research questions (2) and (3):

1. To what extent do secondary mathematics teachers disposition toward errors in the context of their own errors and their students' errors differ and/or coincide?
2. What teacher positional frames were unfolded during class at the moment when errors emerged?
3. How are teachers' dispositions toward mistakes are aligned with teachers' positioning and framing during class?

RESEARCH DESIGN AND RATIONALE

The context as an inherent characteristic of dispositions makes traditional instruments insufficient and inefficient for examining its complexity. It is not enough taking a snapshot for

examining teachers' dispositions (Schussler, 2006). The process for examining dispositions requires theories as a foundation and the appropriate methods and instruments for conceptualizing and assessing them.

The rationale for using a mixed-methods approach was grounded in the fact that, given the complexity in addressing teachers' dispositions toward their own mistakes and their students' mistakes, neither a qualitative approach nor a quantitative one is sufficient by themselves "to capture the details" (Ivankova, Creswell, & Stick, 2006, p. 3). Instead, using both in combination allows in-depth insight into the problem. Also, the rationale for combining both methodological approaches was sampling, since the quantitative approach supported the selection of the case study participants. Another reason was the relevance of having a contextual understanding of the EOQ results (Bryman, 2006).

I conducted this study by applying an explanatory sequential mixed methods design. The sequential explanatory design is composed of two phases--quantitative and qualitative (Creswell et al. 2003). This mixed-methods design is also known as sequential triangulation or integration (Morse, 1991). The word integration brings the idea about one of the most important mixed methods research designs attributes, which are using both methods within the same project to ensure a deeper understanding of the problem. Moreover, it provides the possibility of supporting both methods' instrument validity (Morse, 1991).

The explanatory sequential design is mainly used because the quantitative results are crucial for planning the qualitative phase. First, quantitative data is collected and analyzed. Subsequently, in the second phase, the qualitative part of the study was built on the quantitative part, which in turn, become connected in every part of the study. In other words, the two methods

combination will involve collecting quantitative data first and then explaining the results with in-depth qualitative data analysis (Creswell & Plano-Clark, 2011).

Another important aspect of every mixed-method research design is establishing which approach will have priority (Ivankova, Creswell, & Stick, 2006). For addressing teachers' dispositions toward mistakes, the qualitative phase had more attention throughout the data collection and analysis process, which is to say that study priority was given to the qualitative phase (in mixed methods notation quan → QUAL= Results explanation) (Creswell & Plano-Clark, 2011). Figure 3.1 presents the rationale for using this type of design.

The quantitative phase analysis provided a general overview of this research topic, and the qualitative phase provided an in-depth and exhaustive understanding of the results obtained by the quantitative data collection instrument (Creswell & Plano-Clark, 2011; Ivankova, Creswell, & Stick, 2006; Teddlie & Tashakkori, 2003). By conducting a quantitative pilot study for assessing U.S.-Mexico border mathematics secondary teachers' dispositions toward mistakes, I realized that results were superficial which made it difficult for me to grasp a satisfactory and complete understanding of the topic. It also made me reflect on the idea of using an explanatory sequential mixed methods design.

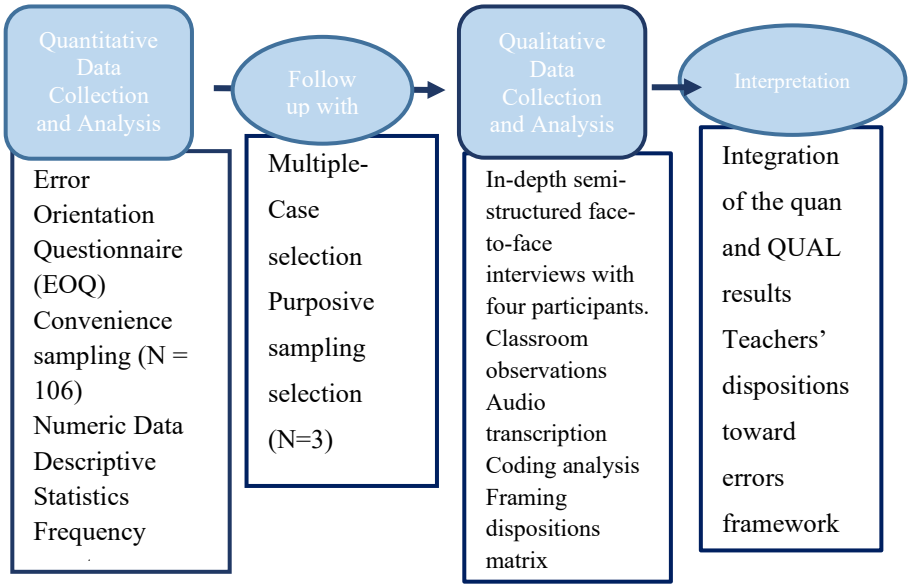

Figure 3.1 Explanatory Sequential Mixed Methods Design Diagram

SAMPLING PROCEDURES, PARTICIPANTS, AND SETTINGS

The explanatory sequential design provides the possibility to manage recruitment and consent in a single stage notwithstanding differences in the rationales for sampling and sample size between the quantitative and qualitative strands since participants for the second round of data collection (qualitative) were part of the first group. To explain the sampling procedures, I will start with the quantitative phase, in which a non-probability sampling technique, namely convenience sampling, was used. I used convenience sampling for both sides of the border, and in both cases, the practical criteria for selecting participants was "easy accessibility, availability at a given time, and the willingness to participate" (Etikan, Musa, & Alkassim, 2016) as I will show below.

The U.S. sample was formed by teachers available at a Texas Western Public University since I had access to teachers from most of the school districts because they were enrolled in grant courses, master's degree courses, and/or Ph.D. courses. For the Mexico sample, participant recruitment was completely different. For explaining the process, I would like to mention that in

Mexico the last Friday of every month there is a meeting in every public and private school of all the different education levels (kindergarten-high school). I took advantage of that meeting to have access to teachers in their own schools. Having access to teachers to apply the survey, interviewing them, and even to conduct observations was easier, since the only thing that was required was a letter explaining the study's purposes and the teacher consent (I knew this information from the pilot study that was previously approved by the IRB, and I conducted in some middle-schools in Mexico near the U.S. border. In such a manner, I asked some middle school and high school principals and assistant principals' permission to apply the survey during their monthly meeting.

In the second phase of the study, for selecting the qualitative phase participants, I used a purposeful sampling strategy. This type of sampling was designed for addressing the research questions. The rationale for selecting cases was selecting those that can provide the most possibilities to focus on depth of information (Teddlie & Yu, 2007). More especially, the purposeful sampling strategy that I used was "Extreme or deviant case sampling [which] involves selecting cases that are information-rich because they are unusual or special in some way, such as outstanding successes or notable failures" (Patton, 2005, p.3). In this sense, two cases were chosen from the highest and lowest scores of the questionnaire applied as part of the quantitative phase, while a third case was chosen taking into consideration the differences between the contexts' scores (G, T, & S).

In the view of a mixed-methods study, which was focused on the QUAL phase, less emphasis was placed on the quan sample size representative, and more emphasis was placed on the saturation of the QUAL sample (Teddlie & Yu, 2007). However, the quantitative sample was considered as representative because based on the data that I collected, it allowed me to answer the quantitative question of this study (Creswell & Plano, 2011). Regarding the saturation of the

QUAL sample, according to Krueger and Casey (as cited by Teddlie &Yu, 2007), researchers should plan three or four interviews or focus groups and then check if saturation has been reached. They explain that saturation is the point when the researcher has heard the range of ideas and he/she is not getting additional information. I achieved the saturation of the QUAL data from two interviews with each of the three teachers, and at least three participant observations.

Informed consent.

The elements of informed consent were explained when participants expressed interest in participating in this study. I explained the study's purposes and procedures and provided participants with the Informed Consent Form (see Appendix A for the informed consent forms). Participants read the form and were allowed to ask any questions about their participation in the study. Prior to the error orientation questionnaire (EOQ) application and to every interview, an informed consent form was presented to each participant with detailed descriptions of the study, and, along with the form, they received a verbal explanation so that the participant could fully grasp the purpose and nature of the interview and the kinds of questions that were asked. Each participant was provided with a copy of the appropriate informed consent document to keep, and additional copies of this document were provided to participants as requested. I kept the signed informed consent documents as described in the security measures section of this document.

This study did not involve more than minimal risk. Interview topics might make participants self-conscious or marginally uncomfortable because of the focus on errors (Schwandt, 2007). However, the possibility of emotional discomfort posed by questions about professional and/or academic experiences is considered minimal. Participants always had the opportunity to skip a question, decline to answer a question, and/or end response to a question.

VALIDITY AND RELIABILITY

Validity and reliability are two concepts that both qualitative researchers and quantitative researchers, should be concerned about during the entire process of the research study (Patton, 2001). Even when reliability is a concept used for evaluating quantitative research, reliability in qualitative research is connected to the examination of trustworthiness (Golafshani, 2003).

Mixed methods validity is constituted by a study design which allows integrating quantitative and qualitative research approaches, instruments, participant selection, and findings. (Bryman, 2008). Validity in mixed methods involves employing strategies that address potential issues in data collection, data analysis, and the interpretations that might compromise the merging or connecting of the quantitative and qualitative strands of the study.

This integration is typically seen as triangulation, which is a strategy for supporting reliability and validity. According to Mathison (as cited in Golafshani, 2003) triangulation has gained relevance on quantitative and qualitative approaches. According to Patton (2001) "triangulation strengthens a study by combining methods. This can mean using several kinds of methods or data, including using both quantitative and qualitative approaches" (p. 247). However, the process to integrate quantitative and qualitative phases represents a challenge per se, that might involve different validity risks.

To address the validity threats in explanatory designs, as a researcher, I made decisions about: a) what quantitative results to explain, b) who to sample for the follow-up, c) what interview questions to ask, and d) how to ensure that the qualitative data indeed explain the quantitative results.

Once I measured and analyzed teachers' dispositions toward errors, I made the first decision to integrate the quantitative and the qualitative phase of my study; I selected the

qualitative sample. Subsequently, I focused on the second study phase with the aim of providing an explanation of the quantitative results. The qualitative phase centered on three teachers' positioning at the specific moment that errors emerge during class. This effort required me to value the multiple realities that participants have in their minds. To acquire valid and reliable multiple and diverse realities, multiple methods of searching or gathering data were designed also by using the quantitative findings.

The qualitative data collection protocols were used to gather open-ended data which adhered with the notion of data triangulation by allowing Damian, Bianca, and Ana assist me in the research question as well as with data collection. Integrating the quantitative and qualitative paradigms, I engaged multiple methods, such as questionnaires, observation, and interviews, what led me to more valid, reliable and diverse construction of realities involve in teachers disposition and positioning toward mistakes.

To improve my findings integration, triangulation included multiple methods of data collection and data analysis and for this study, the connections between quan data about teachers' disposition and QUAL data collection methods for explaining teachers' positions toward mistakes support the triangulation process.

METHODS AND DATA SOURCES

This mixed-methods sequential explanatory multiple-case study design involved two phases (Creswell, Plano Clark, Gutmann, & Hanson, 2003). It focused on the quantitative data collection and analysis because quantitative data helped to identify the types of teachers' dispositions toward errors. In this way, quantitative findings guided the design of the quantitative data collection instruments. Hence, the quantitative data provided me a general picture about

teachers dispositions toward errors, while the qualitative part explained what went on in relation to teachers' dispositions toward mistakes and their positioning in the error context.

Phase I—Quantitative

The main purpose of the quantitative phase of this study was to measure secondary mathematics teachers' disposition toward errors in a U.S.-Mexico border region. I collected data by face-to-face application of the EOQ with the aim of decreasing the number of doubtful responses (Duffy et al., 2005, p.538), since participants had the opportunity to ask me any question about survey questions.

Reliability and validity of the survey scale items modifications for an educational context were established based on a pilot survey administration that was done during the 2016 summer by using frequency distributions and item correlations (Piedmont, & Hyland,1993). Criteria for selecting the participants for the quantitative phase included (1) being a mathematics secondary teacher; (2) at least one year of experience.

Data Collection and Data Analysis

I administered the two context EOQ face-to-face with teachers in the US-Mexico border region. The EOQ was administered in conjunction with demographic questions and open-ended responses in different sites. For the quantitative analysis, I organized participants into two groups based on the side of the border they work (the U.S.-Mexico).

The first analysis of the EOQ consisted of calculating the scores for the questionnaire that did not specify the context (G). Subsequently, I ranked the scores from the highest to the lowest. Non-parametric Pearson correlation was used to analyze the EOQ (Creswell, & Clark, 2012). This procedure was used to determine "the degree of a linear relationship between two variables" (Gravetter, & Wallnau, 2016, p. 514). I analyzed this relationship by using a special version of

the Pearson correlation named as the point-biserial correlation, which "is used to measure the relationship between two variables in situations in which one variable consists of regular, numerical scores, but the second variable has only two values (Gravetter & Wallnau, 2013, p. 542). I conducted the point-biserial by measuring the relationships between the EOQ numerical scores and the binomial variables (e.g., gender, country) from the demographic survey. I also analyzed teachers' demographic information by using cross-tabulations and frequency counts and Pearson correlation.

The first EOQ that was administrated was based in a general context, however, a teacher approached me and told me that it was very difficult answering the questionnaire due to the fact that he was not able to think of something concrete. He argued that having a specific context would be essential to answering a questionnaire about errors since there are differences that depend on who makes the mistake and where the mistake is made. Therefore, a second EOQ version was the result of those comments. I slightly changed this second EOQ version with the aim of contextualizing it by specifying teachers' disposition toward their own errors (T) and the teachers' disposition toward students' mistakes (S).

Research Quantitative Instrument (EOQ).

For the quantitative phase of the study, I used the Error Orientation Questionnaire (Rybowiak, et al., 1999) as the main instrument to collect data from participants teaching mathematics in a U.S.-Mexico borderland area high schools and middle schools. This instrument was designed with the aim of measuring "how one copes with and how one thinks about errors at work" (Rybowiak, et al., 1999, p. 527). According to the authors, this instrument was developed to be used for both practical and theoretical purposes.

I asked the creators of this instrument for permission to use the EOQ and make minor changes to it. I also asked them how they analyzed the results. Dr. Freese, who is one of the authors of this instrument, replied to my email giving me permission to make the necessary changes (M. Freese, personal communication, February 2, 2016). Then, before its application, I adapted this instrument to the teaching context by changing some terminology (e.g., classroom, instead of work).

The EOQ instrument allowed me to measure teachers' dispositions toward errors in a general context. However, some changes were done to the instrument with the aim of adapting it to teachers' dispositions in the context of their own mistakes and in the context of their students' mistakes (see Appendix B). In this sense, I made a distinction between the different contexts where teachers can make mistakes and how their dispositions can be different depending on the context and the person who is making the mistakes.

The EOQ has a total of 37 questions in a 5-point Likert-type scale related to eight different domains. Below is a brief explanation of these 8 domains:

1) Error competence. This domain has been already addressed in education and it has been related to cognitive aspects. Seifried & Wuttke (2017, p. 16) define it as "knowledge about common students' errors and potential causes for students' errors, strategies for handling errors (especially feedback strategies), and error-friendly beliefs."
2) Learning from errors. This domain is also related to the cognitive realm and it is the ability to capitalize on errors (Borasi, 1987; Santagata, 2003).
3) Error risk-taking. It describes flexibility and openness toward mistakes (Rywobiak et al. 1999, p. 534) and "implies that one accepts errors and its consequences in

order to reach higher goals." It is related to the student's courage or eagerness to succeed again before having experienced failure or having made a mistake (Tay et al., 2009).

4) Error strain. According to Rywobiak et al. (1999, p. 543), this domain is related to affective issues since "it is characterized by a generalized fear of committing errors and by negative emotional reactions."

5) Error anticipation. This domain is related to affective and it can be seen from two different perspectives since it can be seen as pessimistic or negative tuned when it is positively correlated to error strain. However, it can be also positively correlated with learning from error or thinking about errors (Rywobiak et al. 1999).

6) Covering up errors. This domain is mainly related to affective issues and it is seen as the strategy of an insecure person when he or she doesn't acknowledge errors (Rywobiak et al. 1999).

7) Error communication. According to Tait-McCutcheon (2008), this domain is related to the conative realm since it is described by students being confident and seeing as valuable discussing and getting agreements when they are working with others.

8) Thinking about errors. The ability to understand and analyze mistakes (Rywobiak et al. 1999).

These eight domains allowed me to collect and measure teachers' disposition toward mistakes from Beyers' (2011) three modes of mental functioning types: cognitive, affective, and conative. Organization of the EOQ domains into the three dispositional cognitive, affective, conative functions was done by contrasting and overlapping each domain questions' keywords

with the dispositional function key words, definition, description, and examples provide by Beyers (2011).

Recognizing the potential limitations of using a tool not constructed for educators, the EOQ quantitative information was used to design the qualitative interview protocol of the study. Hence, the benefits of using a mixed-methods design were in the quantitative phase. The EOQ allows for differentiating contexts and measuring levels of teachers' dispositions toward mistakes in general terms. Whereas, the interviews and the class observations that were conducted during the qualitative phase provided a venue for an in-depth investigation of ambiguous statistical findings.

I organized elements of the disposition construct identified in the EOQ and in the teachers' interviews in the context of the cognitive, affective, or conative mode of mental functioning. This organization was useful in terms of identifying framing teachers' in relation to their productive and/or non-productive disposition toward errors because there are some kinds of dispositions that are desirable and some other that are not (Schussler, 2006) in terms of supporting not only students' learning but in terms of identifying students' as capable or not capable to cope with errors.

Phase II – Qualitative

The emphasis of this research study was on the qualitative phase. Qualitative research enables the researcher to conduct "systematic investigation of social phenomena and human behavior and interaction…in their natural settings" (Litchman, 2013, p. 4). In this case, qualitative methods allowed me to collect the specific data for understanding mathematics teachers' dispositions toward their own mistakes and their students' mistakes and how these dispositions were materialized by their positioning and framing during mistakes episodes in their classrooms.

In this second phase, I used a multi-case study approach, since this methodology provides tools to the researchers to capture the essence of complex phenomena within their context (Meyer,

2001). Even when, according to Yin (2003), there is no formula that tells researchers if they should use a case study as the methodology path for conducting a study, research questions play a crucial role. Taking into consideration the research questions that guide this study allow for in-depth explanations of the phenomenon, a case study approach provided the base for addressing these types of questions. However, that was not the only reason for using this approach. Yin (2003) provides some other reasons for selecting this approach over others, including that it is related to the contextual conditions, which in this case, were relevant for understanding the phenomenon.

The type of case study that was used for conducting the qualitative phase is a multiple-case study. The multiple-case study allowed me to explore differences within and between cases (Baxter & Jack, 2008). In this study, a multiple-case study helped me distinguish between productive and non-productive dispositions toward mistakes across three secondary mathematics teachers.

Research Context

For the multiple-case study, I purposefully selected three participants. Participants were chosen according to their EOQ scores, as explained below.

Criteria for selecting case study participants.

Once EOQ scores were tabulated, three groups were formulated: the teacher with the highest scores, the teacher with the lowest scores, and the teacher which EOQ scores present the biggest difference between the contexts. The higher and the lowest scores were detected, participants were contacted to participate in the second study phase and all of them accepted to participate. Table 3.1 shows teachers disposition toward errors according to the three different contexts.

Table 3.1 Teacher's disposition toward errors

Participant	Disposition
Damian	Productive disposition toward mistakes (G), (T), and (S) contexts.
Bianca	Productive disposition toward mistakes in (S) context and non-productive disposition in (G) and (T) contexts.
Ana	Non-productive disposition toward mistakes in (G), (T), and (S) contexts.

A brief description of teachers' classroom context.

It is important to highlight that even when this study was conducted on both sides of the the U.S.-Mexico border, it does not address cultural, economic, or political issues related to education, nor does it pretend to be part of borderland studies, since it does not address any type of issue contemplated as part of a border phenomenon. This study was conducted on both sides of the border for three specific reasons. First, taking advantage of the geographical situation; second, having a broader overview in the context of error-friendly reforms and curriculums, which have been implemented in both sides of the border; last but not least, my 14 years of experience as a teacher in Mexico and my emerging understanding and interest for the U.S. education system as an educator, a researcher, and a parent.

I had three participants for the qualitative part of this study. One of the three teachers participating in this study was teaching in Mexico and the other two were teaching in the U.S. Consequently, their teaching contexts are not only different in terms of curriculums and reform standards, but also in terms of the physical characteristics of the context. Below there is a brief description of each of three teachers' contexts. To ensure anonymity, I am using pseudonyms for participants.

Bianca is a middle school mathematics teacher. During the time of this study, she had been teaching 8th and 9th grade for 14 years in a very low-income school located in Mexico near the U.S.-Mexico border. Bianca taught seven periods of 50 minutes each every day, and she had about 60 students per class section. In this school, teachers did not have an assigned classroom. Instead, teachers switched between classrooms every period, and students stayed in the same classroom for the whole school day. Only teachers and a small number of students had access to graph calculators, so they usually shared their devices with their peers. The classroom arrangement was very traditional. The teacher's desk was located in front of the classroom, and the students were aligned in six rows of ten desks each. There was minimal space to walk between rows. This did not allow Bianca to pass by students' desks. Outdoors temperature was very similar to the classroom temperature since there were not a/c units.

Damian is a middle school teacher. He had been teaching 8th-grade regular mathematics, remedial mathematics, and algebra for 19 years in a low-income middle school located in the U.S. near the U.S.-Mexico border. Damian taught seven school day periods with about 16 students on average for each period. Damian's middle school was in one of the poorest areas of the city. However, Damian and his students did have access to technology (e.g. graphing calculators, tablets, computer, and projector). Classroom arrangement was set up by small workgroups of three to four students, and Damian was usually walking around the spacious classroom. Classroom temperature and illumination were comfortable.

Ana is a high school teacher. She had been teaching 9th grade for more than 18 years at the same school located in the U.S. near the U.S.-Mexico border. Ana had about 17 students per period and taught for seven periods a day. Ana's high school is a low-income school. It is located in an old neighborhood of the city. Ana and her students had access to technology (e.g. graphing

calculators, tablets, computer, and projector). Classroom arrangement was set up by small workgroups of three students. Ana was usually at the back of the classroom to where all the students' desks are facing; Ana usually stayed at her desk for the whole class. Classroom temperature and illumination were comfortable. This classroom had a lot of decoration details.

