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Methodology To Develop A Minimum Cost Acceptance Sampling Plan For Highway Construction

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METHODOLOGY TO DEVELOP A MINIMUM COST ACCEPTANCE SAMPLING PLAN FOR HIGHWAY CONSTRUCTION

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THESIS

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Abstract

Quality Assurance has always been a part of manufacturing processes. Since World War II, Acceptance Sampling has been introduced and implemented for quality control and improvement. Since statistical methods are applied for quality assurance, especially in acceptance sampling, various methods are becoming popular among the highway agencies over the period of time. As a result, several specifications have been designed to date based upon different statistical theories, each with its own advantages and shortcomings. Acceptance is a form of quality assurance which is employed as an audit tool for decision making to accept or reject the product based on its quality.

A Sampling Plan has to be designed so as to define the number of samples to be tested from a given quantity of product, referred to as a "Lot." Sampling tends to induce certain risks in the process of acceptance due to the fact that 100% inspection is not being carried out. These risks are generally called the Buyer's Risk and Seller's Risk, which are in fact the probabilities of erroneously accepting/rejecting a bad/good lot respectively due to insufficient inspection. Statistical theories provide us the means to either reduce or fix these unavoidable risks by selecting an appropriate "sample size," i.e., number of samples. A sample size has to be selected in a manner that is it practical in the sense of damage to the lot while testing and also the cost of testing which increases with sample size. With the tremendous amount of highway construction each year (e.g., 82 billion dollars of highway and street construction took place in 2008 (Construction Spending, 2008)), sample size and, in turn, the risks associated play important roles in affecting economy, even with a slightest change in terms of the cost of erroneous decisions.

Typically, a sample size of 3 to 7 per lot is used by highway agencies. Considering the aforementioned economical impact, the need arises for the "Practical Sample Size" (PSS) which is a sample size corresponding to minimum total cost of acceptance (i.e., cost of testing and costs incurred due to erroneous decisions). Many theories are designed to find minimum sample size but very little work has been done to find PSS with economic considerations for sampling plans for variables.

This research has been carried out to determine whether current sample size standards are practical (i.e., cost effective). A methodology was developed to find PSS (i.e., to generate practical sampling plan) by using the variable sampling method for unknown mean and standard deviation for

single and multiple acceptance quality characteristics (AQCs); considering decision and sampling costs and historical distribution of lot quality, compared with the existing sample sizes, and recommendations were made. This study shows that the sample sizes calculated with the traditional method of constant risk levels are not always practical. The major advantage of proposed plan is that it does not leave an agency with the decision of evaluating the risks involved in the sampling plan, but instead, provides it with a concrete reason to accept the sampling plan based on decision costs.

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Chapter 1: Introduction

OVERVIEW OF ACCEPTANCE SAMPLING

Quality Assurance (QA) is defined as "All those planned and systematic actions necessary to provide confidence that a product or facility will perform satisfactorily in service" (TRB, 2005) . Quality Assurance includes Quality Control, Acceptance and Independent Assurance. Three types of Acceptance include: 100% Inspection, Sampling or No inspection. The term "Sampling" can be used when estimating the quality of the given lot pertaining to the quality control. Its other use is in acceptance sampling, where the decision to accept or reject constructed lots (i.e., quantity of work, explained in details further in literature review) is based on characteristics of samples. Statistical Acceptance comes into consideration when sampling is selected as an acceptance method. When statistical methods are applied to design the specifications for sampling, the resulting plan is called a "Sampling Plan" which is usually defined by number of samples (i.e., Sample Size) to be inspected and the standard acceptance limits. "…an individual sampling plan has much the effect of a lone sniper, while the sampling scheme can provide a fusillade in the battle for quality improvement." (Schilling, Acceptance Sampling in Quality Control, 1982).

Acceptance Sampling has been implemented as an effective method for quality assurance for many years. As the name suggests, these tasks are conducted during or after the completion of the construction, viz., sampling, testing, interpreting the results and making the decision to either accept, reject or adjust the pay to the contractors, based on the specification limits. Typically each sample is classified as either defective or non-defective by testing it for a certain quality characteristic called an "Acceptance Quality Characteristic" (AQC). The number of defective samples from a lot decides the sentencing of the lot.

It differs from quality control on the basis of the functionality and applications. Quality control (QC) can be described as a process to monitor and maintain the outputs of the manufacturing processes under a specified quality limit set by standards to meet customer satisfaction. Since early civilizations,

there always has been some form of quality control over the manufacturing processes. Quality control is a part of the Quality Assurance (QA) process where the specified standards are implemented to put a check on the production quality. These processes are to be conducted while the manufacturing process is in progress. Another term called Statistical Quality Control (SQC) is also commonly used in the field of quality assurance and is the application of statistical computational methods in the process of quality control. “A process is considered to be in a “state of statistical control”, if the variations among the observed sampling results from it can be attributed to a constant system of chance causes” (Feigenbaum, 1983). Acceptance Sampling is a type of SQC which is used for lot sentencing (Accept/Reject or Pay adjust) or as an audit tool i.e., the purpose is not to estimate the process quality but to make a decision as to accept the lot based on the acceptance standards i.e., consumer’s acceptability requirements.

BRIEF BACKGROUND

The first major step towards modern acceptance sampling plans can be credited indubitably to the research work pioneered by W. A. Shewhart at Bell Laboratories who, according to A. M. Mood, is the “Founder of science of quality control and quality determination,” (Mood, 1943), followed by the development of the first sampling inspection tables by Dodge and Roming, also at Bell Laboratories in 1941 (Dodge & Roming, 1941). During World War II, war standards called Z1.2-1941, Z1.2-1941 and Z1.2-1941 were developed (later published as ANSI Z1.1-1985, ANSI Z1.2-1985 and ANSI Z1.3-1985), (Duncan, 1986, p. pp. 3) under the titles “Guide for Quality Control and Control Chart Method of Analyzing Data” and “Control Chart Method of Controlling Quality During Production.” A few more milestones in the developments in the field of statistical quality control and acceptance sampling are mentioned in works by the Statistical Research Group of Columbia University (Wallis, 1947), the earlier development of Sequential Sampling by Professor A. Wald, recent military standards such as Mil.-std.-105D, Mil-std.-414, ISO std. 2859 and others, AASHTO publications such as AASHTO R 9-90 and Optimal Procedures for Quality Assurance Specifications can be mentioned. A more detailed and interesting history of these developments can be found in (Duncan, 1986, pp. pp. 1-9).

FHWA recently surveyed its division offices – one in each state – asking for information about that state’s materials acceptance methods, including quality control-quality assurance (QC-QA) for hot

mix asphalt. FHWA sees room for improvement in the area of consistent and proactive validations of contractors' tests. Some states have stronger validation programs than others, and FHWA wants to help bolster the QA programs where needed. Some 30 to 35 states use contractors' Quality Control tests for acceptance, and then do an independent validation of some kind. Another 25 to 30 states use contractor tests for acceptance and use a statistical technique called "Percent Within Limits (PWL)" to validate the consistency of the tests. And FHWA estimates that 20 to 25 states do all of their own testing and base their acceptance on those tests. (There is natural overlap among those three groups.) (Brown, 2009).

ADVANTAGES OF ACCEPTANCE SAMPLING

Acceptance Sampling is not a direct measure of quality control since the product is already manufactured and delivered, but if implemented along with the specifications, it can become a part of process control over the period of time, which, in-turn, enables the improvement in the quality of the products in the future.

Advantages:

- Less expensive since there is less inspection
- Less handling of the product, hence reduced damage
- Applicable to destructive testing
- Fewer personnel are involved in inspection activities
- Greatly reduces the amount of inspection error
- Rejection of the entire lot forces a greater pressure on the vendor to improve quality

APPLICATIONS OF ACCEPTANCE SAMPLING

Acceptance sampling is used when:

- testing is destructive
- cost of 100% inspection is extremely high
- 100% inspection is not technologically / time wise feasible
- the vendor has an excellent quality history and some reduction in inspection from 100% is desired

- a program for continuously monitoring the product is necessary, even though the vendor's process is satisfactory

MOTIVATION

With the tremendous amount of highway construction each year (e.g., 82 billion dollars of highway and street construction took place in 2008 (Construction Spending, 2008)), the sample size and, in turn, the risks associated played important roles in affecting the economy even with a slightest change in terms of cost of erroneous decisions.

Typically, a sample size of 3 to 7 per lot is used by highway agencies. Considering the aforementioned economical impact, the need arises for the "Practical Sample Size" (PSS), which is a sample size corresponding to minimum total cost of acceptance (i.e., cost of testing and costs incurred due to erroneous decisions). Many theories are designed to find minimum sample size, but very little work has been done to find the PSS with economic considerations.

This research has been carried out to determine whether current sample size standards are practical (i.e., cost effective). A methodology was developed to find PSS (i.e., to generate practical sampling plan) by using the variable sampling method for unknown mean and standard deviation for single and multiple acceptance quality characteristics (AQCs); considering decision and sampling costs and historical distribution of lot quality, compared with the existing sample sizes, and recommendations were made. This study shows that the sample sizes calculated with the traditional method of constant risk levels are not always practical. This study shows that the sample size should vary according to the cost variations. The traditional method of fixing the buyer's and seller's risks is found to be inadequate in order to meet the PSS requirements.

OBJECTIVES

The major objectives of this thesis are:

1. To develop a methodology based on a mathematical model to recommend the practical sample size. The practical sample size will correspond to the minimum total acceptance cost

while balancing the buyer's and seller's risks associated with an acceptance plan and considering the historical lot quality

2. To compare the results against the existing and comparable methods
3. To develop a user friendly computer program to automate and expedite the calculation of PSS requirements for the given acceptance limits
4. To make recommendations according to the results obtained from this study to update the existing methods

THESIS ORGANIZATION

This thesis is divided into five chapters. Chapter 1 provides a brief introduction to the theory of acceptance and quality control; the objective and motivation to this project are also discussed. The literature review was an important part of this project. It was needed to study the already designed methods and to check for the suitability for the new method to be designed. A thorough review of such literature is provided in Chapter 2. This chapter also introduces the reader to the terminology used in the later portion of this thesis. Chapter 3 postulates a new model for practical sample size determination. It is sub-divided into three parts to formulate models for single AQC for Accept/Reject, single AQC for pay adjustment and multiple AQC analysis for Accept/Reject acceptance plans. Each step in the formulation of all models in the previous chapter is explained in detail in Chapter 4. The newly presented model is then compared with prevalent sampling and acceptance plans. Sensitivity analysis is carried out for an insight into the behavior and inter-relationships of the variables used in the postulated model sampling plan. PSSC, a Microsoft (MS) Excel based tool developed to implement the models discussed earlier, is introduced in the same chapter. Summary and conclusions, along with recommendations for the future research, are included in Chapter 5.

Chapter 2: Literature Review

ACCEPTANCE PLANS

The Transportation Research Circular E-C074 (TRB 2005) defines an acceptance plan as "an agreed-upon procedure for taking samples and making measurements or observations on these samples for the purpose of evaluating the acceptability of a lot of material or construction." A sound statistical sampling plan is essential for unbiased specifications. The types and components of acceptance plans are summarized in the following sections of this report.

Acceptance Plan Categories

Statistical acceptance plans are classified based on specification limits and decision strategy. According to (Freeman & Grogan, 1998) these categories are:

i. Single specification limit, single decision criterion: Single specification limits are used when a material must be controlled above a minimum or below a maximum. An AQL is set and material is either accepted or rejected based on it. There is no pay factor provision.

ii. Single specification limit, dual decision criteria: An AQL and RQL are set. Material at or above AQL is accepted at full or bonus pay while material below RQL is rejected. Material with an estimated quality level between AQL and RQL is usually accepted at reduced pay according to a pay scale.

iii. Dual specification limits, single decision criterion: Dual specification limits are used when a material must be controlled within a range of values. The percent of material between these values is calculated as the Percent within Limit (PWL), i.e., percentage of the total material conforming to the quality standards, and compared to the AQL. Material is then either accepted or rejected. There is no pay factor provision.

iv. Dual specification limits, dual decision criteria: An AQL and RQL are set. Material at or above AQL is accepted at full or bonus pay while material below RQL is rejected. Material with an estimated quality level between AQL and RQL is usually accepted at reduced pay according to a pay scale.

Types of Acceptance Plans

Acceptance plans commonly used by state DOTs can be classified based on how quality is measured, as follows:

- Percent within Limits-based (or percent defective-based) Acceptance Plans: These plans are designed to control the percentage of the lot falling outside specification limits. Thus, product quality is measured using percent within limit (PWL) of each tested quality characteristics. Alternatively, quality is measured using percent defective (PD), where PD = 100-PWL.
- Average-based Acceptance Plans: These plans are designed to control the average value of the tested AQC. Thus, product quality is measured solely based on average value, with no consideration of variability.

PWL-based (or PD-based) plans are considered to be advantageous because they take product variability into account and thus promote uniform quality. Numerous types of Acceptance and Sampling procedures have been developed, each with some advantages and limitations. The sample size and the quality control achieved with each procedure (or "Scheme") varies depending upon the quality measure used, use of upper/lower or both quality levels, single or double decision criteria and on procedure type, i.e., single (decision based on samples taken from a lot independently), double (decision to take next sample depends on first) or multiple or sequential (decision accept/reject/continue depends on previous samples). Generally, sequential and multiple attribute sampling plans lower the sample size (in attribute plans category) and double limit variable sampling plans (in variable plans category) yield the lowest sample size. Table 1 shows the comparison of sample sizes obtained for similar acceptance plans.

Table 1: Comparison of Variables and Attributes Sample Sizes (by Schilling in Juran's Quality Handbook, 1999)

ρ1 = 0.0190; α = 0.05; ρ = 0.0535; β= 0.10	
Plan	**Sample Size**
Single-Sampling attributes	125
Variables:	
σ known	19
σ unknown (s)	52
σ unknown (R of groups of S)	75
Sequential sampling, σ known (ASN at ρ1)	10.3

Figure 1 explains the concepts of PWL and PD for a quality characteristic with a lower limit (e.g., concrete compressive strength). PWL (or PD) of a construction or material lot (i.e., population) can be estimated based on the sample's mean, standard deviation, and size.

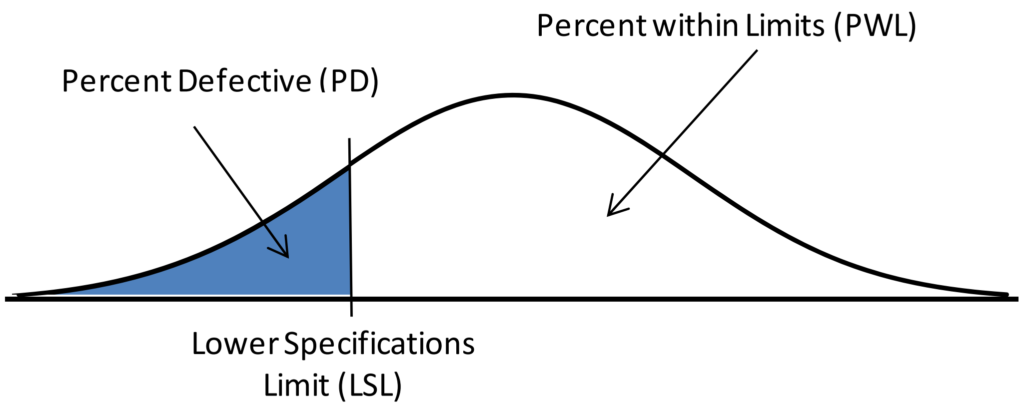


Figure 1: Graphical Representation of PD and PWL for a Single Limit Specification

This methodology is referred to as acceptance sampling by variables. The mathematical procedure used for estimating PWL (or PD) of a lot based on sample data is fully described in the literature (see, for example, the Standard AASHTO 2005, Burati, et al. and Duncan, 1986), provides an in-depth statistical derivation of this methodology.

The selection of a specific sampling plan for the given condition requires experience; in-depth knowledge of plans; careful selection of quality levels and other inputs and judgment of the results. Shilling, in 1990, presented a summary of types of sampling plans and recommended uses in various situations as shown in Table 2 Schilling in (Juran's Quality Handbook).

Table 2: Selection of Sampling Plan

Purpose	Supply	Attributes	Variables
Simple guarantee of producer's ad consumer's quality levels at stated risks	Unique lot	Two-point plan (Type A)	Two-point plan (Type B)
	Series of lots	Dodge-Roming LTPD Two-point plan (Type B)	Two-point plan (Type B)
Maintain level of submitted quality at AQL or better	Series of lots	MIL-STD-105E ANSI/AQSC 1.4 ISO 2859	MIL-STD-414 ANSI/ASQC Z1.9 ISO 3941
Rectification guaranteeing AOQL to consumer	Series of lots	Dodge-Roming AOQL	Use Measurements as go-no go
	Flow of individual units	CSP-1,2,3 Multilevel plan MIL-STD-1235B	
Reduced inspection after good history	Series of lots	Skip-lot Chain	Lot plot Mixed variable-attributes Narrow-limit gaging
Check inspection	Series of lots	Demerit rating	Acceptance control chart
Compliance to mandatory standards	Unique lot	Lot-Sensitive plan	Mixed variable-attributes with c = 0
	Series of lots	TNT plan	
Reliability sampling	Unique lot	Two-point plan (Type B)	MIL-HDBK-108 TR-7
	Series of lots	LTPD plan	TR-7 using MIL-STD-105D switching rules

COMPONENTS OF ACCEPTANCE PLANS

The key components of any acceptance sampling plan are summarized as follows:

- **Lot:** This is the amount of construction or material that may be accepted with pay adjustment or rejected based on the "as-constructed" quality characteristic. A lot represents the amount of material or constructed product (e.g., pavement) produced by essentially the same process, so that the probability distributions of AQCs are likely to be normal. A reasonable way to define a lot is a one-day production. A new lot should be established if there is any reason to believe that a special cause attacked the production process (such as change in concrete mix design, change of material supplier, etc.) and resulted in a significant shift in the mean or standard deviation of any of the AQCs.

  According to (Freeman & Grogan, 1998), "the size of the lot may vary depending on the economics of rejection and on the costs of sampling and testing. The lot size should not be so large that the contractor encounters severe hardship if it is rejected. However, small lots require more sampling and testing, which increases project cost."
- **Sample size:** The sample size refers to the number of tests or measurements for each AQC taken randomly from the lot. Randomness of sampling is a vital assumption upon which the statistical acceptance procedure is based. Random sampling can be defined as a manner of sampling that allows every part of the population (i.e., lot) to have an equal opportunity of appearing in the sample (Ref)
- **Acceptable Quality Level (AQL):** This is the degree of conformance (measured in PWL) or deficiency (measured in PD) at which the state DOT is willing to accept the lot and make full payment to the contractor.
- **Rejectable Quality Level (RQL):** This is the degree of conformance at which the lot is so deficient that replacement or corrective action is warranted.
- **Maximum (or Minimum) Allowable Quality Level (often referred to as M):** A lot is rejected if the sample PWL is less than M (or if the sample PD exceeds M). Traditionally, M is set

between RQL and AQL as an additional reliability to the constructed product and to provide balanced contractor and agency risks (AASHTO 1990).

- **Risks Involved in Acceptance:** The risks involved in acceptance procedure are calculated on the basis of the hypothesis

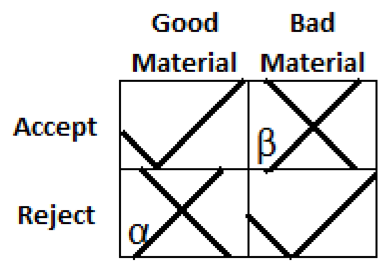


Figure 2: Risks Involved in Acceptance

Operating characteristic (OC) curves should be constructed for each quality characteristic to determine if the buyer’s and seller’s risks associated with the sampling plan are acceptable to the agency and the contractor. The seller’s risk (α) is the risk of erroneously rejecting or assigning a payment decrease to a lot that indeed should be accepted or assigned a pay increase. The buyer's risk (β) is the risk of erroneously accepting or not assigning a payment decrease to a lot that indeed should be rejected or assigned a pay decrease. The seller’s risk represents the contractor’s risk and the buyer’s risk the highway agency’s risk. An example of typical OC curve for accept-reject acceptance plan is shown in Figure 3.

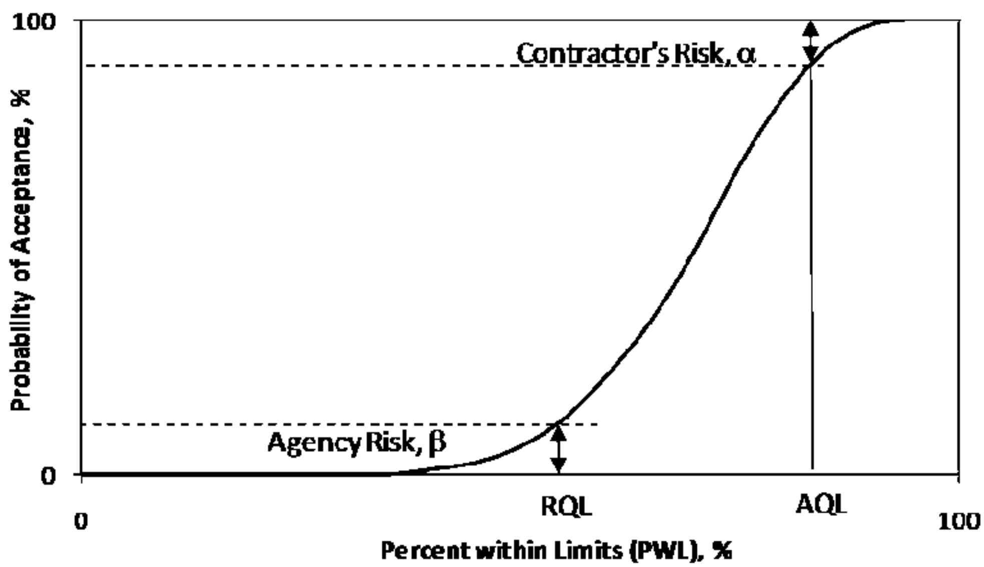


Figure 3: Typical OC Curve for Accept/Reject Acceptance Plan (AASHTO R 9-90)

AASHTO R9-97 (AASHTO) suggests that highway agencies should design acceptance plans that control these risks at suitable levels. It also indicates that these risks can be set based on the criticality of the measured property as it affects safety, performance, or durability as shown in Table 3. The 2005 version of (AASHTO, 2005) does not provide specific recommendations for acceptable risks, but it states "the more critical the application, the lower should be the buyer's risk. But only under rare circumstances should the buyer's risk be lower than the seller's risk."

Table 3: AASHTO Recommended Risk Levels Based on Criticality of Inspected Product (AASHTO R 9-90)

Classification	Buyer's risk at RQL	Probability of acceptance at AQL	Sellers risk at AQL
Critical	0.005	0.950	0.050
Major	0.050	0.990	0.010
Minor	0.100	0.995	0.005
Contractual	0.200	0.999	0.001

A conventional OC curve represents the relationship between PWL (or PD) and probability of acceptance. For quality characteristics with pay adjustment, the probability of acceptance is interpreted as the probability of receiving at least the full payment. The statistical procedure for developing OC curves is well-documented in the statistics and probability literature (see, Duncan 1986). Perhaps one of the most established software tools that can be used for developing OC curves using simulation techniques is the OCPLOT software (Weed, 1996).

Figure 4 shows the sample OC curve for a variables acceptance plan of a single lower-limit quality characteristic (e.g., compressive strength) treated on an accept/reject basis. The curve was generated using the OCPLOT simulation software with the following inputs:

- Sample size (n) = 3
- AQL = 90 PWL
- RQL = 50 PWL
- Minimum allowable PWL (M) = 70 percent

The contractor's risk of not receiving at least the full payment for a lot with a PWL of 90 is 1.2 percent. The agency's risk of paying the full payment for a lot with a PWL of 50 is 6.2 percent. Consulting Table 3 on acceptable risk levels, this acceptance plan is appropriate for what the agency considers critical construction items.

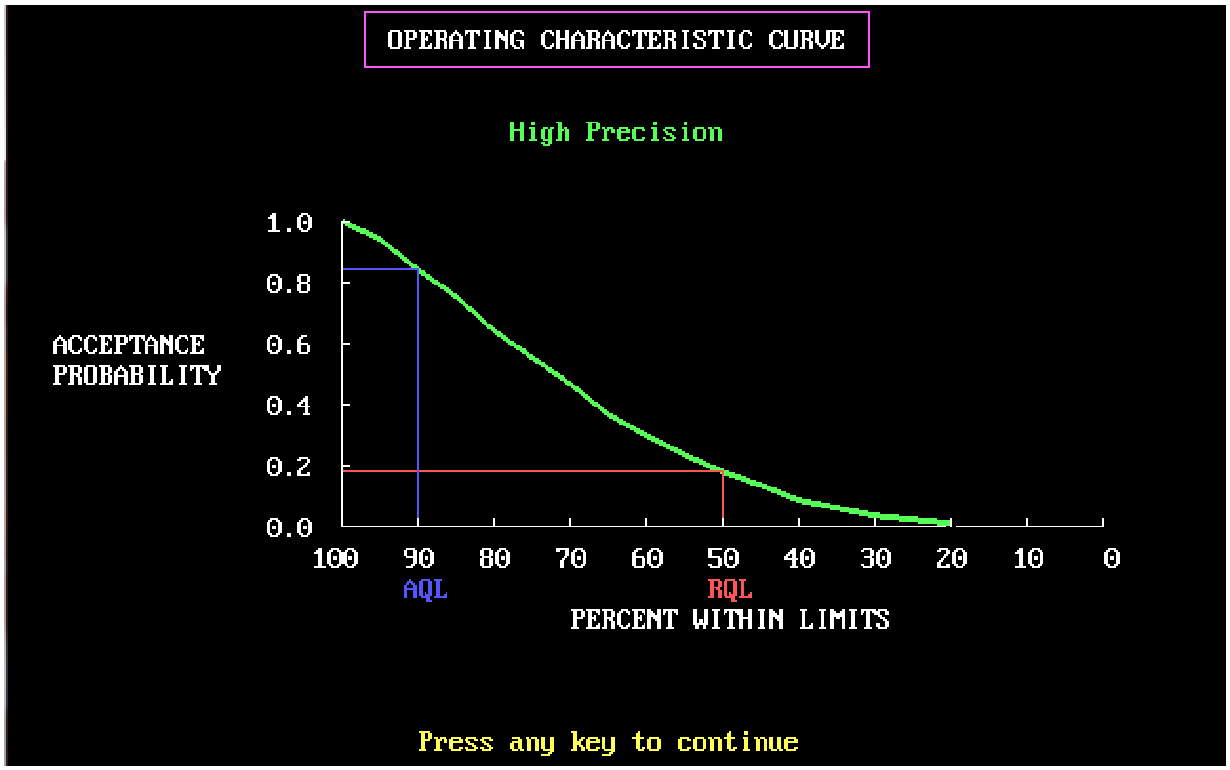


Figure 4: Example of OC curve for an AQC treated on accept/reject basis (from OCPLOT)

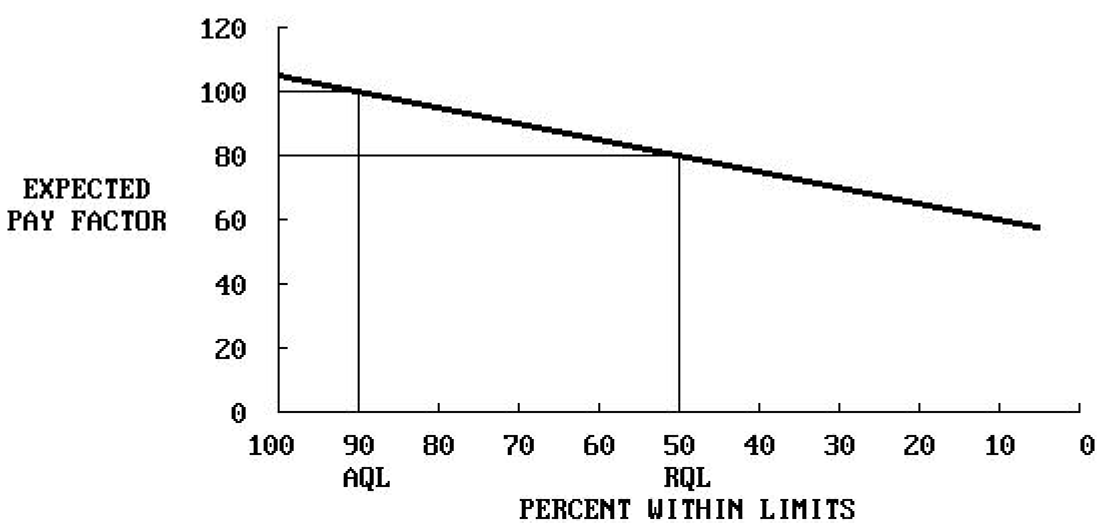


Figure 5: Example EP curve for an AQC treated on pay-adjustment basis (AASHTO R 9-90)

PAY ADJUSTMENT

In many cases, SHAs may not always decide to either accept or reject a lot submitted based on sampling results. Another approach is followed which either pays in full, gives bonus or charges penalty to the contractor based on the quality of lot submitted. It is called as "Pay Adjustment." It is also intended, by penalizing the contractor, to compensate for repairs resulting from reduced quality. OCPLOT by FHWA can be used to generate OC curves, (Weed, 1996).