Data collection

With the aim of providing an in-depth of the case explanation (Baxter and Jack, 2008), I collected data from multiple sources. First, a set of two in-depth semi-structured face-to-face interviews with the three participants were conducted. An in-depth, phenomenological approach to interviewing was used because this interviewing model is comprised of separate interviews with each participant, which enables the researcher to obtain rich data (Seidman, 2013, p.20). These open-ended interviews were audio-recorded with the aim of expediting data transcription and their subsequent analysis. Interviews lasted approximately 30 minutes each. Those took place at a time and place set by participants.

I designed an interview protocol base on the quantitative results. It was separated by contexts since the first interview was thought to address issues about teacher own errors, while the second one was designed to address issues about students' errors. Interview protocols also were designed based on each domain of the EOQ. For example, the interview had questions related to "error risk-taking", "learning from errors", "error competence", to name a few (see Appendix B).

Aside from the interviews, the main data collection method applied in this stage were participant observations (Musante & DeWalt, 2010). Participant observations consisted of 47 total hours and field notes of 11sessions for two months; at least three observations in each participant classroom were conducted. The observation criteria were closely connected with the overall aim of the study. Consequently, data collected from observations consisted of a detailed description of

teachers' activities, actions, reactions, omissions and a broad range of classroom participant interactions that were part of the error episodes (Patton, 2005). More specifically, observations were centered on teachers' positioning and framing of their own errors and their students' errors as well according to the two different error paradigms that were mentioned above. These observations followed an observation protocol grounded in interview data (see Appendix C).

Detailed field notes were written based on the participant observations (Emerson, Fretz & Shaw, 2011). The direct quotes from teachers and/or students' interactions at the moment that errors emerged or when teachers approached me during class to explain to me something about their students or their decisions to handle errors in a determinate way were "jotting" (Sanjek 1990). Promptly after class, I translated observation data into structured analytic memos and organized transcripts as part of the substantial field notes generated throughout the fieldwork. These memos provided a clear idea of "how the process of inquiry is taking shape" (Saldaña, 2015, p.41) in terms of emergent patterns, themes, and concepts that provide the guidance for the research process.

Data Analysis

It is important to highlight that a mixed-methods sequential explanatory multiple-case study data analysis must be performed at two levels: within each case and across the cases (Yin, 2003). For this study, each interview was audio-recorded and transcribed by using Express Scribe transcription software. Subsequently, I analyzed them manually and by using the Nvivo version 11 software. This software was used on the initial line-by-line coding, which according to Charmaz & Mitchell (2001), is a way to begin building research analysis.

My initial examination of data gathered from classroom observation transcripts and analytical memos was based on the ways in which teachers communicate their active dispositions toward mistakes and the ways in which students are expected to participate. Subsequently, to

develop focused codes, I integrated ways of positioning and framing that emerged as part of the initial examination with the two different paradigms that were addressed as part of the literature review, which is errors as resources to learn and errors as deficiencies for learning. The matrix of frames is represented by the italicized text in Table 3.2, as those represent the way that participants operate during the error episodes. Further analysis allowed me to see teachers' positioning and their students' positioning during their interactions in the context of errors (van de Sande & Greeno, 2012). Following that, I re-examined the interviews and classroom observations transcripts, using open coding for analyzing teachers' instructional practices and their students' responses to those that aligned with various ways of framing errors. Those findings are described throughout the next chapter by providing a situated snapshot of each teacher's positioning at the moment of errors and how these positioning facilitated or inhibited using errors productively.

Table 3.2. Teachers' Framing Table

WAYS IN WHICH TEACHERS COMMUNICATED THEIR ACTIVE DISPOSITIONS TOWARD MISTAKES	
Errors as resources for learning **Frame**	**Errors as deficiencies for learning** **Frame**
Understanding and analyzing mistakes, develop critical thinking built-in errors. Ability of capitalizing on errors.	*Understanding errors as learning deficiency. Using errors for diagnosing or remediate learning problems*
Flexibility and openness toward mistakes creating an error-friendly belief.	*Reluctance toward mistakes creating an error-discomforting belief*
WAYS IN WHICH STUDENTS ARE EXPECTED TO PARTICIPATE	
Teacher position their student as capable of coping with errors. Students as competent and qualified to handle their error analysis process and produce mathematical ideas by themselves. **Frame**	**Teacher position their students as not capable of coping with errors** **Frame**
Student as capable of producing mathematical ideas from the analysis of their mistakes	*Students as receivers of mathematical ideas in regard to the correction of their mistakes*

Student as capable of succeeding again after having experienced failure or having made a mistake	*Students as vulnerable participants or/and not capable after having experienced failure or have made a mistake*

SUMMARY

This chapter described the mixed methods design and the rationale for using this specific design. Also, data collection and analysis procedures across the two phases of the study were described. The study employed an explanatory sequential mixed methods design for addressing the U.S.-Mexico borderland secondary mathematics teachers' dispositions toward mistakes by developing a typology based on the ways that teachers positioning themselves and their students during error episodes. The study began with a quantitative instrument application (EOQ) in both sides of a U.S.-Mexico border. Quantitative results were used for selecting the qualitative purposeful sample and for designing the data collection instruments. The research emphasis of this study is on the qualitative phase. In this final qualitative phase, data collection was used based on teacher interviews and classroom observations. To address issues and threats to validity in this explanatory mixed methods design, I used a systematic process that consisted of joint displays to move from quantitative results to qualitative codes and themes. The study attended to ethical considerations throughout the process.

Chapter IV: Findings

In the following chapter, I present both samples' quantitative results and the statistical arguments for considering the U.S. and Mexico samples as a homogenous sample which allowed me to choose the participants for the qualitative phase from both sides of the border, indistinctly. I also present Damian, Ana, and Bianca quantitative results from the three versions of the EOQ and how their disposition toward errors is constituted according to the three dispositional functions (Beyers, 2011). Subsequently, I explain the relationship between teacher's disposition and their positioning during error episodes in class. I address teachers' positioning and frames that Damian, Ana, and Bianca invoked in their work with students and how these frames came to reality in class interactions with errors. In the last section of this chapter, I explain how quantitative and qualitative results became integrated.

QUANTITATIVE RESULTS

Examining teachers' dispositions toward mistakes features in the context of their own errors and their students' errors by measuring them on both sides of the border presented important differences. For example, for US sample (n=44) the percentage of teachers that scored a PED (productive error disposition) < 1, what is to say, a low PED was 19.04%. On the other hand, for the Mexico sample (n = 62) was 35.06%. Another difference in both samples was teachers' years of experience. For the U.S. sample the average was of 9.4 years and for the Mexican samples, the average was 15.6 years. However, no significant correlation between PED and years of experience was found. Regarding gender and students' passing rate, which were the other two variables included in the analysis, no significant correlation to PED, nor a significant difference between the two samples, was found.

It is important to mention that none of the samples demonstrated any significant correlation between the cognitive, the affective, and the conative functions. Another non-parametric test that was performed with the aim of knowing if the samples were homogenous among them was Chi-square test question by question for both samples. Then, 37 Chi-square test of independence were performed to examine the relation between disposition toward errors and the variable side of the border and the results showed that that were no significant association between error disposition and the country variable, $x^2(2, N=104)$, (see Appendix D) for results.

In that sense, and considering exclusively a general context, the quantitative results did not allow me to conclude anything regarding mathematics teacher's dispositions toward mistakes in relation to demographical variables (e.g. gender, country, and teaching experience). However, quantitative data allowed me to realize the relevance of measuring teacher's disposition toward mistakes through different contexts. Then, with the aim of understanding what characterizes the dispositions toward mistakes in the context of their students' mistakes and their own mistakes, two error orientation questionnaires (with and without context) were applied. The first analysis of the EOQ consisted of calculating the scores for the questionnaire that did not specify the context (G). Subsequently, I ranked the scores from the highest to the lowest by country and once I learned that both samples were homogenous, I decided to integrate and report U.S. and Mexico participants' scores as one table (See Appendix E). This purposeful sample (n = 3) results in the two-context questionnaires which allowed me to have a closer look at what characterizes those three teachers' dispositions, according to the three dispositional functions.

DAMIAN'S DISPOSITIONS TOWARD ERRORS: AN ENDURING PRODUCTIVE DISPOSITION THROUGH DIFFERENT CONTEXTS

Damian's EOQ scores provide an outlook of his disposition toward mistakes in the three different contexts. Damian's general context EOQ score was located on the top of the highest percentile; he scored 110 points of a total of 124. Table 4.1 provides an outlook of his scores as compounded by the eight domains of the EOQ in the three different contexts:

Table 4.1 Damian's EOQ scores in G-general context, T-Teacher errors context, S-Student errors context

	Dispositional Cognitive Function			Dispositional Affective Function		Dispositional Conative Function		
	Error competence	Learning from errors	Error anticipation	Thinking about errors	Error strain	Covering up errors	Error risk-taking	Error communication	TOTAL
G	19/20	20/20	21/25	24/25	-5/25	-4/30	18/20	18/20	111/124
T	18/20	20/20	20/25	24/25	-6/25	-2/30	10/20	20/20	104/124
S	19/20	20/20	21/25	25/25	-6/25	-4/30	18/20	18/20	111/124

Damian's EOQ scores overview.

In Table 4.1 (G) scores refers to the first questionnaire that was applied, in which context was not specified. Four months later, I applied a second questionnaire where the context was precise for (T) teacher's errors and student's errors (S) and which is reported as part of the second and third rows in Table 4.1. As it can be noticed, Damian total scores are almost the same. However, although Damian's scores are very closed from each other, these should be reviewed

thoroughly. These similarities provide information about some aspects of Damian's dispositions and at the same time, it also draws attention to certain strands that make them substantially distinct.

First, an aspect to be highlighted regarding Damian's dispositions toward errors total scores is that his disposition in the (G), (T), and (S) are very close despite the time that has passed. More specifically, the (G) and (S) score totals are the same. Which, on one hand, provides a close idea of Damian's enduring disposition that is consistently revealed through the time. And on the other hand, it gives me a hint of how the context might strongly influence Damian's disposition.

Second, the domains specificity might play an important role in Damian's disposition toward errors. In this regard, even when (G) and (S) scores total are identical and, in turn, the scores difference between (G & S) and (T) seems not to be relevant because there are only six points, zooming into domains provided pertinent information to understand Damian disposition toward errors. Consequently, examining how these totals are comprised become crucial to understand the role that context plays in Damian's case.

The six-point difference stems from covering up errors' domain (two-points difference), but mainly from error risk-taking domain (eight-points of difference); at first glance, it seems to be a calculation error. However, I will explain it in detail later. Such a difference in that specific set of disposition provides an idea of how Damian's own mistakes' inclinations are distinct in term of the dispositional mental functions. This fact provides information about how Damian's conative mental function, more specifically to the error-risk taking domain concerning his own mistakes, was what negatively impacted his productive disposition toward mistakes. Having scored ten points of twenty for error risk-taking domain in the context of teacher's errors shows a tendency to avoid showing flexibility and openness toward his own mistakes. In other words, to some extent,

errors in his own context as a teacher are still being perceived as something negative that might be avoided.

A tendency to not recognized error-risk taking usefulness lead him to retain his no-productive disposition toward this specific domain. On the contrary, Damian scored 18 points of 20 in (G & S) contexts for this same domain which shows his tendency to value his students' eagerness and courage of succeeding before having made a mistake. Having almost a perfect score in these two contexts provide an idea of his tendency to the belief that error-risk taking may lead students to increase their levels of persistence and effort.

Regarding covering up errors, a domain of affective mental functions, which is subtracted from the total score (See the Methodology section), Damian scored two negative points of a possible 30 points to be subtracted. It shows a tendency to behave as a confident person who easily acknowledges his own errors. This tendency was consistent in his (G & S) contexts, but with a small difference of two points for both cases.

Concluding, Damian's EOQ scores through the three different contexts provide evidence of his enduring error dispositions toward errors. Congruence between his responses shows a strong core of dispositions. However, as it was explained above, there are some significant differences that might provide information about how Damian's dispositions and positioning may or not may align.

UNWRAPPING RELATIONSHIPS BETWEEN DAMIAN'S DISPOSITION AND HIS POSITIONING

Damian's EOQ responses and scores demonstrate his enduring disposition toward mistakes. The undeniable scores similarities through the different contexts express a deep-rooted disposition, per se. Although, for having an entire panorama of what characterizes Damian's dispositions toward mistakes in his classroom and how his enduring disposition is reflected or not

when his own errors or his students' errors emerged, the EOQ scores become insufficient, considering that EOQ quantitative data provides a limited perspective.

Hence, clarifying convergences and divergences among the three EOQ scores (G, T, and S) and start making connections between his responses and his positioning toward mistakes, asking about what context he thought while answering the first EOQ (G)was a starting point. Even though contexts seemed not to influence Damian's scores, and in turn, his disposition, being aware as a teacher of how dispositions are underlying them might become essential for activating their positioning.

Damian stated that when he answered the first EOQ (G) he was reflecting on his students' mistakes. That fact might suggest that even when Damian's dispositions through the different contexts are very similar, his enduring disposition is linked to his disposition toward his students' mistakes. Then, since Damian's (G & S) scores are almost identical and the difference of those with respect to (S) is only in two domains, I would center the attention on these two.

Damian's positioning and his dispositional affective function.

From the affective mental disposition, a two points difference for covering up errors can be noticed. This domain, like the other for affective dispositions, counts negative points which are deducted from the total (see Methodology). Damian scored a negative four on the context (G & S), while he scored only two negative points in the context of his own mistakes as a teacher. That difference is also reflected during his interview when I asked him about what he does when he makes a mistake during his teaching. He expressed that even when he has a very good relationship with his students and a good sense of humor toward mistakes, what has helped him to establish an error-friendly culture in his classroom, when he makes a mistake, he tries to make his students notice it. When he is not immediately aware of his mistake, he stated:

> I have to go back and re-teach it again and then and maybe apologize ok you know I did this, and it wasn't right, can anybody tell me why it was not right? Because you know kids are going to be wrong in their heads cause what they learn the first time is what usually sticks and if you make a mistake you got to find a way to fix it.

Damian's concern and accountability about what his students learn from the first time might be what makes that difference between (G & S) and (T) covering up errors' domain. With his response, it becomes clear that he makes a difference between teaching and learning errors. He understands that his errors might impact his students' learning opportunities. Damian implicitly stated a connection between teacher's mistakes and students' mistakes; in other words, how teacher's mistakes are one of the root causes of students' misconceptions and mistakes. In this regard, Damian added that it is crucial to creating an error-friendly climate and that best way to doing so is by admitting that you as a teacher also makes mistakes, and even more importantly, encourage your students to correct you when they catch your mistakes.

Damian's positioning and his dispositional conative function.

The biggest difference between (G & S) and (T) Damian's scores are part of his conative dispositional functions, most specifically in the error risk-taking domain. The difference is of eight points, but even more relevant that the difference is that he scored only ten points of a total of 20, which provides an idea of how his non-productive disposition is based almost exclusively in this domain. Error-risk taking describes flexibility and openness toward mistakes (Rywobiak et al. 1999), flexibility and openness that Damian expressed constantly show it to his students' mistakes.

During his interview, he pointed out a difference between error risk-taking in teaching and error risk-taking during learning. He expressed that even when it is important that students know that the teacher makes mistakes, like them, the teacher must be very careful and try not to make that kind of mistakes that can affect students' learning or trust. Damian made an implicit distinction between the types of errors that a teacher can make when he said, "I make mistakes all the time

mathematics it is really easy to miss or misplace a decimal and simple thing like that. But mistakes because I don't know the concept, I think that is not acceptable." In Damian's statement, it can be perceived how he positioned toward the different types of mistakes in the context of teaching. In a way, he established how he, as a teacher, can make computational mistakes given the subject nature, but he is not allowed of making those type of mistakes that could be connected to a lack of content knowledge (e.g. conceptual mistakes).

Regarding his students' mistakes, he believes that it is crucial to motivating his student's risk-taking aptitudes, which can also explain why he scored 18 points of 20. For example, he responded to the same question about error risk-taking:

> And my students, oh, definitely! You want them to get out of their comfort zone because I have algebra kids and I have regular eight grade and I have special Ed inclusion kids. The high-performance students, the algebra kids, they are more perfectionists and when they make a mistake they get upset and affects them. Because, you know, they never come from making a mistake, then they get scared of trying problems. But they need to be able to try and be ok with making mistakes. They need not be afraid of losing their status.

As is evident from the interview excerpt above, Damian's productive disposition toward his students' error risk-taking is demonstrated when he stated, "You want them to get out of their comfort zone." Damian positioned his student as capable of producing mathematical ideas from the analysis of their mistakes and capable to seek and value alternative ways of the error analysis process. Moreover, Damian was drawing attention to important aspects of error risk-taking. Indeed, when Damian was talking about error risk-taking, he made a relevant difference between error risk-taking during teaching and learning and difference between the types of mistakes, but he also made a difference between his type of students and how they perceive their own mistakes. He explained the relevance of supporting those students who are the most advanced in mathematics and who identified themselves as proficient in mathematics to understand that making mistakes is part of his learning.

Moreover, Damian's statement provides a clear idea that students positioned themselves as capable or not capable of doing math in relation to their mistakes by framing their own errors as learning deficiencies. In the same way, Damian's previous statement evidences the connection between correctness and student hierarchical status, which involuntarily makes students and teachers built on frames that have been perpetuated in the math classroom, such is the case of the hierarchical ability frame (Louie, 2017) which one of its significant features is classifying students as high or low. Damian implicitly emphasized errors role for leveling "hierarchies of mathematical ability" that positioned students as smarter than others (Louie, 2017, p. 491). Damian identified how his "high-performance students" get upset or are afraid of making mistakes, that is to say, that he implicitly has identified their non-productive affective disposition toward mistakes, which in turn leads them to frame errors as the cause of their "high-performance" loss of status. Although, Damian's dichotomy between those who should be able of taking advantage of error risk-taking during learning mathematics and positioning all students as capable of succeeding again before having made a mistake might be supporting some of his students' inclinations to see their errors as treats due to the immanent connection reflected on losing their status and error risk-taking.

It is important to highlight that the previous analysis is only taking into consideration Damian's dispositions and positioning toward his own mistakes and his student's mistakes according to his EOQ scores and his interviews. Although, for analyzing how Damian's dispositions toward mistakes are reflected in their classroom teaching, it becomes essential to examine them in conjunction with the way in which he communicated his active disposition toward mistakes, in other words, the way he framed errors in his classroom.

Alignment between Damian's Disposition and Positioning: How Frames were Enacted in the Classroom.

In this section, I first analyze Damian's classroom moment-to-moment interactions when errors emerged as part of a planned activity. Next, I consider the errors as resources to learn framings that emerged during Damian's students' error events and how Damian's enacted frames represented an alignment between his disposition and positioning toward mistakes. Lastly, I analyze Damian's students explicit and implicit responses to the way in which he communicated his active dispositions toward mistakes.

Through all the data from my field notes in Damian's classroom, I perceived a high level of coherence between his dispositions and positioning. Damian's disposition toward mistakes in the context of his own mistakes and his students was aligned with his positioning in the classroom. The filters through Damian frames shaped the moment-to-moment interaction in the context of errors seemed to be represented by his productive dispositions toward errors (G, T, & S).

Damian's embedded productive disposition toward mistakes was enacted using errors as resources to learn frame by applying instructional activities and strategies that involve error's analysis not only as a planned activity but also as part of all his moment-to-moment interactions. In this sense, Damian sent signals to his students from almost exclusively frame --errors as learning resources. Afterward, alignment between Damian's dispositions and positioning was demonstrated by his implicit and explicit communication about the role of errors in learning and how teaching and learning activities, as well as, socio-mathematical norms around students' mistakes established by Damian and his students.

The moment-to-moment teacher-student interaction and peer interaction as was centered on Damian's commitment of capitalizing on errors as he expressed during the interview. During

the interviews, Damian constantly framed errors as the way we learn; he was not only referring to mathematics but life in general by expressing: "Everybody makes mistakes, it is just what you do after that and that doesn't apply not only to math but to life. You are going to make some mistakes, but you got to learn from it and go forward," "Mistakes are part of life is how we learn," and/ or "All make mistakes, then we got to learn from it and just try not to make that same mistake." Those types of statements signaled his understanding of errors as part of all learning processes.

Addressing errors as a planned activity.

Damian's class activities were marked by a high level of student interaction; the same can be observed for his regular class or his algebra class than for his math intervention class, where there are four students only. As he expressed during his interview, he had different types of planned activities for addressing mistakes. Those mistakes could be the ones that emerged during a problem-solving activity, homework revision, or a game. The teacher also prepared erroneous examples to practice a mathematical concept, or he presented problems that were incorrectly solved by one of the students to the whole group for their analysis.

Damian addressed mistakes that student made while solving a problem on the board. He also conducted an activity where students contrast two students' work (with no student name on it), then Damian asked who it did right; students started discussing out loud to detect the mistake and present their arguments about it. The teacher even presented a correct example as an incorrect one, with the aim of having them reflecting on a correct example from a different perspective.

During Damian's class, all students double-checked their mistakes as a daily activity. First, they were reflecting on their mistakes by themselves (first 5 min); next, they discussed their mistakes with their partners even if they were able to correct them or already did it (5 min). Students corrected each other, and when they could not get a consensus, they asked Damian. He

guided his students to understand their mistakes by questioning them or asking them to use a different representation (e.g. geometrical representation). At the end of the class, Damian said, "remember that it is very important to take ownership of your mistakes as part of your daily life because it is the only way to overcome them."

In those type of class activities, the teacher framed errors as the starting point of the daily activities, as concluding the activity, as part of the mathematical-social norms, and as part of his student's real life. Damian's error activities went far beyond the mere act of detecting and correcting mistakes. Damian framed errors as resources to learn because all of his instructional strategies and activities involve error analysis.

The systematic connection between error analysis and learning.

Damian's productive disposition toward errors in the three different contexts (G, T, & S) has been demonstrated through Damian's positioning and in how he positioned his students when he framed errors. He showed his understanding of errors for learning potential and the relevance of developing his student's error competence by encouraging them to show and apply their mathematical knowledge, abilities, skills, and resources not only to overcome errors but even more, to develop their critical thinking from their error-reflection, as it is reported in the following excerpt:

Student B:	(A student raises her hand and Damian approaches her desk). I don't know how to solve it. I am stuck.
Damian:	Try to find another procedure to solve the problem to fix your mistake.
Student B:	But I don't know how. I don't know what happened
Damian:	Try to find your mistake. (Then, Damian says out loud) if you can't fix your mistakes, it is ok, but you need to try to know what is behind them. You need to reflect on what is missing and it is needed to solve the problem, you need to know how you are mistaken to find a way to solve it. You know that you are free to ask for help, but don't ask for help, saying I don't know anything, because that it is not true, so you need to say can you help me

> because I don't know how to do this and that. Try to explain to your partner what you are missing, what you don't know.

The above excerpt contains a sum of different resources to examine how Damian framed errors as learning opportunities. First, the teacher positioned his student as competent to handle her mistake by exhorting her by saying, "try to find another procedure to solve the problem". Second, his perfect scores on learning from errors and thinking about errors which are two of the domains of productive cognitive dispositional functions were powerfully demonstrated when Damian pushed for valorizing learning from error analysis over error correction, stating that "if you can't fix your mistakes, it is ok, but you need to try to know what is behind them".

Error correction was sidelined by Damian and not considered the major concern regarding the idea of reflecting on their errors by becoming aware about what they do not know when he said, "reflect on what is missing and it is needed to solve the problem, you need to know how you are mistaken," "Try to explain to your partner what you are missing, what you don't know." Although, Damian sent a signal that mistakes are something that needs to be "fixed" which might give a sense of teacher framing students' errors as deficiencies, but he stated the opposite by saying that "it is ok" if they cannot correct the error; it demonstrated Damian's productive disposition toward mistakes, since it is the process involved as part of error analysis what Damian emphasized, at the time he understated error correction, per se.

Motivating students to reflect on what they do not know (Mason & Spence, 1999) from the errors that they have made and asking them to explain at the moment they asked for help promoted errors as resources to learn frame. He distinguished between the students' mere ability to detect a mistake, which he characterized as insufficient to understanding errors and to develop the ability

to present arguments about "what is missing" to solve a problem, which is to better understand their own lack of knowledge and to consider ways to overcome with their obstacles.

Damian communicated errors.

Damian error-friendly culture in the classroom was supported by the way he and his students communicate errors. In his case, error communication was depicting as all students' learning the process. Students were the ones who had the main role as participants in the discussion; Damian positioned himself as the one suggesting alternatives and resources rather than the one giving correct answers or procedures, distancing himself from the problem while explicitly highlighted error usefulness on learning. The teacher responded to this as follows:

Student A:	I think, I am wrong cause I am not getting the same answer.
Damian:	Use your calculator to see how the graph that represents your function.
Student A:	Oh! I see I can see my mistake.
Damian:	When you are not pretty sure about your algebraic work or if you made a mistake, you have a way to know it. You can know if you are right or wrong by yourself. You should use the graph of the function to fix your mistakes, to realize if you are right or wrong. You won't need anybody to tell you if you are correct or not, your graph will tell you and see what is not clear. You will see it.

When the student out loud expressed his doubts about his answer, Damian did not approach him, but he framed it as a collective problem by proposing a process they need to go through (Vedder-Weiss et al., 2018). He conveyed confidence in this process by suggesting an alternative representation of the function. Statements such as "You can know if you are right or wrong by yourself" and/or "You should use the graph of the function to fix your mistakes" framed the importance of error self-detection and error self-correction processes. He even went so far as to explicitly asserted, "You won't need anybody to tell you if you are correct or not, your graph will tell you and see what is not clear." That last statement seems to implicitly deny the relevance of discussing error with others by recurring only to the own students' mathematical resources;

however, at the same time, it can provide an idea of how Damian framing was embedded in a productive disposition toward mistakes, which offered the teacher an opportunity to benefit from one student doubts and mistakes to consider mathematical ways to contend with these difficulties that might experience the rest of the class.

Damian states errors' usefulness on learning.

During a workgroup activity, Damian displayed the class roaster with the number of problems that students solved for the last class. The list had every student name next to a column for each problem (for a total of eight). The excel cell for those problems solved correctly was green and yellow for the incorrect ones. All students had access to their own information and the other students' information. It was a common task since without any indication students started working on their own problems, correcting and discussing their mistakes. Those students that got more green cells were supporting their group work members to correct their problems, but also, they were receiving feedback from their peers about their incorrect problems.

During one of my observations, problems included the system of linear equations, Damian remained as an observer until he learned that one of the groups was not even able to identify their mistakes. That group tried to solve a problem by using the substitution method. Damian intervened to explain to them (two students) the elimination method to solve the problem. The students finally were able to solve the problem correctly. Then, Damian responded:

Damian: The good thing about math is that if you are having problems using one method, there are always some other methods that can help you to make sense. For every mistake, you will learn a way that is not useful to solve a problem. Then, you just need to find a method that works for you.

Student A: That it is true because we didn't know that there was another method, and we discovered it because we were failing and failing.

Student B: We know how to use the substitution method, but I believe that we were struggling because of the fractions.

Student A: True, but thanks to that we discovered that there is another method.

Damian: I'm just going to teach it today. But you were first because you're going to help me (laughs).

The above data suggest that Damian has established not only activities that involve error's analysis, but he has established as well socio-mathematical norms that allowed him to openly communicate to those who had erred solving a problem, and in turn, provoked error-friendly attitudes among his students. However, the most overwhelming way that Damian invoked the errors as a resource frame, was not his established activities involving error analysis, or the way that he openly communicated and discussed students' mistakes, not even students' involvement in the error correction process and their level of confidence facing their mistakes, but how he implicitly and explicitly stated errors' usefulness on learning with his students.

Damian gradually introduced the idea of a different method to solve a system of linear equations with a group of students who were struggling to solve that problem. Damian taught them a different method which as he said, he considered an easier method to handle a system of linear equations with fractions. He began by providing an argument about the nature of mathematics activity (Louie, 2017), valorizing the opportunity to use a method that helps students make sense of concepts and ideas since, "if you are having problems using one method, there are always some other methods that can help you make sense."

Damian firmly stated the usefulness of mistakes on learning, not only regarding learning the correct procedures to solve a problem but how "for every mistake, you will learn a way that is not useful to solve a problem." His proposal about learning ways that are not appropriate for someone from the mistakes that person has made, implied how his conative dispositional functions in the context of his students' mistakes were the core of his positioning in this moment-to-moment interaction.

However, in this case, Damian not only activates his conative disposition toward mistakes, but he did the same with his productive cognitive disposition when he framed his student's mistakes as the triggers to introduce the elimination method with those students for the first time in this class. I suggest that when Damian decided to introduce elimination method, he implicitly made a differentiation between the different type of mistakes, even before his student state that "[they] know how to use the substitution method, but [she] believe that [they] were struggling because of the fractions." Damian realized that his students' mistakes were not aligned with any lack of conceptual understanding, instead it was mostly related with fluency of procedures, and at the same time, that specific mistake was aligned with mathematical goals of the lesson "I'm just going to teach it today" shows how Damian selected strategies for problem-solving.

The idea of introducing a new concept, method, or strategy from students' mistakes expanded their mathematical methods and procedures repertoire and presented a unique opportunity for the teachers to support and develop his students' productive dispositions toward mistakes by positioning them as the ones "going to help [him]". Thus, the majority of framing activity that takes place in moment-to-moment interaction reflects and contributes to developing his students' productive dispositions toward mistakes through the three different contexts.

Students rely on others to correct a mistake through discussion.

Damian's students double-checked their mistakes as a daily activity. First, they were handling their mistakes independently (first 5 min). Then, they discussed their mistakes with their small group partners even if they were able to correct them (5 min). During that time, students corrected each other, and if was not possible to get an agreement, they asked Damian who positioned himself as a guide who make questions to help them to understand their mistakes, which usually rerouted students' discussion until they were able to figure out the root cause of their

mistakes and as a result, they solved the problem correctly. Damian's framing involves the activation of his different types of his affective, cognitive, and behavioral dispositional functions (Hammer, Elby, Scherr, & Redish, 2005; Louie, 2017), which in turn, established that way that he positioned himself as a guide. He also privileged his students' positioning as resources to handle their own mistakes and their peers' mistakes at the moment that errors emerged.

In this period, there was also a group not getting the correct answers since they were constantly making mistakes. Damian told them:

Damian:	Go ask Steven for help.
Student X:	Okay
Student R:	Hey Steven, ayúdanos *(Help us)*
Steven:	Ah yo tampoco sabía que hacer allí, miren aquí está mal porque no están haciendo la tabla primero y por eso se pierden. *(ah, I did know how to do it either. Look, it's wrong because you are not doing the table first and that is why you are confused)*
Student X:	¿Si hacemos la tabla ya sale bien? *(So, if we do the table, is it going to be correct?)*
Steven:	Es que la tabla te dice como graficar, o sea lo estas viéndolo *(the table tells you how to graph, so you are like you seen it).* Asi le agarre yo, porque me lo estaba sacando mal y mal. *(that way, I got it, because I was getting it incorrect)*

That conversation seems to be a common student exchange in Damian's classroom; however, I was very surprised because Steven was the student that was struggling in his group and his partners were helping him. Steven came to the place where the group has their box that was used to play a game and which simulated a machine (it is described in detail below) and they told him about how they were struggling to find what they were doing incorrectly. Steven said, "Mario explained to me how to solve one problem and then I was able to fix the other two problems". Students indistinctly positioned themselves as resources and as help-receivers.

During his remediation class, I noticed another way that Damian responded to students' mistakes since he deliberately withheld help, positioning students as capable of independently handle their own errors. For example, at the beginning of the class, Damian was checking students' homework and he started explaining a problem step by step and made some comments about the mistakes that some of them made without mentioning any name. Then, he asked them to solve the rest of the incorrect problems by themselves. Later, Damian had this exchange with a student who was stuck on the revision of a similar problem that he solved incorrectly:

Damian:	How are you doing? Do you get it?
Student A:	No, I don't know
Damian:	(Sticks a note on student's notebook) Remember, what do you need to know to solve this problem? You got it wrong because you didn't know what? I want you to write it down.
Student A:	(starts writing) I already have my list
Damian:	How can you use it?
Student A:	To review my notes.
	(After a few minutes student solved the problem and Damian came back)
Damian:	See, now you know not only how to solve the problem, but possible mistakes that you can prevent.

In the strategy that Damian employed to encourage his student error analysis, he positioned his student as competent to correct his own mistakes and find a way to anticipate and prevent the same type of mistake. The way that Damian positioned his student supported him to persist with his work and might contribute to developing a productive disposition toward mistakes, but more importantly, it might contribute to support him to develop a productive disposition toward mathematics.

It was undeniable that Damian's interactions with the students of his remediation class provided them opportunities to develop a sense of agency and competence. He provided a high level of support, but for the most part, this support takes the form of motivation and peer interaction

encouragement. For example, for those problems that at least one of the four students knew how to solve them, that person was the one who had to explain it to his/her classmates.

The four students of this class were always working as a small group, facing each other and sharing their strategies, for moments, indistinctly one of them took a teaching role, student's participation is balanced and Damian was paying a passive role by standing behind them all the time). Although, when a student was just telling the other three students the procedure to solve a problem without any explanation, Damian interrupted by saying, "Ok, remember that you need to be clear when you are teaching something to your partners." Then, he added, "Remember that you need to make emphasis on the mistakes that they made because they need to understand why that happened."

This classroom data excerpt represents how Damian positioned all his students as capable of coping with errors. First, he did not frame all his students as capable of coping with mistakes, but he framed them as capable of "teaching" mathematics when he asked his remediation class students for showing details clearly. This helped Damian to highlight his remediation students' strengths by offering them the possibility to position themselves as contributors to their peers' learning. By setting an activity where they can "teach" others, Damian was able to position his students' as important resources to support others' learning. As Damian stated, by "mak[ing an] emphasis on the mistakes, because they need to understand," he framed errors as resources to construct mathematical deeper understanding.

In his remediation class, Damian was not only focused on correcting students' mistakes, but he introduced a new topic. He explained it, and he was able to address all questions. Students were discussing how to solve some of the examples he provided, and the teacher was able to check

every student's progress (4 students). When the class was almost over, and they were packing their belongings, Damian asked them:

Damian: How do they feel?
Student A: Good!
Student B: Confident
Damian: Now, you don't only know how to solve these problems, but how to explain them to others, so go and help your partners for the regular class

When the students left the classroom, Damian approached me and said:

> I always try not only to explain to them the concepts that they are having problems with but sometimes one concept ahead, since they are just four, I have that opportunity. I think that sometimes that kind of students only need to feel confident, to feel capable. They are here because they didn't pass their STAAR test last year, but they can learn as same as the other students that passed it. So, if we label them and treat them as students that failed their mathematics test, they will fail again, you know? So, I believe that if I go one topic ahead with them and they go to their regular class and help their classmates, gradually not only they are going to change their perceptions about themselves, but their teachers and their peers as well.

It is clear, that Damian framed all his students, but particularly, his remediation class ones, as capable of succeeding again before having experienced failure or having made a mistake. He showed an honest concern about the relevance of positioning those students as equally important contributors to the group's work despite their standardized test results. Accentuating his students' strength to teach others even during their regular mathematics class, represents by itself a clear way that Damian used to frame all his students as competent and qualified to handle their error analysis process and produce mathematical ideas by themselves.

The idea of "go[ing] one topic ahead with them and they go to their regular class and help their classmates", It was a unique experience in the classroom observations I conducted for teachers to position "low-performance" students this way, as experts who ought to help their "high-performance" peers. It represents a creative form to address his concern about his students' self-

esteem, that once again provided evidence of his deep-rooted productive disposition toward his students' errors.

Students being confident and seeing as valuable discussing and getting agreements when they are working with others

The following student exchange provides evidence of how error-discussion was established as part of Damian's class, which in turn, supported students' confidence to discuss and get agreements when discussing their errors. Students sought resources to handle their mistakes without any previous teacher's indication when they were not even able to recognize their errors.

Student N:	Ay no, nos salió otra vez *(we didn't get it again)*
Student M:	¿Por qué? *(Why?)*
Student N:	Sepa *(don't know)*
Student M:	Let's ask Jenny
Student N:	Good idea. Hey Jenny, help us! We are trying but we don't know.
Jenny:	Let's see. Did you remember when Coach D (how the students called Damian) was explaining how to interpret Distance-time graphs?
Student M:	Kind of.
Student N:	Ok, we just have these two parts of the graph, but what happens when the car was parked? We are not like, we don't need that part.
Jenny:	Why not? Did you stop time?
Student M:	No?
Jenny:	I don't know. Could you?
Student M:	Got it, got it
Student N:	Me too.
Jenny:	Ya ven, ay ta, ya entendieron (*Look here it is, you got it*)

Damian was observing them closely, but he did not intervene. By doing so, he positioned the students who were struggling to solve the problem as free for moving, for seeking help, and the one who helped them as resources for learning. It is important to emphasize that students understand their positions since the ones that asked for help, asked actually for help, not for responses. The one that helped them opened a discussion based on her teacher's explanation

"remember when Coach D was explaining how to interpret distance-time graphs". She did not provide an answer, instead, she made guiding questions "Did you stop time?". Students' exchange provides evidence of their understanding and their positioning according to the established socio-mathematical error norms. These types of students' attitudes are not limited to small groups interactions, instead, they represent a constant in all type of activities. For example, when students were solving a problem on the board and they made a mistake, the other students out-loud started signing "you can do better". The student that made the mistake kept trying until the problem was solved correctly. Damian, and/or the rest of the students, only supported the person answering the problem with hints.

Discussion of errors as part of the socio-mathematical norms.

Steuer, Rosentritt-Brunn, and Dresel (2013) stated that students' positioning toward learning from their mistakes depend not only on individual characteristics, or how teacher position them but also on learning environment features. Moreover, the authors found that students' individual reactions to errors were based on how they perceived the error climate in their classroom. Learning was enhanced when the negative emotional impact of errors was reduced by an error management culture that encompassed practices related to error-communication to sharing knowledge about how to detect and handle them (Keith & Frese, 2005).

Throughout the month, Damian regularly employed an array of instructional practices and strategies to foster students' confidence to openly communicate their errors. At the same time, those activities boosted student inter-dependence on each other as a socio-mathematical classroom norm. This positioning by Damian's students to correct a mistake was through discussion by relying on each other knowledge. They were getting agreements through mathematical error argumentation working with others about how to solve a problem

Damian framed his students as competent and qualified to handle their error analysis process by themselves by offering ways to cope with these challenges. He established a norm that goes beyond a positive attitude toward errors that according to Matteucci, Corazza, & Santagata (2015) is shown by a teacher when he/she informs others about a mistake that a student made to prevent them to make the same mistake. In Damian's classroom, the process of error communication was carried out by students. This communication was not only based on the idea of preventing others to do the same mistakes, but it was built on seeking new alternatives for solving a problem through a conscientious analysis with their peers. It was set on the idea of teacher supporting students being confident despite their mistakes and the nature of those and seeing as valuable discussing and getting agreements about new alternatives to solve a problem or apply a concept.

In other words, error communication in Damian's classroom might be identified as an "error culture" where students are encouraged to identify, discuss, reflect on their errors as tools for learning (Oser & Spychiger, 2005). As a result, error communication concerning Damian's productive disposition was activated by his positioning not only as an individual teacher but also as a primary element of the learning environment for intrinsic features (Steuer, Rosentritt-Brunn, & Dresel, 2013).

Students are encouraged to seek and value alternative ways of the error analysis process.

Through all Damian's classroom observation data students played an active role not only in regards to their learning but to their peers' learning also. They were in charge of error handling processes during the different type of classroom teaching and learning strategies that Damian applied. For example, a game in which students were working on groups and each of those groups has a box. The box had two small holes with the labels: outcome and income. One of the students

of each group took turns to be the one who is inside the box. The student who was inside the box had the problem correct answer and he/she was the one who checked the other three students' work. Every group had its own set of problems and I can see that the groups were working at their own pace. All the students were talking and discussing their procedures and joking about their mistakes. At the same time, the group members were able to see what other groups were doing and freely communicate with them. Damian was walking and observing them, he was not only observing their mathematical procedures and answers but also students' interactions; he was very focused on all group members participation. Damian detected that one of the groups was getting all their problems wrong. He observed them for a while and the following interaction took place.

Damian:	What does it happen, guys?
Students:	We don't know (smiling and shrugging).
Damian:	Go and see who can help you, you can't stay like that
Student A:	(laugh) Okay, ve tu para allá a ellos les esta saliendo todo bien" *(you go with that group, they are getting everything right)*
Student B:	Yo voy con estos (pointing the group next to them) se me hace que ya hicieron estos *(I go with those; I think that they are done with these ones)*
Student C:	Ay si, ni que fuera tan listos, deja ver (laughing). *(oh yes, I don't think they are that smart, let me see).*
	Then, when these two students were talking another group.
Student B:	Hey, explíquenos, what are we doing wrong" (he has the sheet of paper that they use to solve the problem).
Student K:	Mira aquí, que fue lo que paso *(look here, what is happening)* compare ours with yours. What are you missing?
Student B:	We didn't multiply the fraction
Student C:	Si, es eso y por eso ya salió todo mal *(yes, it is the reason and then everything was wrong)*. Lo mismo paso aquí, bueno más o menos *(the same happened here, well more or less).*
Damian:	Tell them that they can do better, asking them to think a little bit more.
Student K:	Ay ya se, si saben pero no se concentran, por eso se equivocan y les sale mal *(it is true, you know, but you don't focus and that is why you got them wrong).*
Student C:	Ok, we already know what we are doing wrong, it is not that hard

At first glance, Damian's participation seems to contain one framing alignment problem at the moment he positioned his students as the ones who need help to remediate their mistakes by saying, "Go and see who can help you, you can't stay like that". Notwithstanding, Damian's efforts to position students as resources for each other and at the same time, positioning those students who made mistakes as capable were evident in this data. The teacher invited students to rely on others to correct mistakes through discussion, which in turn, represents students being confident about their mathematical ability despite their errors. Damian's positioning all his students at the same level of mathematical ability and even at the same level of understanding became evident not only when he asked the student who was supporting the ones that made the mistakes to asked those to make a bigger effort, but when the student framed his classmates errors not as a deficiency of learning, but as a lack of attention.

SUMMARY

In any given instance, there is no guarantee that all students will generate mathematical ideas from the discussion of every error that they make. But over time, as Damian routinely positions all students as competent to generate mathematical ideas from the analysis of their own mistakes and their peers' mistakes, it will foster students' possibilities to discuss the type of mistakes that ought to promote their mathematical understanding. Damian's alignment between his productive disposition toward mistakes and framing errors as tools for learning was constantly evident.

BIANCA'S DISPOSITIONS TOWARD ERRORS: WHEN THE CONTEXT MATTERS

Bianca's disposition toward mistakes was measured first by using the Error Orientation Questionnaire. She scored 50 points of a total of 124 for her disposition in the context of her own errors and 95 in the context of her students' errors. Her disposition toward mistakes in the context

of her students' mistakes score was in the highest percentile as was described above. Table 4.2 provides an outlook of her scores as compounded by the eight domains of the EOQ:

Table 4.2 Bianca's EOQ scores in G-general context, T-Teacher errors context, S-Student errors context

	Dispositional Cognitive Function				Dispositional Affective Function		Dispositional Conative Function		
	Error competence	Learning from errors	Error anticipation	Thinking about errors	Error strain	Covering up errors	Error risk-taking	Error communication	TOTAL
G	11/20	17/20	17/25	17/25	-18/25	-23/30	16/20	15/20	52/124
T	11/20	18/20	16/25	17/25	-20/25	-23/30	16/20	15/20	50/124
S	17/20	20/20	18/25	16/25	-8/25	-6/30	18/20	20/20	95/124

Bianca's EOQ scores overview.

A review of the mathematics teacher's disposition toward errors in two contexts revealed that Bianca had a considerably higher disposition score toward her students' mistakes, and quite surprisingly a low disposition score toward her own mistakes. In Table 4.2, (G) scores refers to the first questionnaire that was applied, in which context was not specified. The second and third rows (T) and (S) are the scores for the second questionnaire that was applied four months later in which context was precise for (T) teacher's error and student's errors (S). On the contrary, between (T) and (S) no consistent pattern has been found. It is clear that (T) and (S) scores reflect the biggest disparity between Bianca's disposition toward error, as it is as well (T) and (G) almost identical scores despite the time that has passed. Such similarity between (T) and (G) scores reflect the hegemonic nature of Bianca's core disposition toward errors. This similarity reflects a deeper

personal disposition. In this way, this similarity reflects her personal context is the root of her dispositions toward errors.