When evaluating acceptance plans with price adjustment schedules, the concept of OC curves can be extended to give a graphical representation of the relationship between the quality of the lot and the probability of the lot being assigned various pay factors. These are called Expected Payment (EP) curves. An EP curve represents the mathematical pay expectation, or the average pay the contractor can expect to receive over the long run for submitted lots of a given quality (TRB, 2005). An example EP curve is shown in Figure 5. In the pay adjustment schedule shown, lot at AQL receives full (100%) payment while the minimum amount of pay is at RQL = 80%. Any PWL above AQL receives bonus (Max. at 100 PWL = 102%) and below RQL receives penalty or reduced pay (Min. at 0 = 60%).

Decisions for such types of acceptance plans are based on the EP curve for single AQC. In case of multiple AQCs, several OC curves are plotted, as % probability >= PF vs. PWL for selected sample size for different PWL levels are drawn to make a decision. EP curves can be either stepped or continuous. An example of stepped and continuous payment schedules is shown in Figure 6 , (NCHRP-346, 2005)

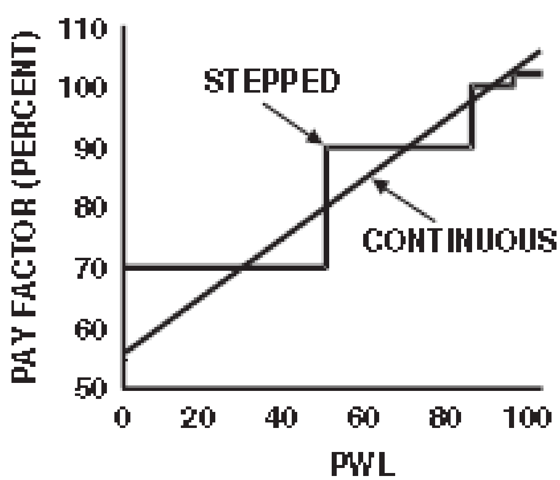
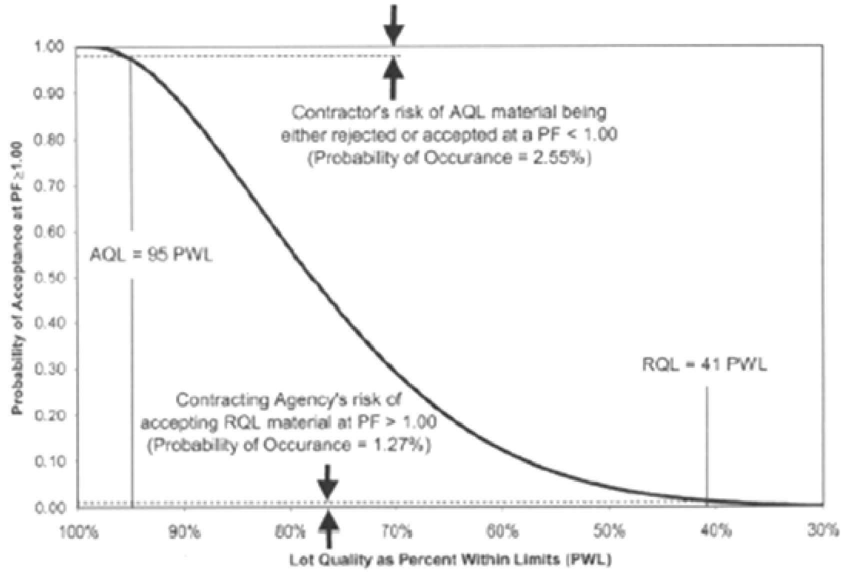


Figure 6: Example of Stepped and Continuous payment Schedules (AASHTO R 9-90)

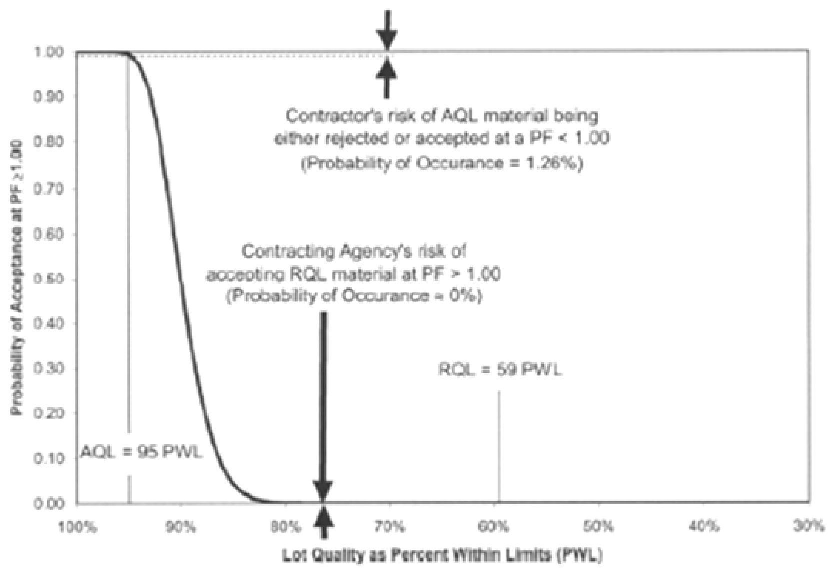
FACTORS AFFECTING SAMPLE SIZE

To develop a practical sample size calculation procedure, it is critical to identify key factors that affect and are affected by the sample size and then capture the interdependencies among these factors in an integrated procedure. These key factors are listed below. (These parameters were discussed earlier in the "Components of Acceptance Plans."):

- Acceptable, Rejectable, and Allowable Quality Levels (AQL, RQL, and M)
- Seller's (contractor's) risk (α)
- Buyer's (agency's) risk (β). The relation between risks and sample size is shown in Figure 7, (Hand A. & Epps, 2006).
- Product variability
- Specifications limits (tolerance)
- Cost of Sampling and Testing: Obviously, the more sampling and testing performed, the greater the project cost will be.



Typical OC curve showing risk for n = 5 [after Muench and Mahoney (5)].



OC curve showing risk for n = 50 [after Muench and Mahoney (5)].

Figure 7: Inter-relationship between Sample Size and Risks (Hand A. & Epps, 2006)

CURRENT PRACTICES

AASHTO standards

AASHTO standards are the recommended standard practices for State Highway Agencies (SHAs) for quality control and assurance. These standards were formulated based on the "AASHTO Road Test, 1992" results and recommendations. This test was basically aimed at the built-up highway quality to investigate whether it conforms to what has been the intention of the construction processes and the earlier construction quality specifications. It was found that in many of the cases the quality intended was falling into the region below the lower quality level. Hence, further steps were taken to change this scenario. National Quality Initiative (NQI) task force was a joint venture of the Federal Highway Association (FHWA), ASHTO and several other agencies. It is described in details by (Weed, 1996).

The Acceptance Specification development process in AASHTO's publication (Optimal Procedures for Quality Assurance Specifications) is provided to SHAs as a guide and as a means to establish the roots of overall Total Quality Management throughout the United States. The general procedure to reach the sampling and acceptance plan is shown in Figure 8, Figure 9 and Figure 10, (Burati, Weed, Hughes, & Hill).

The general recommendations regarding design of an acceptance plan are as follows:

1. Select and Evaluate Quality Characteristics based on Data available
2. Decide Quality Measure: PD, PWL or AAD
3. Define Upper and Lower Quality Levels: AQL, RQL
4. Select Sample Size: n, L
5. Select Minimum Quality Level: m
6. Calculate Risks and Plot OC Curve using OCPLOT
7. Cross-check risks with Table 3
8. If risks calculated in step 6 were acceptable, select the plan or else repeat steps 3-6.
9. Select Pay Factor Equation
10. Plot EP curve

11. Make decision to accept this plan. If unacceptable, repeat step 9 or from step 3-10.

This procedure is an improvement to the basic methodology developed for R 9-05 (AASHTO, 2005), which is based on U.S. Department of Defense specifications (Military Standard 414 – Sampling Procedures and Tables for Inspection by Variables for Percent Defective).

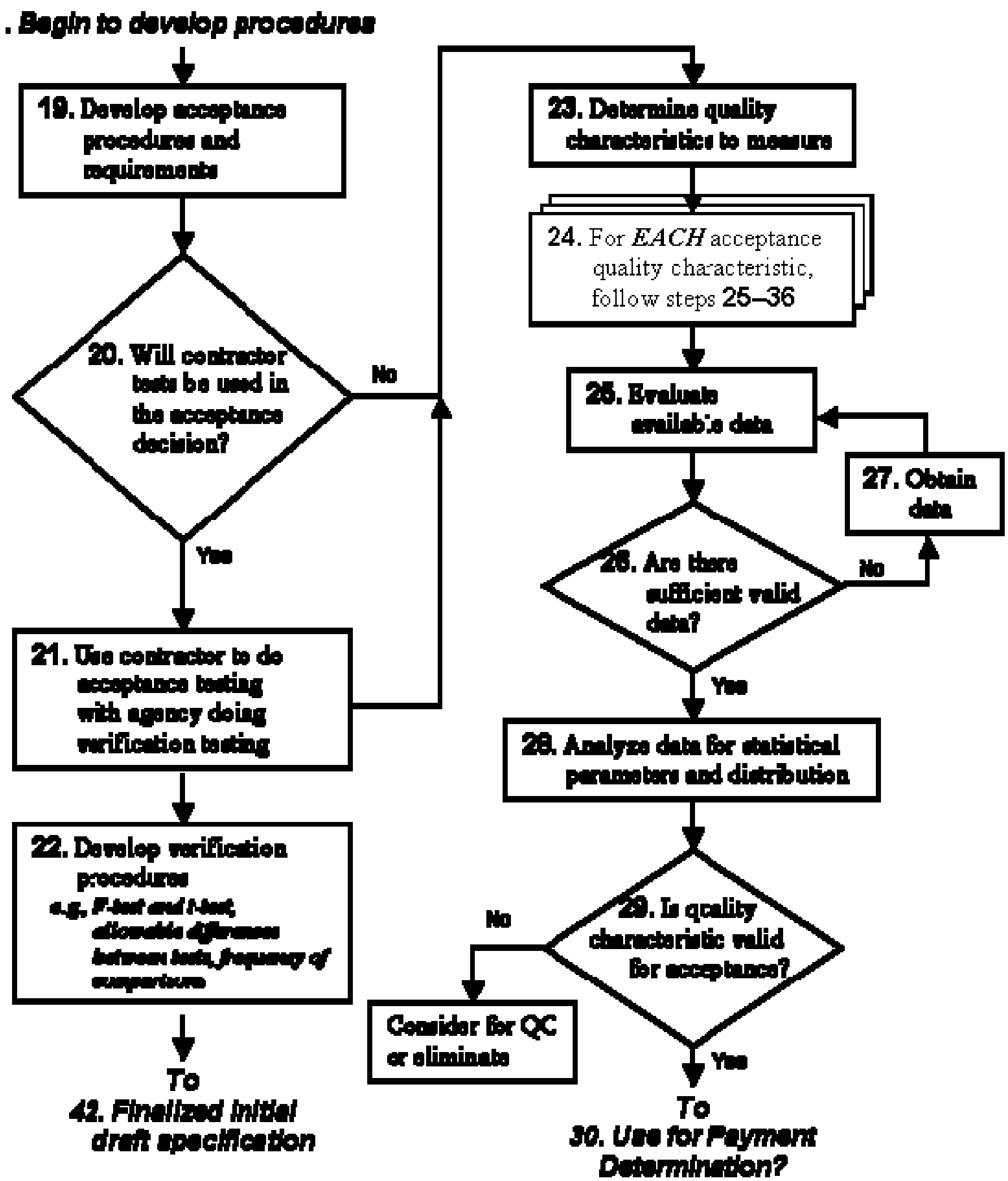


Figure 8: AASHTO Specification Development Procedure_1 (Burati; Weed, 2003)

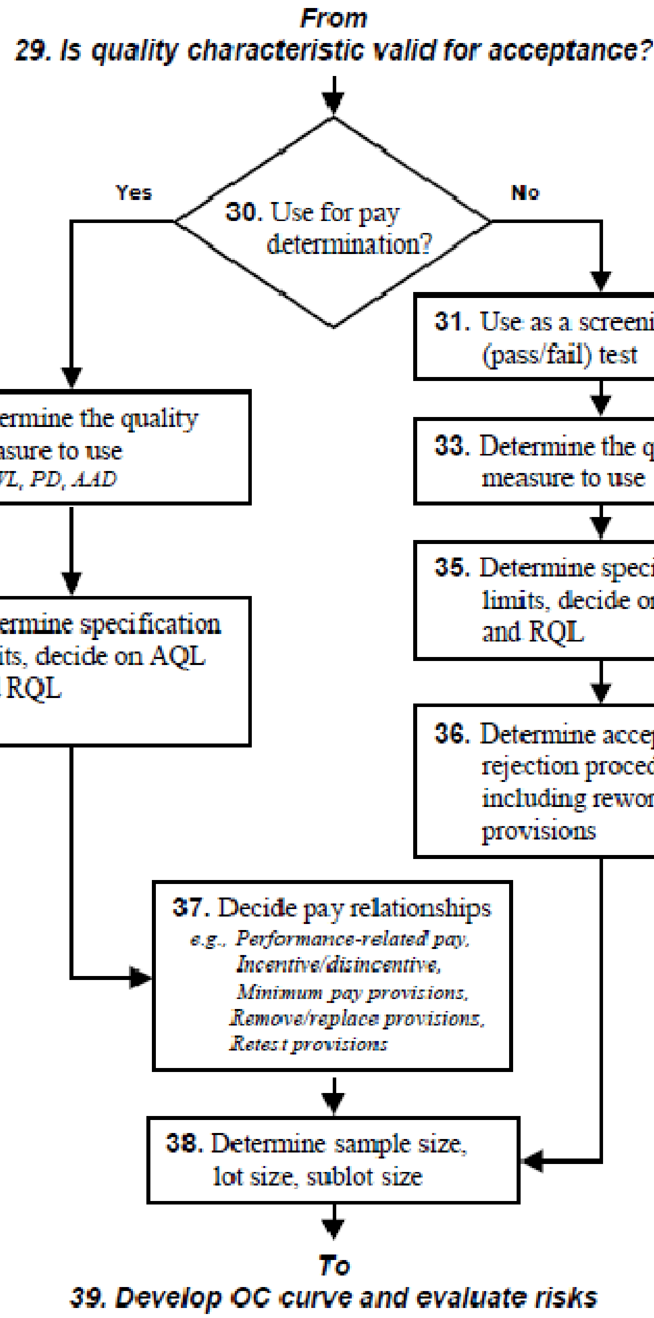


Figure 9: AASHTO Specification Development Procedure_2 (Burati; Weed, 2003)

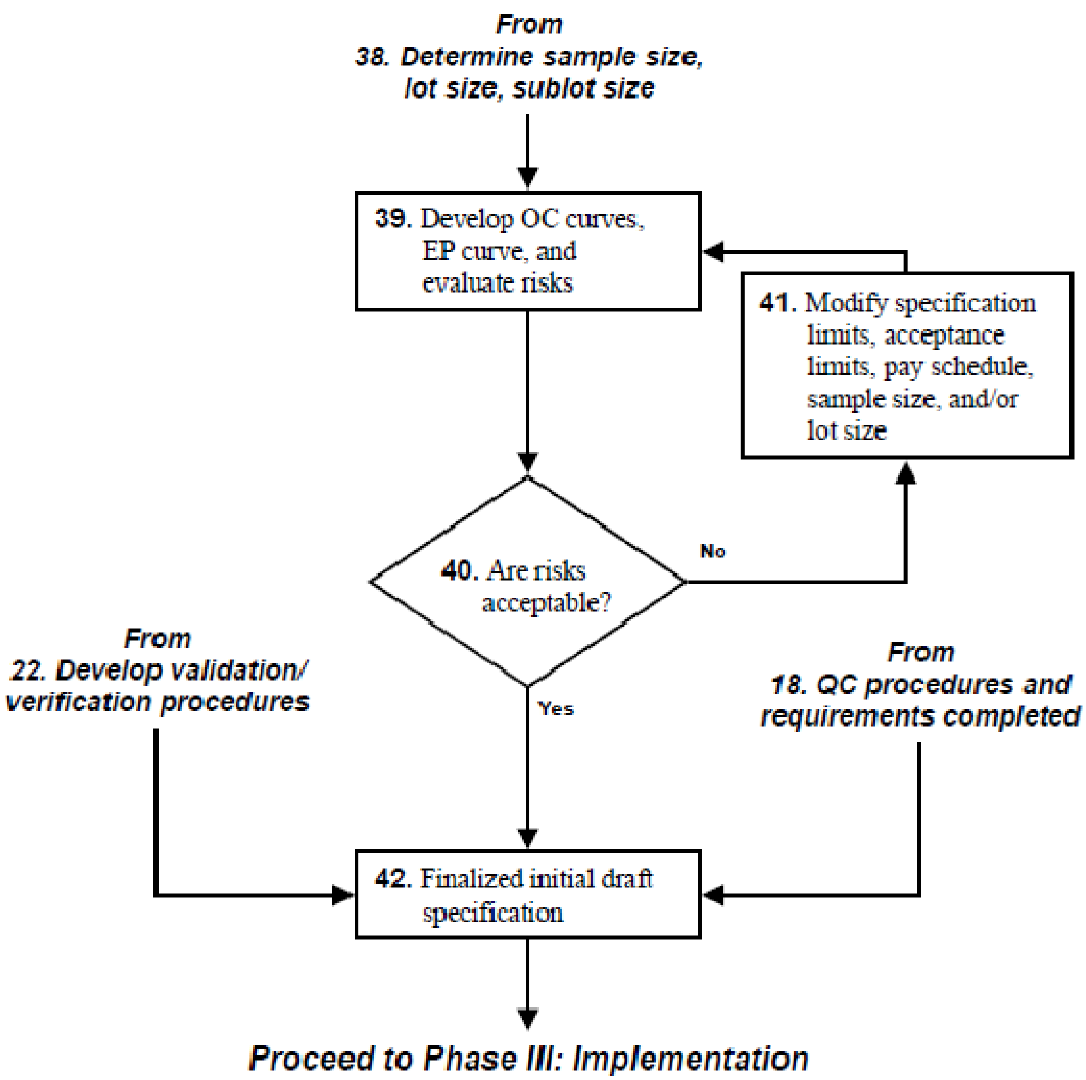


Figure 10: AASHTO Specification Development Procedure_3 (Burati; Weed, 2003)

These guidelines are followed by SHAs with variations in some factors, such as quality measure and quality levels. Obviously, the sample size specified by SHAs also varies according to the quality levels required. A summary of various credentials used by SHAs is discussed in NCHRP report (State Construction Quality Assurance Programs, 2005). Compaction, asphalt content and ride quality are the top preferences by most of the agencies for acceptance criteria for HMA, Figure 11, using PWL/PD and range as a quality measure, Figure 12. In case of PCC pavements, most popular AQCs are air content, slump and thickness, Figure 13, and SHAs are not in consensus as to the use of quality measure since use of PWL/PD, average and ranges is nearly equal, Figure 14.

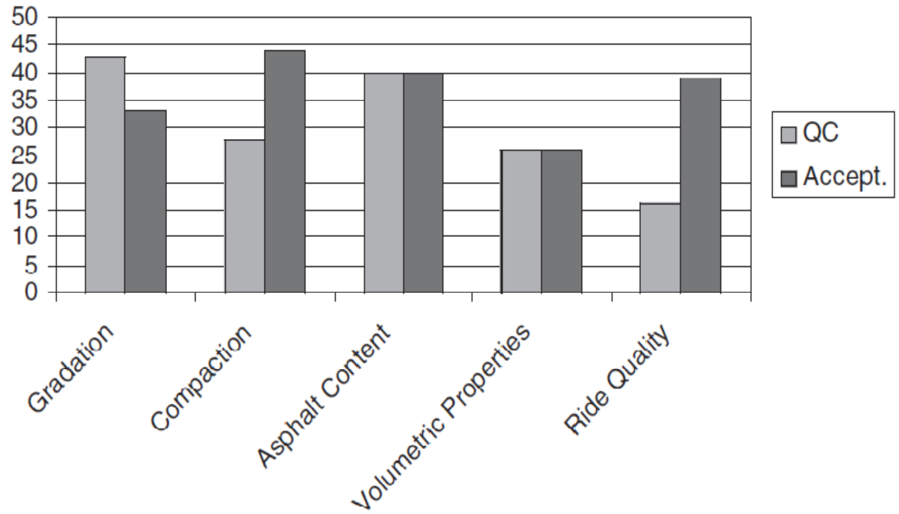


Figure 11: Attributes most often used for QC and acceptance of HMA by SHAs (NCHRP-346, 2005)

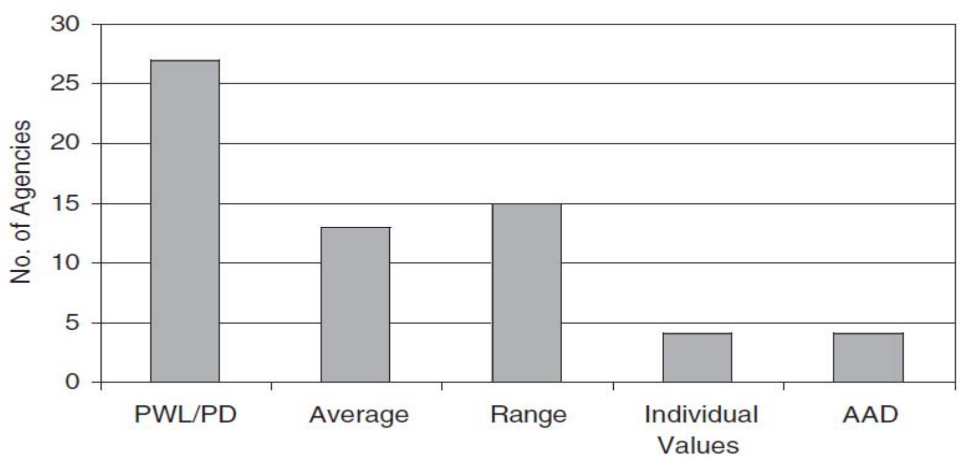


Figure 12: Quality measures most often used for acceptance of HMA by SHAs (NCHRP-346, 2005)

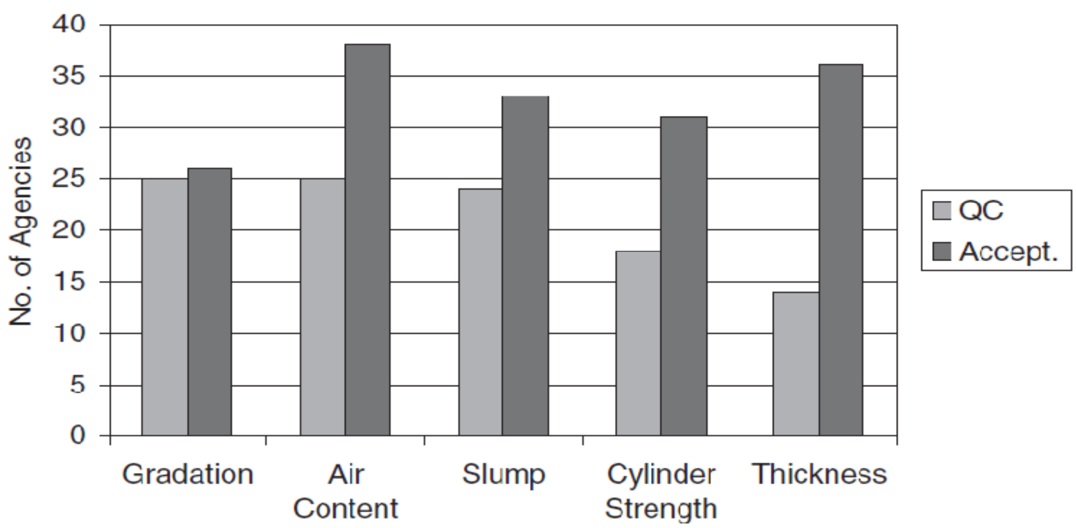


Figure 13: Attributes most often used for QC and acceptance of PCCP by SHAs (NCHRP-346, 2005)

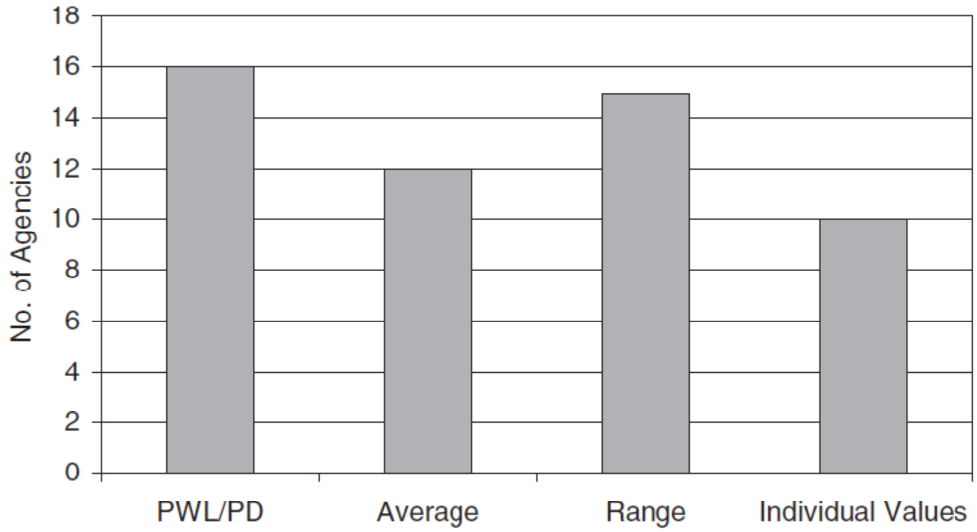


Figure 14: Quality measures most often used for PCCP by SHAs (NCHRP-346, 2005)

PRACTICAL SAMPLE SIZE: RELATED WORK

The sample size (per lot), required to meet allowable tolerance in the mean (typically stated in the construction specifications), considering both the seller's risk (α) and buyer's risk (β), can be computed as follows:

$$n = \left[\frac{(Z_\alpha + Z_\beta)\sigma}{e}\right]^2 \qquad \text{Eq. 2.1}$$

Where,

n = minimum sample size

Z = standard normal distribution value for the required confidence

σ = standard deviation of the population

e = tolerable error allowed by the specifications

α = seller's risk, and

β = buyer's risk.

In addition to the above formula, highway agencies often determine sample size based on practical considerations (primarily time and personnel constraints).

While Eq. 2.1 is widely used in many statistics textbooks and is adopted by (ASTM, 2009) it has major limitations:

1. It does not consider cost.
2. It is not applicable to PWL (or PD) acceptance plans.

This study seeks to address these shortcomings by determining the practical sample size from the agency's economic stand point (i.e., sample size that minimizes the agency's cost).

Past efforts made to address sample size issues are illustrated in the following sections.

A model was developed by McCabe, AbouRizk and Gavin (1999) to account for concerns about confidence in the sample standard deviation obtained from a small sample size. The model is a conservative approach for estimating the population standard deviation (and thus the sample size) by resorting to a modified sample variance, which actually represents a confidence interval of the standard deviation that accounts for variance between lots being evaluated, instead of a mere deterministic and uncertain value, as shown in equation below.

$$\sigma' = \overline{\sigma} \pm Z_{\alpha/2}\sigma \; ; \quad \sigma = (\overline{\sigma}/2)c \qquad \text{Eq. 2.2}$$

Where

σ' = modified population standard deviation.

c: Gamma function parameter as a function of n (sample size).