Then, Bianca's dispositions are not only relevant but, in fact, stand at the core of handling errors during her math class. The difference between (G) and (T), on one hand, and (S) on the other, presents a conflict that may influence the interactions between Bianca and her students in the context of errors. This disparity may be responsible for practices based on Bianca being more influenced by her experience as a pupil or her teaching routines than her understanding of the mathematics reform.

Such a set of disposition also provides an idea of how her own mistakes define her inclinations in term of the dispositional mental functions. In Table 4.2, also can be observed how the results' disparity is mostly provoked by her dispositional affective function scores. This fact provides an indication of how her affective response to mistakes impacts her productive disposition toward mistakes. Given that, error strain and covering up errors scores that are subtracted from the total score (See the Methodology section), it shows how errors are perceived by Bianca as a threat.

Excavating into her dispositional affective functions allows seeing how dispositional functions are constituted. Bianca shows a tendency to experience a lot of stress, concern, and embarrassment toward mistakes in the context of her own mistakes (T) and not have the tendency to experience those type of emotions and feelings on the context of her students' mistakes. In other words, errors in her own context as a teacher are perceived as a source of stress and embarrassment, mainly. Bianca's inclination to believe that her own errors in relation to her mathematics teaching and learning as a negative result, it is convergent to her tendency to view errors in a general context, as the similarity in the scores between (G) and (T) show it.

Unwrapping Relationships between Bianca's Disposition and her Positioning

Bianca's EOQ responses and scores manifest that disposition toward her own errors underlay her most constant disposition; however, these data provide only a partial answer. Understanding what characterizes Bianca's dispositions toward mistakes in the context of her students' mistakes and her own mistakes and how her changeless disposition was reflected in their classroom teaching was not possible by only measuring how her disposition was revealed.

With the aim of clarifying convergences and divergences among the three EOQ scores (e.g., G, T, and S) and start making connections between her responses and her positions toward mistakes, asking Bianca about what context did she think while answering the first EOQ (e.g., G) was crucial. That is because, as was presented in the quantitative data, EOQ scores are influenced by the context. Then, Bianca's response, "I was thinking about what I used to do [in the classroom-MA], I answered the questionnaire thinking about my students, about the mistakes they make" provides a vague connection between her changeless disposition toward mistakes (G & T) and her teaching practices.

The quantitative data provided evidence of how (G) and (T) cores are almost identical for the three different dispositional functions. However, her position expressed a connection between (S & G) by saying that she was thinking about what her students' mistakes when she answered (G) EOQ provides an idea of how her positioning is not restrictive to her affective disposition toward mistakes, but it was influenced by her conative and cognitive dispositions as well. That apparent connection among her non-productive dispositions and her teaching practices when errors emerged was moving from professional reform-based discourses down to her disposition.

In the same way, when she was asked if there were opportunities for her students to discuss their problems and mistakes with their peers, Bianca replied, "It is the rule, is what they

[authorities-MA] ask us to do." The underlying assumption was Bianca's idea of how she positioned herself during class which not only may be considered as part of her teaching experience, beliefs, and knowledge, but it also reflected her understanding about math reforms; this provided evidence of her cognitive productive disposition toward mistakes. On the other hand, Bianca's non-productive dispositions toward mistakes (G) (T) was caused by her affective dispositional function. Nonetheless, Bianca stated that she was fully convinced about the purpose of errors in mathematics learning, supporting mathematics learning experiences of all her students.

Bianca's positioning and dispositional cognitive function.

Bianca perceived errors as something that makes her students have a better understanding as she expressed during her interview: "It is easier to learn when they make a mistake than when they know it very easily since the first time," "when they see that they were wrong and how they were wrong, sometimes the concept becomes clearer than when I explain or when a partner explains to them," or "An error makes you pay more attention, that it is the reason why they learn better." Several aspects of these statements are noteworthy to understand how Bianca's productive dispositional cognitive function is rooted. First, these express Bianca's understanding of errors as tools for reflection and analysis. Second, Bianca did not only assign a positive status to errors, but she expressed in some way how errors were an important condition for having a deeper understanding. Third, she expressed a systematic connection between students' errors, analysis, and learning.

Furthermore, according to Bianca's position, errors were not seen as a learning deficiency. It is clear that she did not see errors intrinsically as springboards, but the analysis that can take place in handling them. This is further supported by the fact that Bianca was not centering her explanations or clarifications about a specific problem as the main resource for learning; instead,

she was focusing on how students' error detection and error analysis had a direct influence on their learning. More specifically, Bianca positioned herself as a teacher who believes that for learning is not necessary to get the right answer on the first attempt but to reflect on what is done and how it is done, or on what needs to be done and how it needs to be. In that sense, she established that errors provide an opportunity to reflect on a problem instead of solving it mechanically. She emphasized thinking about errors and learning about errors:

> When they make a mistake, I feel that they are learning better, because for example, when I send them to review their notebook, to review the book or what we underlined, or what we wrote about what we learned, or some classmate says, remember that we saw this and this. Then, they start thinking and they usually say "oh, it is true", so they notice it by themselves. They review step by step their own procedure, and they say "oh, it's true, that's where it went wrong. So, I should have done this, instead" and they erase and correct it by themselves and then say, "because this and this" and, then I say, "now it is clear."

As is evident from the interview excerpt above, Bianca was drawing attention to important aspects of errors as learning tools. As she explained, "I send them" becomes clear that, usually, she was the one who detects the error first. However, the error was not mentioned at all "they notice it by themselves," as a result, the following step after Bianca let their students know about their mistakes was students reviewing all the information that they were supposed to have about the concept that they were working on.

According to Bianca, her students used all possible resources like books, their notes, class' reflections, even their peers to learn what the error was—to detect the error. Bianca's statements were a public expression of her productive dispositional cognitive function by taking away the attention from the error, per se, opening the possibility of starting a reflection from an error.

Moreover, even when Bianca expressed to be the one in charge of detecting her students' mistakes, it was the student who needed to find the error, reflect on it, analyze it and correct it. Indeed, Bianca focused her attention on student's error detection as a result of a process review,

“they review step by step their own process and they say oh, it's true, that's where it went wrong” which worked as a first step to correct it. As a part of that practice, Bianca expressed that the process of using errors as resources did not start, nor stopped, with student’s error correction. “They review step by step their own procedure, and they say oh, it's true, that's where it went wrong. So, I should have done this, instead.” To the contrary, it was a process that needed a certain level of argumentation of the reason of why an error was made “because this and this and, then I say, now it is clear until student realizes the nature of his/her mistake.” When she expressed how her students provided some argumentation about their mistakes, it seemed that a glimmer of analysis can be found.

That error classroom practices which Bianca expressed to promote, encompassed her productive disposition toward mistakes in the context of her students’ mistakes (S). Bianca positioned herself as a teacher who had an inclination from a cognitive perspective to believe that working on mistakes was not only a process to detect it, erase it, and correct it, but as Bianca concluded, there was a brief analysis shown by the student argumentation. Even more, it can be used as a teacher’s evaluation tool when a student expressed “now, it is clear.” Therefore, Bianca favored the use of errors in her classroom as a whole process which involved practices of error detection, error analysis, resources for error analysis, error correction, error argumentation, and error as a tool for teacher learning evaluation and student self-evaluation.

However, as it can be seen on Bianca’s EOQ error competence, a domain related to cognitive aspects showed a low score which provides an idea of a non-productive disposition associated with this specific domain. That domain is mostly focused on knowledge about students’ types of errors. Therefore, even when Bianca recognized the relevance of thinking and learning from errors, she did not express anything about understanding the different nature of her students’

mistakes. Hence, Bianca did not make any differentiation between types of errors. Mainly, her use of errors was limited to procedures and correct answers. That in fact, did not only compromise Bianca's ability to capitalizing on errors and framing errors as resources for learning but it could turn what at first sight was an instructional strategy to capitalize on errors into a student analysis focused on the procedure to find only the correct answer. In this sense, a calculation error might be different from a conceptual one, or an error in reasoning; as a consequence, having a student who made a mistake in a procedure only might not be enough for his/her mathematical learning and understanding. Although overall, it is true that all mistakes are important, it is also crucial that a teacher emphasizes conceptual and reasoning error with the potential for supporting students' critical thinking.

In addition, it is important specifying that error reflection was not limited to an individual process between the student who made a mistake and his/her mistake or one-to-one analysis (teacher-student), but it was open to discussion with other students (e.g., student's peers as resources to correct a mistake), however, it is essential that teacher distinguishes among errors, something that did not happen in Bianca's classroom. The main reason is that some of the students' errors (e.g., computational mistakes) would not represent a worthy use of class-time, such it was in this case. In this way, a productive cognitive disposition toward mistakes also involves distinguishing among types of errors, which seemed not to be included on Bianca's objective assessment of her students' errors.

Bianca's positioning and her dispositional affective and conative functions.

In Bianca's case, error strain, which is a domain of affective dispositional function, intertwined with error communication, which is part of conative dispositional functions. Because error communication goes beyond the mere fact that a student is being informed or not by her or

his teacher about an error, it involves attitudes and negative emotional reactions. These emotional reactions set how teacher positions him or herself and how her students are also positioned by her at the context of mistakes. In other words, communicating an error is setting the rules of how it needs to be treated. For this reason, in order to assess errors' role in learning examining Bianca cognitive disposition would be not enough since aspects from the affective and conative dispositional functions play an important role, too.

There was a strong consistency between Bianca's affective dispositional functions and her response to the kind of feelings that she had experienced when she had made a mistake in front of her students. When Bianca was asked about the type of feeling that she experienced when she makes a mistake, she emphasized more than once,

> "I feel shame, I feel embarrassed, yes, I feel that, but only for a little time and then you have to recognize it and talk to them, I am honest and tell them that what I did was not right,"
> "I recognize my mistakes. Even when it is embarrassing,"
> "When I am wrong, I feel shame, and when they (students) are wrong I feel that I am not doing well."

These statements are remarkable for two major reasons. First, underpinning her feeling of shame and embarrassment, affective disposition was the core of who Bianca was. These dispositions are an acceptance of the negative status of errors as natural, valuing them as deficiencies on learning. This meant Bianca's shame and embarrassment feelings were deep underlying influences on her non-productive affective dispositions toward mistakes. Second, in the last Bianca's statement, she drew her attention to the difference between her own mistakes and her students' mistakes which she also reported on her EOQ scores.

Bianca's affective dispositional modes were the key for understanding the discrepancies between her disposition toward errors in (T) and (S), Bianca's underlying feelings about mistakes

were negative, no matter the context. Those discrepancies between Bianca's affective disposition toward her own mistakes and her students' mistakes might be elucidated by her last statement "when they (students) are wrong I feel that I am not doing well." Her productive affective disposition toward her student mistakes, even when it was involved a negative feeling was because Bianca positioned herself as responsible for her students' mistakes. In a certain way, it is not about who is making the mistake, but who is responsible for it.

Bianca positioned her students as learners, and according to her statements, that position allowed them to make mistakes, without any ownership. In this sense, she positioned herself as a source of knowledge which limited her from learning using her own errors. Bianca's position and the way that she positioned her students provides an explanation of the important gap between the (G & T) and the (S) affective dispositional scores. Statements as "You are learning, it is ok to be wrong" shows a strong coherence between her affective disposition which forms a justificatory base of her positioning. In this way, when Bianca talked about her disposition toward her own mistakes and her student's mistakes, her positioning fluctuated between mistakes as learning deficiencies and teacher's knowledge weakness. This fluctuating positioning provides an understanding of the challenge that teachers face in order to integrate a different error conception.

TENSIONS BETWEEN BIANCA'S POSITIONING AND DISPOSITIONS: HOW FRAMES WERE ENACTED IN THE CLASSROOM

In this section, I first analyzed Bianca's classroom moment-to-moment interactions when errors emerged as part of plan activity. I then considered the errors as resources to learn framings that emerged during the different error episodes experienced and how Bianca's enacted frames represented a tension between her disposition and positioning toward mistakes. Lastly, I analyzed

Bianca's students explicit and implicit responses to the way in which she communicated her active dispositions toward mistakes.

Bianca's disposition toward mistakes in the context of her students was not aligned with her positioning in the classroom. On the other hand, the filters through Bianca's frames shaped the moment-to-moment interaction in the context of errors seemed to be her dispositions (T & G) instead. Bianca's efforts to enact using errors as resources to learn frame by applying instructional activities and strategies that involve error's analysis were presented. However, Bianca sent signals to her students from both opposite frames—errors as learning resources and errors as learning deficiencies. Consequently, tensions between those two frames revealed Bianca implicit and explicit communication about the role of errors in learning and how activities around students' mistakes were constructed by Bianca and her students.

The moment-to-moment interaction between Bianca and her students not only was centered on Bianca's wish for capitalizing on errors but as she expressed during the interview her need of student self-correction given a large number of students in her classroom. Consequently, Bianca's framing of students' errors during teaching shifted not only based on how it was affected by her dispositions but also on how it was affected by the context of her teaching.

Addressing errors as a planned activity.

Bianca's first activity in the classroom, which she repeated during the seven periods, was asking her students about homework. Students immediately responded by making a line to present their homework. This opening task was recognized by her students as part of the starting point of their class routine. This way, she started checking each student's notebook, problem by problem by writing a checkmark on those which were correct, and she left on blank those which were incorrect. Then, as soon as students returned to their desks, they started working on those problems

that did not have a checkmark. Bianca did not even say a word to them, nor a simple indication, she was just checking students' problems and gave them their notebooks back immediately. As part of this process, they would go directly to work on their incorrect problems. Hence, Bianca without saying a word communicated to her students what she expected her students to do, to know, and even, explain their mistakes.

I saw that process used by Bianca to check her students' homework daily. It was planned and established since students followed it without any additional direction. In this introductory activity, she framed errors as a starting point of the daily activities. However, this activity might also be perceived as a teacher error detection which gives the impression that the teacher frames errors as learning deficiency by positioning herself as the one who has control over the process. In this same vein, Bianca positioned herself as the one with the possibility to know and differentiate what was wrong from what it was right by centering on an outcome or problem response and as a result positioned her students as the ones who need to remediate their mistakes by correcting them.

This planned activity where students knew what they needed to do opened a possibility for teacher and student to address errors. Since, in the first place, students were involved in their error correction but not exclusively. Second, this was a routinely established activity in terms of time and resources. Regarding time, Bianca assigned ten minutes of her daily class to work on errors. In terms of resources, a student who made a mistake was allowed to use his/her peers as the main resources to correct it through discussion. The following data provides a clear idea of how Bianca handled her introductory activity.

> Bianca: You have 10 minutes, remember please, please, please, don't limited yourself to just copying the correct answer, I know that I always repeat this, but instead ask how to do it, because as you know if you are randomly chosen and you already fixed the problem on your notebook, but you still making the same mistakes. It is going to count as a negative point.

The systematic framing of errors and low grades.

Bianca's words give an idea of how she went back and forward between framing errors as learning tools and framing them as a learning deficiency of learning. Bianca pushed for valorizing learning over correctness which was evidence of her attempts to enact errors as a resource frame: she reflected on how she centered her attention on knowing the process but not the correct answer. Although, she sent signals that mistakes are something that needs to be "fixed" which, at the same time, provides evidence of how Bianca framed her students' errors as deficiencies. In addition, she explicitly communicated the traditional link between mistakes and negative grades (e.g., "it is going to count as a negative point"). Emphasizing the student's lack of reflection, discussion or attention when an error persisted after an opportunity to "fix it," Bianca implicitly sent her students a signal of perceiving errors as a threat.

After ten minutes passed, Bianca called those students who got an incorrect problem to present it on the board by talking about the error that they made and how it was corrected. If students were not capable of providing the right procedure and answer they were penalized with a bad grade or "negative points" as Bianca said. Because, even when the student's mistake is communicated, attention is focused on the fact that a mistake was done and need to be corrected or it will result in a bad grade. That connection between mistakes and grades provides evidence not only about Bianca's positioning toward errors in terms of deficits, and at the same time, how she positioned her students as vulnerable by framing them as "weak on math."

Statements as "you are not ready" when a student was not able to present a correct procedure, it is an example of how Bianca framed her students' mistakes as flaws of their ability to learn mathematics. The following examples of how Bianca framed her students' errors indicates

that even when teachers made efforts to incorporate errors to the teaching and learning activities, she still signals poor student's grades as the consequence of their errors.

> "If you don't fix your mistake, it is going to count as a negative point."
> "In this second attempt, you have a valuable opportunity for correcting your mistakes and get a better grade. Because, if you don't fix your problems you will fail."

Whether or not Bianca's intention to frame her students' mathematical mistakes as resources to learn and at the same time position her students as capable of coping with errors, these statements do not assist her or her students to handle mistakes as learning tools. Even more, those type of statements might influence her students to see their mistakes as flaws of their ability to learn mathematics.

Bianca discussed mistakes openly, however she set them as the result of lack of ability and practice.

Discussing mistakes openly in Bianca's classroom was a teacher-directed activity of keeping individual students responsible for their mistakes with limited involvement of other students. During Bianca's math class it was common to hear her saying, "See, you are just staring at your problems because you don't even have an idea about what it is wrong, that is not going to fix your mistake, find a classmate that knows it," "ask someone that got it right to explain you", and "ask who can help you to fix your mistake." Those statements provide an idea of how Bianca expected that those students who made an error participate in a different position from those students who had answered their problems correctly. Bianca supported error communication from a perspective that positioned those students who made a mistake as not capable of coping with errors by themselves and those other students that correctly answered their problems like the one who was able to "fix" a mistake.

It represents a tension between handling errors as tools for developing students' critical thinking and handling errors for remediating a learning deficiency. That tension was observed while Bianca was focused on mistakes, specifically on the fact that a mistake was made and on fixing it. Consequently, in this activity, there was no differentiation between computational mistakes and other types of mistakes (e.g., conceptual mistakes, reasoning mistakes). Therefore, mistakes were not addressed by students for achieving a deep understanding, but for getting a correct answer. There was no flexibility and openness toward the students' mistakes to communicate and involve students in the analysis of those type of mistakes to support all students' learning.

Even though Bianca had a good intention, the activity did not support each student's mathematical learning, it was directed only to those students who made a mistake. This intention was not aligned with understanding mistakes as significant resources for learning, for helping students to gain conceptual understanding by examining meaningful mistakes. Another example was when during a monthly evaluation, Bianca gave her students a second attempt with the aim of correcting the mistakes that they made in their test and she expressed to all with a loud voice as the following:

> Remember that now you are trying to fix your computational only, during class you had the opportunity to fix those that you made because you don't know. Because if you don't know what it is wrong because you don't understand, don't waste your time and mine as well. If you don't know what it is wrong, how can you correct it? so pay attention and ask at the right moment.

In this excerpt, Bianca differentiated between the type of errors by not framing those computational errors as learning resources for students, instead, she made an implicit statement about knowing and error correction. The excerpt also provides an idea of how the error handling process was based on valorizing correctness in the first place. There was not an analysis of

mathematical ideas or concepts, nor a reflection on why a specific mistake may arise due to the complexity of the topic. Errors were framed as tools to highlight students' lack of attention and as a consequence, as she stated students' lack ability.

Bianca's positioned students as capable of coping with errors, however, the discussion about errors was based on routine problems.

Bianca's planned activity to addressing errors influenced students' responses toward their own mistakes in different ways. For example, in spite of Bianca's fluctuating positioning toward error and the opposite error frames that she enacted as a result of that fluctuating positioning, students tended to respond favorably to Bianca's efforts to enact a frame that positioning students as capable of coping with errors. The following discussion between three students provides an idea of students' initiative to handle their mistakes, how they got involved into the error correction process, and how they relied on others to correct a mistake through discussion during the ten assigned minutes to correct mistakes.

Student A: Did you got problem #3 correct?
Student B: Yes, I did.
Student A: May I borrow your notebook to see where the problem is?
Student B: Yes, go-ahead
Student A: How did you get this?
Student B: Let me see

At that moment, another student asked student B if she got problem #3 correct. That problem was asking to solve for x. It was a linear equation, but the result was a fraction. Then the student who correctly answered the problem said,

Student B: Let's do it together, because you both have the same problem
Student A: Ok, I almost get it
Student B: So, you explain it
Student A: It is because I did everything fine the first time, but then I changed it because x is equal to a fraction and I was confused.
Student B: (laughs) a fraction is also a number.

Student A: (laughs) yes, but it looks odd, it was the first problem with fractions, that is why I decided to change it because I thought that it was wrong and then it was wrong (laughs).

Student C: No, I don't know how to do it, because the teacher always explains problems with the x here. (his problem was that he could manipulate the equity since x was at the right of the equal sign)

Student A: I got confused also when x is in the right like we are accustomed to seeing it in this side (pointing the other side of the equality) but look just switch it and take care of the signs.

Students B: It is an equal sign. And what does it mean?

Student C: So, it is the same, the only difference is that (silence) so it is the same if x is here or here (pointing the two sides of the equality)

Student A: Yes, for example, x=5 or 5=x, is it the same?

Student C: Okay! I got it!

Sometimes, the conversation represents error correction as sharing and comparing answers between the expert student and those who need help, and at times, as sharing student understanding of mathematical ideas. This was true not only for this specific group of students but for all the rest of the class as it is stated on data. That type of error had its root on the type of examples that the teacher offered to them. Having similar structure examples to discuss during class, lead to reduce a mathematical concept to a process that needs to be mechanized. Presenting routine problems to students represents a lack of opportunities for students to understand and explore a mathematical concept, which in turn will lead students' reasoning mistakes.

Discussion of errors as part of Bianca's classroom socio-mathematical norms, although some students passively wait for peer correction of their mistakes.

Even when students were not arranged to work in groups due to the space and the number of students, they freely moved around to discuss their mistakes with their classmates. Students discussed errors from their homework and from their classroom work; those errors were not exclusively computational mistakes. For example, in the previous case, student B showed evidence

of reasoning correctly, however, her error was due to the reasoning difficulty that fractions involve. On the other hand, student C error was due to his misunderstanding the underlying concept of equality, which at the same time did not allow him to be reasoning correctly.