$\overline{\sigma}$: Deterministic estimate of population standard deviation.

Velivelli (2002) applied this conservative approach to determine the sample size for concrete strength in the FAA P-501 specifications for airfield concrete pavement. This has resulted in a relatively high sample size of 5 beams for flexural strength and 7 cylinders for compressive strength.

In 1970, Juran discussed economics of quality and effect of nonconformity on the cost (Juran J.M, 1970). According to Juran, as the quality increases in terms of number of defectives the total cost, which is the total of losses due to defectives and cost of production, decreases. He further states that "the optimum (number of defectives), is reached when 'perfectionism' begins to set in." This optimum can

be modified either through improved quality control and a reduced number of defectives or by loosening the inspection levels (i.e., quality specifications). Figure 15 describes Juran's model of effect of quality on economics. Optimum is reached at minimum total cost.

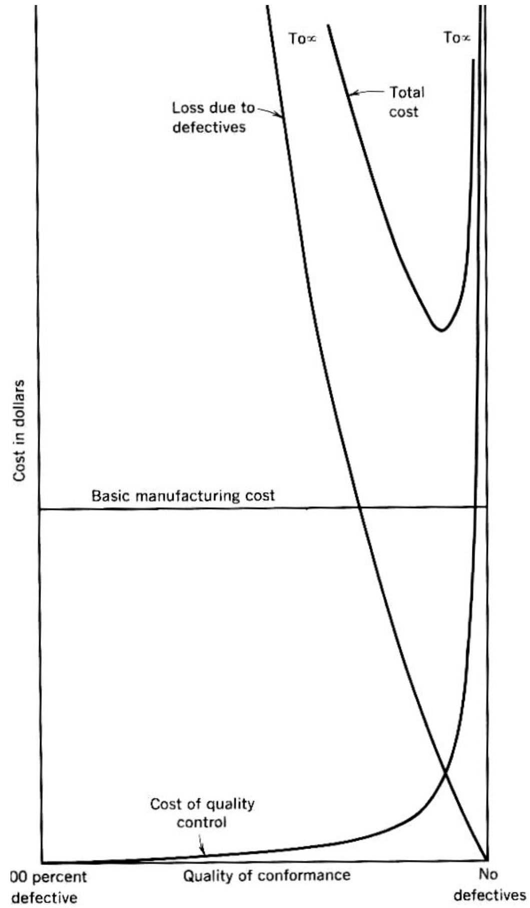


Figure 15: Economics of Quality of conformance by Juran

Interpretation of Juran's analysis as applied to the Acceptance Sampling Plan can be inferred by further explaining the term "quality of conformance." In general, quality of conformance depends on AQL, RQL and sample size for a given acceptance and sampling plan. In order to improve the overall quality of construction in a state, the State Highway Agency's (SHAs) decide not to "loosen" the acceptance criteria, i.e., not to compromise the quality. Hence, the major factor affecting the cost related to quality of conformance would be sample size, and, for the given quality levels such as AQL, RQL for a range of sample size the optimum should decrease as the sample size increases. Juran also suggests that in order to fix an optimum quality level historical data may be used as reference and further explains the procedure to improve the quality of the products

One of the studies that recognized the importance of determining the practical sample size with respect to cost was done in the field of manufacturing by South (1982). In this paper, the author emphasized introducing the cost of erroneous decisions as a critical measure to evaluate and/or to validate acceptance plans. According to the author, the current specifications do not provide concrete methodology to choose the alpha and beta (producer's and seller's risks); hence, the judgment as to the effectiveness of the sampling plan regarding minimizing the risks is based on the agency's decision and perception of the acceptance plan. This paper, in order to provide a criteria of judgment of the selected sampling plan, presents an innovative method to develop a sampling plan based on minimization of the total cost which is the sum of cost of erroneous decisions (i.e., accepting a bad lot and rejecting a good lot) and cost of sampling and testing and provides an "insight into how the cost of sampling relative to error cost affects the practical sample size and the appropriate values for alpha and beta."

The acceptance method designed in this paper assumes that the submitted lot has PD either equal to the AQL or the Lot Tolerance Percent Defective (LTPD). The author also points out that this assumption was made in order to make this method simpler and easier to understand. It was also recommended to use more complex methods once this method is mastered. Having stated the aforementioned, the author also mentions that using more complex methods might not always prove to be worth the extra cost and complexity in order to reach the solution.

The cost of erroneous decisions for submitted lots was calculated as the product of percentage of bad lots (lots of unacceptable quality) received and cost incurred from accepting a bad lot, provided that no sampling was done. In order to reduce the cost incurred due to acceptance of a bad lot, the author suggests sampling. When the sampling method is followed, the cost due to accepting a bad lot should also be considered. Then the expected cost due to errors was calculated as:

$$EC = (\alpha) * (C1) * (K) + (\beta) * (\mathrm{C2}) * (1 - \mathrm{K}) + (\mathrm{n}) * (\mathrm{C3}) \qquad \text{Eq. 2.3}$$

Where, EC = expected error and sampling cost

α = the probability that a good lot will be rejected

β = the probability that a bad lot will be accepted

C1 = the extra costs incurred when a good lot is rejected

C2 = the extra costs incurred when a bad lot is accepted

K = the probability that the lot being sampled is a good lot

n = the number of items sampled

C3 = the cost to sample one item

Good lot = a lot with the percent defective items equal to the AQL

Bad lot = a lot with the percent defective items equal to the LTPD.

In order to implement this sampling plan, calculations for a range of sample size ranging from 0 to 250 were carried out. The sample size corresponding to the minimum EC was selected as the practical sample size. The following observations were made from further analysis:

1. Since the sample size selected has the minimum total cost, this plan was named "Minimum Cost Sampling Plan," which means that "given the values of the parameters any other sampling plan will have a higher expected cost."
2. The greater the probability that the given lot is good; the lesser is the required sample size.
3. It was proved that fixing alpha and beta risks to 0.05 and 0.01 does not always correspond to the minimum expected error and sampling cost (EC).
4. Although larger sample size might cost more in testing, when the decision cost is considered the testing cost is greater, but the total cost is much less, which is due to the decrease in expenses that might have incurred due to an erroneous decision.
5. The author suggests using hyper-geometric distribution instead of binomial distribution (used in this method to calculate beta and alpha risks) if the sample size obtained is a significant percentage of the lot size, so as to consider lot size as an additional parameter for the minimum cost sampling plan design.

The Practical Sample Size Calculation Method (PSSCM) developed under this thesis uses the basic methodology outlined by South. Some of the limitations to this method and the recommendations

in this paper were rectified and taken into consideration while developing the sampling plan. Further details are explained in Chapter 3: Determination of the Practical Sample Size of this thesis.

Since the beginning of Acceptance Sampling, attribute sampling was in use for many years until Shainin (1950) developed the Lot-Plot method. Variable sampling plans were found to require less sample size than attribute sampling plans. Further developments in these plans were, (Wetherill & Chiu, 1975):

1. Bowker and Goode (1952): Schemes based on computed OC curves and normality assumption. Procedures are given for σ known and unknown.
2. Lieberman and Resinkoff (1955): Theory and tables for inspection by variables, which was then later used in MIL-STD-414 (1957).
3. Duncan (1965): Detailed explanation of various cases in sampling by variables with examples and solutions.

The plans developed considering economic aspects include plans by:

1. Stange (1964, 1966) presented Minimax Scheme, which assumed minimax function instead of using prior distribution. (A prior distribution function is the distribution of the lot submitted. In general, it is assumed to be Normal, but in some plans, in order to estimate the quality more precisely, different distributions are used.)
2. Grundy et al (1998): Developed economic solutions with prior normal distribution. The quality measure used was "Averages."

PRIOR DISTRIBUTION

In 1943, Mood stressed a new approach to sampling plans by using the concept of prior distributions which is not considered in the design of Dodge-Roming plans (Mood, 1943).

According to Mood, a Sampling inspection of lots may take one of two courses: (a) Item inspection, in which a lot is accepted or completely inspected on the basis of one or more samples drawn from the lot, (b) Lot inspection, in which a lot is accepted or rejected on the basis of one or more samples drawn from the lot.

The current practices previously mentioned in this chapter use the first approach. The latter is, in fact, considered seriously by very few researchers. This approach believes in the use of the information which becomes available once the sampling is started i.e., if 'n' is decided as the sample size and first few samples are taken out, the rough idea of the distribution of the lot quality can be obtained which can be used to estimate lot quality more precisely and can be used for further inspection. One of the important theorems proved by Mood, Theorem II, shows that if the prior distribution is approximated before sampling is continued, at one point there is no further information that can be obtained by further sampling, hence it can be used to minimize the sample size.

The theories that have been developed based on this methodology include research by Wetherill, (Wetherill G. , 1959). In this paper, a sequential plan developed for sampling by variable is developed based on use of prior distribution. Here, a variation of Mood's theory is used where, instead of acquiring the distribution during inspection, the distribution of the lot quality is assumed to be known from past experience and is used to calculate sample size. The sampling cost function is assumed to be linear. Loss function is calculated based on the Bayesian method. It is postulated that the "Most economical scheme will be the one for which the sum of the average total costs of inspection and of the rejection of lots of normal quality is minimum."

Several other studies and the plans developed are either intended for attribute sampling plans or use Bayesian methods. A few of the prominent studies are by Champernowne (1953), "System of Single Sampling Inspection Based on Prior Distributions and Costs" (Hald A. , 1960) and "Single Sampling Inspection Plans with Specified Acceptance Probability and Minimum Average Costs" (Hald A. , 1964), "An Application of Least Cost Acceptance Sampling Schemes" by Griffiths (1965), "Bayesian Single Sampling Attribute Plans for Continuous Prior Distributions" (Hald A. , 1968) and recently by Lam Yeh (1990).

The traditional method of using the historical data for judgment and selection of plans is shown in Table 4 (Schilling, Acceptance Sampling in Quality Control, 1982):

Past Results	Quality History: Little	Moderate	Extensive	Criterion
Excellent	AQL Plan	Cumulative Results Plan	Demerit Rating or Remove Inspection	Almost no (<12) lots rejected
Average	Rectification or LTPD Plan	AQL Plan	Cumulative Results Plan	Few (<102) lots rejected
Poor	100 Percent Inspection	Rectification or LTPD Plan with Cumulative Results Criterion	Discontinue Acceptance	Many (≥ 102) lots rejected
Amount	Less than 10 lots	10-50 lots	More than 50 lots	

Table 4: Progression of Sampling Plans: Extent of Quality History (Schilling, 1982)

NOTE ON USE OF HISTORICAL DATA

Optimal Procedures for Quality Assurance Specifications (2003) give a word of caution while acquiring and using historical data as:

- Care must be taken when using historical data because they may not always be unbiased. In fact, historical data may frequently be biased. To be valid, the historical data must have been obtained using a random sampling procedure. That is, the sampling locations should have been selected at random, and not systematically or by the judgment of the inspector. When judgment is used for sample selection, bias can be introduced because there may be a tendency to select a sample location where the material looks questionable in an effort to ensure that "bad" material is not incorporated into the project. On the other hand, there could be a tendency to select a sample location where the material looks particularly good in an effort to ensure that the material is accepted. Either of these will provide a biased estimate of the actual properties associated with past construction.

- Another potential problem with historical data is the past process of selecting and testing a second sample when the first sample failed to meet the specification requirements. If the second sample met the specifications, and as a result the first sample was disregarded and its value not recorded, then the historical data will tend to underestimate the actual variability associated with the process being measured. The validity of historical data must be scrutinized thoroughly before deciding to use them to establish QC limits.
- Since the specification and/or acceptance limits will be generic, i.e., agency-wide, the data must not only have been obtained in a manner consistent with their use in the specification, but they must also be broad-based. That means they must have come from production/construction that represents different geographical areas of the State, different contractors with different operations, and projects of different sizes, to mention just some of the considerations. The data must have been obtained by a random sampling procedure and have been sampled and tested in the same manner with the same type equipment as will be required in the new acceptance plan.

RESEARCH NEED FOR PRACTICAL SAMPLE SIZE

1. Current practices do not take into consideration the variability in material quality. If the costs are taken into consideration, the variability is taken indirectly into account by involving the unit bid price and testing costs which are usually directly related to the quality of the material.
2. Many of the decisions depend on the judgment of SHAs, viz., choosing and balancing alpha and beta risk levels with no firm guidance as to this selection, ((South, 1982), (Duncan, 1986)).
3. There is no direct control over the costs associated with the erroneous decisions.
4. Current practices do not consider prior distribution of data (i.e., assumption is made that the process is not controlled statistically).

Chapter 3: Determination of the Practical Sample Size

PRACTICAL SAMPLE SIZE CALCULATION METHOD (PSSCM)

PRACTICAL SAMPLE SIZE – DEFINITION

The objective of this thesis is to investigate the practical sample size, which is a sample size corresponding to the minimum cost of acceptance for an agency while taking historical lot quality into consideration. Figure 16 shows a general method to find the practical sample size 'n' with respect to the total cost of lot acceptance. The total cost ($Cost_{(D+T)}$) is the numeric sum of the agency's cost of sampling and testing and ($Cost_T$), and costs incurred due to erroneous acceptance decisions ($Cost_D$), i.e., costs corresponding to buyer's risk (β Risk or Agency's risk or Type II error) and seller's risk (α or Contractor's risk or Type I error). This method follows the basic principle used by Hald, (June 1964) where the expected loss per batch is expressed as follows: R = (cost of sampling) + (loss of wrong decisions). Hald used the Bayesian method and prior distributions to find practical sample size by minimizing costs for attribute sampling plans. PSSCM follows a simple and effective approach for variables sampling plans.

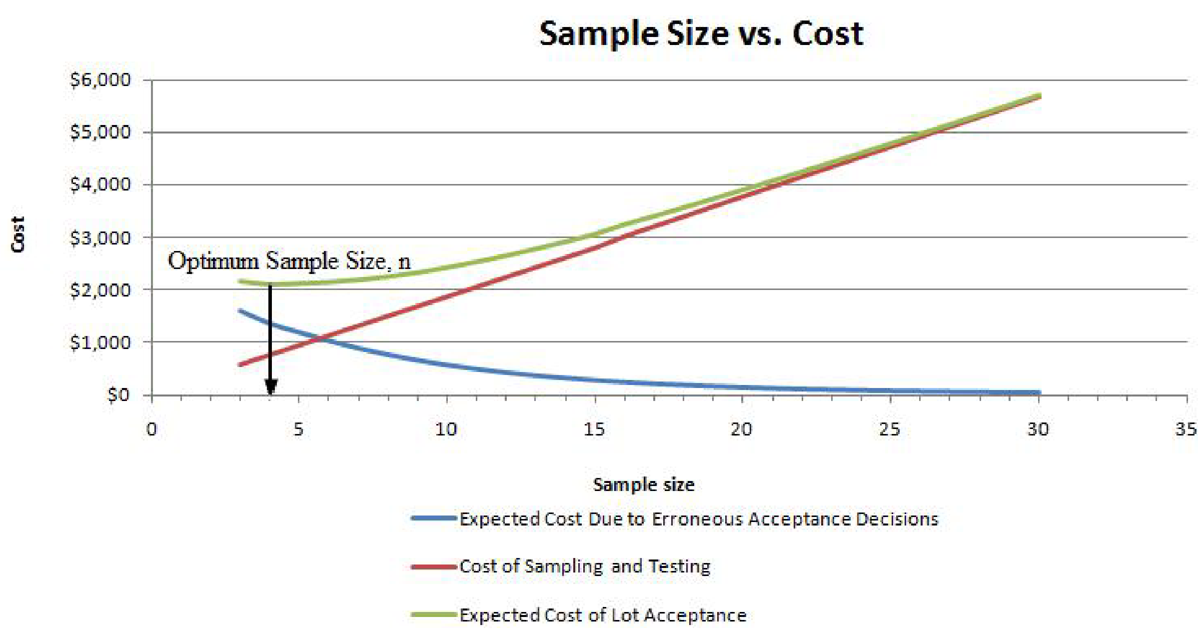


Figure 16: Practical Sample Size at Minimum Acceptance Cost

BASIC APPROACH

As discussed earlier in the literature review, the traditional approach (AASHTO R 9-90) to determine a sample size is to select AQL, RQL, 'm' and sample size and to simulate an OC and/or EP curve using OCPLOT. Based on OC Curve, the sample size is accepted if the probability of acceptance at given AQL and RQL is justified and the decision is based on either the risks or an EP curve.

In this study another statistical approach by (Wallis, 1947) and (Duncan, 1986) is used as shown in Figure 17. According to (Wallis, 1947), every sampling plan basically has three major phases:

1. Plan of Action, i.e., set of values on basis of which to accept or reject the lots.
2. Amount of inspection required by the plan, i.e., to determine sample size, and
3. OC curve, i.e., proportion of submitted lot of various qualities that will be accepted and rejected if the plan is accepted.

The same phases are implemented in the following procedure.

Since the AASHTO method is a trial-and-error method to fix the sample size and the approach by Duncan assumes the alpha and beta risks as 0.05 or 0.1, these methods could not be directly implemented to determine the practical sample size. The calculation of the practical sample size essentially requires the calculation of α and β risks corresponding to given AQL, RQL and minimum sample PWL (m) so that these risks can be used to calculate the decision cost associated with the sample size in consideration. Also, the method by Duncan needs an input 'k' which is equivalent to 'm,' i.e., a quality index corresponding to the PWL at 'm'. Therefore, formulae provided by this approach were solved in terms of Z_α and Z_β instead from n and k. Also, this approach is modified to incorporate the conversion of 'm' to 'k,' to calculate α and β risks for several of the sample sizes and to calculate the decision costs associated as shown in Figure 18: Practical Sample Size Calculation Method (PSSCM)-Basic Approach.

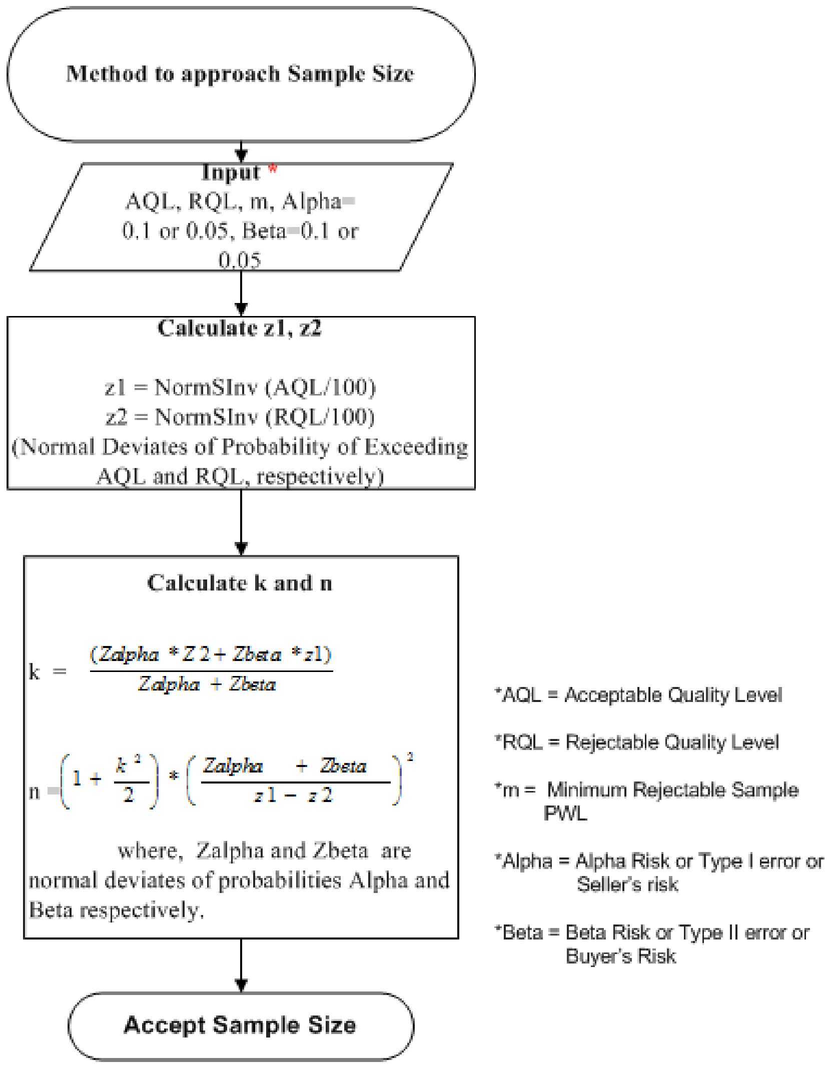


Figure 17: Statistical Approach by Wallis and Duncan

Modifications to the acceptance method designed by South were made in order to improve the method with respect to:

- Variables method used instead of attributes sampling method to yield lower sample size.
- Calculation of Risks: Use of a more precise method developed by Wallis and modified by Duncan to calculate Alpha and Beta risks.
- Lot size was added as an input to PSSCM in order to take into consideration the effect of lot size on the cost due to erroneously accepting a bad lot.

- Instead of assuming (or using) probability that a lot is good or bad, it is proposed to use the historical percentages (i.e., probabilities) of having a lot with the current AQC at PWL equal to or below RQL (P_{BR}), between RQL and AQL (P_{RA}) or above AQL (P_{AA}), since these percentages can be easily obtained from the construction quality databases of the states.

Other advantages of including historical percentages in the analysis are:

- It introduces the effect of variability in construction and material quality over the current acceptance plan.
- It reduces the required sample size a considerable extent for an AQC with a fairly good historical record, whereas increases for bad quality records, hence yields realistic results for sample sizes.
- It should also be noticed that the sample size required while using variable sampling plans with mean and standard deviation unknown usually yields higher sample sizes as compared to the plans using known statistical parameters (Duncan, 1986, p. 275). The use of historical percentages indirectly induces the similar effect of known mean and standard deviation while keeping the actual calculations much easier and less complex than other plans.
- If instead of using actual historical percentages, the desired percentages are used to achieve desired material quality or to reduce variability in it as a part of quality assurance and improvement to the overall quality in a state. I.D. Hall also commented about the use of prior distributions in reference to (Mood, 1943),

  "To operate a scheme that persuades producer to change prior distribution in a desired way should, I feel, be a fundamental aim of inspector" (Stephens, 2001, p. pp. 15).
- On the other hand, it limits the current plan only to those states where sound historical databases are available. Also, the sample sizes cannot be directly compared to the existing plans without having historical data.

The advantages of using the formula by Wallis and single sided method for variables with unknown mean and standard deviation include:

- Less number of inputs required
- Less assumptions
- Requires less computation and minimum computer resources
- Ease in application to the minimum total cost approach

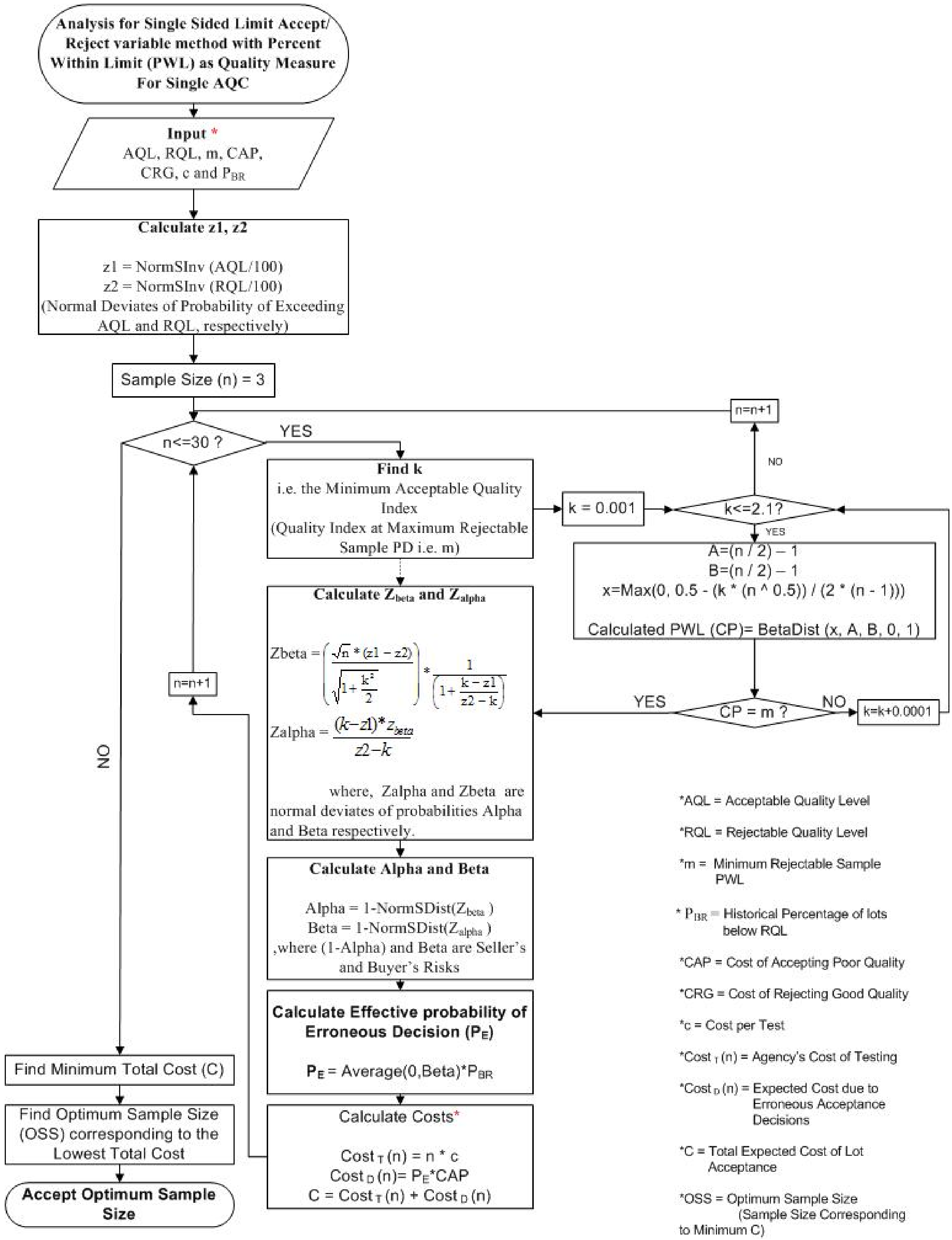


Figure 18: Practical Sample Size Calculation Method (PSSCM)-Basic Approach

STEP 1 MODEL: SINGLE AQC IN ACCEPT/REJECT ACCEPTANCE PLAN

Inputs:

The inputs are accepted from user as Percentage within Limits (PWL); however, for the calculation purposes, these are converted into Percent Defective (PD). The inputs are AQL, RQL and m expressed as percentage.

Hence, for calculations: we know the following variables which are accepted as inputs:

- AQL (Acceptable Quality Level), PWL
- RQL (Rejectable Quality Level), PWL
- m (minimum acceptable sample PWL),PWL
- L (Lot Size), Tons or Sq. Yd.
- b (unit bid price), $/Ton or $/Sq. Yd.
- Cost of Accepting Poor Quality (CAP), $/Ton or $/Sq. Yd.
- Cost of Rejecting Good Quality (CAP), $/Ton or $/Sq. Yd.
- Cost of Testing and Sampling (c), $
- For Pay Adjustment: a (%) and b
- P_{BR}: Historical percentage (i.e., probability) of having a lot with the current AQC at PWL equal to or below RQL
- $P_{RA:}$ Historical percentage of current AQC at PWL between RQL and AQL
- $P_{AA:}$ Historical percentage of current AQC at PWL equal to or above AQL

Calculation of Risks:

a) Calculations start with the sample size (n) = 3 and are repeated up to 30 (increment by 1). Z_1 and Z_2 are the normal deviates of the probability of exceeding AQL and RQL respectively. Here,

$$\text{AQL (PD)} = 100 - \text{AQL (PWL)} \qquad \text{Eq. 3.1}$$

and

$$\text{RQL (PD)} = 100 - \text{RQL (PWL)} \qquad \text{Eq. 3.2}$$

Hence, Z_1 and Z_2 were calculated as,

$$z_1 = \phi^{-1}(AQL/100)$$
$$z_2 = \phi^{-1}(RQL/100) \qquad \text{Eq. 3.3}$$

Where, ϕ^{-1} is the inverse of the standard normal cumulative distribution function (sometimes called as a "Quantile function").

In order to evaluate these values one may use the standard charts for cumulative probabilities of the normal probability distribution (Wallis, 1947, p. pp. 31)or using NormSInv: a Microsoft Excel function which returns the inverse of the standard normal cumulative distribution, AQL and RQL are expressed as Percent Defective.

b) In order to find 'k' from given 'm,' try and error method was used. Several k values were tried to calculate the Percent Defective so as to get the same percent defective as that of given m. The following formula from (AASHTO R 9-90) was used:

$$P.D. = \int_{x=0}^{x = Max\ [(0,1/2-(Q\sqrt{n})/(2(n-1)))]} \beta(a,b,x)\,dx \qquad \text{Eq. 3.4}$$

Where: Q = k, a = b = n/2-1

The function mentioned above is the same as that of the Microsoft Excel function BetaDist, hence, the same was used as below:

P.D. = BetaDist(x, a, b, 0, 1), where BetaDist returns Cumulative Beta probability density function and

$$x = Max\ [(0,1/2 - \frac{k\sqrt{n}}{2(n-1)}] \qquad \text{Eq. 3.5}$$

c) Now that we know k, Z_1, Z_2, we can calculate $Z\alpha$ and $Z_{ß}$ which are the normal derivates of the probabilities α and β, as:

$$Z_{\beta} = \left(\frac{\sqrt{n} * (z_1 - z_2)}{\sqrt{1 + \frac{k^2}{2}}} \right) * \frac{1}{\left(1 + \frac{k - z_1}{z_{2-k}} \right)}$$ Eq. 3.6

$$z_{\alpha} = \frac{(k - z_1) * z_{\beta}}{z_2 - k}$$ Eq. 3.7

These formulae derived from the formulae offered by W. Allen Wallis (Wallis, 1947) for finding 'n' and 'k' (which is the function of m) for given AQL, RQL, α and β were used,

$$k = \frac{Z_{\alpha} * Z_2 + Z_{\beta} Z_1}{Z_{\alpha} + Z_{\beta}}$$ Eq. 3.8

$$n = \left(1 + \frac{k^2}{2}\right) * \left(\frac{Z_{\alpha} + Z_{\beta}}{Z_{\alpha} - Z_{\beta}}\right)^2$$ Eq. 3.9

d) Since, Z_{α} and $Z_{\beta}$are also the normal derivates of the probabilities α and β, respectively the probabilities were then calculated by using a function called NormSDist which returns the standard normal cumulative distribution function of Z_{α} and Z_{β}.Hence,

$$\alpha = \phi(Z_{\alpha})$$ Eq. 3.10

And,

$$\beta = \phi(Z_{\beta})$$ Eq. 3.11

Hence, the Buyer's Risk = β and Seller's Risk was found to be = (1- α). The same procedure is repeated for the range of sample sizes (here, n = 3 to 50).

(Note: The values for Q and k for any sample size and PD can also be obtained from the bales in Appendix A of (AASHTO R 9-90) or can be located using Larson's Nomograph (see Figure 19:

Larson's Nomograph). The equations are used to achieve more precision in calculations and to use these equations to automate the calculations using the computer program. The use of the other two methods is described in detail in Calculation of Alpha and Beta Risks (pp. 46) of this thesis.)

Calculation of Costs:

a) Expected Cost of Lot Acceptance:

$$C = \mathrm{Cost}_{T_{(n)}} + \mathrm{Cost}_{D_{(n)}} \qquad \text{Eq. 3.12}$$

Where, $Cost_T(n)$ is agency's cost of testing and $Cost_D(n)$ is agency's expected cost due to erroneous acceptance decisions.

b) Cost of Sampling and Testing:

$$\mathrm{Cost}_{T_{(n)}} = c_{(n)} \qquad \text{Eq. 3.13}$$

Where, c (i) is the cost of testing I number of samples for current AQC

c) Expected Cost Due to Erroneous Acceptance Decisions:

$$\mathrm{Cost}_{D_{(n)}} = P_E * CAP + \alpha_n * CRG$$

$$\mathrm{Cost}_{D_{(n)}} = PE * CAP \qquad \text{Eq. 3.14}$$

, where PE = Effective Probability of Erroneous Decision

= (Probability of erroneously accepting a bad lot at PWL at or below RQL) * Probability that the PWL of AQC is at or below RQL (Using the law of intersection of probability for independent events as: P (A and B) = P (A) * P (B), (Duncan, 1986, p. pp. 31))

$$PE = ((0 + \beta_n)/2) * P_{BR} \qquad \text{Eq. 3.15}$$

(Note: Probability of erroneously accepting a bad lot at PWL at or below RQL is assumed to be equal to the mean beta risk = average of 0 and beta risk for the simplified calculations).

$$\mathrm{CAP} = \text{lot size} * \text{bid price} = \mathrm{L} * \mathrm{b} \qquad \text{Eq. 3.16}$$

STEP2 MODEL: SINGLE AQC IN PAY ADJUSTMENT ACCEPTANCE PLAN

The calculations for α and β risks for single AQC remain the same in pay adjustment acceptance plans as that of accept/reject plans, and can be calculated using Eq. 3.1 through Eq. 3.11.

The cost calculations for pay adjustment plans are based on the pay equations, i.e., a (%) and factor b.

$$\mathrm{Cost}_{\mathrm{D}(\mathrm{n})} = \mathrm{delta\_pf} * \mathrm{PE} * \mathrm{CAP} \qquad \text{Eq. 3.17}$$

, where delta_pf = (pf_aql – pf_rql)/100

pf_aql = a + b * (100-AQL) and pf_rql = a + b * (100-RQL)

, P_E can be calculated using Eq. 3.15 and

CAP from Eq. 3.16.

STEP 3 MODEL: MULTIPLE AQCS IN ACCEPT/REJECT ACCEPTANCE PLAN

Calculation of Beta_Multiple:

The Beta risk for multiple AQCs (Beta_multiple) is considered as the probability that at least one of the AQCs will be at or below RQL (i.e., the union of all such events). It is assumed that two AQCs are independent i.e., Event that AQC1 is at RQL does not affect the PWL of AQC2 (in case of two AQCs) and the same rule can be extended for multiple AQCs.

Solving for case of two AQCs:

1. Probability that AQC1 is at or below RQL: Possible combinations would be
   - (AQC1 at or below RQL) AND (AQC2 at or below RQL)
   - (AQC1 at or below RQL) AND (AQC2 between RQL and AQL)
   - (AQC1 at or below RQL) AND (AQC2 between AQL and 1 (i.e., total probability))
2. Probability that AQC2 is at or below RQL: Possible combinations would be
   - (AQC2 at or below RQL) AND (AQC1 at or below RQL) will not be taken into account since it has been considered already.
   - (AQC2 at or below RQL) AND (AQC1 between RQL and AQL)

- (AQC2 at or below RQL) AND (AQC1 between AQL and 1 (i.e., total probability))

Calculating individual probabilities:

i. (AQC1 at or below RQL) AND (AQC2 at or below RQL)

= [{(0 + Beta for AQC1)/2}* $P_{BR(AQC1)}$] AND [{(0 + Beta for AQC2)/2}* $P_{BR(AQC2)}$]

= [{(0 + Beta for AQC1)/2}* $P_{BR(AQC1)}$] ∩ [{(0 + Beta for AQC2)/2}* $P_{BR(AQC2)}$]

P(i) = [{(0 + Beta for AQC1)/2}* $P_{BR(AQC1)}$] * [{(0 + Beta for AQC2)/2}* $P_{BR(AQC2)}$]

ii. (AQC1 at or below RQL) AND (AQC2 between RQL and AQL)

P(ii) = [{(0 + Beta for AQC1)/2}* $P_{BR(AQC1)}$] * [{(Beta for AQC2+(1-Alpha for AQC2))/2}* $P_{RA(AQC2)}$] , since producer's (seller's) risk = 1 – alpha.

iii. (AQC1 at or below RQL) AND (AQC2 between AQL and 1)

P(iii) = [{(0 + Beta for AQC1)/2}* $P_{BR(AQC1)}$] * [{((1-Alpha for AQC2)+1)/2}* $P_{AA(AQC2)}$]

iv. (AQC2 at or below RQL) AND (AQC1 between RQL and AQL)

P(iv) = [{(0 + Beta for AQC2)/2}* $P_{BR(AQC2)}$] * [{(Beta for AQC1+(1-Alpha for AQC1))/2}* $P_{RA(AQC1)}$]

v. (AQC2 at or below RQL) AND (AQC1 between AQL and 1)

P(v) = [{(0 + Beta for AQC2)/2}* $P_{BR(AQC2)}$] * [{((1-Alpha for AQC1)+1)/2}* $P_{AA(AQC1)}$]

Hence, the probability of erroneously accepting bad lot when either of AQCs is at or below RQL will be the union of events i to v.

Since probabilities i to v are mutually exclusive (i.e., no two of these probabilities can exist at the same time for the same lot), the union will not include the intersection of any two events, (Duncan, 1986, p. pp. 26).

Important Note: If $P(\beta|P_{xx})$ were calculated so that these two events are dependant, the calculations would have to follow Bayesian method and since P_{xx} are directly related to prior distributions, complex theories reviewed in literature review of this thesis should have to be applied.

Beta_multiple = P (event 1 or 2)

= i or ii or iii or iv or v

= i U ii U iii U iv U v

$$\text{Beta\_multiple} = P(i) + P(ii) + P(iii) + P(iv) + P(v) \qquad \text{Eq. 3.18}$$

The same method can be applied for the calculation of Beta_multiple for more than two AQCs.

Cost Calculations for Multiple AQCs:

For multiple AQCs, the following costs are calculated for all the combination of the sample sizes ranging from 3 to 30 for all the AQCs.

a) Expected Cost of Lot Acceptance:

$$C = \text{Cost}_T(\text{multiple}) + \text{Cost}_D(\text{multiple}) \qquad \text{Eq. 3.19}$$

Where, Cost T (n) is agency's cost of testing and Cost D (n) is agency's expected cost due to erroneous acceptance decisions.

b) Cost of Sampling and Testing:

$$\text{Cost}_T(\text{multiple}) = \text{Cost}_T\,(\text{AQC1}_{(i)}) + \text{Cost}_T\,(\text{AQC1}_{(j)}) + \ldots\ldots + \text{Cost}_T\,(\text{AQCn}_{(k)}) \qquad \text{Eq. 3.20}$$

Where, i, j, k are the sample sizes for a particular combination of all AQCs.

c) Expected Cost Due to Erroneous Acceptance Decisions:

For Accept/Reject:

$$\mathrm{Cost_D(multiple)} = \mathrm{beta\_multi} * \mathrm{CAP} \qquad \text{Eq. 3.21}$$

ASSUMPTIONS IN CALCULATION

1. Lot distribution is assumed to be normal.
2. AQCs are independent, i.e., Event that AQC1 is at RQL does not affect the PWL of AQC2 (in case of two AQCs). But, in case of two AQCs being considered in the same plan, the selection of the lot depends on PWL of both or either of AQCs.
3. Random sampling is assumed so that the PWL can be safely assumed to be normally distributed throughout the lot.
4. Minimum sample size for any given lot and acceptance limits is assumed to be 3. The reason is the limitation of the normal distribution function that it cannot be evaluated at PWL=<50. Also, beta distribution function fails at a=b=n/2-1 = 2/2-1 = 0 (for n=2).

Chapter 4: Practical Sample Size Calculation Model

CALCULATION OF ALPHA AND BETA RISKS

Step1 Model

In this chapter, implementation of PSSCM for calculation of PSS through each step for the following inputs is explained in detail:

1. Number of AQC = 1
2. AQC = Asphalt binder content (P_b) and gradation
3. AQL = 90 (PWL)
4. RQL = 50 (PWL)
5. m = 70 (PWL)
6. Lot size = L = 2000 tons
7. Unit Bid Price = b = $100.00/ton
8. Historical percentage of lots:
    a. P_{BR} = 5%
    b. P_{RA} = 85%
    c. P_{AA} = 10%
9. Cost of testing and sampling for each sample size from Table 12, pp. 88.

(**Note:** Historical percentages are probabilities of lot falling below RQL, between AQL and RQL or above AQL. These can be easily calculated from the data fetched from construction quality databases maintained by SHAs. Many of SHAs have been maintaining such databases for almost a decade. Further information can be obtained from FHWA-HRT-07-019, (Rao, et al., 2006). Virtually, for every observed (known)/assumed distribution, we can calculate area under the curve from the PDF of that distribution (using mean and standard deviation obtained from quality database) which will yield percentage of lots required for following calculations.)

Calculation for Practical Sample Size:

1. Convert PWL to PD:

   aql = 100 – 90 = 10 PD

   rql = 100 – 50 = 50 PD

   m = 100 – 70 = 30 PD

   CAP = L * B = 2000 * 100 = $200000.00

2. Calculation for ‘k’:

   a. Using (AASHTO R 9-90) Appendix C (pp. 838),

      Solving Quality Index (Q) for n = 3 and Estimated Lot Percent Defective for Selected Sample Sizes = m = 30

      Using linear interpolation formula: $$y = y_0 + (x - x_0)\frac{y_1 - y_0}{x_1 - x_0}$$

      Hence, from Table 5,

      y = 0.68 + (30.00-29.96) * {(0.66-0.68)/(30.63-29.96)}

      = 0.68 + 0.04 * (-0.02/0.67)

      = 0.68 – 0.0012

      ~= 0.679 = k

Table 5: Calculation of 'k' using AASHTO R 9-90

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TABLE C Estimation of Lot Percent Defective by Standard Deviation Method *Continued*

Variability—Unknown Procedure | Standard Deviation Method

Quality Index (Q)	Estimated Lot Percent Defective for Selected Sample Sizes												
	3	4	5	6	7	8	9	10	15	20	30	50	100
0.65	30.97	28.13	27.39	26.92	26.66	26.49	26.37	26.28	26.07	25.98	25.90	25.05	25.82
0.66	30.63	28.00	27.06	26.60	26.33	26.16	26.04	25.96	25.74	25.66	25.59	25.53	25.49
0.67	30.30	27.67	26.73	26.27	26.00	25.83	25.72	25.63	25.42	25.33	25.26	25.21	25.17
0.68	29.96	27.33	26.40	25.94	25.68	25.51	25.39	25.31	25.10	25.01	24.94	24.89	24.86
0.69	29.61	27.00	26.07	25.61	25.35	25.19	25.07	24.99	24.78	24.69	24.62	24.57	24.54
0.70	29.27	26.67	25.74	25.29	25.03	24.86	24.75	24.67	21.64	24.38	24.31	24.26	24.23

b. Using Larson's Nomograph:

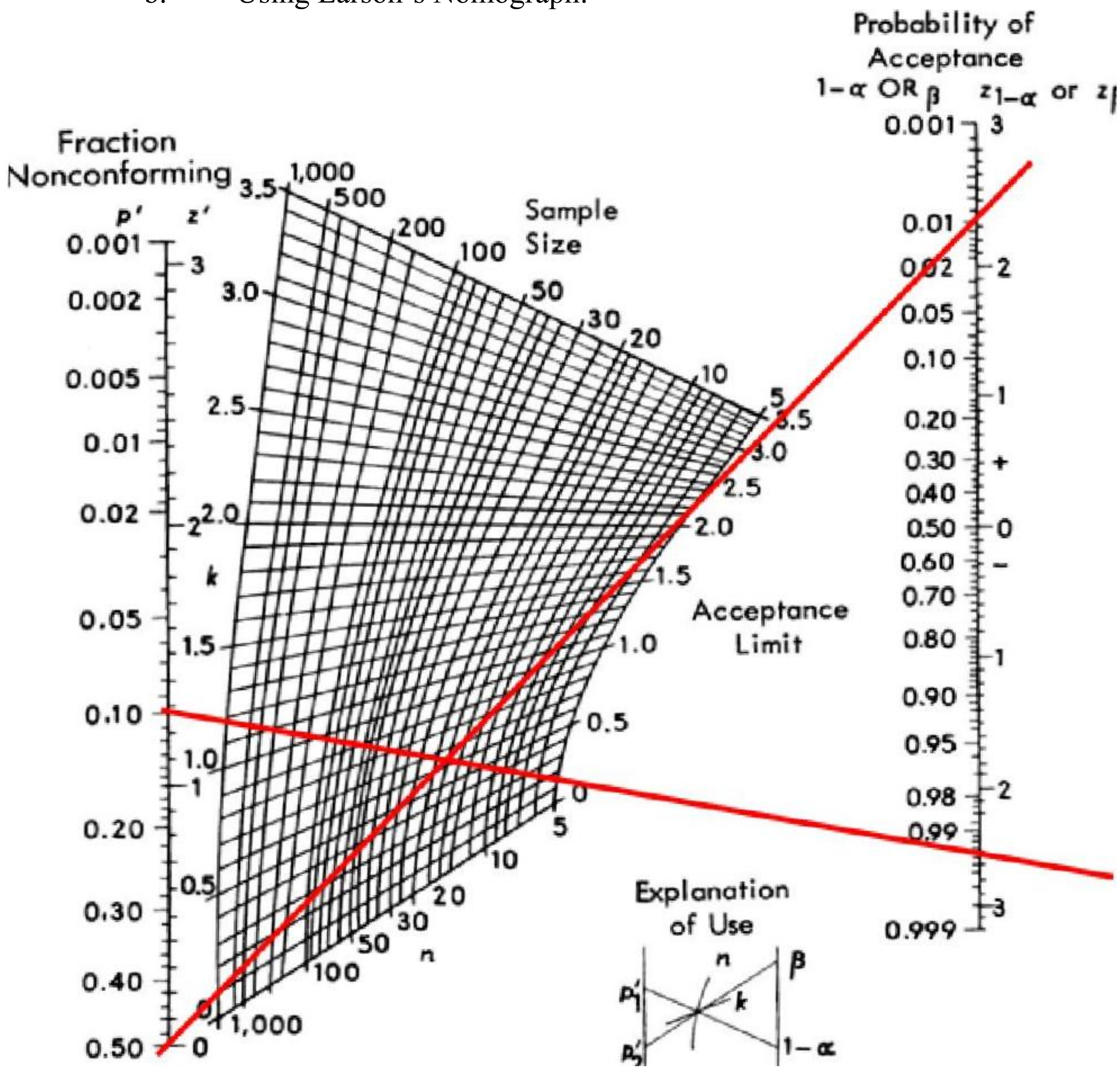


Figure 19: Larson's Nomograph

Values of 'k' can be roughly estimated using Larson's nomograph (Jacobson, November 1949) as shown in Figure 19: Larson's Nomograph.

i. To begin with, values of fraction non-conforming (i.e., AQL or RQL in PD) are marked on the left axis.

ii. Lines connecting values of PD and sample size are drawn.

iii. Point where these lines intersect and meet with curves for 'k' is approximated as 'k.'

Using the current sample size '3,' it is difficult to locate exact 'k' value on the nomograph since the minimum values on axis for 'n' is 5. An approximate value of 'k' comes close to slightly above 0.6.

c. Using A.J. Wallis formula for k and n (see Eq. 3.8 and Eq. 3.9)

From Eq. 3.3,

Z_1 = 1.28 and Z_2 = 0.00

Or using Microsoft Excel function NormSINV,

Z_1 = 1.282 and Z_2 = 0.00

To solve for 'k' from 'm,' try and error method can be used so that value of k matches with 'Q' at PD = m as explained in Eq. 3.4. For example, let's assume k = 0.1 for the first trial.

Hence, from Eq. 3.4,

a = b = n/2-1 = 3/2-1 = 0.5

Q = k = 0.1

x = maximum of 0 and {0.5 - (0.1 * (3 ^ 0.5)) / (2 * (3 - 1))}

= maximum of 0 and 0.4891

= 0.4891 ≠ (m/100 = 30/100 = 0.30), hence try next approximation for 'k'

This calculation might take a long time to solve manually. But if instead a VBA macro or Microsoft Excel's add-in 'Solver' is used, a precise answer can be obtained. The values obtained for all the sample sizes from 3 to 30 as calculated by solver add-in are shown in Table 6.

3. From Eq. 3.6 and Eq. 3.7, (solving for sample size, n =3)

Z_α = (((3 ^ 0.5) * (1.282 - 0)) / ((1 + (0.5 * 0.679 ^ 2)) ^ 0.5)) / (1 + ((0.679 – 1.282) / (0 – 0.679)))

= (2.2204/1.1092) * (1/(1.8882))

= 1.060

And,

Z_β = ((0.679-1.282)*1.060)/(0-0.679)

= 0.941

Values obtained for all other sample sizes are:

Table 6: Alpha and Beta values for example sampling plan

n	k	z_alpha	z_beta	Alpha	Beta
3	0.679	0.941	1.060	0.1733	0.1591
4	0.600	1.255	1.105	0.1048	0.1346
5	0.572	1.471	1.186	0.0706	0.1179
6	0.558	1.648	1.272	0.0497	0.1017
7	0.550	1.803	1.357	0.0357	0.0874
8	0.545	1.943	1.439	0.0260	0.0750
9	0.542	2.072	1.518	0.0191	0.0645
10	0.539	2.193	1.594	0.0142	0.0555
11	0.537	2.307	1.666	0.0105	0.0478
12	0.536	2.415	1.736	0.0079	0.0413
13	0.535	2.518	1.804	0.0059	0.0357
14	0.534	2.618	1.869	0.0044	0.0308
15	0.533	2.713	1.932	0.0033	0.0267
16	0.532	2.805	1.993	0.0025	0.0231
17	0.532	2.894	2.052	0.0019	0.0201
18	0.531	2.980	2.110	0.0014	0.0174
19	0.531	3.064	2.166	0.0011	0.0152
20	0.530	3.145	2.221	0.0008	0.0132
21	0.530	3.225	2.274	0.0006	0.0115
22	0.530	3.302	2.327	0.0005	0.0100

Table 6 (Continued)

n	k	z_alpha	z_beta	Alpha	Beta
23	0.529	3.378	2.378	0.0004	0.0087
24	0.529	3.452	2.428	0.0003	0.0076
25	0.529	3.524	2.477	0.0002	0.0066
26	0.529	3.595	2.525	0.0002	0.0058
27	0.529	3.665	2.573	0.0001	0.0050
28	0.528	3.733	2.619	0.0001	0.0044
29	0.528	3.800	2.665	0.0001	0.0039
30	0.528	3.866	2.710	0.0001	0.0034

4. Since, Z_α and Z_β are the normal derivates of the probabilities Alpha and Beta using normal distribution charts or Microsoft excel function NormSDIST, we get: α = 0.1733 and β = 0.1591. Values corresponding to other sample sizes are shown in Table 6. Values for α and β can also be approximated using (AASHTO R 9-90) Appendix B or using Larson's nomograph. In Figure 19: Larson's Nomograph the intercepts made by red lines onto the axis to the right (Axis for Probabilities of Acceptance) represent (1 – α) and β for AQL and RQL respectively.

5. Now, in order to calculate costs, effective probability of erroneous decisions must be calculated as: $P_E = ((0 + \beta_n))/2) * P_{BR}$

$= (0.\ 0.15907/2) * 0.05$

$= 0.00397675$

Therefore,

$Cost_D = P_E * CAP = 0.0039775 * 200000.00 = \$795.35$

It should be noted that the values of β, α, P_E, etc., are not rounded in order to calculate precise costs.

$Cost_{T_n} = C_n = \$588.42$ (from Table 12)

And, C = &795.35 + $588.42 = $1,383.77

Similarly, the values for all sample sizes are calculated as shown in Table 7.

(The values in table 7 are based on an interview of a commercial laboratory manager from Austin, Texas in 2008. Austin's prices were adjusted to represent the national average prices using a ratio of 0.803 (R.S. Means 2008). A 0.803 City Index indicates that sampling and testing prices in Austin, TX are 80.3 percent of the national average prices.)

Table 7: Costs Associated with Decisions and Testing for Single AQC Accept/Reject (Example)

n	$Cost_D$	$Cost_T$	$Cost_{(D+T)}$
3	$795.35	$588.42	$1,383.77
4	$673.24	$773.23	$1,446.47
5	$589.54	$958.03	$1,547.57
6	$508.51	$1,142.84	$1,651.35
7	$436.90	$1,327.65	$1,764.55
8	$375.21	$1,512.45	$1,887.66
9	$322.49	$1,697.26	$2,019.75
10	$277.51	$1,882.07	$2,159.58
11	$239.13	$2,066.87	$2,306.00
12	$206.33	$2,251.68	$2,458.01
13	$178.26	$2,436.49	$2,614.75
14	$154.19	$2,621.30	$2,775.49
15	$133.51	$2,806.10	$2,939.61
16	$115.73	$3,024.91	$3,140.63
17	$100.41	$3,213.96	$3,314.37
18	$87.19	$3,403.02	$3,490.21
19	$75.77	$3,592.08	$3,667.85
20	$65.90	$3,781.13	$3,847.03

Table 7 (Continued)

n	CostD	CostT	Cost(D+T)
21	$57.35	$3,970.19	$4,027.54
22	$49.95	$4,159.25	$4,209.19
23	$43.52	$4,348.30	$4,391.83
24	$37.95	$4,537.36	$4,575.31
25	$33.10	$4,726.42	$4,759.52
26	$28.89	$4,915.47	$4,944.36
27	$25.23	$5,104.53	$5,129.76
28	$22.04	$5,293.59	$5,315.62
29	$19.26	$5,482.64	$5,501.90
30	$16.83	$5,671.70	$5,688.54

6. The practical sample size (PSS) corresponds to the minimum total cost $C_{(D+T)}$ in Table 7. Hence, PSS for this example scenario is: n_o = 3 and Corresponding α and β risks are: 0.1733 and 0.1591, respectively.
7. Hence, the final sampling plan (PWL) would be defined as:
   AQL= 90, RQL=50, n = 3, k = 0.679, lot size = 2000 tons, bid price = $100.00.
8. OC curve similar to that of OCPLOT can be generated by performing the same calculations for continuously varying either AQL or RQL and keeping 'm' constant. The values of the corresponding PWLs against the beta or 1- α values for the practical sample size will yield OC curve. Figure 20 shows an example OC curve generated using PSSCM for varying AQL and RQL, m = 70, lot size = 400 tons, bid price = $100.00 and with the same cost data as in previous example.

N	k
4	0.600

	PWL	PA	
RQL	50	0.135	β
	55	0.191	
	60	0.262	
	65	0.346	
	70	0.445	
	75	0.555	
	80	0.672	
	85	0.789	
AQL	90	0.895	1- α
	95	0.973	

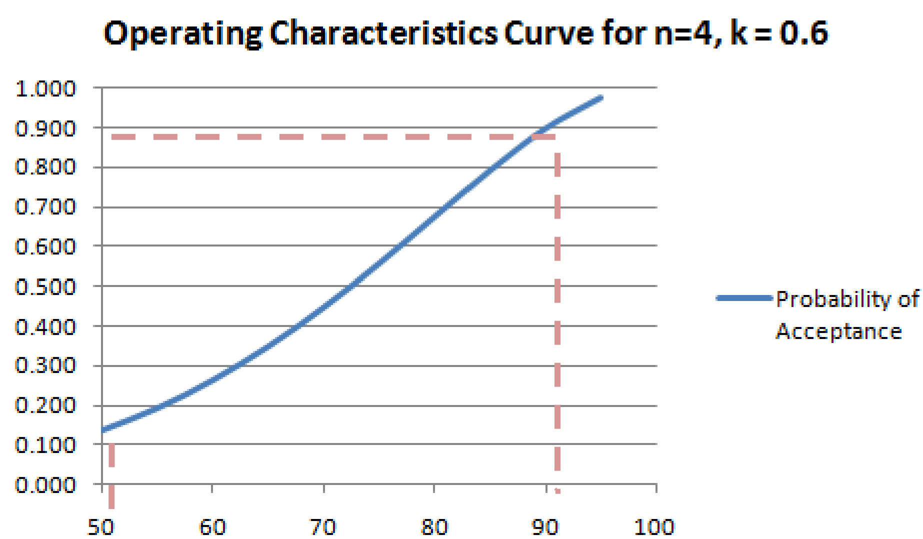


Figure 20: Operating Characteristics Curve for n=4, k = 0.6 using PSSCM

From Figure 20, any given combination of α and β risks AQL and RQL could define a new sampling plan with the same lot size, unit bid price and testing and sampling costs, i.e., 4 might not be always the practical sample size. However, unlike traditional sampling plans, with PSSCM it is not possible to imbalance the risks because it tends to shift the minimum cost of acceptance to another PSS.