Bianca's students relied on others to correct a mistake through discussion. It can be said, that students were the main resource to correct mistakes. That correction process, at times, represents students as capable of producing mathematical ideas, since even when detection was only at the teacher's domain, discussion and correction which are crucial aspects on error handling process were on students. However, other times, Bianca's classroom correction process represents an activity where some students are positioned as receivers of those mathematical ideas since during all my observations was common to find the same students helping or supporting others just to communicate and evaluate their answers. This can be read in the following example:

Student D: Hey, let me compare my homework. Oh, I got x=-2
Student E: I got it correct. It is x=5
Student D: Ah let me copy it.

SUMMARY

Tensions between handling errors as tools for developing students' critical thinking and handling errors for remediating a learning problem was constant through all Bianca's classroom data. Bianca's enactive frames showed a point of converging between her dispositions toward mistakes (G & T), even when her framing of mistakes was in the context of her students' (S) mistakes; the frames that were enacted converge with a non-productive disposition toward mistakes in most of the data. In this sense, Bianca's positioned in relation to errors inevitably shifted as she engaged in her students' treatment of errors. In this sense, even when according to her EOQ (S) she expressed a productive disposition toward her students' mistakes, her persistent dispositions (T & G) represented the core of her teaching in the context of errors.

All her efforts from going outside that core shown by including activities for addressing mistakes as part of her daily activities and the fact that her students can recognize activities around errors as part of the classroom socio-mathematical norms, in the end, were not enough for taking her out from the core of her disposition. Then, when errors emerged in Bianca's classroom, her immanent non-productive disposition toward errors (T & G) continuously became activated by framing errors as learning deficiencies. Bianca's error fluctuating positioning is not only reflected on her teaching, but also on how she positioned here students toward mistakes.

ANA'S DISPOSITIONS TOWARD ERRORS: AN ENDURING NON-PRODUCTIVE DISPOSITION THROUGH DIFFERENT CONTEXTS

Ana's scores through the three different EOQ questionnaires present undeniable similarities, which in turn, provide an idea of her entrenched disposition toward mistakes. Ana's general context EOQ score was located as one at the lowest percentile, she scored 68 points of a total of 124. The following table provides a summary of her scores by the eight domains of the EOQ in the three different contexts:

Table 4.3 Ana's EOQ scores in G-general context, T-Teacher errors context, S-Student errors context

	Dispositional Cognitive Function				Dispositional Affective Function		Dispositional Conative Function		TOTAL
	Error competence	Learning from errors	Error anticipation	Thinking about errors	Error strain	Covering up errors	Error risk-taking	Error communication	
G	14/20	20/20	17/25	18/25	-19/25	-17/30	14/20	17/20	66/124
T	14/20	20/20	17/25	19/25	-20/25	-12/30	14/20	18/20	66/124
S	15/20	20/20	19/25	18/25	-24/25	-18/30	14/20	19/20	63/124

Ana's EOQ scores overview.

In Table 4.3, as it was specified for the other two cases, (G) scores refers to the first questionnaire that was applied, in which context was not specified. In the same way, four months later, Ana answered the second questionnaire where the context was precise for (T) teacher's errors and student's errors (S) and which is reported as part of the second and third rows in Table 4.3. As can be noticed, there is a slight difference between them. However, I performed an in-depth

reviewed with the aim of detecting and understanding the source of her non-productive disposition toward errors in the three contexts. At first glance, the similarities between Ana's scores provide information about a strong non-productive disposition toward mistakes; also, it draws attention to certain strands that make them substantially distinct or identical.

First, Ana's disposition toward errors total scores in the (G), (T), and (S) contexts are very close despite the time that has passed, which in turn, it provides evidence of how her entrenched non-productive disposition toward errors is persistently revealed. More specifically, the (G) and (T) score totals which are identical. However, even though there is no difference between those two total scores, there is a difference in how those scores are compounded which needs to be stressed.

Zooming in on EOQ domains becomes crucial to understand how Ana's disposition toward errors is constituted. In this regard, even when (G) and (T) scores total are identical, they are not homogeneously constituted. For example, the most relevant difference is amongst the covering up errors which is a domain of the affective dispositional function. For this specific domain, it is also a significant difference with the (S) context. There are five points of difference for the former domains and six for the latter. Such a difference in that specific set of disposition provides an idea of how Ana's own mistake inclinations are distinct in term of the dispositional affective functions.

Unwrapping Relationships between Ana's Disposition and her Positioning

Ana's EOQ scores through the three different contexts demonstrate a firmly fixed disposition toward mistakes, however, as noted previously EOQ quantitative data provide a limited perspective. In this sense, delving into each EOQ domain to identify similarities and differences between the three different contexts became essential. At first glance, contexts seem not to

influence Ana's disposition, although domains divergences might provide information about how Ana enacts her active dispositions in her classroom.

During Ana's interview, she stated that when she was answering the first EOQ she was thinking without a context in mind. However, when she was thinking about her own mistakes; she said, "I imagined myself taking a test or baking a cake, errors in general." That fact might suggest that even when Ana's disposition through the different contexts is very similar, her enduring disposition is linked to her disposition toward her own mistakes. Then, since Ana's (G & T) total scores are identical and the noteworthy difference between them is only in one domain, I would center the attention on this. Quantitative data provided evidence of how (G) and (T) scores are almost identical for the three different dispositional functions. However, her position expressed an implicit connection between (G & T) by mentioning mistakes that are made by her.

Ana's positioning and her dispositional affective function.

The biggest difference between (G), (S), and (T) Ana's scores is part of her affective dispositional functions, most specifically in the covering domain which are about six points of difference between (G & S) with regards to (T) and for the error strain the difference between (G & S) with respect to (T) is 5 and 4 points respectively. Even when that difference is significant, even more significant is the fact that both of those domains are domains that are counts as negatives and her (T) scores are impressively low, which provides an idea of how her non-productive disposition is based almost exclusively in error strain domain. According to Rywobiak et al., (1999) error is identified by a generalized fear, shame, anxiety, and/or worry of making mistakes. During Ana's interviews, she constantly expressed and named those types of negative emotions.

She expressed, "a lot of the times, we are afraid to try new things, so you avoid errors," "I feel ashamed when I make a mistake," or "fearing about mistakes is normal;" these types of

statements explain her low (T) score for the error strain domain. Regarding her affective disposition toward her students' mistakes, she expressed "I pointed out [mistakes-MA] and I try to correct them without letting them down or without frustrating them because I know just overall nobody is comfortable of making mistakes." Experiencing fear and frustration is an inclination that is deep-rooted as a part of Ana's disposition. In a way, she did not only establish how she, as a teacher, positioned herself by taking risks or not on her teaching but also, she established how her students participated or not on their error handling processes.

As it is evident from the interview, Ana's non-productive disposition toward her own mistakes and her students' mistakes is demonstrated when she stated, "nobody is comfortable making mistakes." Indeed, when Ana is talking about her own mistakes and she expressed "I get embarrassed," and "my students' self-esteem is the most important thing for me, and I know that paying attention to their errors might affect it," it can be noticed that her major concern was avoiding her students feeling a similar embarrassment as a result of their mistakes. Her assumption that her students might share the same emotions as her supports a schema of participation where she positions herself as the only source of knowledge.

Moreover, Ana's statement provides a clear idea of how she positioned her students as not capable of doing mathematics in relation to their mistakes by framing their own errors as learning deficiencies which are a cause of embarrassment. Ana framed those students who made mistakes as non-capable of succeeding again before having experienced failure or having made a mistake, instead of by her statements; she supported those students' inclinations to see their errors as treats. However, for analyzing how Ana's disposition toward mistakes was reflected in her classroom teaching, it became essential to analyze them in conjunction with the way in which she

communicated her active disposition toward mistakes; in other words, how she framed errors in her classroom.

Alignment between Ana's Disposition and Positioning: How Frames were Enacted in the Classroom.

In this section, I analyzed Ana's classroom moment-to-moment interactions when errors emerged as part of a planned activity by considering the ways in which the teacher communicated her active disposition toward mistakes frames. Through all the data from my field notes in Ana's classroom, there is a high level of coherence between her disposition and positioning. Ana's disposition toward mistakes in the context of her own mistakes and her students were aligned with her positioning in the classroom. Ana's frame shaping the moment-to-moment interaction in the context of errors were represented by her non-productive dispositions toward errors through the different contexts (G, T, & S).

Ana's deep-rooted non-productive disposition toward mistakes was enacted using an errors as learning deficiencies frame by avoiding any contact to mistakes. In this sense, Ana sent signals to her students from almost exclusively frame --errors as learning deficiencies. Then, alignment between Ana's dispositions and positioning was demonstrated by her implicit and explicit communication about the null role of errors in learning and how she covered those who err and how she privileged correctness over learning in every moment-to-moment interaction.

Ana corrects errors by herself.

The connection between Ana's disposition toward mistakes expressed during her interviews and the way she positioned herself during the error episodes that took place in her classroom were clear. For example, when I asked Ana what she does when she makes a mistake, she said, "I try to correct it as soon as possible." To illustrate, during one of my three observations,

when she was solving a problem that some students solved incorrectly, she also made a mistake since she forgot a negative sign. I knew that it was one of the problems that seven students have solved incorrectly because, at the beginning of the class, she had mentioned one by one the names of those students who got correct answers in their homework problems. She never mentioned the specific reason why she had picked that problem.

Her students did not say anything, even when it was clear that at least one of them might be aware of her mistake because more than ten names were mentioned at the beginning of the class to indicate that those students got the correct answer. However, when she went back to her desk, she realized that her answer did not match with the correct one and immediately she went back to the board and checked her procedure. Right away, she noticed that she missed a sign. Facing the board, she immediately said with a tight voice, "I know that I made a mistake and where I made it, so I am gonna fix it, right now." Her students remained quiet; they did not say anything.

That episode represents how Ana's non-productive disposition toward her own mistakes converge with her positioning in class. Even when half of the students had the correct answer, Ana did not use her students' mistakes, neither her own mistake as an opportunity to learn or discuss the problem. This could have been used as an opportunity to discuss the concepts involved in the process, or at least, the source of her simple mistake. Instead, as she mentioned, she detected her mistake; she "fixed it" immediately. While it is true that she did not try to cover her mistake which is providing an idea of her productive score (T) on covering up errors domain, it is also true that her error-strain non-productive disposition represents the core of the way of how she approached her own mistakes; when she informed her students about her mistake, not only in terms of how she corrected it by herself but on the way she framed errors; this was evident not only by saying that she made it, she detected, and then she will correct it, but also by her physical position

(Moghaddam, Harré, & Lee, 2008) toward her students. Communicating mistakes without physically facing her students and using a tight tone of voice represent a way of framing mistakes as something negative, per se. Ana framed errors as deficiencies of learning and teaching with different communication-acts. No negative words were needed, nor an attempt to cover a mistake. It was that urgency for "fixing" what was wrong and her intrinsic shame the strongest public expressions of her disposition that were shown by her positioning and her students positioning in that circumstance which involved her error (Herbel-Eisenmann et al., 2015).

During my observations, Ana started lecturing by retaking those examples that some students had answered incorrectly. Regards the errors that were made by Ana's students, the story was not different in terms of who has the duty and the right for correcting mistakes, neither is it in terms of how her non-productive disposition and her positioning converged. Ana solved the problems without saying anything about the kind of mistakes they made. She did not even ask them about which part of the problem they had struggled with or had not understood; instead, she solved the problems without interruptions. In other words, she solved them without giving time to the students to discuss the mistakes they had made or without asking them anything; no form of teacher-student interaction was evident.

During the three days that I had the opportunity of observing her class, I was able to see Ana's practice to correct her student's mistakes. It was a practice where the interaction was one-to-one, Ana and the student who made a mistake participate in the correction practice; the student's participation was passive in all the cases.

Through all my observations, Ana's students were arranged in groups of three or four; as soon as she assigned them some problems, she started walking around the aisles. The classroom was completed in silence in every moment, a silence that I had never experienced before in a

classroom, I did not hear any voice. Students were working on their worksheets; those worksheets were the main part of students' daily activities. After a while though, as part of Ana's classroom practices while students are working on their own, Ana started moving very slowly to check them, and I observed that she suddenly stopped on some of her students' tables; She stopped with a student that had made a mistake, then she started whispering to him by saying "look, this is wrong" at the time she was pointing her student notebook. They did not keep eye contact, instead, both were staring at the notebook. The student erased what, then he started writing the procedure that Ana dictated him. Immediately after, she left that student table and stopped with another one. In that case, she took the student's mechanical pencil to erase the student's procedure and rewrote the correct procedure and answer. At that moment, she did not even say a word.

Personalizing mistakes by isolating them for the rest of the group.

From the previous data, it becomes evident not only the way Ana corrected mistakes by herself, but also how she expected her students to participate after they made a mistake. Ana framed her students' mistakes as problems of learning that need to be fixed promptly and, at the same time, she positioned her students as not capable of coping with errors. During her interview when I asked her about her procedures after students make a mistake, she said:

> I pointed it out and I try to correct them without letting them down or without frustrating them cause I know just overall nobody is comfortable of making mistakes although we make ourselves strong like we can do this but yeah nobody is comfortable and I try not to embarrass them in any way if I'm walking around I'll walk away from the student and I say ok I see that some of you are doing this, so this is what is supposed to be done instead.

In fact, Ana framed mistakes as a cause of frustration and discouragement, consequently, her students cannot even think or reflect on their mistakes. As she said, she did not allow them to get frustrated., The source of her non-productive disposition lies in the affective domain; she perceived making mistakes in a close connection to frustration, embarrassment, discomfort, and/or

disappointment. Her idea of avoiding personalizing mistakes, in turn, provoked not only an error personalization but its isolation.

Moreover, even when, according to her interview, her strategy for communicating mistakes, walked away from the students that made a mistake in order to prevent the other students to learn who made the specific mistake, that still not reflecting a productive disposition in any possible way. Moreover, whispering or not expressing a word about a mistake exclusively detected and corrected by her, represents by itself a way of how she isolated a student mistake from the rest of the class, no matter more than one student made exactly the same mistake. For example, Ana started talking with a student to inform her that something was wrong.

Subsequently, Ana told her the correct procedure. In the same small group (as noted previously students were arranged in small groups of three or four students) there was another student with the same type of mistakes; Ana claimed that she corrected them immediately by writing directly on her students' notebooks. Ana detected the same type of error for the fourth time, however, on that occasion, she started asking some questions to the student about the process that she used to solve the problem. That moment, Ana was pointing to her student notebook and talking with a soft voice, almost whispering. While in the previous three instances, she detected and corrected her students by giving them the correct procedure without explanations; she instead tended to question them. There was something considerably different in that error detection and correction process; Ana guided the student to find her mistakes and to correct them. As soon as she concluded reviewing the problems, she said, "please be careful with the angles." At the end of the class, I learned why that last error correction was significantly different for the last student; Ana announced "Congratulations to the best student of the week, Dalia." Dalia was the student that Ana was guiding with questions to detect and correct her mistake.

Ana's non-productive affective disposition toward mistakes was associated with the way she framed her students' mistakes; it was evident for me that as she was trying to avoid an embarrassment, she almost whispered or preferred not to talk and correct the mistake by writing on her student notebook the correct procedure. Personalizing mistakes goes beyond the idea of detecting and correcting them by isolating them; personalizing error strategies is an unconscious process for establishing who can be capable of coping with their mistakes and who does not.

Explicitly valorizing correctness.

During my three Ana's classroom observations, I noticed that students promptly took their daily worksheets as they entered the classroom. Ana had two desks: One at the front of the classroom and another one in its back part. Students' desks were oriented to the back. Ana was sitting in the back part of the class in front of her students watching the door without saying a word. Taking the worksheet at the beginning of the class seems to be an institutionalized practice since all the student took their sheet without saying anything every single day that I observed.

Ana's class was completely quiet. Nobody was talking; you could not hear a sound. Students were focused on the worksheet to the point that I feel that they did not even notice my presence. I have never experienced that silence in a classroom. It was completely unusual for me. Nobody talked for 20 minutes, not even Ana. She was on her desk writing something, the one that was close to the door, writing notes; students were working individually without saying a word. They were seated in groups of four, however, they did not even look at each other during this first 20 minutes. 20 minutes later, Ana stood up and asked her students if they already had sent her their problem responses to which students rapidly agreed.

Right away, Ana moved to the other desk and using her laptop displayed at the screen in front of the students the first problem response; it was a multiple-choice worksheet and, on the

screen, she displayed the one that was correct. Afterward, Ana mentioned a total of ten names and said, “Good job, you got it right” That day’s worksheet had a total of 12 problems and Ana repeated the same procedure with every question which included naming those students who were answering correctly, followed by the same phrase: “Good job, you got it right.” However, she did not mention anything about those who got their problems wrong.

Through that data, multiple issues around errors were identified. For example, how the teacher was focusing on answers without caring about procedures. Moreover, how the teacher emphasized correctness by mentioning students’ names and praising them with the mentioned phrase. With this activity Ana framed errors as something that did not even deserve to be mentioned, as something that needs to be covered. At the same time, Ana positioned those students who solved their problems correctly as more capable, as the only ones who did a “good job” by getting the correct answer. She actively displayed her non-productive disposition toward mistakes by valorizing both correct answers and the students who obtained them.

If it is true that she did not explicitly position those students who did not get the correct answer as incompetent to cope with their errors, it is her affective disposition that is at the core of her positioning, and in turn what led her framing during the error episodes. Ana’s disposition to cover up mistakes was guiding the way on how she addressed or not her students’ mistakes. It became clear that she had the inclination to think that it is disadvantageous to make one's mistakes public, more importantly, that what needs to be public is correctness.

Students passively wait for their teacher’s correction of their mistakes.

Ana left aside students from judging, reasoning, making connections about their own errors and their peers’ errors, i.e., she constrained their learning from errors. Ana positioned her students as receivers of mathematical ideas in regard to the correction of their mistakes, in regard to all the

learning possibilities that a productive error might offer. In Ana's classroom, students were not encouraged to seek ways of correcting their errors; they did not show any type of initiative, nor were involved in the error detection and/or correction process. Instead, they waited until Ana detected their mistakes. The following data provides an idea of how Ana positioned herself at the moment that an error emerged or is detected by her, and in turn, how she positioned her students.

During the third day of observations, Ana repeated the same process that I observed in the two previous days. She started talking very softly, almost whispering, then pointed a part of her student notebook. She did not make any question to the student, she approached him and said: "Look, this is not right;" then, she took the student's pencil and she erased what the student had written. Thereupon, Ana started writing down on the student's notebook the right procedure without saying a word. When she finished solving the problem, she asked him, "Do you have any questions?" the student just shook his head to say no.

Even when that practice is a recurrent one in Ana's classroom, students' lack of participation should draw attention to understanding not only Ana's disposition but how the way that she framed errors inhibited her students' initiative to cope with their own errors. Students did not rely on themselves to correct even a simple computational mistake. They were not even confident of communicating their mistakes when they were working individually, but arranged in small groups, even more, they acted as passive participants that needed to wait for their teacher to know if they are right or wrong. However, it was not possible to suggest or notice something else, since even when their silence expressed their non-productive disposition toward mistakes, more inferences were not possible due to the limited data.

SUMMARY

Ana's enactive frames showed a close tie between her dispositions toward mistakes and her position during the (G & T); even when her framing of mistakes was in the context of her students (S) mistakes, the frames that were enacted converge with a non-productive disposition toward mistakes in most of the data. Through all data, was possible to verify how Ana's non-productive dis/position aligned with error as learning deficiencies frame. It was also evident that the core of her affective non-productive disposition toward mistakes permeated not only the way she framed mistakes but how her students frame them as well.

QUALITATIVE GENERAL FINDINGS

The error handling practices that were identified for each of the three teachers were used for generating a matrix of frames (Table 4.4). Those practices are displayed as bulleted items.

Table 4.4 Ways of framing errors

WAYS IN WHICH TEACHER COMMUNICATE THEIR ACTIVE DISPOSITIONS TOWARD MISTAKES	
Errors as resources to learn frame	**Errors as deficiencies of learning frame**
Understanding and analyzing mistakes, develop a critical thinking built-in error. Ability of capitalizing on errors. • Instructional strategies and activities involve error's analysis • Errors are addressed as a planned activity • Teacher communicates and anticipates errors • Teacher differentiates between different types of mistakes • Systematic connection between error analysis and learning	*Understanding errors as learning deficiency. Using errors for diagnosing or remediate learning problems* • Focusing on correctness as established activity • Personalizing mistakes by isolating them for the rest of the group • Teacher corrects errors by him/herself • Teacher avoids and prevents errors • Discussing solution errors to routine problems • Explicitly valorizing speed and correctness (Louie) • Systematic connection between error and low grades
Flexibility and openness toward mistakes creating an error-friendly belief. • Teacher discusses mistakes openly • Explicitly states errors usefulness on learning • Discussion of errors as part of the socio-mathematical norms	*Reluctance toward mistakes creating an error-discomforting belief* • Teacher covers up mistakes • Set errors as the result of lack of ability and practice • Focusing discussions exclusively on answers (Louie)
WAYS IN WHICH STUDENTS ARE EXPECTED TO PARTICIPATE	
Teacher position their student as capable of coping with errors. Students as competent and qualified to handle their error analysis process by themselves frame	**Teacher position their students as not capable of coping with errors frame**
Student as capable of producing mathematical ideas from the analysis of their mistakes • Students are encouraged to seek and value alternative ways of the error analysis process	*Students as receivers of mathematical ideas in regard to the correction of their mistakes* • Students passively wait for their teacher and/or peer correction of their mistakes

• Students show initiative • Students are involved in the error correction process	• Students see their mistakes as flaws of their ability to learn mathematics
Student as capable of succeeding again after having experienced failure or having made a mistake • Students rely on others to correct a mistake through discussion • Students being confident and seeing as valuable discussing and getting agreements when they are working with others.	*Students as vulnerable participants or/and not capable after having experienced failure or have made a mistake* • Students reluctance to communicate their mistakes

These qualitative results encompass the different ways that Damian, Bianca, and Ana positioned themselves and their students at the moment that errors arose by framing them as tools or as deficiencies for learning. Besides, how these teachers expected their students to participate by positioning them as capable to cope with their own and their peer's mistakes or as weak pieces of the learning context due to their mistakes, from which they should stay away. Table 4.5 shows teachers positioning differences and how for Ana's and Damian's cases, one of the framings roundly prevails over the other, while Bianca's positioning fluctuates between both.