Step2 Model

Since, all the calculations for α and β risk for the Step1 Model and Step2 Model are the same, values of α and β risks remain the same.

Calculations for costs:

Consider the pay factor equation: PF = 55 + 0.5 * PWL (a= 55, b = 0.5)

Hence, pf_aql = 55 + 0.5 * (100-10) = 100.00

And pf_rql = 55 + 0.5 * (100-50) = 80.00

(Since AQL and RQL were converted to PD, values in PWL should be used to calculate pay factors).

Delta_pf = (100-80)/100 = 0.2

Hence, $Cost_{D(n)}$ = delta_pf * P_E * CAP, $P_{E\,=}$ 0.0039775 and CAP= 200000.00

$Cost_{D(3)}$ = 0.2 * 0.0039775 * 200000.00 = $159.10

$Cost_T$ and $C_{(T+D)}$ or C can be calculated similar to model 1 and PSS corresponding to the minimum C is found out to be 3.

For the current acceptance plan, an Expected Pay Curve can also be plotted as shown in **Figure 21**. EP Curve is a curve plotted for quality level against expected payment.

Using the PF equation used: PF = 55 + 0.5 PWL, for an array of values for PWL PF is calculated. EP curve represents actual payment assigned to the lot PWL after testing a lot. Maximum payment is at PWL = 100 while minimum payment is at PWL = RQL.

Table 8: Costs Associated with Decisions and Testing for Single AQC Pay Adjustment (Example)

n	$Cost_D$	$Cost_T$	$Cost_{(D+T)}$
3	$159.07	$588.42	$747.49
4	$134.65	$773.23	$907.88
5	$117.91	$958.03	$1,075.94
6	$101.70	$1,142.84	$1,244.54

Table 8 (Continued)

n	CostD	CostT	Cost(D+T)
7	$87.38	$1,327.65	$1,415.03
8	$75.04	$1,512.45	$1,587.49
9	$64.50	$1,697.26	$1,761.76
10	$55.50	$1,882.07	$1,937.57
11	$47.83	$2,066.87	$2,114.70
12	$41.27	$2,251.68	$2,292.95
13	$35.65	$2,436.49	$2,472.14
14	$30.84	$2,621.30	$2,652.14
15	$26.70	$2,806.10	$2,832.80
16	$23.15	$3,024.91	$3,048.05
17	$20.08	$3,213.96	$3,234.04
18	$17.44	$3,403.02	$3,420.46
19	$15.15	$3,592.08	$3,607.23
20	$13.18	$3,781.13	$3,794.31
21	$11.47	$3,970.19	$3,981.66
22	$9.99	$4,159.25	$4,169.24
23	$8.70	$4,348.30	$4,357.01
24	$7.59	$4,537.36	$4,544.95
25	$6.62	$4,726.42	$4,733.04
26	$5.78	$4,915.47	$4,921.25
27	$5.05	$5,104.53	$5,109.57
28	$4.41	$5,293.59	$5,297.99
29	$3.85	$5,482.64	$5,486.49
30	$3.37	$5,671.70	$5,675.07

In case of PSSCM, since probability of lot being bad is incorporated in the calculation (P_E) the sample size required is less for a fair quality of lot.

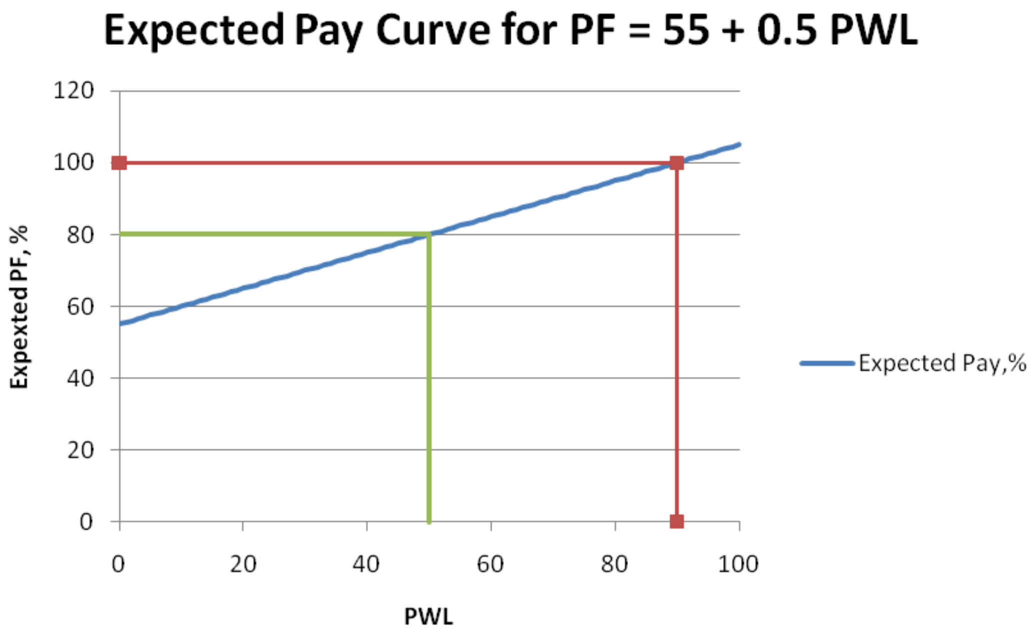


Figure 21: Expected Pay Curve for PF = 55 + 0.5 PWL

Step3 Model

Continuing the calculations from the Step1 Model, we already have α and β risks calculations for AQC1. Let's consider another AQC, AQC2 such that

AQL = 85.00, RQL = 50.00, m = 75.00, L = 2000.00, B =100.00, a = 55.00, b = 0.50, P_{BR} = 10%, P_{RA} = 85.00%, P_{AA} = 5.00% and cost of testing and sampling as shown in Table 13: Costs of Testing and Sampling for example Sampling Plan (Density from Cores (Gmb), $), pp. 89.

The values calculated for α and β risks are shown in Table 9. (Note: PSS for individual AQC2 = 4.)

Table 9: Risk Calculations for Step3 Model (Example)

n	k	z_alpha	z_beta	Alpha	Beta	$Cost_D$	$Cost_T$	$Cost_{(D+T)}$
3	0.816	0.330	1.225	0.3707	0.1214	$1,213.69	$649.32	$1,863.01
4	0.750	0.506	1.325	0.3064	0.0926	$925.56	$827.40	$1,752.96
5	0.723	0.625	1.439	0.2661	0.0751	$750.87	$1,005.48	$1,756.35

Table 9 (Continued)

n	k	z_alpha	z_beta	Alpha	Beta	CostD	CostT	Cost(D+T)
6	0.709	0.717	1.552	0.2366	0.0603	$602.88	$1,183.56	$1,786.44
7	0.701	0.795	1.662	0.2132	0.0483	$483.01	$1,361.64	$1,844.65
8	0.696	0.865	1.766	0.1936	0.0387	$387.17	$1,539.73	$1,926.90
9	0.692	0.928	1.865	0.1768	0.0311	$310.80	$1,717.81	$2,028.61
10	0.690	0.986	1.960	0.1621	0.0250	$249.92	$1,895.89	$2,145.81
11	0.688	1.040	2.051	0.1491	0.0201	$201.31	$2,073.97	$2,275.28
12	0.686	1.092	2.138	0.1375	0.0162	$162.42	$2,252.05	$2,414.47
13	0.685	1.141	2.223	0.1270	0.0131	$131.24	$2,430.14	$2,561.38
14	0.684	1.187	2.304	0.1175	0.0106	$106.20	$2,608.22	$2,714.42
15	0.683	1.232	2.382	0.1089	0.0086	$86.05	$2,786.30	$2,872.35
16	0.682	1.276	2.458	0.1011	0.0070	$69.80	$3,079.45	$3,149.25
17	0.682	1.317	2.532	0.0939	0.0057	$56.68	$3,271.92	$3,328.60
18	0.681	1.358	2.604	0.0873	0.0046	$46.08	$3,464.38	$3,510.46
19	0.681	1.397	2.674	0.0813	0.0037	$37.49	$3,656.85	$3,694.34
20	0.680	1.435	2.742	0.0757	0.0031	$30.53	$3,849.31	$3,879.84
21	0.680	1.472	2.809	0.0705	0.0025	$24.88	$4,041.78	$4,066.66
22	0.680	1.508	2.874	0.0658	0.0020	$20.29	$4,234.25	$4,254.54
23	0.679	1.543	2.937	0.0614	0.0017	$16.56	$4,426.71	$4,443.27
24	0.679	1.577	3.000	0.0573	0.0014	$13.52	$4,619.18	$4,632.70
25	0.679	1.611	3.061	0.0536	0.0011	$11.05	$4,811.64	$4,822.69
26	0.679	1.644	3.120	0.0501	0.0009	$9.03	$5,004.11	$5,013.14
27	0.679	1.676	3.179	0.0468	0.0007	$7.39	$5,196.58	$5,203.96
28	0.678	1.708	3.237	0.0438	0.0006	$6.04	$5,389.04	$5,395.09
29	0.678	1.739	3.293	0.0410	0.0005	$4.95	$5,581.51	$5,586.46
30	0.678	1.770	3.349	0.0384	0.0004	$4.05	$5,773.97	$5,778.03

Calculating individual probabilities: (n1 =3, n2=3)

i. (AQC1 at or below RQL) AND (AQC2 at or below RQL)

P(i) = [{(0 + 0.1733)/2}* 0.05] * [{(0 + 0.1214)/2}* 0.1]

= 0.0043325 * 0.00607 = 0.000026298275

ii. (AQC1 at or below RQL) AND (AQC2 between RQL and AQL)

P(ii) = [{(0 + 0.1733)/2}* 0.05] * [{(0.1214 +(1-0.3707))/2}* 0.85]

= 0.0043325 * 0.3706425 = 0.00160580863125

iii. (AQC1 at or below RQL) AND (AQC2 between AQL and 1)

P(iii) = [{(0 + 0.1733)/2}* 0.05] * [{((1-0.3707)+1)/2}* 0.05]

= 0.0043325 * 0.0092675 = 0.00004015144375

iv. (AQC2 at or below RQL) AND (AQC1 between RQL and AQL)

P(iv) = [{(0 + 0.1214)/2}* 0.1] * [{(0.1733+(1-0.1591))/2}* 0.85]

= 0.00607 * 0.431035 = 0.00261638245

v. (AQC2 at or below RQL) AND (AQC1 between AQL and 1)

P(v) = [{(0 + 0.1214)/2}* 0.1] * [{((1-0.1591)+1)/2}* 0.1]

= 0.00607 * 0.007955 = 0.00004828685

Beta_multiple = P(i) + P(ii) + P(iii) + P(iv) + P(v)

= 0.000026298275 + 0.00160580863125 + 0.00004015144375 + 0.00261638245 + 0.00004828685

= 0.00433672765

~= 0.005

Similarly, the beta multiple values for all sample sizes can be calculated.

$Cost_D$ = Beta_miltiple * CAP = 0.005 * 200000.00 = $1000.00

In this case, $Cost_T$ = $Cost_T$(AQC1(n)) + $Cost_T$(AQC2(n))

= 588.4 + 649.3 = $1237.74

Hence, $C_{(D+T)}$= $2237.74

These steps for calculating beta_multiple and costs are repeated for every combination of all the sample sizes of each AQC. In this case, a total of 784 (28*28) combinations need to be solved in order to decide PSS corresponding to minimum total cost. But since it is observed that in most of the cases the PSS does not exceed 15, only 169 (13*13) combinations are solved. The results are given in Table 14, (pp. 90).

Comparison of Calculated risks with OCPLOT

Risks calculated using OCPLOT (Weed, 1996) and PSSC (Practical Sample Size Calculator, developed to implement PSSCM) were compared for the following acceptance criteria:

Method: Accept/Reject (pass/Fail)

Precision: High

AQL = 90

RQL = 50

m = 70

Limit Type: Single Sided

Table 10: Comparison of Risks Calculated by OCPLOT and PSSC

n	PSSC		OCPLOT	
	Alpha	Beta	Alpha	Beta
3	0.173	0.159	0.149	0.175
4	0.105	0.135	0.089	0.153
5	0.071	0.118	0.061	0.141
6	0.050	0.102	0.045	0.109
7	0.036	0.087	0.030	0.094
8	0.026	0.075	0.028	0.079
9	0.019	0.064	0.018	0.069
10	0.014	0.056	0.013	0.060
11	0.011	0.048	0.009	0.057
12	0.008	0.041	0.006	0.045
13	0.006	0.036	0.005	0.043
14	0.004	0.031	0.005	0.035
15	0.003	0.027	0.003	0.029

The results show that:

1. The difference between α and β values for the current acceptance criteria follow a very similar pattern, but the values are not exactly same. The difference in these values is relatively higher for the smaller sample size.

2. Generally, the smaller the risks calculated, the smaller is the sample size required. Hence, the sample size obtained using PSSC would have been less as compared to the sample size used according to OCPLOT to obtain similar risks, if PSSC would use only risk as a decision criteria.
3. As per table, the recommended values for risks for a major project are $\alpha = 0.01$ and $\beta = 0.05$. The sample size required to adjust these values for the given plan is nearly 12 (see Figure 24). If the same plan is analyzed using PSSCM, since the decision is based on cost criteria, the sample size comes out to be 3 for lot size = 2000 tons (see page 88, cost data used represents national average values). It should be note here that sample size of 9 or higher is only required for lot sizes >8000 tons using PSSC.

   The detailed comparison results are shown in Table 10, and Figure 22, Figure 23, Figure 24, and Figure 25.

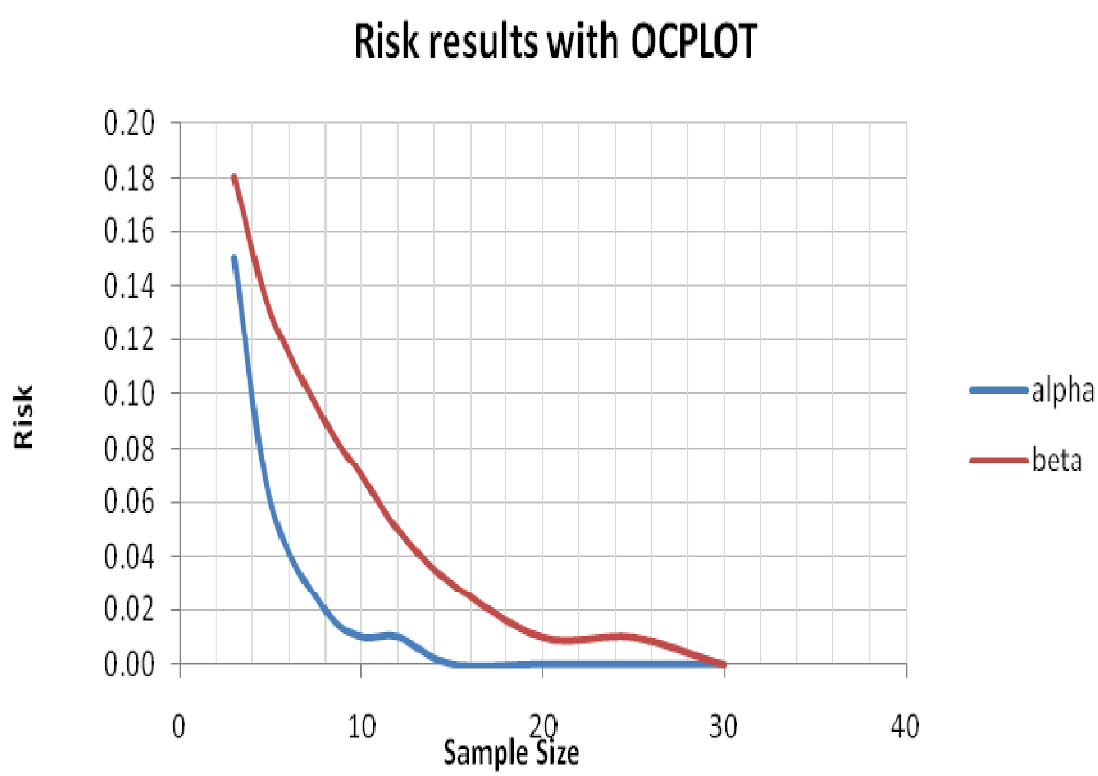


Figure 22: Risk Results with OCPLOT

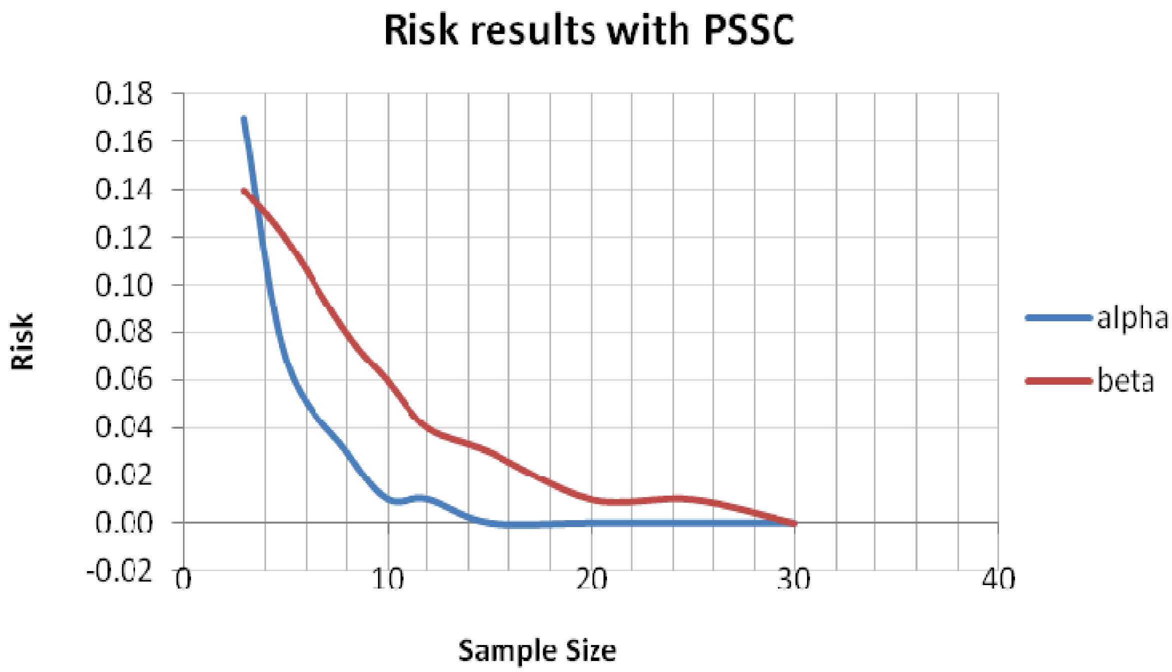


Figure 23: Risk Results with PSSC

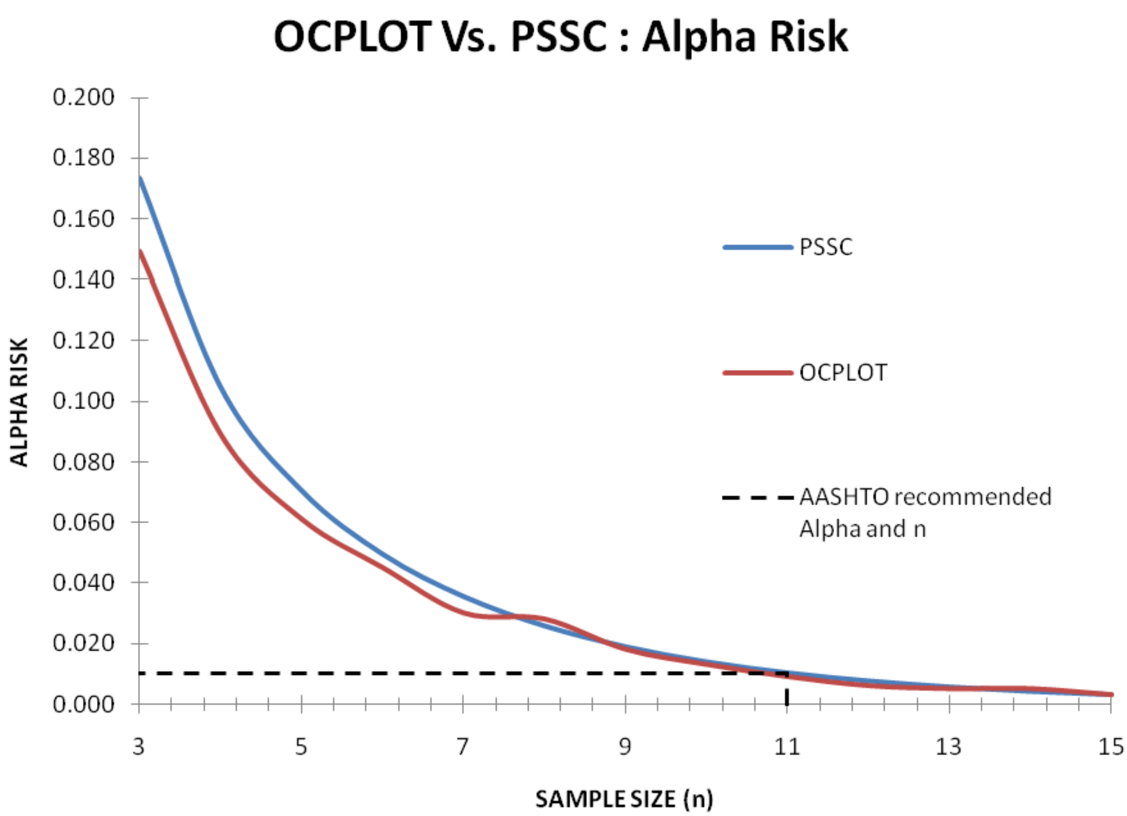


Figure 24: OCPLOT vs. PSSC: Alpha Risks

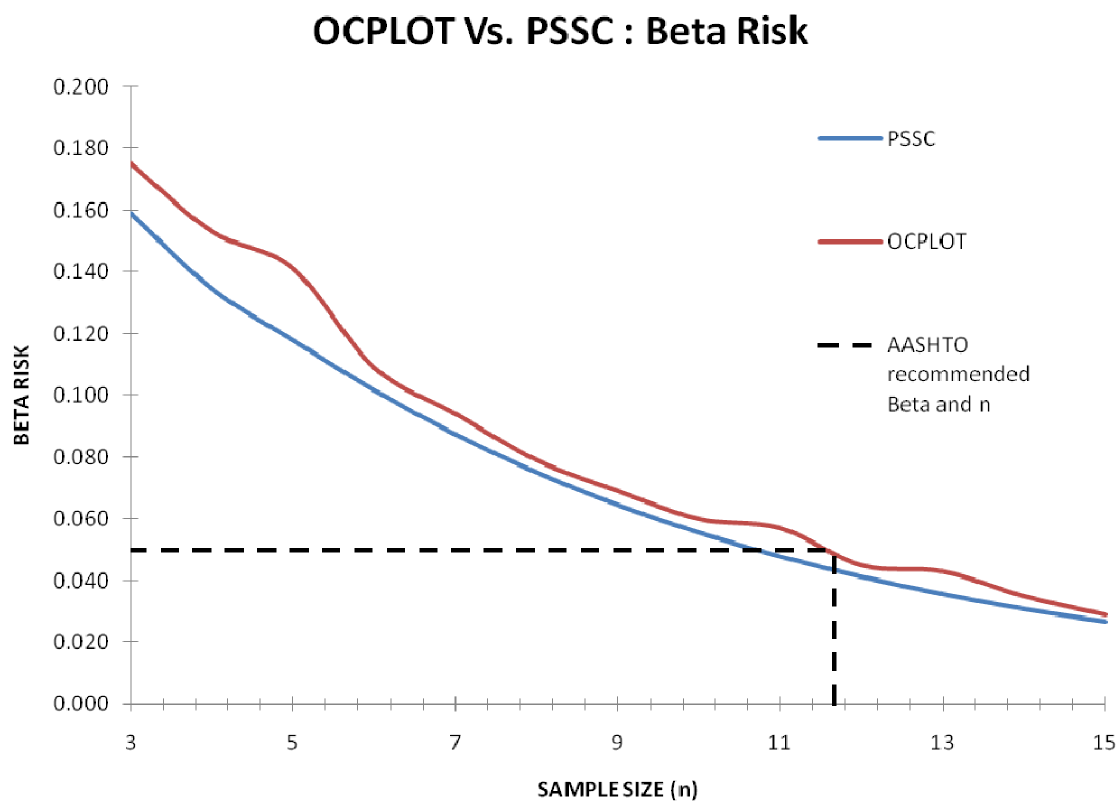


Figure 25: OCPLOT vs. PSSC: Beta Risks

COMPARISON OF AASHTO AND PSSCM ACCEPTANCE PLANS:

According to Duncan, "The risk involved in sampling is thus not a point of difference in the comparison of various types of plans. Meaningful comparisons are only made between plans that have roughly the same OC curve", (Duncan, 1986, p. pp. 314).

Hence, plans generated by OCPLOT and PSSCM could be compared using one of the methods used by Duncan, which is called a two point method (p_1' and p_2'), where two points on OC curve i.e., 1-α and β match (approximately) and other points do not deviate much.

Following plan is selected:

PSSC: AQL= 90, RQL=50, n = 4, k = 0.600 (i.e., m = 70), lot size = 4000 tons, bid price = \$100.00, PBR= 5%, P_{RA}= 85% and P_{AA} = 10%.

OCPLOT: AQL= 90, RQL=50, n = 4 (Single limit)

Figure 26 shows comparison of two plans.

k	n	AQL	RQL	m	L	b	%H
0.6	4	90	50	70.00	4000	100	5,85,10

	PWL	PA (PSSC)	PA (OCPLOT,4)	
RQL	50	0.135	0.149	α
	55	0.191	0.219	
	60	0.262	0.291	
	65	0.346	0.375	
	70	0.445	0.488	
	75	0.555	0.578	
	80	0.672	0.698	
	85	0.789	0.814	
AQL	90	0.895	0.911	1- α
	95	0.973	0.977	
	100	1.000	1.000	

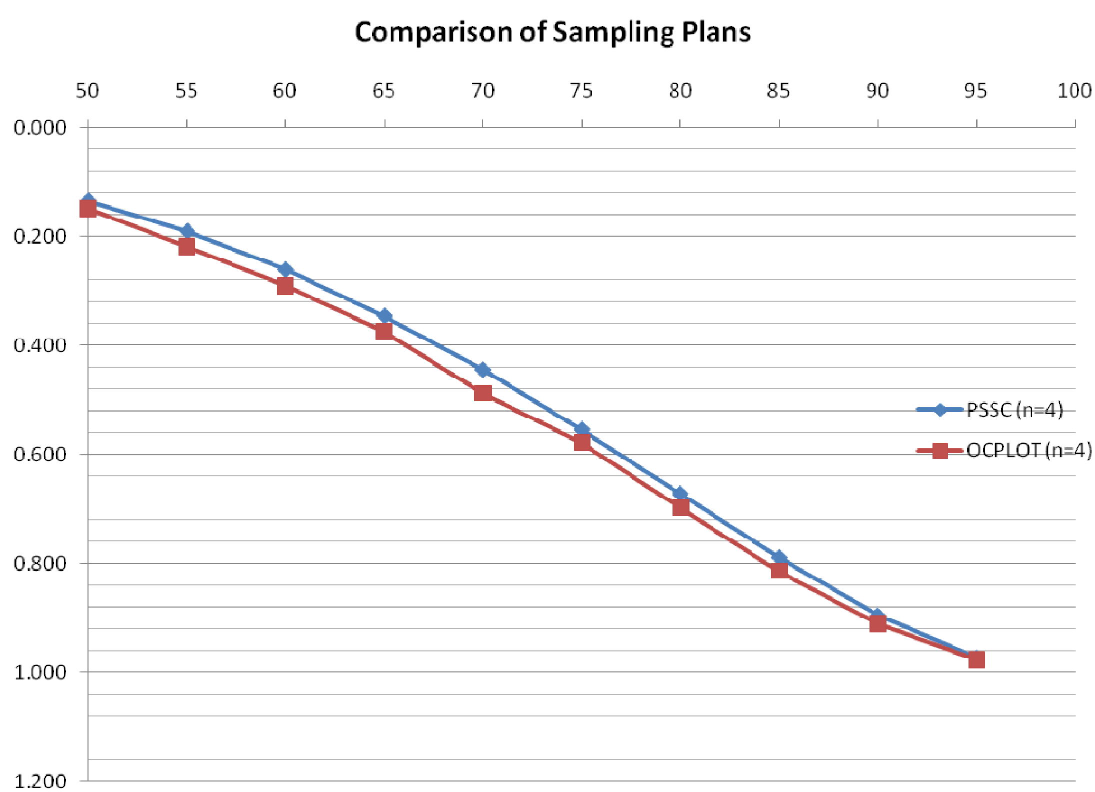


Figure 26: Example of Comparable and Similar Sampling Plans

From Figure 26, it can be noted that two OC curves almost match at points corresponding to PA = 90 and 50 and other points on the curve do not deviate much. Hence, these plans are comparable. Since, the sample size required by both plans is the same i.e., n=4 and the acceptance criteria are also same, these two plans are identical. Hence, in this case, the sample size required by the AASHTO method would be the practical sample size.