Table 4.5 Damian, Bianca, and Ana positioning toward errors

Errors as resources to learn Frame		Errors as deficiencies of learning Frame	
Damian, Bianca	Addressing errors as a planned activity	Bianca	Systematic connection between error and bad grades
Damian	Systematic connection between error analysis and learning	Bianca	Teacher sets them as the result of lack of ability and practice
Damian	Teacher error communication	Bianca	Error discussion based on routine problems
Damian	Teacher explicitly states errors usefulness on learning	Bianca	Students passive wait for peer correction of their mistakes
Damian, Bianca	Students rely on others to correct a mistake through discussion	Ana	Teacher corrects errors by herself
Damian	Students being confident and seeing as valuable discussing and getting agreements when they are working with others	Ana	Personalizing mistakes by isolating them for the rest of the group
Damian, Bianca	Discussion of errors as part of the socio-mathematical norms	Ana	Explicitly valorizing correctness
Damian	Students are encouraged to seek and value alternative ways of the error analysis process.	Ana	Students passive wait for teacher correction of their mistakes
Damian, Bianca	Teacher discusses mistakes openly		
Damian, Bianca	Teacher positioning of student as capable of coping with errors		

However, both, the quantitative and the qualitative findings revealed just a fragment of either teacher's disposition toward mistakes or their framing during class. Then, due to the nature of this mixed-methods explanatory sequential design study, the following section describes the extent the quantitative and qualitative results cohere through narrative both key aspects (Fetters, Curry, & Creswell, 2013; Tashakkori, & Creswell, 2007a).

INTEGRATION OF QUANTITATIVE AND QUALITATIVE RESULTS

A qualitative follow-up of quantitative results enhances the interpretation of qualitative results and vice versa, what is to say that the quantitative phase and the qualitative phase inform each other, not only in terms of findings, but during all the research process (Tashakkori, & Creswell, 2007b). However, as a sequential explanatory design, the qualitative component was given more priority. Then, the sequential analysis of quantitative and qualitative data allowed me to integrate data, and as a result achieve a multidimensional understanding teacher's error framing (Onwuegbuzie, & Leech, 2004).

The alignment between teachers' disposition and their invoked frames was found in most of the cases. Then, convergences and divergences between their measured disposition toward errors in the context of their own mistakes and their students' mistakes and their framing by each case will be discussed as part of the mixed methods finding integration.

Damian sustained errors as learning tools frame through all the contexts.

Damian's case showed that using errors as springboards was possible since his disposition toward errors transcended to his classroom practices. For example, Damian's error approach enabled all his stents to participate at the same level in the process of detection, analysis, correction, and explanation of their own errors and their peers' errors, having opportunities to learn not only from their own mistakes, but from somebody else's errors (Borasi, 1987; Kilpatrick, 1987). Damian's instructive practices were thought and designed with the aim of encouraging students to reconsider their thinking structures (Engler et al., 2004). Subsequently, Damian teaching practices reflected his productive disposition toward mistakes from a cognitive perspective, when he considered his student's mathematical knowledge, and at the same time, his conative and affective productive disposition was reflected in the way that he framed mistakes.

In Damian's classroom, students' and teachers' errors were leveraged by all of them to provide not only learning opportunities but to support students' productive disposition toward errors and toward mathematics. I found that ways to frame errors in his classroom influenced students' attitudes towards learning from mistakes (Steuer & Dresel, 2011; Tulis, 2013). Damian's students replicated their teacher error approach in a way that error went unnoticed by students themselves; this happened because instead of looking at errors as something that might affect their grade or their learning, they perceived errors as a natural part of the learning process.

Damian enacted error as a tool for learning frame, and at the time, he encouraged his students to move forward to analyze conceptual mistakes instead of focusing on computational mistakes, only (Santagata, 2005. These findings demonstrated Damian's disposition influenced his positions during the error episodes (Stooksberry et al., 2009). Conjointly, it was demonstrated that Damian's error management in his classroom influenced his students' error management as well, by framing them as a natural part of their learning and positioning themselves as qualify to actively participate in the error analysis process.

Bianca aimed to move toward framing errors as learning tools.

Bianca provided a problematized account of her error handling activities, connecting and addressing student difficulties, her teaching, and the error-status. That afforded some productive disposition, which was evidenced by: (1) errors addressed as a planned activity; (2) errors discussed as part of the socio-mathematical norms; and (3) errors discussed openly. Yet, this productive disposition was undermined by unproductive frames: establishing a systematic connection between error and low grades; setting errors as the result of lack of ability and practice; centering error discussion on routine problems, only. Similarly, the way Bianca positioned her

students and how they positioned themselves in a passive role waiting for the peer correction of their mistakes.

Despite Bianca's understanding of mathematical reform that proposes a new status for errors in student math learning and her commitment to using errors productively in her classroom which was expressed during her interviews, her attempts to apply teaching strategies that incorporate error analysis, at times evidenced a tension between the two opposite error paradigms. Furthermore, without consistent, deliberate attention to teacher's framing, much of Bianca's teaching practices had the unintentional and inadvertent effect of perpetuating correctness as a paramount (Louie, 2017). Bianca's case provided an example about the difficulties that teachers face when error positive status is emphasized at a reform level.

Ana sustained errors as learning deficiencies frame.

Ana evidenced her non-productive disposition toward mistakes through what she did/not in her classroom. During her interviews, she expressed more than once that her major concern as a teacher was her students' self-esteem which according to her it might be in risk due to the errors that they could make and the way that she openly communicated them. However, during the interviews, she categorically expressed to understand the role of errors as tools for her students' mathematical learning from a cognitive perspective. Nevertheless, during the practice, her error framing aligned with her self-reported disposition; in other words, her teaching practices remained to reproduce errors as learning deficiencies frame which represents a connection non-productive affective disposition. As Tsamir, Rasslan, and Dreyfus (2006) state Ana rejected the use of error-based tasks and even more she preferred to cover her students' mistakes because she believes that errors may cause students embarrassment and frustration.

Evidently, teacher correcting errors by herself, as Ana did, limited her students' learning mathematical resources, knowledge, level of understanding, and abilities. Ana personalized mistakes by isolating them from the rest of the group. Doing so, she positioned herself as the only resource to "fix" her students' errors and positioning them as the most vulnerable participants. Yet, Ana's frames suggested that reflecting on student errors might be just as emotionally threatening if the mistake is framed as a group or peer analysis. I, therefore, contend that framing errors as deficiencies of learning and students as not capable of coping with their errors might not be the result of teachers' cultural practices and beliefs as commonly held (e.g. Santagata, 2005), but may also result from their dispositions toward mistakes, more specifically their affective dispositions.

Teachers' positions and dispositions

Comparisons between observations and interviews were performed as part of the analysis and mixed-methods integration. Table 4.6 shows teachers dis/position towards errors according to the three different contexts. The table provides a general outlook of teachers' dispositions and positioning. It also examines case similarities and differences. In this sense, the researcher could make decisions about how it would be more convenient to report findings and the pertinence of reporting all and each of the three cases as a part of the case study.

Table 4.6 Teachers Dispositions and Positioning toward mistakes comparative outlook.

Participant	Disposition	Positioning
Damian	Productive disposition toward mistakes (G), (T), and (S) contexts.	Productive
Bianca	Productive disposition toward mistakes in (S) context. Non-productive disposition in (G) and (T) contexts.	Fluctuating positioning between productive and non-productive.
Ana	Non-productive disposition toward mistakes in (G), (T), and (S) contexts.	Non-productive

Chapter V: Discussion

In this chapter, a brief description of how errors have been treated by researchers, teachers, and mathematics educators is presented with the aim of contextualizing how this study's research questions were addressed; understanding of the convergences and divergences with research trends in how errors are framed by teachers are discussed below. Subsequently, a brief summary of how this research study was conducted with the aim of achieving the main objectives settled by the research questions. Next, a summary of the study results and their interpretation are provided by a discussion of the study objectives considering the major findings. As a part of the discussion, the relevance of identifying teachers' disposition toward mistakes and the powerful effect on the way that teachers position themselves and their students in the specific context of mistakes is suggested. Last, I discuss the impact this research has had on me as a practitioner and as a researcher, which has led me to provide certain recommendations for future research.

Errors have been studied and addressed by researchers and interpreted by teachers during practice from different and even contrasting views. For example, from a student perspective, I still have a clear memory of errors as an element for diagnosing learning problems; in other words, errors have been seen from a deficit perspective in my own education. Since the idea of using errors as learning tools appears in the mathematics teaching scene in the middle of the 1980, it has become a solid part of mathematics education research. Paradoxically, this study reveals that in some cases, teaching practice is yet far from ideal.

For more than three decades, researchers have assessed the suitability and impact of using errors for promoting students learning (Borasi, 1987; Kilpatrick, 1987); thus, they have been assigning a new role for errors based on the understanding that students build their own knowledge. Although, as previous research has stated, and it has been also evidenced by this study, teacher

negative feelings that emerge during error episodes represent a major problem to implement instructional reforms (Lannin, Barker, & Tonwsend, 2007; Schleppenbach et al., 2007; Steuer, Rosentritt, & Dresel, 2013; Tulis, 2013).

A discussion about how addressing mathematics teachers' disposition toward mistakes in light of how teachers' position themselves and their students during class time, and in turn, how this all relates back to practice, will be also discussed as a part of this chapter. Measuring teachers' disposition toward mistakes provided an idea of what characterizes the dispositions toward mistakes in the context of their students' mistakes and their own mistakes. Understanding how teachers framed errors when these occurred and the way they positioned themselves and their students, supported the process to answer this study research question about how teachers' dispositions toward mistakes are reflected in their classroom teaching.

QUANTITATIVE RESULTS DISCUSSION: TO WHAT EXTENT SECONDARY MATHEMATICS TEACHERS' DISPOSITION TOWARD ERRORS IN THE CONTEXT OF THEIR OWN ERRORS AND THEIR STUDENTS' ERRORS DIFFER AND/OR COINCIDE?

Dispositional cognitive functions.

Teachers' disposition toward mistakes in the context of their own errors and their students' errors was measured with the aim of examining teacher's tendency or inclination for perceiving, recognizing, conceiving, and judging mathematical errors (Beyers, 2011). In the following figures 5.1 and 5.2 the cognitive disposition of Damian, Bianca, and Ana and how the latter two teachers' low scores for error competence and error anticipation provide an idea of their limited knowledge for handling errors (Seifried & Wuttke, 2017) and, at the time, it represented a certain lack of ability for thinking about errors in a critical way (Rywobiak et al. 1999).

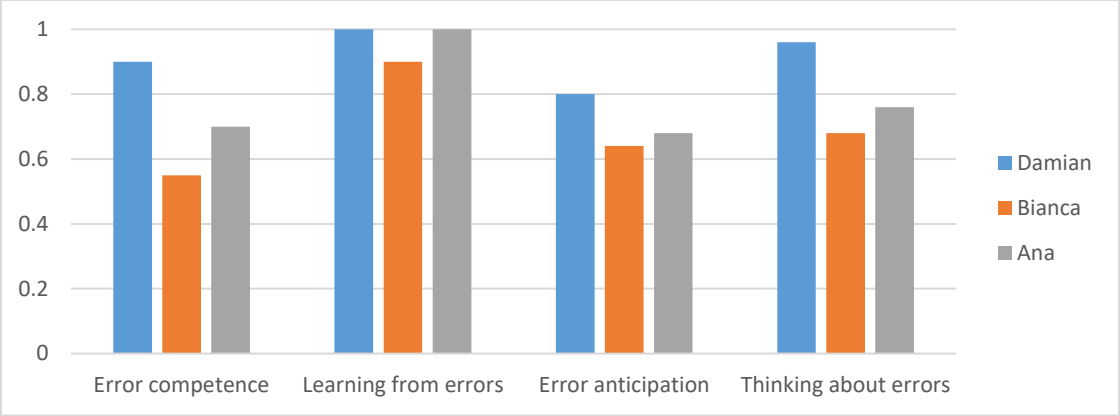

Figure 5.1 Damian, Bianca, and Ana cognitive disposition toward their own errors

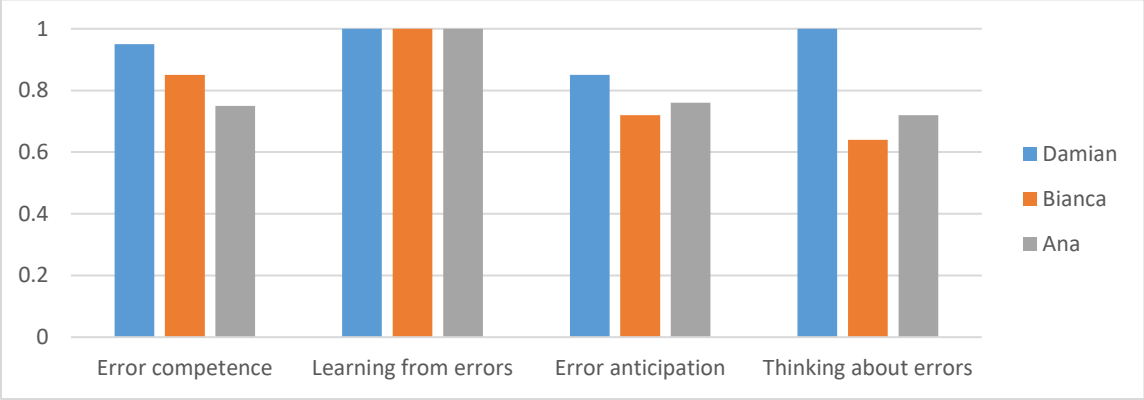

Figure 5.2 Damian, Bianca, and Ana cognitive disposition toward their students' errors

Dispositional affective function.

Even when there are some differences between Damian's, Bianca's, and Ana's cognitive disposition, it is their affective disposition that sets Bianca's and Ana's non-productive disposition toward mistakes in both contexts. In other words, it is their tendency or inclination to have or experience particularly negative attitudes, beliefs, feelings, emotions, moods, or temperaments with respect to mathematics errors what mainly characterize their non-productive disposition (Beyers, 2011). It is relevant to mention that for this specific function both domains are negatives, thus, the higher the score, the lower the productive affective disposition toward errors. Consequently, Damian's productive disposition is then reflected by his low scores in both domains. Then, his affective disposition is characterized by his confidence and positive emotional reactions

to face his own and his students' errors. The only contextual difference is seen as part of his tendency to believe that errors must not be covered, especially in the context of his own errors.

On the other hand, we have Bianca's case that her non-productive disposition toward mistakes is highly determined by a generalized fear of making mistakes, by negative emotional reactions when these emerge in both contexts. The same could not be said, however, for talking about covering up mistakes, since she did not tend to cover her mistakes, nor her students' ones. The case of Ana is like Bianca's, in the sense that she presented a tendency to experience negative emotions toward her mistakes and her students'. For Ana's case, covering her errors' domain scores in both contexts is what characterizes her non-productive disposition toward errors. Figures 5.3 and 5.4 contrast how different is a productive affective disposition from the non-productive disposition of Bianca and Ana in the context of their own mistakes and their students' mistakes and Damian's productive one.

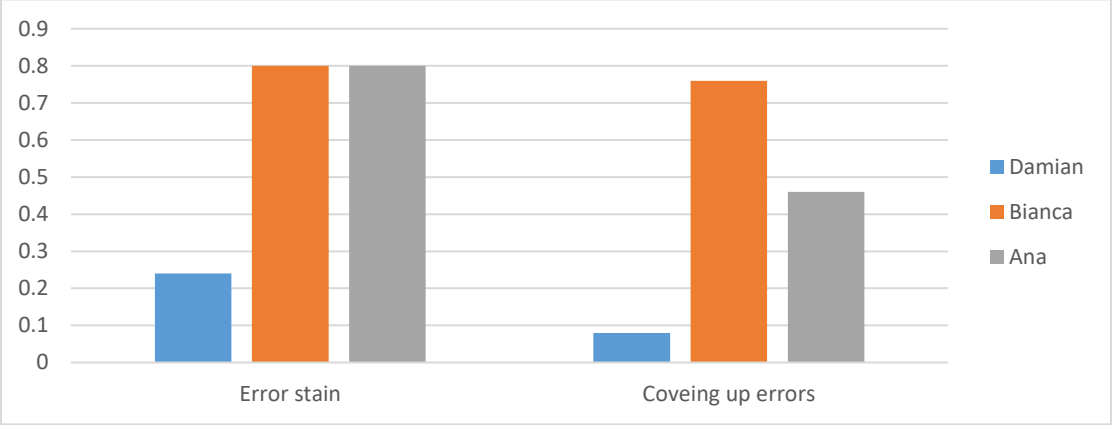

Figure 5.3 Damian, Bianca, and Ana affective disposition toward their own errors

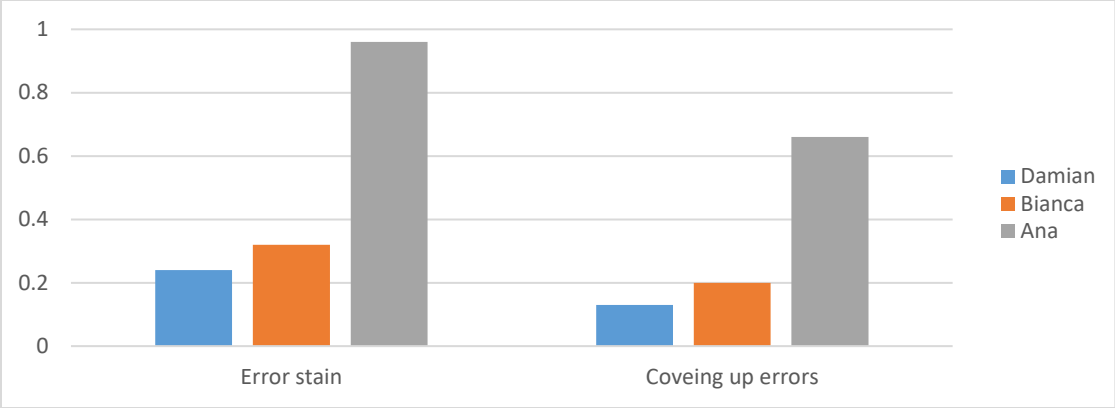

Figure 5.4 Damian, Bianca, and Ana affective disposition toward their students' errors

Dispositional conative function.

Teachers' tendency or inclination to purposively strive or to exercise diligence, effort, or persistence in the face of mathematical errors in both contexts was also measured (Beyers, 2011). Information about conative disposition scores through error risk-taking and error communication domains is provided. Damian's inclination to avoid taking the risk in the context of his own mistakes is one of the factors that negatively influence his productive disposition toward mistakes in both domains. Also, Damian's disposition is characterized by his tendency to communicate his students and his own errors. Bianca's conative disposition toward errors, in general, is characterized by being confident and seeing as valuable discussing and getting agreements when they are working with others (Tait-McCutcheon, 2008). In Ana's case, her tendency to take risks regarding mistakes in both contexts is low. She saw students discussing and getting agreements when they are working with others as valuable.

Figures 5.5 and 5.6 show a comparative approach between Damian, Bianca, and Ana. In these can be noticed a relevant difference between teachers' disposition, especially talking about error risk-taking in the context of teacher's errors.

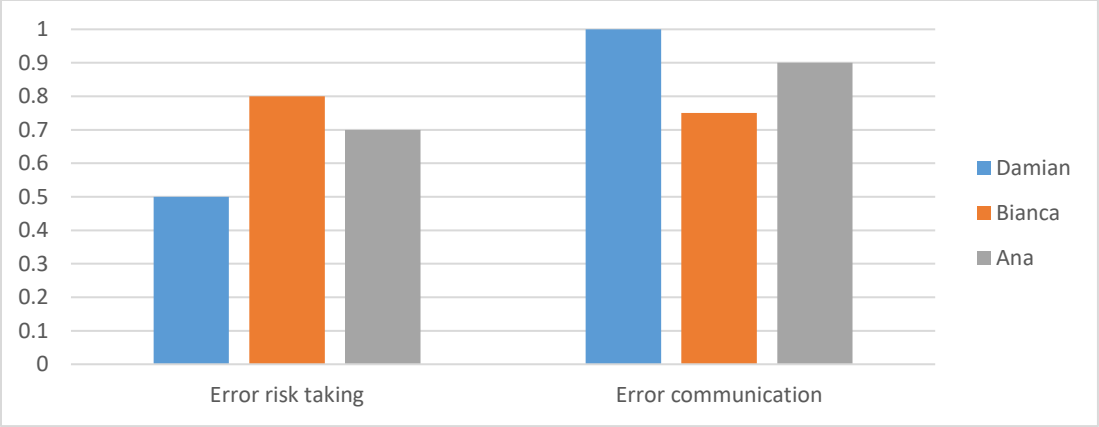

Figure 5.5 Damian, Bianca, and Ana conative disposition toward their own error

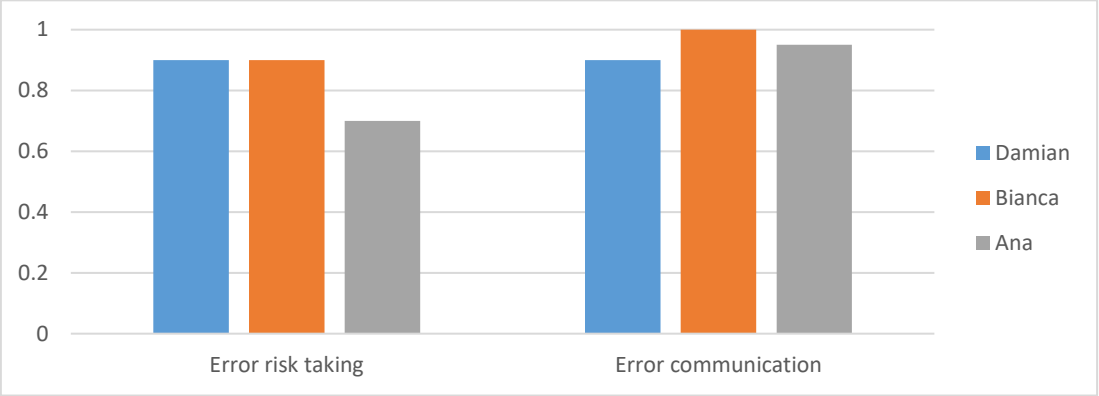

Figure 5.6 Damian, Bianca, and Ana conative disposition toward their students' errors

QUALITATIVE RESULTS DISCUSSION: WHAT TEACHER POSITIONAL FRAMES UNFOLDED DURING CLASS AT THE MOMENT THAT ERRORS EMERGED?