Note that although for a given AQL, RQL, 'm' and 'n,' two methods yield same sample size, the AASHTO method does not consider past performances of submitted lots. Also, the AASHTO recommendations guide does not specify exactly as to the change in the sample size according to the lot size. In other words, if SHA decides the lot size and the sample size required is coming out to be very high, specifications leave this decision up to the agency to decide whether to increase the lot size or to keep the sample size obtained by calculations. While, in case of PSSCM, since lot size and historical percentages (%H, in general) are taken into consideration, the sample size obtained by OCPLOT for same quality levels might not be the practical sample size. For example, let us consider following sampling plan:

k	n	AQL	RQL	M	L	b	%H
0.709	6	90	50	75.00	3000	100	10,80,10

As shown in Figure 27, these two methods match in sampling plans for sample size = 6. But, consider a case where %H was (5, 80, 15) i.e., better than that of considered initially, i.e., (10, 80, 10), the sample size calculated with the AASHTO method would be still same, but PSS changes to no = 4 and the plans do not match anymore.

Table 11: Comparison of Sampling Plans by AASHTO Method and PSSCM

	PWL	PA (PSSC,6), (10,80,10)	PA (OCPLOT,6)	PA (PSSC,4),(5,80,15)	
RQL	50	0.060	0.070	0.093	β
	55	0.101	0.114	0.135	
	60	0.159	0.188	0.190	
	65	0.239	0.268	0.260	
	70	0.343	0.383	0.345	

	PWL	PA (PSSC,6), (10,80,10)	PA (OCPLOT,6)	PA (PSSC,4),(5,80,15)	
	80	0.614	0.643	0.564	
	85	0.763	0.782	0.694	
AQL	90	0.895	0.902	0.826	1- α
	95	0.980	0.980	0.907	

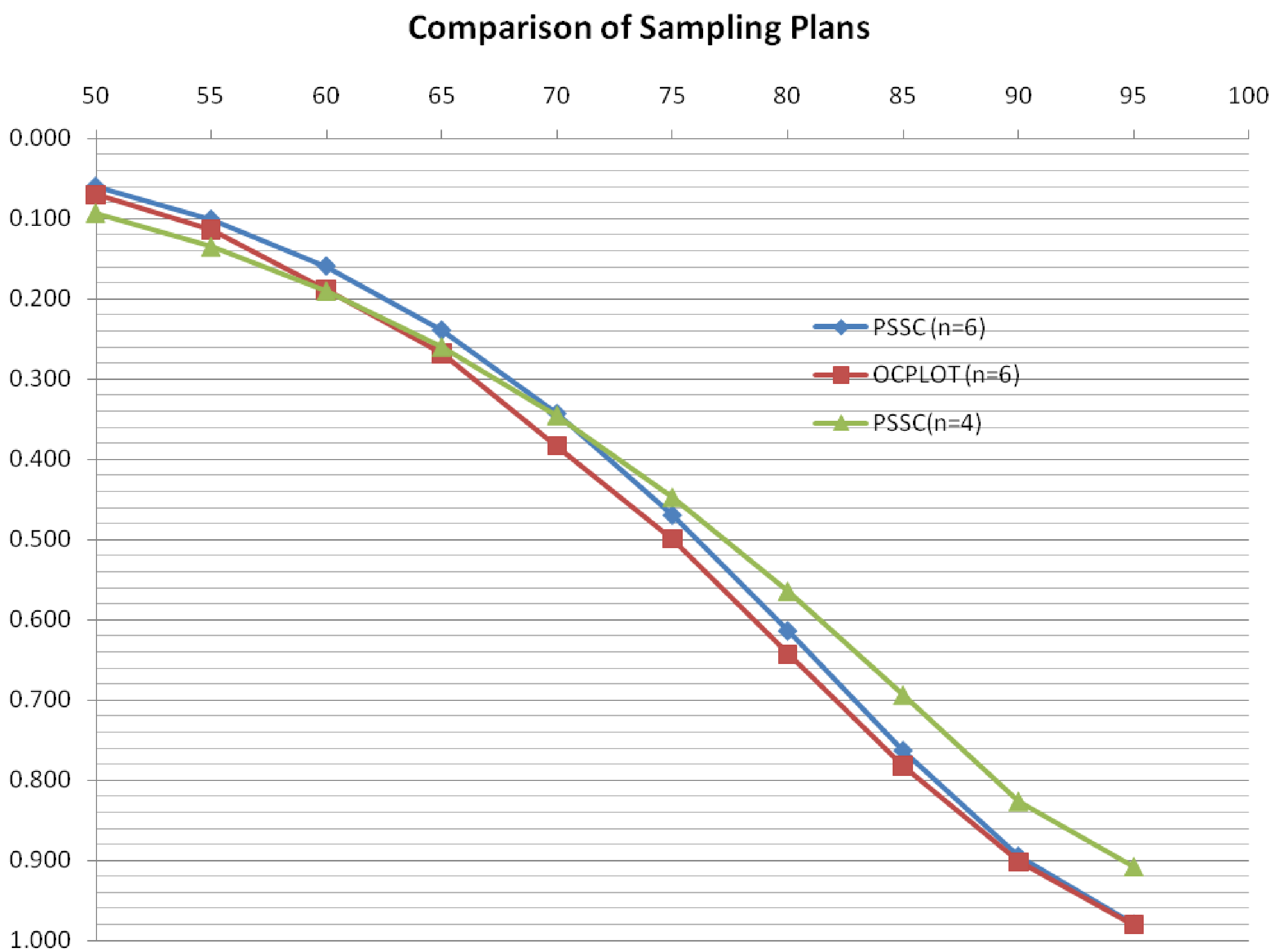


Figure 27: Comparison of AASHTO and PSSCM Sampling Plans

SENSITIVITY ANALYSIS

In order to identify the behavior of PSS relative to the factors such as Lot Size, RQL and minimum sample PWL, i.e., 'm,' sensitivity analysis was carried out. PSSC was modified to simulate these behaviors by changing the value of each factor while keeping other factors constant, from lower limit to upper limit. Note that axes for 'n' in the following charts represent PSS for each iteration of simulation, for single AQC, with change in the variable input. The data calculated for such simulation is too large and hence is not included in this document. The excel workbooks used for this analysis are included in the attached CD.

1. Effect of Lot Size

The following values were used for analysis:

- Lot Size = 2000 To 20000 (Step 100),
- %H = ($P_{BR\,=}$ 5, $P_{RA\,=}$ 85, $P_{AA\,=}$ 10)
- AQL = 90 PWL
- RQL = 50 PWL
- M = 70 PWL
- b = $100.00
- Data used for cost of sampling is used from Table 12 appendix for AQC1.

The results obtained are shown in Figure 28 through Figure 30, which indicate following observations:

- Given that all other factors in analysis remain constant, sample size varies directly as lot size.
- From the regression analysis, a trend-line drawn to approximate the behavior of PSS curve indicates that this is logarithmic relationship, which can be further confirmed by plotting lot size on a $\log_{10}$ scale. A straight line obtained proves that the practical sample size follows logarithmic curve as lot size varies. (Figure 29).

- Hald, in his paper (Hald A. , 1960) while using hyper-geometric function and cost function to find practical sampling plan found the similar effect of sample size and lot size. According to Hald, "Sample size should increase proportional to the logarithm of lot size if the compound binomial prior distribution has a discrete limit for large lots." Although PSSCM does not assume any particular prior distribution, %H can be modified according to the distributions.
- Study of effect of Lot Size on Total Cost of Acceptance (Figure 30) also varies logarithmically (R^2=0.9996).

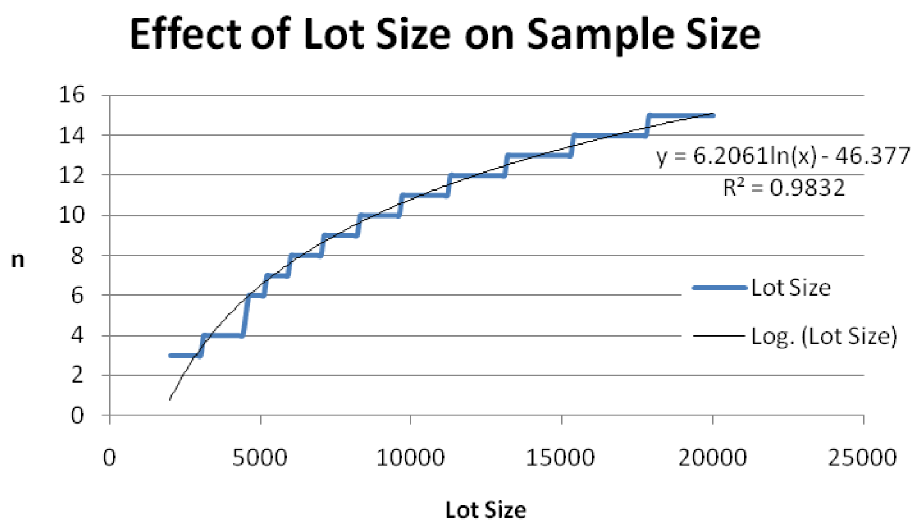


Figure 28: Effect of Lot Size on Sample Size (Single AQC)

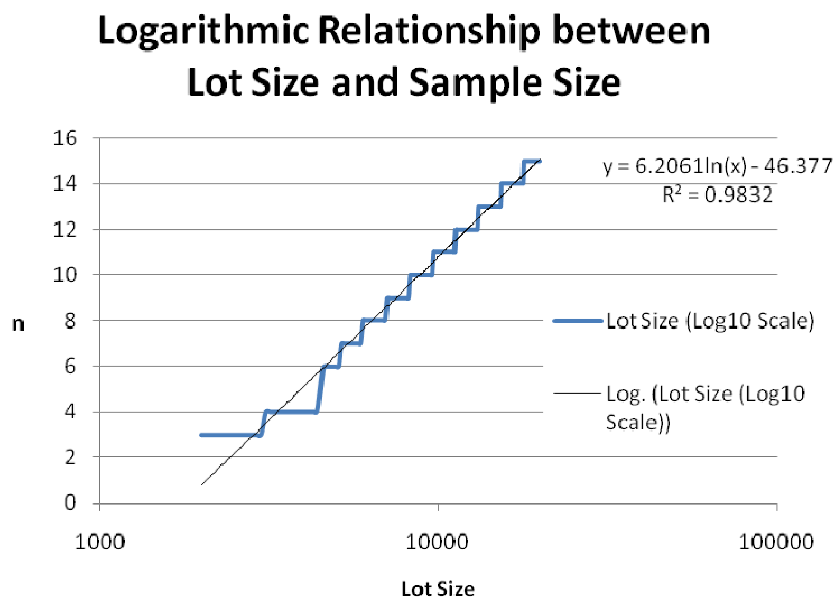


Figure 29: Logarithmic Relationship between L and n

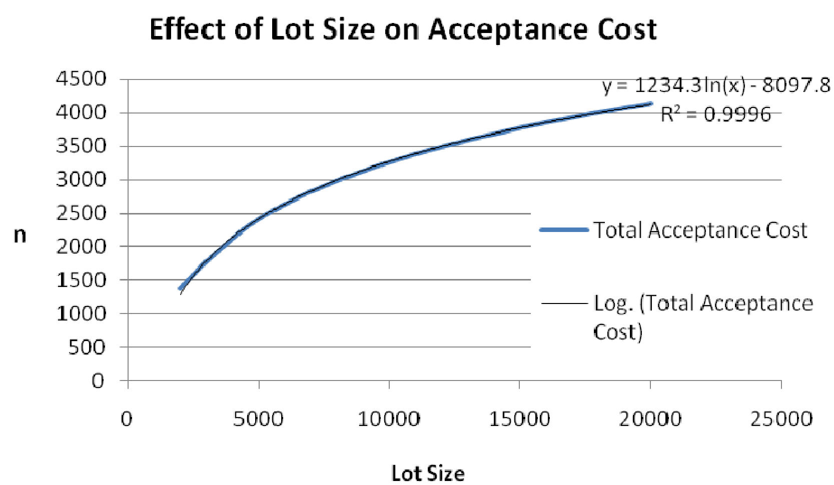


Figure 30: Effect of Lot Size on Total Acceptance Cost

2. Effect of Minimum Acceptable PWL (m):

In order to thoroughly understand an effect of minimum sample PWL on sample size, lot size and m both, were input as variables while keeping all other factors constant.

The results show that:

- From Figure 31, a relationship between lot-size, sample size can be noted to follow the similar trend as seen in last section, even for varying 'm.'

- For lower lot sizes, sample size remains almost constant over a relatively larger range of lots.
- Values of ‘m’ less than 75 PWL do not deviate more than 2-3 sample sizes, even for lot sizes greater than 6000 tons.
- These variations can be studied in more detail from a 3D view presented in Figure 32. For the lots greater than 4000 tons, PSS follows a bell shaped distribution over a range of values of m.

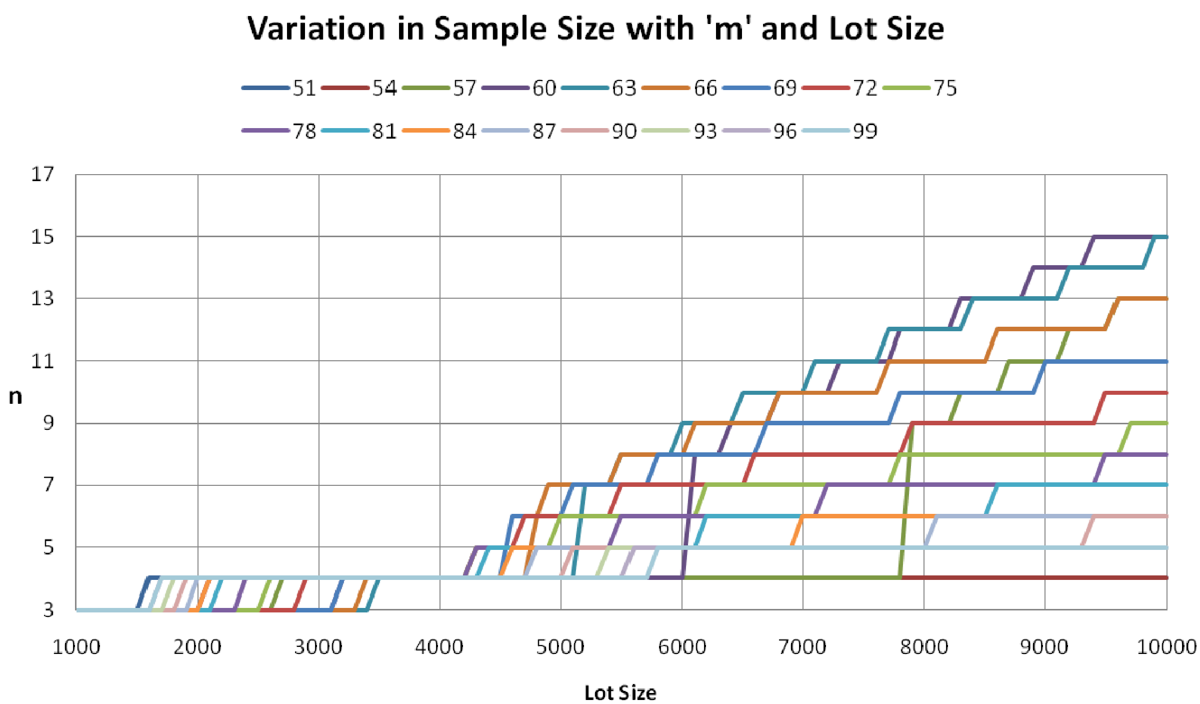


Figure 31: Variation in Sample Size with 'm' and Lot Size

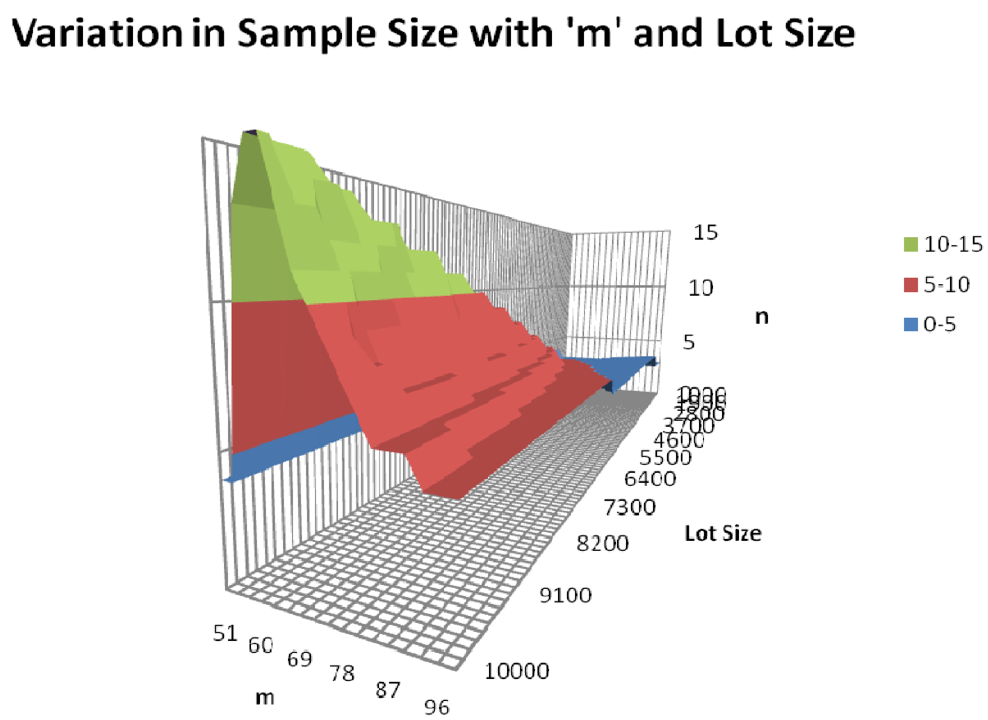


Figure 32: Variation in Sample Size with 'm' and Lot Size (3D)

3. Effect of Rejectable Quality Level (RQL):

Generally, RQL is kept constant while different sample sizes are evaluated to incur calculated amount of beta risk within an acceptable limit. Since, in this particular case the decision is not only based on risks but the costs too, the usually observed variation of sample size with variable RQL is a little modified. From Figure 33 and Figure 34, for lower lot sizes (<4000 tons), sample size for almost all RQL, the practical sample size remains constant. For the lot sizes >4000 tons, PSS required increases with decrease in PWL. In other words, the larger the percent defective (PD), the larger the sample size required, proportional to increase in lot size.

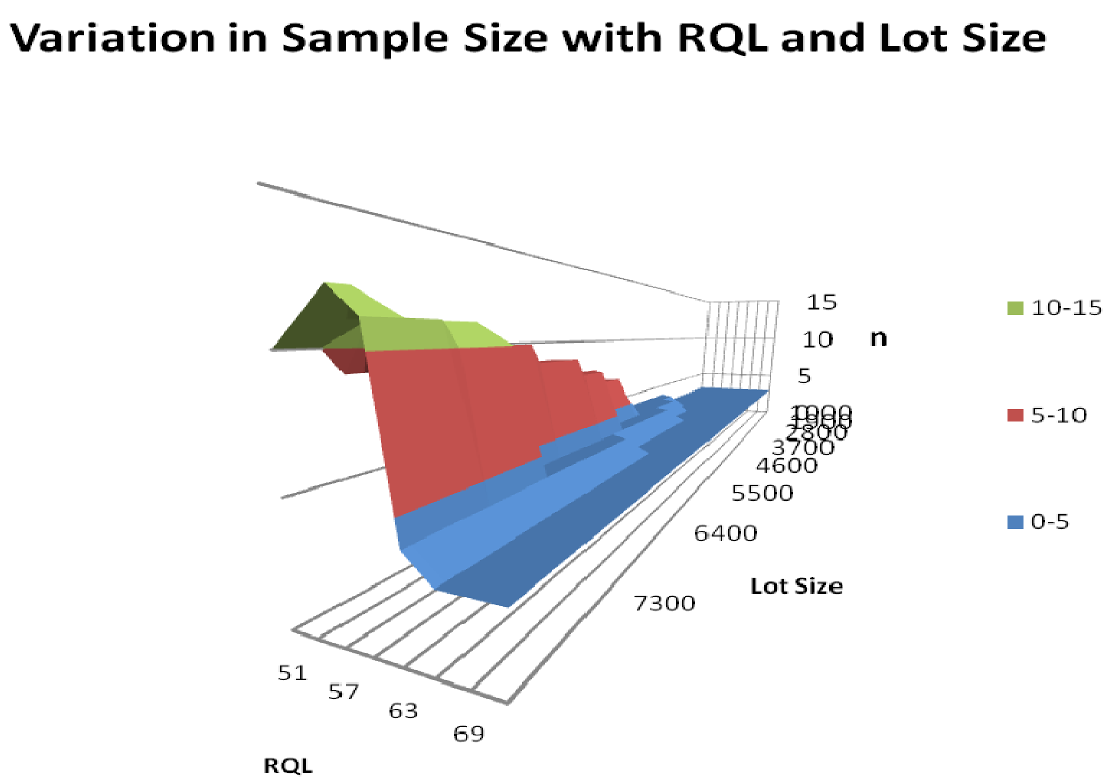


Figure 33: Variation in Sample Size with RQL and Lot Size (3D)

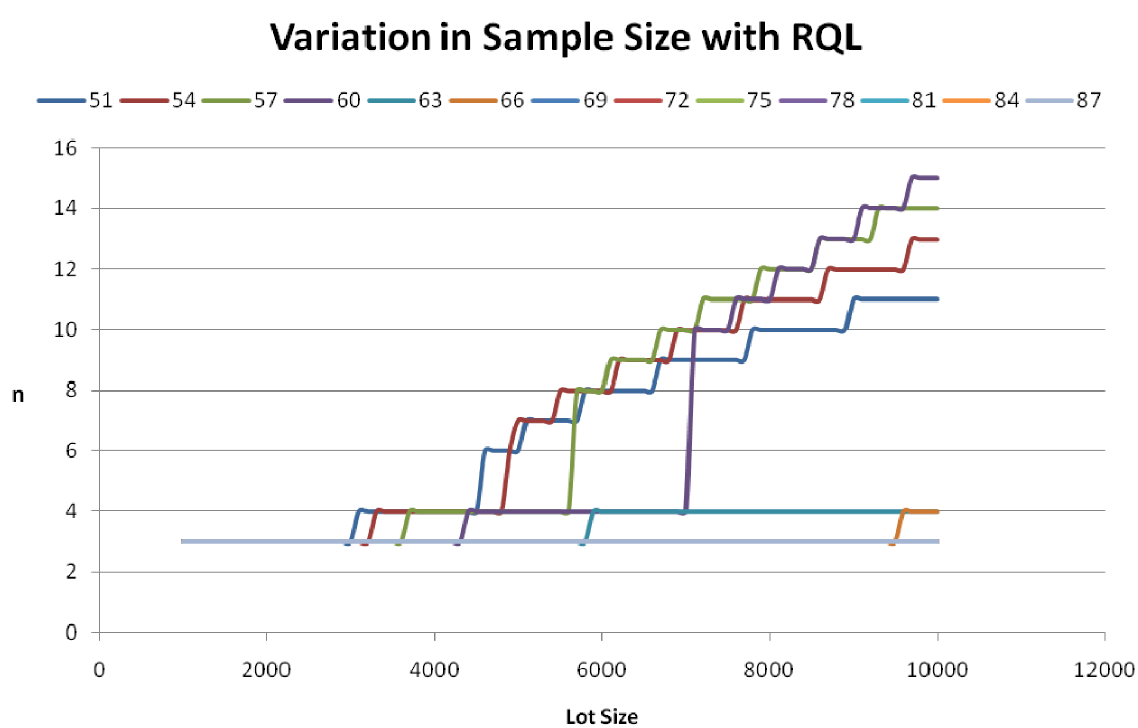


Figure 34: Variation in Sample Size with RQL (2D)

4. Combined Effect of m and Rejectable Quality Level (RQL):

Variation in requirement of PSS can be further analyzed against the combined effect of m and RQL while keeping AQL, %H and cost data constant. In the last section (3), m =70 was used. To visualize the effect of change in 'm' over the RQL-L-n relationship scrutinized in last section, simulations for varying RQL, n and lot-size were carried out in similar way, but this time using m=80. The difference between the practical sample size requirements is plotted as shown in Figure 35. As it could be expected, for lower values of RQL (RQL<65 PWL) the PSS required goes further down and the difference plotted on y-axis (n_i-n_j) is negative. Variation in PSS becomes steeper for higher RQL values, and becomes even sharper with an increase in lot sizes. This anomaly is explained in next section.

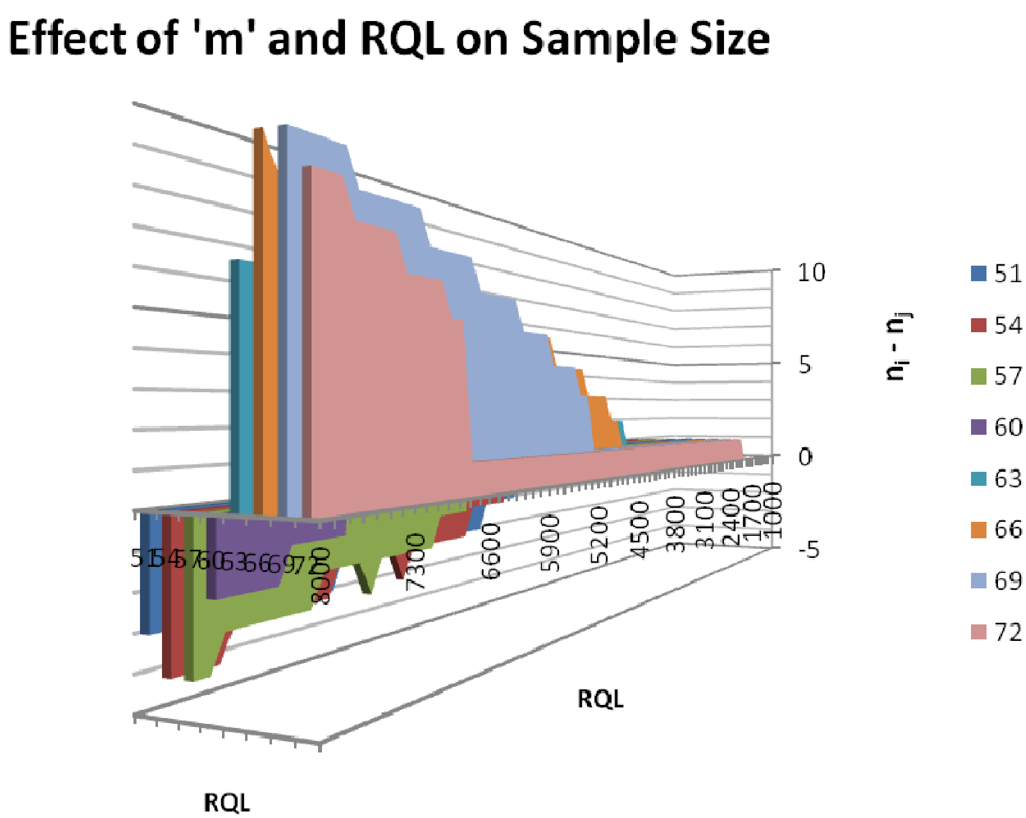


Figure 35: Effect of 'm' and RQL on Sample Size

5. **Effect of Historical Percentages:**

Effects of historical percentages are an indirect measure of taking prior distributions of lots into account. Generally, the more the historical percentage of lots falling below RQL, or the lesser the amount of lots above AQL, the more is the sampling required, since P_E will induce an increase in beta risks and so the PSS required. The effect of historical percentages on the sample size is tested for the sampling plans simulated in the last section (section 4). The target was to record the changes in PSS requirements for each plan by changing the constant %H from (5, 85, 10) to (10, 80, 10) i.e., considering a poorer history of lot quality and keeping the same m= 80 from last section. Results are shown in Figure 36. Comparison of results from section 4 and obtained by using new % H is plotted as a difference of PSS on y-axis. The results clarify the anomaly of a sudden increase in PSS requirements for the last sampling plan. It should be noted that in Figure 36, as the lots were given more scrutiny considering the poorer past probabilities, the change in sample size requirements is comparatively gentle corresponding to the increase in lot size and RQL.

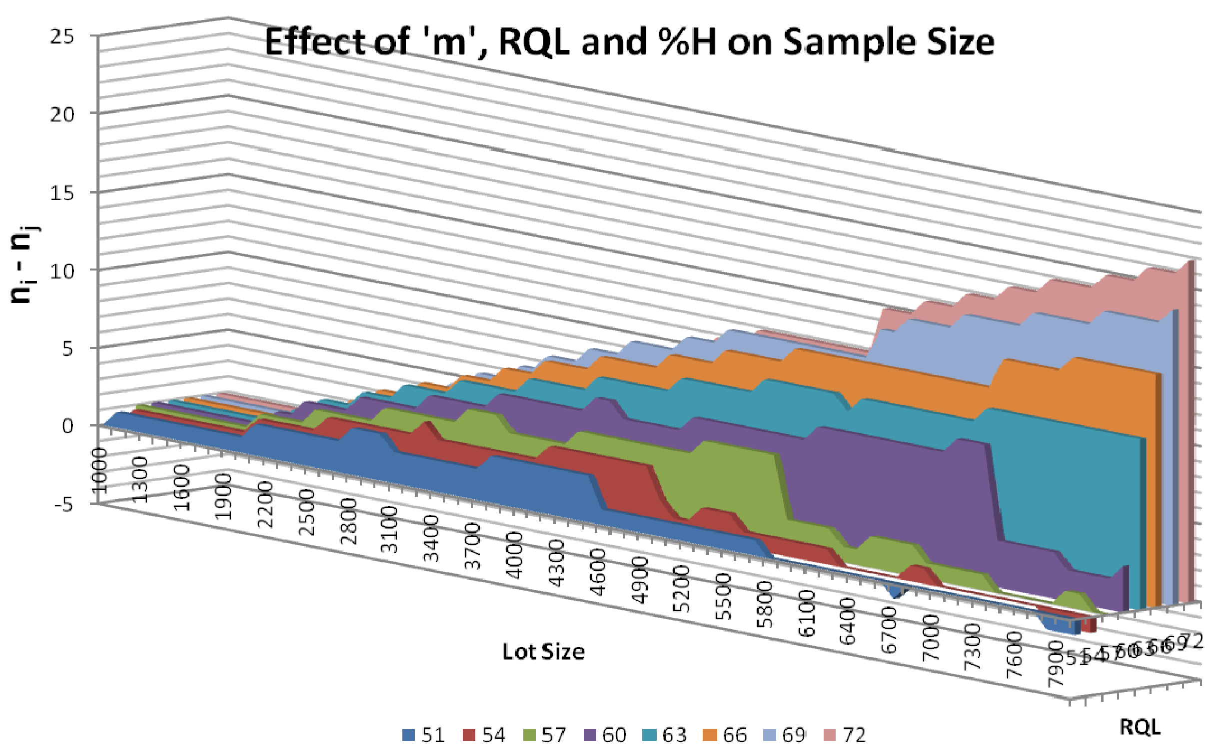


Figure 36: Effect of 'm', RQL and %H on Sample Size

PRACTICAL SAMPLE SIZE CALCULATOR

Practical Sample Size Calculator (PSSC) was developed to implement PSSCM.

Purpose:

Purpose of designing PSSC is:

1. To automate the calculations for methodology of finding PSS developed for model 1 through 3 for PSSCM.
2. For faster calculations of complex functions and iterative processes.
3. Existing software for calculating risks, OCPLOT, is incompatible with newer operating systems such as windows vista. PSSC may be used to calculate risks as an option for OCPLOT.

Design/Interface

Simple, user friendly interface is designed in Microsoft Excel.

1. Main form, shown in screenshot Figure 37, is used to collect input data, analyze and display outputs in a tabbed dialog.
2. Two separate forms are used to input Historical Percentages and Costs of Sampling and Testing data (shown in Figure 38and Figure 39).
3. Input boxes for AQL, RQL, m and other values are pre-validated.
4. Same interface is duplicated on Sheet1 of the workbook to be able to run analysis without using forms.
5. Output is displayed on the 'Results' tab for sample sizes and risks (Figure 40).
6. Graphical representation of Cost analysis and Risks analysis is shown in the next tabs (Figure 41, Figure 42). Additional graphs are plotted for each AQC with the results for multiple AQCs (Figure 43, Figure 44). Detailed output is stored in spreadsheet (Figure 45).

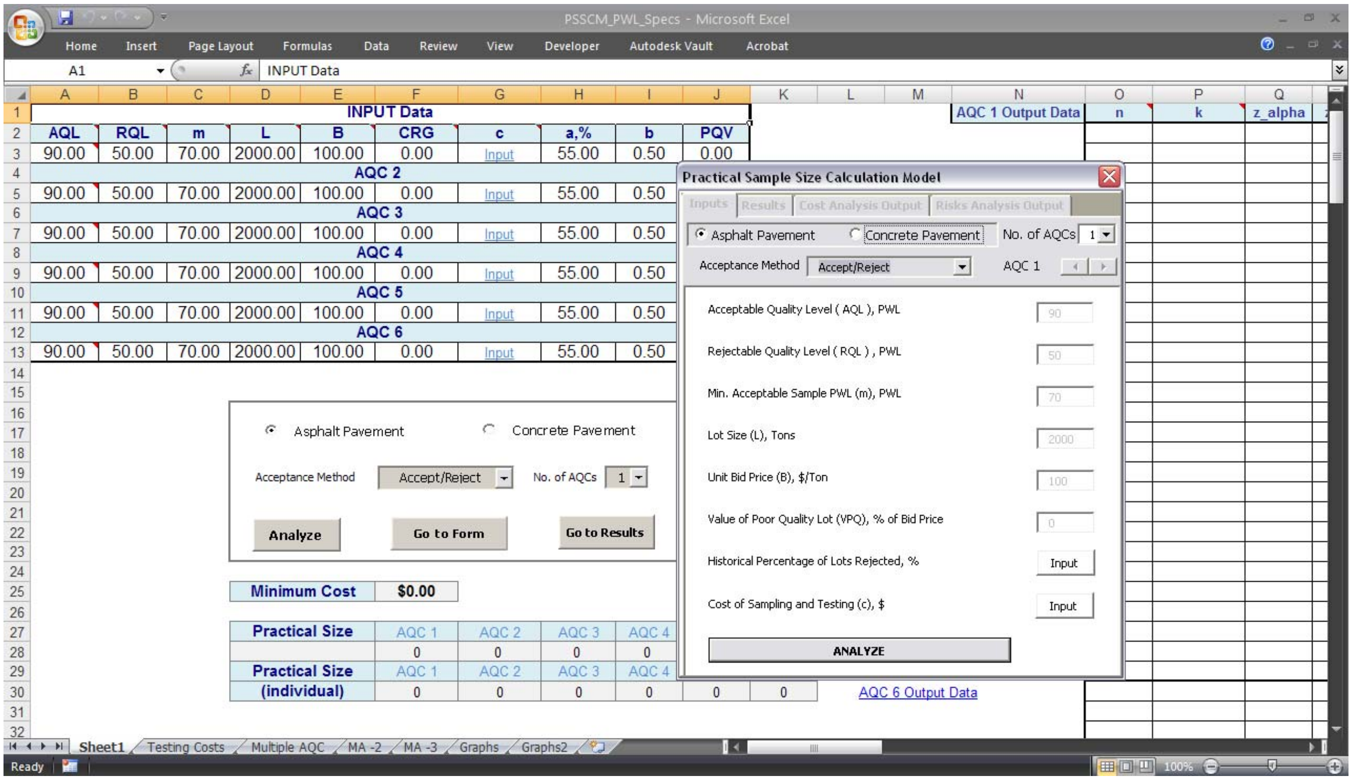


Figure 37: PSSC: Main Screen

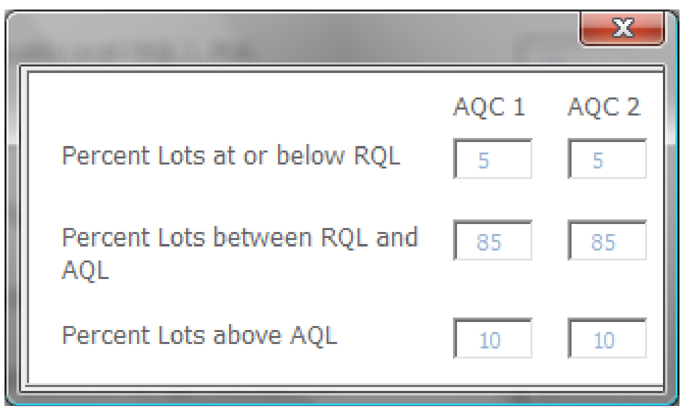


Figure 38: PSSC: Input form for Historical Percentages

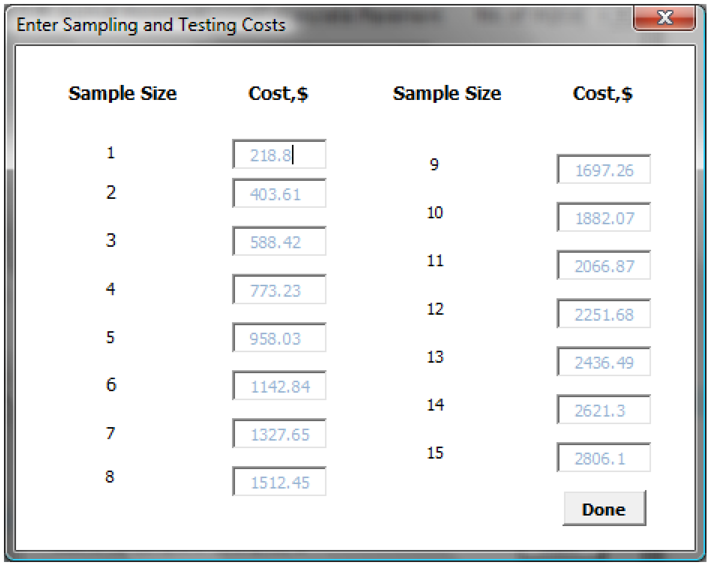


Figure 39: PSSC: Input Form for Cost Data

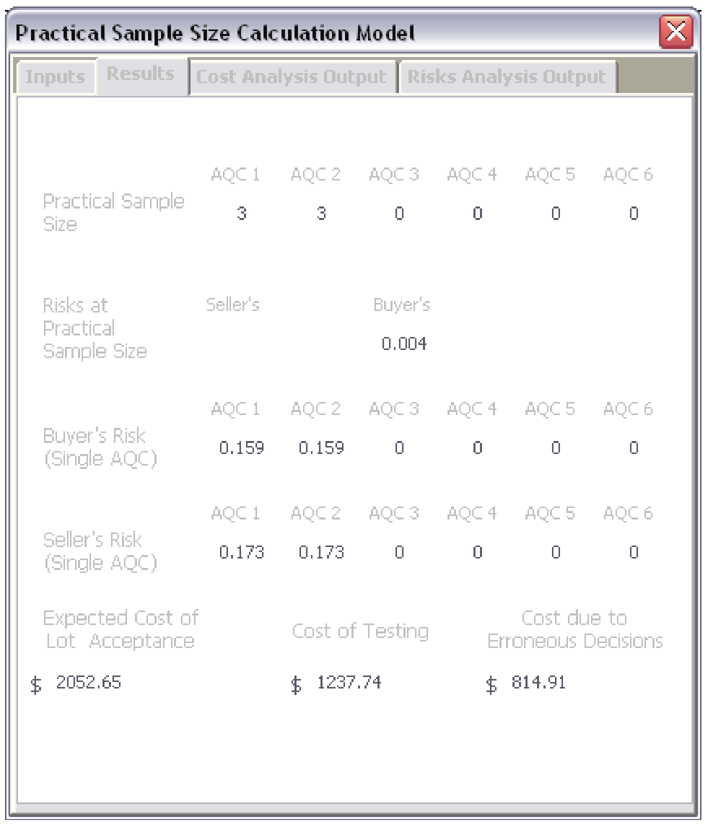


Figure 40: PSSC: Output Screen

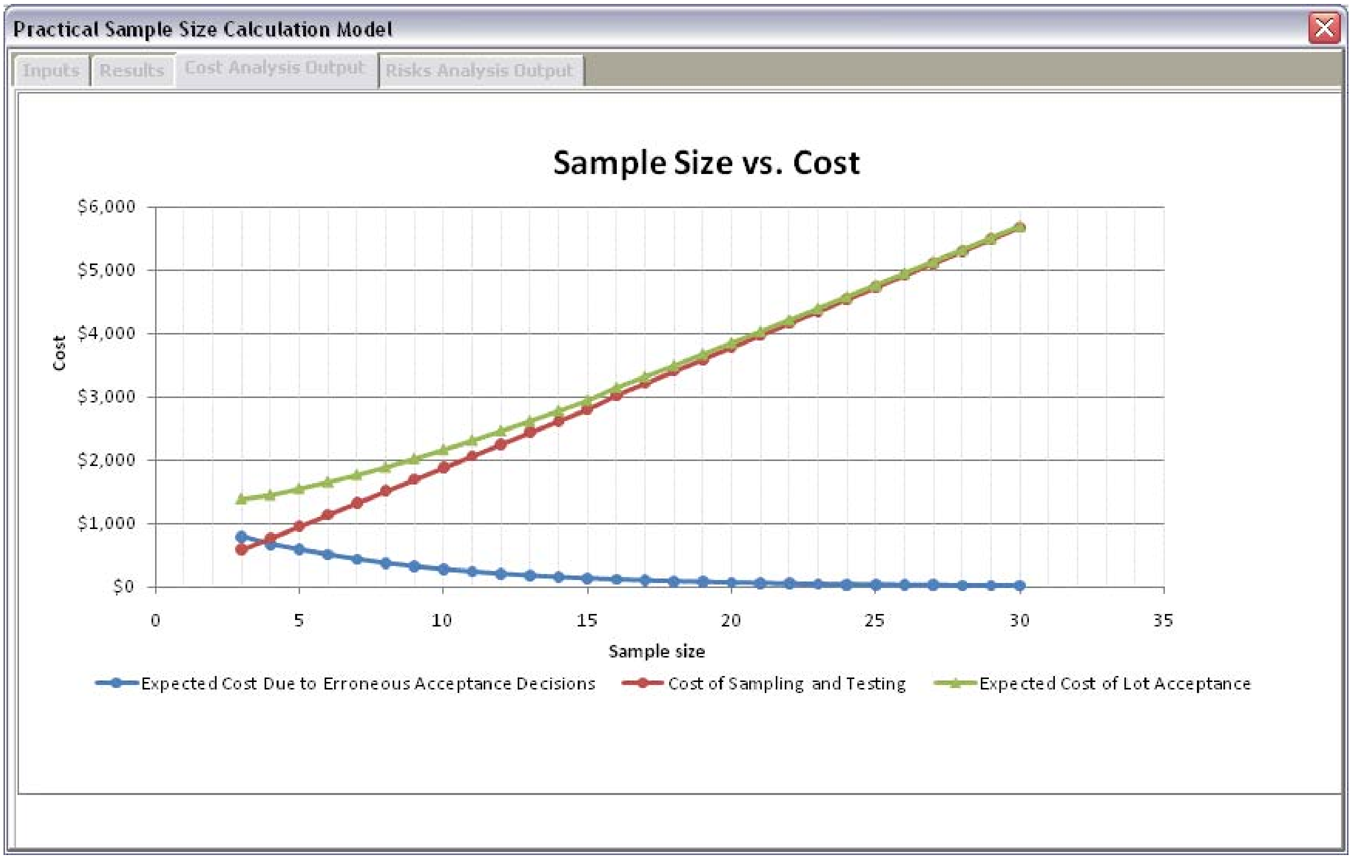


Figure 41: PSSC: Sample Size Vs. Cost Output

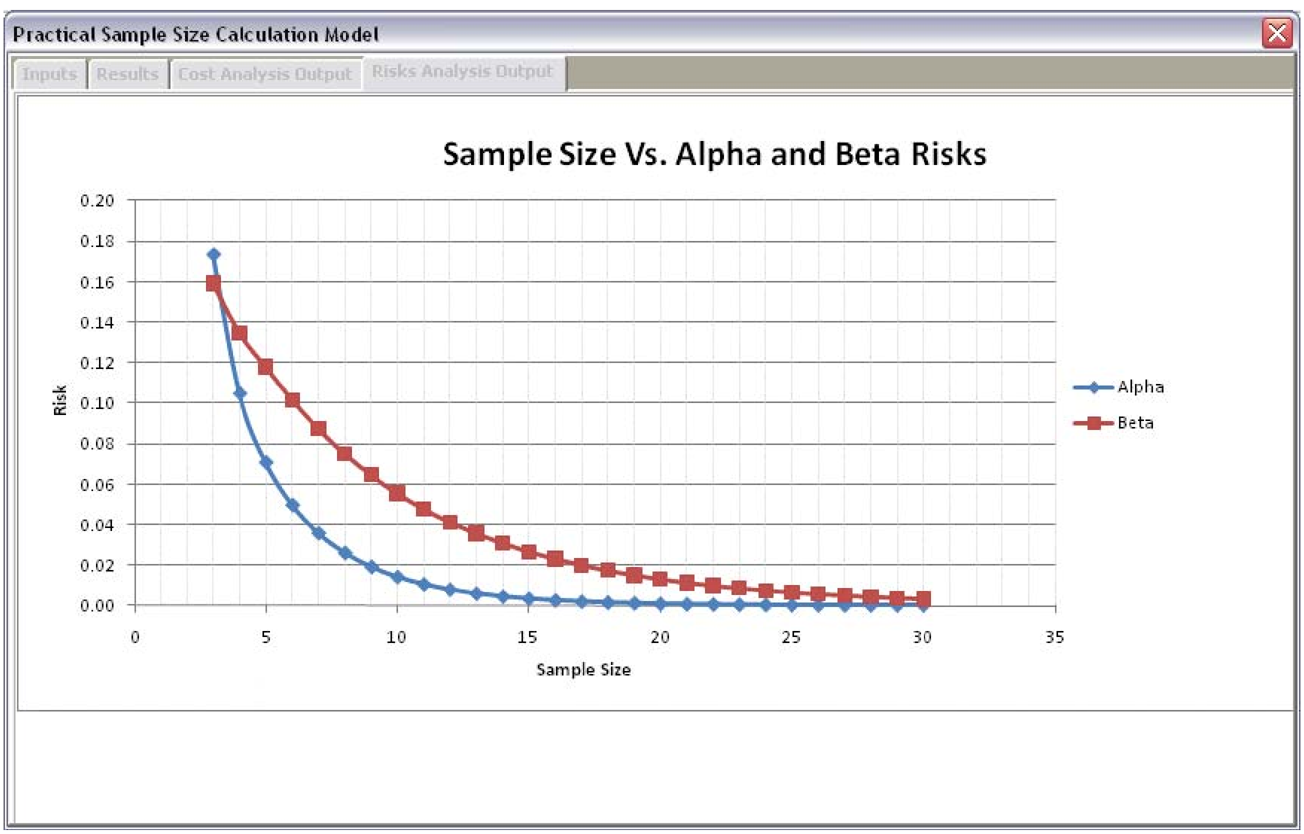


Figure 42: PSSC: Sample Size Vs. Alpha and Beta Risks Output

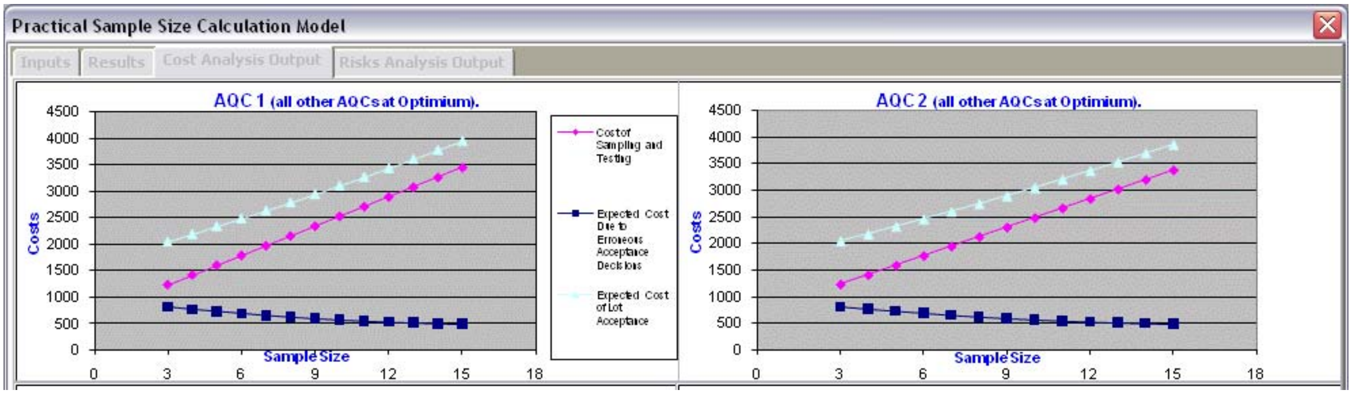


Figure 43: PSSC: Multiple AQCs Costs Analysis Output

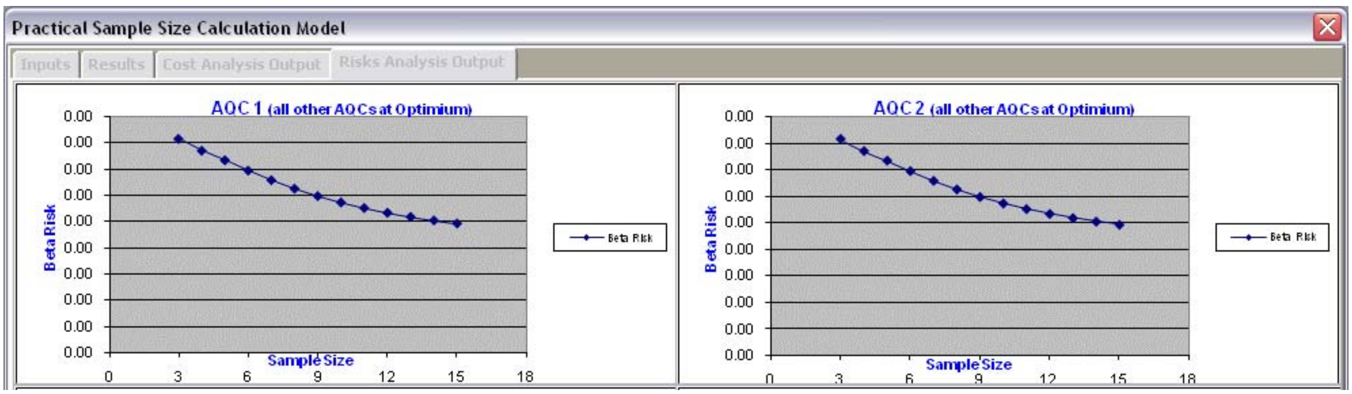


Figure 44 PSSC: Multiple AQCs Risks Analysis Output

n	k	z_alpha	z_beta	Alpha	Beta	$Cost_D$	$Cost_T$	$Cost_{(D+T)}$
3	0.679	0.941	1.060	0.1733	0.1591	$795.35	$588.42	$1,383.77
4	0.600	1.255	1.105	0.1048	0.1346	$673.24	$773.23	$1,446.47
5	0.572	1.471	1.186	0.0706	0.1179	$589.54	$958.03	$1,547.57
6	0.558	1.648	1.272	0.0497	0.1017	$508.51	$1,142.84	$1,651.35
7	0.550	1.803	1.357	0.0357	0.0874	$436.90	$1,327.65	$1,764.55

Figure 45: Single AQC Analysis Detailed Output

How to Use

Following are the basic steps to run analysis:

1. Select type of Pavement (Asphalt/Concrete)
2. Select Type of Analysis (Accept/Reject or Pay Adjustment)
3. Select Number of AQCs to be analyzed

4. Click on “Input” buttons to input cost data and historical percentages for each AQC.
5. In case of two AQCs, input all data for first AQC, then click right arrow to go to next AQC data.
6. Click on “Analyze” to run analysis.

System Requirements

1. OS: Windows XP/Vista (x86 and x64)
2. PSSC does not require much space, except for where data from multiple AQC analyses is saved.
3. Minimum 500 MB RAM.
4. PSSC is compatible with MS Excel 2003 and 2007.
5. Solver add-in must be installed on the computer in order for analysis to run.

Setup

Set-up details are provided in Appendix: PSSC Installation (pp. 94).

Chapter 5: Summary, Conclusions and Recommendations

Summary and Conclusions

Acceptance Sampling has been in practice for more than 75 years. Since the beginning, researchers and decision makers have been continuously seeking a rational method to optimize the resources needed for quality assurance, which includes improvement to the acceptance sampling plans. Over all these years, a lot of variations in the acceptance schemes, from selection of attributes to variables, quality measures from LTPD to AAD, use of known mean and standard deviations or assuming unknown, from single limit plans to sequential sampling plans and so on, were seen. Some of these methods, theories or plans have been implemented and followed successfully, while some ideologies, such as use of prior distributions, Bayesian methods, optimization methods, minimum cost sampling plans could not find a place in the main stream. The major limitation in implementing the latter is that these methods are either mathematically too complex to be used to develop generic specifications, need such an input which is difficult to estimate (e.g., in case of Bayesian theories process curve and loss functions are needed and cannot be obtained easily.) or do not consider use of OC curves. But amongst all these contradictions and difficulties, an important problem remains unsolved i.e., developing practical sampling plans.

As discussed earlier in the literature review, many theories have been developed to find minimum sample size and very few for practical sample size. Amongst theories for optimization, few are mathematically simple, fewer are applied for variables sampling plans or cost effective plans, and very rarely do any combine all three. This research addresses this very problem. A new approach, which is a combination of a few different, but effective methods, is molded into a single, simple and fairly accurate method which yields the practical sample sizes using acceptance by variables procedure to minimize the total cost (incurred due to erroneous decisions and sampling and testing) using very little information about the quality of lots in the past.

This particular combination was used because:

1. Variables plans generally yield lower sample sizes;

2. Use of variables plan provides better information on process quality and considerable cost savings is achieved;
3. Use of erroneous decision costs provide better judgment regarding selection of sample size than decisions solely based on risks; and
4. Use of historical percentages of lot quality help decide the practical sample size i.e., more sample size may be required for poor quality history and vice versa.

Considering the above advantages of this unique combination, the methodology presented in this thesis was designed using formula by A. Wallis and a method of acceptance sampling by variables to control fraction non-conforming when standard deviation is unknown was used. The practical sample size was targeted at minimum total cost of acceptance.

The results obtained were compared to the risks calculated by well established software OCPLOT. The results of comparison of sampling plans generated by OCPLOT and PSSCM (Practical Sample Size Calculation Method) show that:

1. Sampling plans for fairly liberal acceptance criteria, for lower sample sizes and good history of lot quality match based on OC Curves. Sample size for such plan was observed to be the same for both methods. Hence, it can be concluded that such sample size obtained with OCPLOT (i.e., following AASHTO recommended method) might be the practical sample size.
2. Since AASHTO specifications do not base judgments based on historical data and costs, the practical sample size obtained by PSSCM may not always correspond to the same sample size by AASHTO's currently specified method.