Explaining what teachers' framings emerged when errors arose during class was the purpose of the qualitative data collection and analysis which were based on two sets of interviews and a different number of classroom observations in all teacher classrooms. About the interviews, it is important to emphasize that all teachers expressed not only an understanding of the positive role that errors play on student mathematical learning which has been introduced and stated by math-reforms, but a personal commitment to use their students' errors as tools for learning. Although, during the error episodes that took place in the classroom moment-to-moment interaction some of them remained entangled and replicating errors as deficiencies of learning

frames. Then, during class observations, I was focused on how teachers framed their own mistakes and their students' mistakes, with the aim of knowing how teachers' disposition was reflected when errors emerged.

The way that teachers framed errors and how they positioned themselves and their students was interpreted by contextualizing them according to errors as resources to learn and errors as learning deficiencies; these two opposite error mathematics education paradigms that have been led all around the world by mathematics teachers though all levels of education and have been well documented by researchers, as well. In this way, teacher error handling practices did not approach from a deficit perspective by underestimating the efforts that teachers made for accomplishing reform requirements and minimizing the challenges that teachers face in their attempt to transform their practice.

Teachers' framing and positioning were examined in the light of the paradigm that stresses error importance from a remediation perspective, from error pattern diagnosis (e.g., Ayres, 2001; Brousseau, 2006; Brown & Quinn, 2006; Del Puerto, Minnaard, & Seminara, 2006; Ketterlin-Geller & Yovanoff, 2009; Koriakin et al., 2017; Livy & Vale, 2011; Marshall, 1983; Peng & Luo, 2009; Pochulu, 2009; Radatz, 1979; Resnick et al., 1989; Rico, 1995; Rodríguez-Domingo, Cañadas, Molina, & Castro, 2012; Socas, 2007; Tariq, 2008; Tirosh, 2000) and the paradigm that supports the idea of treating errors as tools that promote student mathematical deeper understanding and create learning opportunities (Borasi, 1987/1994; Booth, Lange, Koedinger, & Newton, 2013; Bray & Santagata 2014; Engler et al., 2004; Heemsoth & Heinze, 2014; Isotani et al., 2011; Kramarski & Zoldan, 2008; McLaren et al., 2012; Melis, 2005; Santagata 2005; Schleppenbach, Flevares, Sims, & Perry, 2007; Tsovaltzi et al., 2010; Tulis, 2013; Tulis, Steuer & Dresel, 2017; Zimmerman, Moylan, Hudesman, White, & Flugman, 2011).

SUMMARY

In this study, I identified two opposite error frames that teachers used to address errors in their classrooms. One of these frames provided students autonomy and support for using errors as tools for their learning. The other, instead, provided an idea of students' incapacity to cope with their own mistakes. Findings indicated that framing errors as tools for learning—involving students in error analysis in small groups—were mainly identified as part of teacher productive affective disposition.

Non-productive affective disposition toward errors was related to framing errors as learning deficiencies. However, showing a good understanding of how errors support student learning by being capable of capitalizing on them as part of the daily teaching practices are not covered by teacher's productive cognitive disposition, in other words having a productive cognitive disposition does not guarantee their capacity to frame error productively. This finding provides support for stating the relevance of conceptualizing teacher's disposition toward errors and having the basis for understanding the root of teacher's error positioning and framing. And consequently, achieve the main purpose of this study which is constructing a deeper understanding of teachers' productive and non-productive dispositions toward mistakes on three different domains---cognitive, affective, and conative (Beyers, 2011) and how those are reflected in their classroom positioning and framings.

LIMITATIONS

There are several limitations to this study, as the small number of cases may not yield results that are generalizable to a larger population. The data collected is a snapshot of experienced secondary mathematics teachers' disposition toward mistakes in the context of their classroom. In this sense, I intended to conduct an explanatory study to shed light on some of the relationships

between the domains of teacher's disposition toward mistakes and their framing when errors emerged during class.

There were limitations to the study that were inherent to the methodology. For example, the first stage of this study which was the quantitative part was based on self-reported data, which has known limitations (Linn & Miller, 2005). The EOQ instrument was adapted to the education context which might not accurately account for the experiences the participants had in their classroom context.

Regarding the qualitative part of this study, two important things need to be emphasized. First, even when two participants were teaching at middle school level and the other at high school level, the three of them were teaching algebra, which in one sense might give some uniformity to the sample, but at the same time, it would limit the possibilities of having access to a wider type of errors (e.g. conceptual errors) related to a more demanding mathematical content. Second, the participants were selected to represent diversity among teacher's disposition; however, not all teachers showed the same openness to be observed. Then, while Damian allowed me to observe him during all his periods, as many times as I needed, Ana only allowed me to observe one period three days, which also represents a limitation.

Another important limitation of this study is that students' disposition toward mistakes were addressed indirectly since I did not measure their disposition toward errors in any possible way. Nor did I interview them. The only data that was used regarding students were the one collected from the participant observations. Besides, my own assumptions and understanding were an important limitation as well.

IMPLICATIONS AND RECOMMENDATIONS

The findings presented in chapter four raised important implications and recommendations for mathematics teachers, administrators, and researchers. The process of characterizing teacher's disposition might have great implications and recommendations for policymakers, as well. Standards and policy creators, and the people in charge of overseeing that those policies are being applied, need to be familiar with the issues of practice associated with teacher's dispositions in order to understand how they will be required to adjust and/or modify not only the polices, but the strategies of how those can be implemented according to what characterized teachers' disposition.

As stated previously in the literature review, the way that teachers respond to errors is directly influenced by their dispositions (Wagner & Herbel, 2009); for this reason, even when teachers consider themselves as reform-oriented in terms of how errors play a positive role on student learning, it can be seen that some of those teachers' classroom handling of error practices remain too entangled in error non-productive frames. Furthermore, I found that it is teachers' affective disposition that is the most influential for teachers framing errors as deficiencies. This finding along with this literature implies that teachers need to be aware of their own disposition features and how those are frame during their class. If teachers are exhibiting dispositions that (a) are aligned with using errors as learning tools or (b) aligned with errors as learning deficiencies, then teacher's positions and the way that they positioning their students need to be part of administrators issues to attend by teacher professional development.

The findings revealed in this study show the relevance of teacher's disposition on their teaching practice and the importance of defining and assessing teacher's disposition prior to a teaching approach implementation. As a result of this study, many opportunities for reflection at different levels need to be considered. First, I recommend that administrators become familiar with

mathematics teachers' disposition toward errors and the reforms standards concerning errors in understanding the role of them in student learning. Teacher's disposition assessment might not be limited to teacher education programs, but in-service teacher training and professional development should include a serious and structured reflection about teachers' disposition and its connection to their classroom practices.

I recommend administrators then, rather than focus on complying with pre-established standards, they should focus on teachers' and students' responses to those standards in practice; alternatively, classroom observations need to be performed not only by administrators but also by other teachers whose debriefing of those observations might become crucial. This study contributes a framework for assessing and analyzing teachers' dis/position toward mistakes highlighting two error paradigms ---errors as tools and errors as deficiencies--- that are constantly present across all levels of the mathematics classroom. Then, these two frames and the practices that are encompassed on them might support future research to understand relevant issues about not only how teachers perceive errors, but their students as well, and the outcomes related to teacher's error framing.

I give my recommendations for further research based on the limitations of this study. First, add an explanation about the context of the EOQ to make it clearly related to education. Second, additional field studies should be conducted at different mathematics domain courses (e.g. geometry, pre-calculus, calculus). In addition to a qualitative methodology, a large pool of quantitative surveys should be distributed to students to examine if there is any correlation between teacher's and students' disposition toward mistakes.

CONCLUSIONS

This research study was important because it provided a snapshot of the role of errors in the specific cases of three secondary mathematics teachers. This portrait provided insight into the overall process of defining mathematics teachers' disposition toward errors. How teacher's disposition toward errors are related to the way teachers frame their own and their students' errors during their class practice. These findings can assist administrative and principals as they seek to understand their teacher's disposition toward errors and the role that this has on their teaching practices. Third, this study revealed the type of error dispositions function that is closely related to framing errors as learning deficiencies and in turn, limited students' access to learning from mistakes. This finding can assist teachers, principals, and professional development designers for assessing teacher's affective disposition toward errors. Fourth, this study filled the gap in the research on defining and assessing teacher's dispositions toward mistakes, but even more relevant, it provides a framework to examine teacher's framing during their teaching practices at the moments that errors emerge.

Furthermore, this study provides an understanding of aligning or disrupting error frames to implement learning strategies from a productive error approach with the aim of offering opportunities for all students. This finding requires collective action from schools and districts to change teachers' disposition toward errors. Then, I suggest that the way that teachers handle their students' mistakes might go beyond developing a productive disposition toward mistakes, but supporting students to develop their disposition toward their mathematical errors by focusing on their mathematical ability as not fixed, understanding that effort rather than ability matter (Beyers, 2011).

With the aim of successfully developing a productive disposition toward errors, I consider essential that all those who are involve in the process of teaching and learning mathematics understand that all mathematics dispositions are encompassed by cognitive, affective, and conative factors and not only cognitive factors which are the ones that commonly weighted over the other two (Beyers 2011; Connell & Wellborn, 1990; Csikszentmihalyi, & Schiefele,1995). This study has shown that conative and affective factors play a constituent role of the learning and learning from errors process (Connell & Wellborn, 1990; Csikszentmihalyi, & Schiefele,1995).

It is equally important to develop a productive disposition toward errors, not only by teachers but students as well; it should become crucial that teachers see all their students as capable to learn and not evaluate their mathematical ability according to the number of errors that they make or the speediness to solve a problem correctly. Mathematics teachers need to divest of their narrow understanding of mathematical ability that gives rise to a non-productive disposition toward mistakes.

A narrow understanding of the mathematical ability that leads to making a difference between those students that make mistakes and those who are commonly correct. A narrow conception about the mathematical ability that according to Louie (2017) leads teachers and students to think that students' role is memorizing and executing the right procedure and finding the correct answer as soon as it is possible and which can be only achieved by those students that have a natural and fixed mathematical aptitude. Those conceptions conduct teachers to avoid and destroy opportunities for reasoning and experimentation by introducing mathematics as a set of steps to be memorized to then routinely apply them. Such is the case of Bianca and Ana who have consciously routinized their instruction on behaving according to them due to their years of experience teaching the same class and that textbooks have not experienced any change. Both

restrained their students' learning by never posing non-routine problems. Bianca and Ana did not framed errors and mathematics as something to be explored.

Having and providing a heuristic approach for guiding students to a complex way of mathematical reasoning might be according to Goldin (2004) named and practiced as simply as "trial and error." Teachers might be focused on creating and offering a safe environment for failure and discovery where students are encouraged to take risks and teachers is supporting transparency of their own and their students' mistakes as part of the mathematical learning (Luria, Sriraman, & Kaufman, 2017). Teachers need to empower themselves, and more importantly, their students' cognitive, affective, and conative structures with the aim of making every student feel capable to cope with their mistakes and being successful to awaken the energy of learning and explore mathematics from different perspectives.

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Glossary

Disposition: “The core that affects, guide, and support teachers’ external behaviors, thoughts, and the context of their teaching” (Schussler, 2006, p.258).

Cognitive Disposition: “Tendency or inclination to engage (or not) in a particular cognitive mental process associated with perceiving, recognizing, conceiving, judging, reasoning, and so on, in mathematics” (Beyers, 2011, p.23).

Affective Disposition: “Tendency or inclination to have or experience particular attitudes, beliefs, feelings, emotions, moods, or temperaments with respect to mathematics” (Beyers, 2011, p. 23).

Conative Disposition: “Tendency or inclination to purposively strive or to exercise diligence, effort, or persistence in the face of mathematical activity” (Beyers, 2011, p.23).

Communication Acts: Speech, gestures, physical positions, and stances. (Herbel-Eisenmann et al., 2015)

Positional Framing: “The ways in which participants position themselves and each other when more than one person is in the activity of interaction” (Greeno, 2009, p.269).

Error competence: This domain has been already addressed in education and it has been related to cognitive aspects. Seifried & Wuttke (2017, p. 16) define it as “knowledge about common students’ errors and potential causes for students’ errors, strategies for handling errors (especially feedback strategies), and error-friendly beliefs.”

Learning from errors: This domain is also related to the cognitive realm and it is the ability of capitalizing errors (Borasi, 1987; Santagata, 2003).

Error risk-taking: It describes flexibility and openness toward mistakes (Rywobiak et al. 1999, p. 534) and “implies that one accepts errors and its consequences in order to reach higher goals.” It is related to the student’s courage or eagerness of succeeding again before having experienced failure or having made a mistake (Tay et al., 2009).

Error strain: According to Rywobiak et al., this domain is related to affective issues since "it is characterized by a generalized fear of committing errors and by negative emotional reactions" (1999, p. 543).

Error anticipation: This domain is related to affective and it can be seen from two different perspectives since it can be seen as pessimistic or negative tuned when it is positively correlated with error strain. However, it can be also positively correlated with learning from error or thinking about errors (Rywobiak et al. 1999).

Covering up errors: This domain is mainly related to affective issues and it is seen as the strategy of an insecure person when he or she doesn't acknowledge errors (Rywobiak et al. 1999).

Error communication: According to Tait-McCutcheon (2008), this domain is related to the conative realm since it is described by students being confident and seeing as valuable discussing and getting agreements when they are working with others.

Thinking about errors: The ability to understanding and analyzing mistakes, develop critical thinking built-in errors (Rywobiak et al. 1999).

Appendix A

Informed Consent (Quantitative Phase)

University of Texas at El Paso (UTEP) Institutional Review Board
Informed Consent Form for Research Involving Human Subjects

Protocol Title: Teachers' Dispositions toward Mistakes in Teaching and Learning: A Study of Secondary Mathematics Teachers in a U.S.-Mexico Border

Principal Investigator: Mariana Alvidrez

UTEP: Teaching, Learning, and Culture Ph. D.

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1. Introduction

You are being asked to take part voluntarily in the research project described below. Please take your time making a decision and feel free to discuss it with your friends and family. Before agreeing to take part in this research study, it is important that you read the consent form that describes the study. Please ask the study researcher or the study staff to explain any words or information that you do not clearly understand.

2. Why is this study being done?

You have been asked to take part in a research study of mathematics teachers' dispositions toward mistakes. You are being asked to be in the study because you are in service mathematics teacher. If you decide to enroll in this study, your involvement will last about one semester (Fall 2018).

3. What is involved in the study?

If you agree to take part in this study, the researcher will:
Apply a questionnaire. This instrument was designed with the aim of measuring "how one copes with and how one thinks about errors at work" (Rybowiak, et al., 1999, p. 527). The EOQ has a total of 37 questions in a 5-point Likert type scale. A date, time and location for the individual interview will be determined to accommodate the participant.

4. What are the risks and discomforts of the study?

This study does not involve more than minimal risk.

5. What will happen if I am injured in this study?

The University of Texas at El Paso and its affiliates do not offer to pay for or cover the cost of medical treatment for research related illness or injury. No funds have been set aside to pay or reimburse you in

the event of such injury or illness. You will not give up any of your legal rights by signing this consent form. You should report any such injury to *Dr. Mourat Tchoshanov or Mariana Alvidrez* and to the UTEP Institutional Review Board (IRB) at (915-747-7693) or irb.orsp@utep.edu.

6. Are there benefits to taking part in this study?

There will be no direct benefits to you for taking part in this study. This research may help us to understand teachers' beliefs about errors.

7. What other options are there?

You have the option not to take part in this study. There will be no penalties involved if you choose not to take part in this study.

8. What Are My Costs?

There are no direct costs. You will be responsible for travel to and from the research site and any other incidental expenses.

9. Will I be paid to participate in this study?

You will not be compensated for taking part in this research study.

10. What if I want to withdraw, or am asked to withdraw from this study?

Taking part in this study is voluntary. You have the right to choose not to take part in this study. If you do not take part in the study, there will be no penalty or loss of benefit. If you choose to take part, you have the right to skip any questions or stop at any time. However, I encourage you to talk the researcher so that she knows why you are leaving the study. If there are any new findings during the study that may affect whether you want to continue to take part, you will be told about them.

The researcher may decide to stop your participation without your permission, if she thinks that being in the study may cause you harm.

11. Who do I call if I have questions or problems?

If you have questions or concerns about your participation as a research subject, please contact the UTEP Institutional Review Board (IRB) at (915-747-7693) or irb.orsp@utep.edu.

12. What about confidentiality?

The records of this study will be kept private. In any sort of report, we make public we will not include any information that will make it possible to identify you. Research records will be kept in a locked file; only the researcher will have access to the records.

13. Authorization Statement

I have read each page of this paper about the study (or it was read to me). I know that being in this study is voluntary and I choose to be in this study. I know I can stop being in this study without penalty. I will get a copy of this consent form now and can get information on results of the study later if I wish.

Select an option:

Yes, I agree ( )

No, I do not agree ( )

Participant Name: _____ Date: _____

Participant Signature: _____ Time: _____

Researcher Signature: _____ Date: _____

Informed Consent (Qualitative part)

University of Texas at El Paso (UTEP) Institutional Review Board
Informed Consent Form for Research Involving Human Subjects

Protocol Title: Middle and high school Mathematics Teachers' Beliefs about Errors and their error handling practices.

Principal Investigator: Mariana Alvidrez

UTEP: Teaching, Learning, and Culture Ph. D.

---

1. Introduction

You are being asked to take part voluntarily in the research project described below. Please take your time making a decision and feel free to discuss it with your friends and family. Before agreeing to take part in this research study, it is important that you read the consent form that describes the study. Please ask the study researcher or the study staff to explain any words or information that you do not clearly understand.

2. Why is this study being done?

You have been asked to take part in a research study of mathematics teachers' dispositions toward mistakes. You are being asked to be in the study because you are in service mathematics teacher. If you decide to enroll in this study, your involvement will last about one semester (Fall 2018).

3. What is involved in the study?

If you agree to take part in this study, the researcher will:
Make you an interview that will last approximately 30 minutes. Notes will be written during the interview. Interviews will be audio recorded only if you agree. You will be contact for three different interviews which will be taking approximately 30 minutes each and will be spaced one week from each other. A date, time and location for the individual interview will be determined to accommodate the participant.

4. What are the risks and discomforts of the study?

This study does not involve more than minimal risk

5. What will happen if I am injured in this study?

The University of Texas at El Paso and its affiliates do not offer to pay for or cover the cost of medical treatment for research related illness or injury. No funds have been set aside to pay or reimburse you in the event of such injury or illness. You will not give up any of your legal rights by signing this consent

form. You should report any such injury to *Dr. Mourat Tchoshanov or Mariana Alvidrez* and to the UTEP Institutional Review Board (IRB) at (915-747-7693) or irb.orsp@utep.edu.

6. Are there benefits to taking part in this study?

There will be no direct benefits to you for taking part in this study. This research may help us to understand teachers' beliefs about errors.

7. What other options are there?

You have the option not to take part in this study. There will be no penalties involved if you choose not to take part in this study.

8. What are my costs?

There are no direct costs. You will be responsible for travel to and from the research site and any other incidental expenses.

9. Will I be paid to participate in this study?

You will not be compensated for taking part in this research study.

10. What if I want to withdraw, or am asked to withdraw from this study?

Taking part in this study is voluntary. You have the right to choose not to take part in this study. If you do not take part in the study, there will be no penalty or loss of benefit. If you choose to take part, you have the right to skip any questions or stop at any time. However, I encourage you to talk the researcher so that she knows why you are leaving the study. If there are any new findings during the study that may affect whether you want to continue to take part, you will be told about them.

The researcher may decide to stop your participation without your permission, if she thinks that being in the study may cause you harm.

11. Who do I call if I have questions or problems?

If you have questions or concerns about your participation as a research subject, please contact the UTEP Institutional Review Board (IRB) at (915-747-7693) or irb.orsp@utep.edu.

12. What about confidentiality?

The records of this study will be kept private. In any sort of report, we make public we will not include any information that will make it possible to identify you. Research records will be kept in a locked file; only the researcher will have access to the records. Only the IRB-sanctioned researcher will have access to audio recordings and to the transcribed interview data, which will be stored on a password-protected computer that only researcher will have access to. All data will be stored using pseudonyms for the name of each participant. Audio recording device containing the interviews will be kept in a locked storage

container in a locked room. Researcher will destroy the file after it has been transcribed, which she anticipates will be within two months of its recording.

13. Authorization Statement

I have read each page of this paper about the study (or it was read to me). I know that being in this study is voluntary and I choose to be in this study. I know I can stop being in this study without penalty. I will get a copy of this consent form now and can get information on results of the study later if I wish.

Select an option:

Yes, I agree to be audio recorded ( )

No, I do not agree to be audio recorded ( )

Participant Name: _____ Date: _____

Participant Signature: _____ Time: _____

Researcher Signature: _____ Date: _____

Appendix B

Error Orientation Questionnaire

To what extent do the following statements apply to you? Circle your choice, please.