Sensitivity analysis was carried out to identify behavior of the practical sample size as affected by other variables included in analysis. Results show that PSS generally follows expected behavior proportional to RQL and m. Effect of lot size was studied over sample size for variations in RQL, m and historical percentages. The results show that PSS varies proportional to the logarithm of lot size.

Considering the fact that the AASHTO method specifies sampling frequency for the sample sizes which are usually decided based on decision errors (or risks), it is suggestive that the sample size to lot size proportion might be inappropriate for the given quality limits of acceptance plan.

Microsoft Excel based tool PSSC was also developed to aid in calculations and in generating results for PSSCM.

To summarize the major outcomes of this research:

1. A mathematically simple but effective method is developed to find the practical sample size which gives a close-form solution and minimizes total cost of acceptance.
2. It is investigated that the current sample size requirements are not always practical. Changes should be made in current specifications to incorporate effects of costs due to erroneous decisions and use of historical data.
3. Reconsideration on recommended sample sizes should be done with regards to the lot size.

RECOMMENDATIONS FOR FUTURE RESEARCH

1. The sampling method developed in this research does not consider costs due to erroneously rejecting a good lot since approximation of these costs is highly difficult. Researches directed towards the aforementioned goal, in addition to PSSCM, might lead to a more precise method to find the practical sample sizes.
2. An assumption has been made in PSSCM, while analyzing multiple AQCs, that two AQCs are statistically independent. In practical, this might not be always the case which follows the need to research for PSS requirements given that AQCs are interdependent.
3. Calculations for historical percentages given the distributions and to generate OC and EP curves can be added to PSSC so that it can help compare the plans with other existing methods, thus avoiding manual calculations.

References

AASHTO. (1998). AASHTO R 9-90. Standard Recommended Practice for Acceptance Sampling Plans for Highway Construction, 816-874.

AASHTO, R. 9.-0. (2005). Standard Recommended Practice for Acceptance Sampling Plans for Highway Construction. American Association of State Highway and Transportation Officials.

ASTM, E.-0. (2009). Standard Practice for Calculating Sample Size to Estimate, With Specified Precision, the Average for a Characteristic of a Lot or Process.

Bureau, U. C. (2008). Construction Spending. Retrieved July 15, 2009, from U.S. Census Bureau: http://www.census.gov/const/www/c30index.html

Besterfield, D. (2004). Quality Control (7th Edition ed.). Pearson Prentice Hall.

Brown, D. (2009, June Wednesday, 17th). Online Magazine of Asphalt Institute, "FHWA Works to Support Statistical Quality Assurance Techniques". Retrieved July 2009, from http://www.asphaltmagazine.com/singlenews.asp?item_ID=1745&comm=0&list_code_int=mag01-int

Burati, J. L., Weed, R. M., Hughes, C. S., & Hill, H. S. (2003). Optimal Procedures for Quality Assurance Specifications. Clemson University, Civil Engineering Department. Office of Research, Development, and Technology Federal Highway Administration (FHWA).

Burr, I. (1976). Statistical Quality Control Methods. New York and Basel: Marcel Dekker.
Champernowne, D. (1953). The economics of Sequential Sampling Procedures for Defectives. Journal of the Royal Statistical Society. Series C (Applied Statistics), 2 (2), pp. 118-130.
Dodge, H., & Roming, H. (1941). Single Sampling and Double Sampling Inspection Tables. Bell System Technical Journal, 1-61.

Duncan, A. (1986). Quality Control and Industrial Statistics. IRWIN.

Feigenbaum, A. (1983). Total Quality Control (3rd Edition ed.). New York: McGraw-Hill.
Freeman, R., & Grogan, W. (1998). Statistical Acceptance Plan for Asphalt Pavement Construction. U.S. Army Corps of Engineers.

Griffiths, B., & Rao, A. (1965). An Application of Least Cost Acceptance Sampling Schemes. Mathematical Mthods of Operations Research, 9 (1), pp. 8-17.

Hald, A. (1968). Bayesian Single Sampling Attribute Plans for Continuous Prior Distributions. Technometrics, 10 (4), pp. 667-683.

Hald, A. (1964). Single Sampling Inspection Plans with Specified Acceptance Probability and Minimum Average Costs. Institute of Mathematical Statistics.

Hald, A. (1960). The Compound Hypergeometric Distribution and a System of Single Sampling Inspection Based on Prior Distributions and Costs. Technometrics, 2 (3), pp. 275-340.

Hald, A.; Copenhagen University (Denmark). (June 1964). Bayesian single sampling attribute plans for discrete prior distribution. Technical rept. no. 5

Hand A., J., & Epps, J. (2006, September). Fundamentals of Percent Within Limits and Control-Quality Assurance Compaction Specifications. Transportation Reseach Circular No. E-C105, pp. pp. 140-162.

Jacobson, L. (November 1949). Nomograph for Determination of Variables Inspection Plan for Fraction Defective. In I. American Society for Quality Control, Industrial Quality Control (p. 24).

Juran J.M, G. F. (1970). Quality Planning and Analysis. New York: McGraw-Hill.

Juran J.M., G. A. (1999). Juran's Quality Handbook (5th Edition ed.). The University of Michigan: McGraw-Hill.

McCabe, B., AbouRizk, S., & Gavin, J. (1999). Sample Size Analysis for Asphalt Pavement Quality Control. Journal of Infrastructure Systems, 5 (4), pp. 118-123.

Messina, W. (1987). Statistical Quality Control for Manufacturing Managers. Wiley-Interscience.

Mood, A. (1943). On the Dependence of Sampling Inspection Plans upon Population Distributions. The Annals of Mathematical Statistics, 14 (4), pp. 415-425.

NCHRP-346. (2005). State Construction Quality Assurance Programs. Transportation Research Board, National Cooperative Highway Research Program. Washington, D.C.: Transportation Research Board.

Rao, C., Darter, M., Smit, A., Mallela, J., Smith, K., Von Quintus, H. L., et al. (2006). Advanced Quality Systems: Guidelines for Establishing and Maintaining Construction Quality Databases. Federal Highway Administration. McLean, VA: FHWA, Turner-Fairbank Highway Research Center.

Schilling, E. (1990). ACCEPTANCE SAMPLING - PAST, PRESENT, AND FUTURE. Statistical Sampling: Past, Present and Future - Theoretical and Practical, ASTM STP 1097, 5-13.

Schilling, E. (1982). Acceptance Sampling in Quality Control. In STATISTICS: textbooks and monographs (Vol. 42, p. 2). CRC Press.

South, J. (1982, October). Selecting an Acceptance Sampling Plan that Minimizes Expected Error and Sampling Cost. Quality Progress, 18-22.

Stephens, K. (2001). Handbook of Applied Acceptance Sampling: Plans, Principles and Procedures. American Society for Quality.

TRB. (2005). Glossary of Highway Quality Assurance Terms. Transportation Research Circular Number E-C074, Third Update.

Velivelli, K., Gharaibeh, N., & Nazarian, S. (2002). Sample Size Requirements for Seismic and Traditional Testing of Concrete Pavement Strength. Transportation Research Record, Journal of the Transportation Research Borad, 1813, pp. 21-27.

Wallis. (1947). Use of Variables in Acceptance Inspection for Percent Defective. In C. U. Statistical Research Group, Selected Techniques of Statistical Analysis for Scientific Industrial Research and Production and Management Engineering (pp. 17, 22-25, 31). McGraw-Hill Book Company.

Weed, R. (1996). Quality Assurance Software for the Personal Computer. New Jersey: New Jersey Department of Transportation.

Wetherill, G. B., & Chiu, W. K. (1975). A Review of Acceptance Sampling Schemes with Emphasis on the Economics Aspect. International Statistical Review / Revue Internationale de Statistique, 43 (2), pp. 191-201.

Wetherill, G. (1959). The Most Economical Sequential Sampling Scheme for Inspection by Variables. Journal of the Royal Statistics Society. Series B (Methodological), 21 (2), pp. 400-408.

Yeh, L. (1990). An Optimal Single Variable Sampling Plan with Censoring. Journal of the Royal Statistical Society, Series D (The Statistician), 39 (1), pp. 53-66.

Appendix

Cost Data

Table 12: Costs of Testing and Sampling for example Sampling Plan (Asphalt Binder Content (Pb) & Gradation (Tested at Plant), $)

Sample Size	Asphalt Binder Content (Pb) & Gradation (Tested at Plant), $
1	218.8
2	403.6
3	588.4
4	773.2
5	958.0
6	1142.8
7	1327.6
8	1512.5
9	1697.3
10	1882.1
11	2066.9
12	2251.7
13	2436.5
14	2621.3
15	2806.1

Cost of Sampling and Testing (National Average, 2008)

Table 13: Costs of Testing and Sampling for example Sampling Plan (Density from Cores (Gmb), $)

Sample Size	Testing Cost (Density from Cores (G_{mb}), $)
1	293.1
2	471.2
3	649.3
4	827.4
5	1005.5
6	1183.6
7	1361.6
8	1539.7
9	1717.8
10	1895.9
11	2074.0
12	2252.1
13	2430.1
14	2608.2
15	2786.3

Cost of Sampling and Testing (National Average, 2008)

DETAILS OF CALCULATIONS

Table 14: Calculation of Beta_multiple and Costs for 2 AQC Plan (Example)

n1	n2	Beta_multiple	$Cost_T$	$Cost_D$	$Cost_{(D+T)}$
3	3	0.0046	1237.74	910.31	2148.05
3	4	0.0039	1415.82	775.42	2191.24
3	5	0.0035	1593.90	694.11	2288.01
3	6	0.0031	1771.98	623.53	2395.51
3	7	0.0028	1950.06	566.22	2516.28
3	8	0.0026	2128.15	520.70	2648.85
3	9	0.0024	2306.23	484.85	2791.08
3	10	0.0023	2484.31	456.75	2941.06
3	11	0.0022	2662.39	434.76	3097.15
3	12	0.0021	2840.47	417.61	3258.08
3	13	0.0020	3018.56	404.27	3422.83
3	14	0.0020	3196.64	393.94	3590.58
3	15	0.0019	3374.72	385.97	3760.69
4	3	0.0045	1422.55	892.52	2315.07
4	4	0.0037	1600.63	749.38	2350.01
4	5	0.0033	1778.71	663.01	2441.72
4	6	0.0029	1956.79	588.39	2545.18
4	7	0.0026	2134.87	527.83	2662.70
4	8	0.0024	2312.96	479.66	2792.62
4	9	0.0022	2491.04	441.65	2932.69
4	10	0.0021	2669.12	411.74	3080.86
4	11	0.0019	2847.20	388.24	3235.44
4	12	0.0018	3025.28	369.82	3395.10
4	13	0.0018	3203.37	355.39	3558.76
4	14	0.0017	3381.45	344.13	3725.58
4	15	0.0017	3559.53	335.36	3894.89
5	3	0.0044	1607.35	872.96	2480.31
5	4	0.0036	1785.43	725.92	2511.35
5	5	0.0032	1963.51	637.12	2600.63
5	6	0.0028	2141.59	560.64	2702.23
5	7	0.0025	2319.67	498.58	2818.25
5	8	0.0022	2497.76	449.19	2946.95
5	9	0.0021	2675.84	410.15	3085.99
5	10	0.0019	2853.92	379.37	3233.29
5	11	0.0018	3032.00	355.14	3387.14
5	12	0.0017	3210.08	336.07	3546.15
5	13	0.0016	3388.17	321.09	3709.26
5	14	0.0015	3566.25	309.34	3875.59
5	15	0.0015	3744.33	300.15	4044.48
6	3	0.0042	1792.16	847.03	2639.19
6	4	0.0035	1970.24	697.87	2668.11
6	5	0.0030	2148.32	607.74	2756.06
6	6	0.0027	2326.40	530.30	2856.70
6	7	0.0023	2504.48	467.48	2971.96
6	8	0.0021	2682.57	417.45	3100.02
6	9	0.0019	2860.65	377.86	3238.51

Table 14 (Continued)

n1	n2	Beta_multiple	CostT	CostD	Cost(D+T)
6	10	0.0017	3038.73	346.59	3385.32
6	11	0.0016	3216.81	321.92	3538.73
6	12	0.0015	3394.89	302.47	3697.36
6	13	0.0014	3572.98	287.13	3860.11
6	14	0.0014	3751.06	275.05	4026.11
6	15	0.0013	3929.14	265.56	4194.70
7	3	0.0041	1976.97	821.50	2798.47
7	4	0.0034	2155.05	671.09	2826.14
7	5	0.0029	2333.13	580.16	2913.29
7	6	0.0025	2511.21	502.19	3013.40
7	7	0.0022	2689.29	438.96	3128.25
7	8	0.0019	2867.38	388.57	3255.95
7	9	0.0017	3045.46	348.65	3394.11
7	10	0.0016	3223.54	317.09	3540.63
7	11	0.0015	3401.62	292.14	3693.76
7	12	0.0014	3579.70	272.42	3852.12
7	13	0.0013	3757.79	256.83	4014.62
7	14	0.0012	3935.87	244.52	4180.39
7	15	0.0012	4113.95	234.81	4348.76
8	3	0.0040	2161.77	798.15	2959.92
8	4	0.0032	2339.85	646.99	2986.84
8	5	0.0028	2517.93	555.56	3073.49
8	6	0.0024	2696.01	477.30	3173.31
8	7	0.0021	2874.09	413.85	3287.94
8	8	0.0018	3052.18	363.26	3415.44
8	9	0.0016	3230.26	323.15	3553.41
8	10	0.0015	3408.34	291.39	3699.73
8	11	0.0013	3586.42	266.26	3852.68
8	12	0.0012	3764.50	246.35	4010.85
8	13	0.0012	3942.59	230.59	4173.18
8	14	0.0011	4120.67	218.10	4338.77
8	15	0.0010	4298.75	208.22	4506.97
9	3	0.0039	2346.58	777.40	3123.98
9	4	0.0031	2524.66	625.78	3150.44
9	5	0.0027	2702.74	534.04	3236.78
9	6	0.0023	2880.82	455.64	3336.46
9	7	0.0020	3058.90	392.07	3450.97
9	8	0.0017	3236.99	341.37	3578.36
9	9	0.0015	3415.07	301.15	3716.22
9	10	0.0013	3593.15	269.27	3862.42
9	11	0.0012	3771.23	244.00	4015.23
9	12	0.0011	3949.31	223.97	4173.28
9	13	0.0010	4127.40	208.07	4335.47
9	14	0.0010	4305.48	195.46	4500.94
9	15	0.0009	4483.56	185.45	4669.01
10	3	0.0038	2531.39	759.18	3290.57

Table 14 (Continued)

n1	n2	Beta_multiple	CostT	CostD	Cost(D+T)
10	4	0.0030	2709.47	607.28	3316.75
10	5	0.0026	2887.55	515.36	3402.91
10	6	0.0022	3065.63	436.89	3502.52
10	7	0.0019	3243.71	373.29	3617.00
10	8	0.0016	3421.80	322.54	3744.34
10	9	0.0014	3599.88	282.25	3882.13
10	10	0.0013	3777.96	250.29	4028.25
10	11	0.0011	3956.04	224.93	4180.97
10	12	0.0010	4134.12	204.80	4338.92
10	13	0.0009	4312.21	188.81	4501.02
10	14	0.0009	4490.29	176.09	4666.38
10	15	0.0008	4668.37	165.98	4834.35
11	3	0.0037	2716.19	743.27	3459.46
11	4	0.0030	2894.27	591.23	3485.50
11	5	0.0025	3072.35	499.20	3571.55
11	6	0.0021	3250.43	420.72	3671.15
11	7	0.0018	3428.51	357.11	3785.62
11	8	0.0015	3606.60	306.35	3912.95
11	9	0.0013	3784.68	266.02	4050.70
11	10	0.0012	3962.76	234.02	4196.78
11	11	0.0010	4140.84	208.60	4349.44
11	12	0.0009	4318.92	188.40	4507.32
11	13	0.0009	4497.01	172.33	4669.34
11	14	0.0008	4675.09	159.53	4834.62
11	15	0.0007	4853.17	149.34	5002.51
12	3	0.0036	2901.00	729.42	3630.42
12	4	0.0029	3079.08	577.32	3656.40
12	5	0.0024	3257.16	485.24	3742.40
12	6	0.0020	3435.24	406.78	3842.02
12	7	0.0017	3613.32	343.19	3956.51
12	8	0.0015	3791.41	292.43	4083.84
12	9	0.0013	3969.49	252.10	4221.59
12	10	0.0011	4147.57	220.06	4367.63
12	11	0.0010	4325.65	194.60	4520.25
12	12	0.0009	4503.73	174.35	4678.08
12	13	0.0008	4681.82	158.22	4840.04
12	14	0.0007	4859.90	145.36	5005.26
12	15	0.0007	5037.98	135.10	5173.08
13	3	0.0036	3085.81	717.39	3803.20
13	4	0.0028	3263.89	565.28	3829.17
13	5	0.0024	3441.97	473.17	3915.14
13	6	0.0020	3620.05	394.75	4014.80
13	7	0.0017	3798.13	331.21	4129.33
13	8	0.0014	3976.22	280.47	4256.69
13	9	0.0012	4154.30	240.13	4394.43
13	10	0.0010	4332.38	208.08	4540.46
13	11	0.0009	4510.46	182.59	4693.05

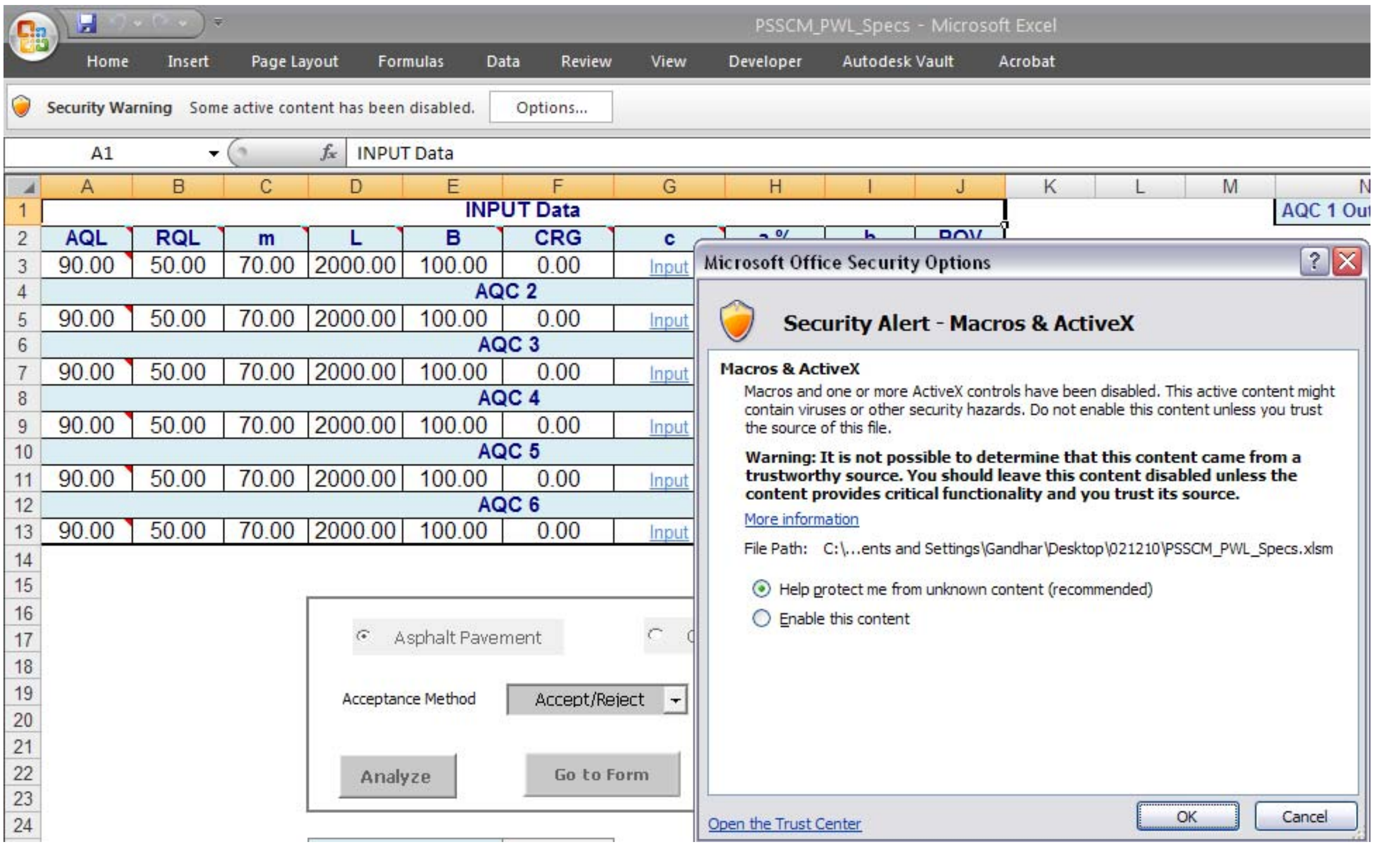
Table 14 (Continued)

n1	n2	Beta_multiple	CostT	CostD	Cost(D+T)
13	12	0.0008	4688.54	162.30	4850.84
13	13	0.0007	4866.63	146.12	5012.75
13	14	0.0007	5044.71	133.21	5177.92
13	15	0.0006	5222.79	122.90	5345.69
14	3	0.0035	3270.62	706.95	3977.57
14	4	0.0028	3448.70	554.86	4003.56
14	5	0.0023	3626.78	462.75	4089.53
14	6	0.0019	3804.86	384.38	4189.24
14	7	0.0016	3982.94	320.88	4303.82
14	8	0.0014	4161.03	270.16	4431.19
14	9	0.0011	4339.11	229.84	4568.95
14	10	0.0010	4517.19	197.78	4714.97
14	11	0.0009	4695.27	172.27	4867.54
14	12	0.0008	4873.35	151.95	5025.30
14	13	0.0007	5051.44	135.73	5187.17
14	14	0.0006	5229.52	122.79	5352.31
14	15	0.0006	5407.60	112.43	5520.03
15	3	0.0035	3455.42	697.88	4153.30
15	4	0.0027	3633.50	545.84	4179.34
15	5	0.0023	3811.58	453.73	4265.31
15	6	0.0019	3989.66	375.42	4365.08
15	7	0.0016	4167.74	311.97	4479.71
15	8	0.0013	4345.83	261.29	4607.12
15	9	0.0011	4523.91	220.97	4744.88
15	10	0.0009	4701.99	188.91	4890.90
15	11	0.0008	4880.07	163.39	5043.46
15	12	0.0007	5058.15	143.04	5201.19
15	13	0.0006	5236.24	126.80	5363.04
15	14	0.0006	5414.32	113.82	5528.14
15	15	0.0005	5592.40	103.43	5695.83

PSSC INSTALLATION

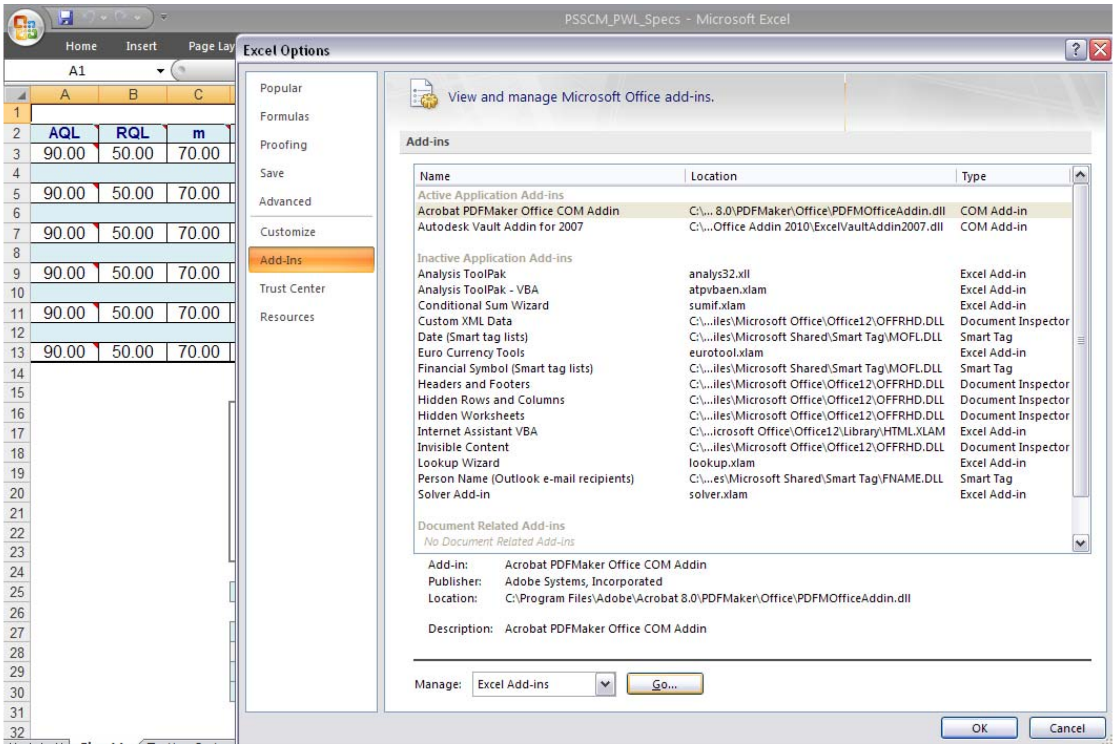
PSSC needs Solver add-in to be installed on computer for analysis purposes. Solver add-in is usually included in MS Excel package. Please follow the screenshots below to install this add-in.

1. Click on “Options…” button displayed on the start-up at the bottom of the tab strip in the Excel. Select “Enable this content”.

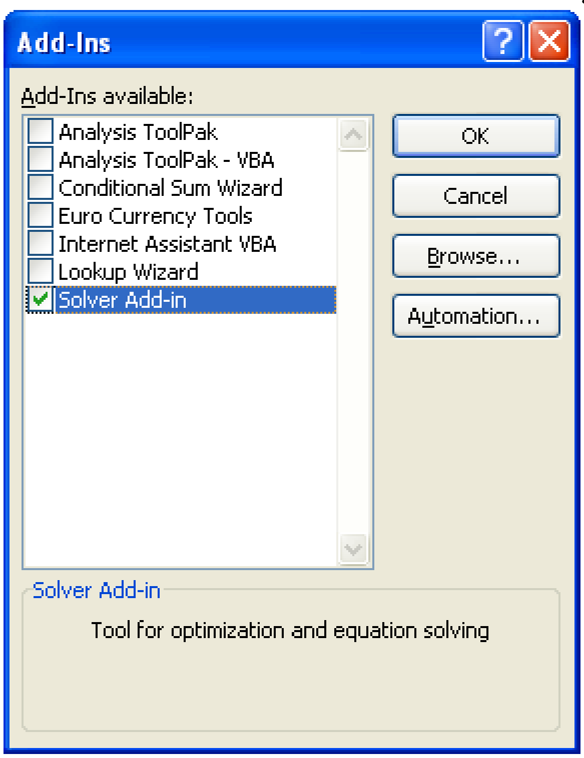


2. Install the built in Solver add-in in the Excel by clicking on the “Excel Options” button from the “Home” menu in Excel.

3. From the “Add-ins” section, click on “Go…” button to select an add-in to install.



4. Select “Solver Add-in” from the next dialog box. Wait until Excel finishes installing add-in.



Curriculum Vita

Gandhar Wazalwar was born on August 15, 1984 in Katol (District: Nagpur), India. He is youngest child of Mrs. Manisha A. Wazalwar and Mr. Anant B. Wazalwar. He graduated from the Benarasidas Ruyia High School and Jr. College, Katol, India, in the summer of 2002 and entered the Bapurao Deshmukh College of Engineering, Sewagram, India in the fall of 2002. After receiving a bachelor's of engineer's degree in Civil engineering from Nagpur University, Nagpur in 2006, he entered the Graduate School at The University of Texas at El Paso in fall 2007. He was teaching assistance for one semester under the mentoring of Dr. Nasir Gharaibeh. He gained extensive research experience under the mentoring of Dr. Nasir Gharaibeh and Dr. Carlos Chang Albritres. His research work included "Methodology to Develop a Minimum Cost Acceptance Sampling Plan for Highway Construction".

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