#	Question	Not at all	A bit	Neither a bit, nor a lot	A lot	Totally
1	I don't let go of the goal, although I may make mistakes	1	2	3	4	5
2	I find it stressful when I err	1	2	3	4	5
3	It is better to take the risk of making mistakes than to 'sit on one's behind'	1	2	3	4	5
4	Mistakes assist me to improve my work	1	2	3	4	5
5	I anticipate mistakes happening in my work	1	2	3	4	5
6	I would rather keep my mistakes to myself	1	2	3	4	5
7	If I cannot rectify an error by myself, I turn to my colleagues	1	2	3	4	5
8	When a mistake occurs, I analyze it thoroughly	1	2	3	4	5
9	If it is at all possible to correct a mistake, then I usually know how to go about it	1	2	3	4	5
10	I feel embarrassed when I make an error	1	2	3	4	5
11	In carrying out my task, the likelihood of errors is high	1	2	3	4	5
12	It can be useful to cover up mistakes	1	2	3	4	5
13	My mistakes have helped me to improve my work	1	2	3	4	5
14	Whenever I start some piece of work, I am aware that mistakes occur	1	2	3	4	5
15	When I have done something wrong, I ask others, how I should do it better	1	2	3	4	5
16	I often think: 'How could I have prevented this?'	1	2	3	4	5
17	Why mention a mistake when it isn't obvious?	1	2	3	4	5
18	When I have made a mistake, I know immediately how to correct it	1	2	3	4	5
19	I am often afraid of making mistakes	1	2	3	4	5
20	After I have made a mistake, I think about how it came about	1	2	3	4	5
21	Mistakes provide useful information for me to carry out my work	1	2	3	4	5
22	I'd prefer to err, than do nothing at all	1	2	3	4	5
23	I do not find it useful to discuss my mistakes	1	2	3	4	5
24	If something goes wrong at work, I think it over carefully	1	2	3	4	5
25	When I do something wrong at work, I correct it immediately	1	2	3	4	5
26	While working I am concerned that I could do something wrong	1	2	3	4	5
27	If I make a mistake at work, I 'lose my cool' and become angry	1	2	3	4	5
28	I expect that something will go wrong from time to time	1	2	3	4	5
29	Employees who admit to their errors, make a big mistake	1	2	3	4	5
30	When I make a mistake at work, I tell others about it in order that thay do not make the same mistake	1	2	3	4	5
31	My mistakes help me to improve my work	1	2	3	4	5
32	If one wants to achieve at work, one has to risk making mistakes	1	2	3	4	5
33	If I cannot manage to correct a mistake, I can rely on others	1	2	3	4	5
34	To get on with my work, I gladly put up with things that can go wrong	1	2	3	4	5
35	After a mistake has happenned, I think long and hard about how to correct it	1	2	3	4	5
36	Most of the time I am not astonished about my mistakes because I expect them	1	2	3	4	5
37	It is a disadvantageous to make one's mistakes public	1	2	3	4	5

Cuestionario de Orientación al Error

¿En qué medida se aplican las siguientes declaraciones a usted? Circule su elección, por favor.

#	Preguntas	Totalmente en desacuerdo	En desacuerdo	Ni de acuerdo ni en desacuerdo	De acuerdo	Totalmente de acuerdo
1	No dejo de lado el objetivo, aunque pueda cometer errores	1	2	3	4	5
2	Cometer errores me resulta estresante	1	2	3	4	5
3	Es mejor tomar el riesgo de cometer errores que 'no intentar'	1	2	3	4	5
4	Los errores me ayudan a mejorar mi trabajo	1	2	3	4	5
5	Prevengo errores que suceden en mi trabajo	1	2	3	4	5
6	Prefiero mantener mis errores para mí	1	2	3	4	5
7	Si no puedo rectificar un error por mi cuenta, me dirijo a mis colegas	1	2	3	4	5
8	Cuando un error ocurre lo analizo a fondo	1	2	3	4	5
9	Si es posible corregir un error, entonces por lo general sé cómo hacerlo	1	2	3	4	5
10	Siento vergüenza cuando cometo un error	1	2	3	4	5
11	Al llevar a cabo mi trabajo, la probabilidad de cometer errores es alta	1	2	3	4	5
12	Puede ser útil ocultar los errores	1	2	3	4	5
13	Mis errores me han ayudado a mejorar mi trabajo	1	2	3	4	5
14	Cada vez que empiezo algun trabajo, soy consciente de que se pueden cometer errores	1	2	3	4	5
15	Cuando hago algo mal pregunto a otros, como debería hacerlo mejor	1	2	3	4	5
16	A menudo pienso: '¿Cómo podría haber evitado esto?'	1	2	3	4	5
17	¿Por qué mencionar un error cuando no es obvio?	1	2	3	4	5
18	Cuando he cometido un error, sé inmediatamente cómo corregirlo	1	2	3	4	5
19	Tengo a menudo miedo de cometer errores	1	2	3	4	5
20	Después de que he cometido un error, pienso en cómo se produjo	1	2	3	4	5
21	Los errores me proporcionan información útil para llevar a cabo mi trabajo	1	2	3	4	5
22	Yo prefiero errar (equivocarme), que no hacer nada en absoluto	1	2	3	4	5
23	Discutir mis errores no me parece útil	1	2	3	4	5
24	Si algo va mal en el trabajo, lo analizo cuidadosamente	1	2	3	4	5
25	Cuando hago algo mal en el trabajo, lo corrijo inmediatamente	1	2	3	4	5
26	Mientras estoy trabajando me preocupa que pueda hacer algo mal	1	2	3	4	5
27	Si cometo un error en el trabajo, pierdo mi buen humor y me enfado	1	2	3	4	5
28	Espero que algo salga mal de vez en cuando	1	2	3	4	5
29	Los empleados que admiten sus errores, cometen una gran equivocación	1	2	3	4	5
30	Cuando cometo un error en el trabajo, le digo a otros acerca de él con el fin de que ellos no cometan el mismo error	1	2	3	4	5
31	Mis errores me ayudan a mejorar mi trabajo	1	2	3	4	5
32	Si se quiere obtener logros en el trabajo, uno tiene que correr el riesgo de cometer errores	1	2	3	4	5
33	Si no puedo corregir un error, puedo confiar en otros	1	2	3	4	5
34	Para continuar con mi trabajo, felizmente supero mis errores	1	2	3	4	5
35	Después de que he cometidó un error, pienso cuidadosamente cómo corregirlo	1	2	3	4	5
36	La mayoría de las veces no me asombro de mis errores porque me los esperaba	1	2	3	4	5
37	Hacer los errores de uno públicos es una desventaja	1	2	3	4	5

Error Orientation Questionnaire

To what extent do the following statements apply to you? Circle your choice for every context, please.

		In the context of my own learning					In the context of students' learning in my own classroom				
		Not at all	A bit	Neither a bit, nor a lot	A lot	Totally	Not at all	A bit	Neither a bit, nor a lot	A lot	Totally
#	Questions	1	2	3	4	5	1	2	3	4	5
1	I don't let go of the goal, although mistakes are made	1	2	3	4	5	1	2	3	4	5
2	When mistakes are made, I find it stressful	1	2	3	4	5	1	2	3	4	5
3	It is better to take the risk of making mistakes than to 'sit on one's behind'	1	2	3	4	5	1	2	3	4	5
4	Mistakes assist in making improvements	1	2	3	4	5	1	2	3	4	5
5	I anticipate mistakes	1	2	3	4	5	1	2	3	4	5
6	I would rather keep mistakes to myself	1	2	3	4	5	1	2	3	4	5
7	If I cannot rectify an error by myself, I turn it to my colleagues	1	2	3	4	5	1	2	3	4	5
8	When a mistake occurs, I analyze it thoroughly	1	2	3	4	5	1	2	3	4	5
9	If it is at all possible to correct a mistake, then I usually know how to go about it	1	2	3	4	5	1	2	3	4	5
10	I feel embarrassed when an error is made	1	2	3	4	5	1	2	3	4	5
11	In solving a problem, the likelihood of errors is high	1	2	3	4	5	1	2	3	4	5
12	It can be useful to cover up mistakes	1	2	3	4	5	1	2	3	4	5
13	In the past, mistakes have helped to make improvements	1	2	3	4	5	1	2	3	4	5
14	Whenever a problem or task is solved, I am aware that mistakes occur	1	2	3	4	5	1	2	3	4	5
15	When something is done wrong, I ask others, how it could be corrected?	1	2	3	4	5	1	2	3	4	5
16	I often think: 'How could it be prevented?'	1	2	3	4	5	1	2	3	4	5
17	Why mention a mistake when it isn't obviuos?	1	2	3	4	5	1	2	3	4	5
18	When a mistake is made, I know immediately how to correct it	1	2	3	4	5	1	2	3	4	5
19	I am often afraid of mistakes	1	2	3	4	5	1	2	3	4	5
20	After a mistake has been made, I think about how it came about	1	2	3	4	5	1	2	3	4	5
21	Mistakes provide useful information to carry out my learning and teaching	1	2	3	4	5	1	2	3	4	5
22	To make an error is better than do nothing at all	1	2	3	4	5	1	2	3	4	5
23	I do not find it useful to discuss mistakes	1	2	3	4	5	1	2	3	4	5
24	If something goes wrong, I think it over carefully	1	2	3	4	5	1	2	3	4	5
25	When something is done wrongly, I correct it immediately	1	2	3	4	5	1	2	3	4	5
26	I am concerned that something could be done wrongly	1	2	3	4	5	1	2	3	4	5
27	If a mistake is made, I 'lose my cool' and become angry	1	2	3	4	5	1	2	3	4	5
28	I expect that something will go wrong from time to time	1	2	3	4	5	1	2	3	4	5
29	People who admit to their mistakes, make a big mistake	1	2	3	4	5	1	2	3	4	5
30	When a mistake is made, I tell others about it in order that they do not make the same mistake	1	2	3	4	5	1	2	3	4	5
31	If one wants to advance, one has to risk making mistakes	1	2	3	4	5	1	2	3	4	5
32	If it could not be managed to correct a mistake, I can rely on others	1	2	3	4	5	1	2	3	4	5
33	To get on task, I gladly put up with things that can go wrong	1	2	3	4	5	1	2	3	4	5
34	After a mistake has happened, I think long and hard about how to correct it	1	2	3	4	5	1	2	3	4	5
35	Most of the time I am not astonished about mistakes because I expect them	1	2	3	4	5	1	2	3	4	5
36	It is disadvantageous to make one's mistakes public	1	2	3	4	5	1	2	3	4	5
37	Mistakes help to improve teaching and learning	1	2	3	4	5	1	2	3	4	5

Cuestionario de Orientación al Error

¿En qué medida se aplican las siguientes declaraciones a usted? Circule su elección dependiendo de cada contexto, por favor

		En el contexto de mi propio aprendizaje					En el contexto del aprendizaje de los alumnos en mi salón de clases				
#	Preguntas	Totalmente en desacuerdo	En desacuerdo	Ni de acuerdo ni en desacuerdo	De acuerdo	Totalmente de acuerdo	Totalmente en desacuerdo	En desacuerdo	Ni de acuerdo ni en desacuerdo	De acuerdo	Totalmente de acuerdo
1	No dejo de lado el objetivo, aunque se puedan cometer errores	1	2	3	4	5	1	2	3	4	5
2	Cuando se comenten errores, me resulta estresante	1	2	3	4	5	1	2	3	4	5
3	Es mejor correr el riesgo de cometer errores que 'no intentar'	1	2	3	4	5	1	2	3	4	5
4	Los errores ayudan a hacer mejoras	1	2	3	4	5	1	2	3	4	5
5	Anticipo los errores	1	2	3	4	5	1	2	3	4	5
6	Prefiero mantener los errores para mí	1	2	3	4	5	1	2	3	4	5
7	Si no puedo rectificar un error por mi cuenta, me dirijo a mis colegas	1	2	3	4	5	1	2	3	4	5
8	Cuando un error ocurre lo analizo a fondo	1	2	3	4	5	1	2	3	4	5
9	Si es posible corregir un error, entonces por lo general sé cómo hacerlo	1	2	3	4	5	1	2	3	4	5
10	Siento vergüenza cuando se comete un error	1	2	3	4	5	1	2	3	4	5
11	Al resolver un problema, la probabilidad de cometer errores es alta	1	2	3	4	5	1	2	3	4	5
12	Puede ser útil ocultar los errores	1	2	3	4	5	1	2	3	4	5
13	En el pasado, los errores han contribuido en hacer progresos	1	2	3	4	5	1	2	3	4	5
14	Siempre que esta resolviendo un problema, soy consciente de que se pueden cometer errores	1	2	3	4	5	1	2	3	4	5
15	Cuando algo se hace mal, le pregunto a otros ¿Cómo se podria corregir?	1	2	3	4	5	1	2	3	4	5
16	A menudo pienso: '¿Cómo se podria haber evitado esto?'	1	2	3	4	5	1	2	3	4	5
17	¿Por qué mencionar un error cuando no es obvio?	1	2	3	4	5	1	2	3	4	5
18	Cuando se ha cometido un error, sé inmediatamente cómo corregirlo	1	2	3	4	5	1	2	3	4	5
19	A menudo le temo a los errores	1	2	3	4	5	1	2	3	4	5
20	Después de que se ha cometido un error, pienso en cómo se produjo	1	2	3	4	5	1	2	3	4	5
21	Los errores me proporcionan información útil en el proceso enseñaza-aprendizaje	1	2	3	4	5	1	2	3	4	5
22	Es preferible equivocarse, que no hacer nada en absoluto	1	2	3	4	5	1	2	3	4	5
23	No me parece que sea útil discutir los errores	1	2	3	4	5	1	2	3	4	5
24	Si algo sale mal, lo analizo cuidadosamente	1	2	3	4	5	1	2	3	4	5
25	Cuando algo es hecho de forma equivocada, lo corrijo inmediatamente	1	2	3	4	5	1	2	3	4	5
26	Me preocupa que algo se haga mal	1	2	3	4	5	1	2	3	4	5
27	Si un error se comete, pierdo mi buen humor y me enfado	1	2	3	4	5	1	2	3	4	5
28	Espero que algo salga mal de vez en cuando	1	2	3	4	5	1	2	3	4	5
29	Las personas que admiten sus errores, cometen una gran equivocación	1	2	3	4	5	1	2	3	4	5
30	Cuando se comete un error, informo a otros con el fin de que ellos no cometan el mismo error	1	2	3	4	5	1	2	3	4	5
31	Si alguien quiere progresar, ese alguien tiene que correr riesgos	1	2	3	4	5	1	2	3	4	5
32	Si no es posible corregir un error, se puede confiar en otros	1	2	3	4	5	1	2	3	4	5
33	Para continuar con las actividades, felizmente sobrellevo las cosas que pueden salir mal	1	2	3	4	5	1	2	3	4	5
34	Después de que se ha cometidó un error, pienso cuidadosamente cómo corregirlo	1	2	3	4	5	1	2	3	4	5
35	La mayoría de las veces no me asombro de los errores porque me los esperaba	1	2	3	4	5	1	2	3	4	5
36	Es una desventaja hacer públicos los errores de alguien	1	2	3	4	5	1	2	3	4	5
37	Los errores ayudan a mejorar el proceso enseñanza-aprendizaje	1	2	3	4	5	1	2	3	4	5

Appendix C

Interview Protocols

Interview 1

1. When you commit a mistake during your teaching (e.g. solving a problem on the board, explaining or introducing a new concept to students) what do you do? How do you approach it?
2. What is the role of your errors, if any, in your teaching? What do they afford you?
3. Do you think error risk taking can impact your teaching? If - yes, how?
4. Do you anticipate mistakes in your teaching practice? Why or why not?
5. What kind of feelings do you experience when you make a mistake in front of your students?
6. Do you confess to your students when you make a mistake during your teaching? Why or why not?

Interview 2

1. When your students make a mistake (e.g. solving a problem, explaining or introducing a new concept) what do you do? How do you approach it?
2. What is the role of students' mistakes, if any, in learning math/science?
3. Have you ever witnessed students fear to make mistakes? If - yes, share the story.
4. Do you usually anticipate your students' mistakes? Why or why not?
5. What kind of feelings do you experience when your students make a mistake?
6. Do you make your students' mistakes public? Why or why not?
7. What do you tell your students when they make a mistake?

Observation Protocol

Date of the observation _____
Name of the teacher_____
Location of the class_____
Grade level_____ Class_____
Observation no. _____
Start time_____ End time_____

Lesson design and implementation

	Never occur				Always occur
1) The instructional strategies and activities involve students' error analysis	1	2	3	4	5
2) The lesson plan is designed to engage students' in error analysis	1	2	3	4	5
3) In this lesson, students are encouraged students to learn from mistakes	1	2	3	4	5
4) Students are allowed to think about mistakes	1	2	3	4	5
5) This lesson encourages students to seek and value alternative ways from the error analysis process	1	2	3	4	5
6) The lesson allows students understand that mistakes provide useful information	1	2	3	4	5
7) In this lesson, teacher is aware of mistakes	1	2	3	4	5
8) When mistakes occur, teacher knows how to correct them	1	2	3	4	5
9) Teacher is anticipating the students' mistakes	1	2	3	4	5
10) There is a time dedicated for discussion about mistakes	1	2	3	4	5
11) Errors are addressed as planned activity	1	2	3	4	5
12) Teacher discusses common errors before they might occur	1	2	3	4	5

13) Teacher corrects the mistakes by himself 1 2 3 4 5

14) Teacher differentiates between the different types of mistakes

14.1 computational and procedural mistakes 1 2 3 4 5

14.2 conceptual mistakes 1 2 3 4 5

14.3 reasoning mistakes 1 2 3 4 5

Classroom error climate

1) There is a climate of respect when someone makes a mistake 1 2 3 4 5

2) Teacher finds stressful to err 1 2 3 4 5

3) Teacher finds stressful when students make a mistake 1 2 3 4 5

4) Teacher feels embarrassed when she/he makes a mistake 1 2 3 4 5

5) Teacher shows positive attitude toward his/her own mistakes 1 2 3 4 5

6) Teacher mentions his/her mistakes openly 1 2 3 4 5

7) Teacher discusses his/her own mistakes openly 1 2 3 4 5

8) Teacher is patient addressing students mistakes 1 2 3 4 5

9) Active participation to discuss errors is encouraged 1 2 3 4 5

10) Students were involved in the error correction processes 1 2 3 4 5

11) Students are involved in the communication of their ideas to correct a mistake 1 2 3 4 5

12) Students rely on others to correct a mistake 1 2 3 4 5

13) Students are afraid of making mistakes 1 2 3 4 5

14) The metaphor “from mistakes we learn” is very characteristic of this classroom 1 2 3 4 5

Recording salient events

Time	Description of the event

Appendix D

Question	Chi-square statistic	P value	p < .05
1	0.8035	0.9379	not significant
2	0.8698	0.9292	not significant
3	0.744	0.9457	not significant
4	2.3154	0.6779	not significant
5	0.2947	0.9901	not significant
6	0.4684	0.9765	not significant
7	1.7432	0.7828	not significant
8	0.4484	0.9783	not significant
9	0.6588	0.9563	not significant
10	0.953	0.9168	not significant
11	1.7535	0.7809	not significant
12	0.3931	0.983	not significant
13	3.3705	0.4978	not significant
14	0.5643	0.9669	not significant
15	0.2215	0.9943	not significant
16	1.3946	0.8451	not significant
17	3.2924	0.5101	not significant
18	1.378	0.848	not significant
19	0.8681	0.929	not significant
20	0.2328	0.9721	not significant
21	3.9124	0.271	not significant
22	0.0203	0.9999	not significant
23	0.7948	0.9391	not significant
24	0.584	0.9648	not significant
25	0.4653	0.9264	not significant
26	1.058	0.787	not significant
27	4.6617	0.1983	not significant
28	0.5011	0.9186	not significant
29	0.5097	0.9178	not significant
30	0.4472	0.9784	not significant
31	0.0799	0.9992	not significant
32	1.1728	0.7595	not significant
33	0.3086	0.9584	not significant
34	3.7386	0.2911	not significant
35	1.3411	0.8543	not significant
36	0.5996	0.9725	not significant
37	1.1773	0.8818	not significant

Appendix E

Participant id-CODE	EOQ G Score	Qualitative phase participant		
542-031	**111**	**Damian**	**EOQ (T) 104**	**EOQ(S) 111**
542-045	110			
542-125	110			
O-02	110			
542-033	109			
217-232	109			
D-06	109			
R-08	109			
542-034	108			
R-04	108			
542-043	107			
J-04	107			
R-05	107			
542-140	106			
R-07	106			
542-142	105			
R-03	105			
542-100	104			
R-11	104			
R-02	104			
542-197	103			
V-03	102			
O-04	102			
V-07	102			
542-046	101			
217-140	101			
P-03	101			
P-01	100			
D-21	100			
R-13	99			
D-01	99			
A-03	99			
217-132	98			
542-004	98			
542-092	98			
542-129	98			

R-01	98
542-044	97
D-08	97
O-06	97
O-07	97
542-091	95
J-03	94
R-03	94
V-10	93
O-01	93
R-07	93
O-05	93
V-01	93
P-05	92
A-01	92
217-044	91
A-05	91
R-08	90
542-137	89
R-09	89
V-05	89
A-04	88
R-06	87
542-038	86
D-20	86
R-04	85
V-06	85
D-02	85
O-03	84
J-01	84
O-08	83
P-02	83
R-01	83
217-223	82
R-12	82
J-06	80
J-10	80
R-05	80
V-08	80
542-151	79
V-02	78

D-07	78			
J-07	78			
A-06	77			
R-06	76			
542-124	75			
J-09	75			
O-21	74			
542-146	73			
217-216	73			
A-02	73			
217-222	72			
542-167	72			
217-213	72			
542-101	72			
542-149	72			
217-202	71			
217-209	71			
542-107	70			
217-220	70			
V-09	70			
O-09	70			
542-023	70			
217-218	69			
217-219	69			
217-201	69			
542-136	**66**	**Ana**	**EOQ (T) 66**	**EOQ (S) 63**
J-02	**52**	**Bianca**	**EOQ (T) 50**	**EOQ (S) 95**

Vita

Mariana Alvidrez earned a Bachelor degree in Economics from La Universidad Autónoma de Ciudad Juárez in 1998 and a Bachelor degree in Education from La Secretaría de Education in Mexico in 2006. In 2014, she received her Master of Science degree in Mathematics education from La Universidad Autónoma de Ciudad Juárez. She joined the Teaching, Learning, and Culture doctoral program in 2015.

Dr. Alvidrez has presented in the U.S. and international conferences including 2016 AMTE and 2017 and 2019 Math-Edu in Russia. She also participated in the Congreso Iberoamericano de Docentes in Spain. Her work has being published in 2017 and 2019 Math-Edu proceedings.

Mariana Alvidrez worked as a elementary teacher for 4 years until she started her own Montessori private school where she worked for 10 years more. While pursuing her doctoral degree, Dr. Alvidrez worked as research assistant for the Department of Education in collaboration with the Department of Mathematics.

Dr. Alvidrez dissertation, “From Mistakes We Learn: Variations in Teacher Disposition toward Errors in Mathematics Classroom,” was supervised by Dr. Mourat Tchoshanov.

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