Using Prediction And Classroom Voting Via Clickers To Address Secondary School Students' Overreliance On The Representativeness Heuristic

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USING PREDICTION AND CLASSROOM VOTING VIA CLICKERS
TO ADDRESS SECONDARY SCHOOL STUDENTS’ OVERRELIANCE ON THE
REPRESENTATIVENESS HEURISTIC

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USING PREDICTION AND CLASSROOM VOTING VIA CLICKERS
TO ADDRESS SECONDARY SCHOOL STUDENTS’ OVERRELIANCE ON THE
REPRESENTATIVENESS HEURISTIC

by

Tami Kay Dashley, B.S.

THESIS
Presented to the Faculty of the Graduate School of
The University of Texas at El Paso
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for the Degree of
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Department of Mathematical Sciences
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Abstract

Students often have misconceptions in their probabilistic reasoning—one such misconception is the overuse of the representativeness heuristic, in which one determines that one event is more likely than another event based on how representative the event is of some aspect of its parent population. A research study was conducted to address high school students’ overuse of the representativeness heuristic using three groups: (i) a prediction-and-voting group in which students were taught a lesson on probability with the use of prediction and classroom voting teaching methods, (ii) a prediction-only group where students were taught the lesson using only prediction, and (iii) a comparison group in which students were taught without the use of prediction or classroom voting. Analyses of pre- and post-test data indicate that while all three student groups showed improvement in their use of the representativeness heuristic; the students in the prediction-and-voting and prediction-only groups did not do statistically better than the comparison group in terms of replacing their overuse of the representativeness heuristic with correct probabilistic reasoning. Test data analyses also revealed that the prediction-and-voting group was the only group that did not show a tendency to inappropriately answer “equally likely” to probabilistic problems. This indicates that prediction and classroom-voting teaching methods might foster deeper levels of thinking and improved judgment in students, as opposed to indiscriminately applying newly learned ideas.
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Chapter 1

Introduction

Probability, the study of likelihood and uncertainty, plays a critical role in the decisions we make, the conclusions we reach, and the explanations we provide. Indeed, many of us are able to apply it in considering the likelihood of events such as the outcome of an election, the upcoming weather conditions, the risk of contracting a certain disease, the state of the financial market, and determining the reliability of evidence in criminal trials based on our knowledge of probability. In fact, an extensive array of research and literature has been devoted to the study of how people perceive, process, and evaluate the probability of uncertain events (Kahneman & Tversky, 1972; Shaughnessy, 1992; Garfield & Ben-Zvi, 2007). One particular area of research is based on the study of one’s reasoning with respect to probability which is more formally known as heuristics (Fischbein & Gazit, 1984). As Konold (1995) noted, everyday reasoning relies on reasoning about probabilities, which often results in one using heuristics to judge probability. While the use of heuristics can be useful to estimate complicated probabilities, the reliance on heuristics in reasoning about probability may lead to misconceptions in how one thinks about probability (Hirsch & O’Donnell, 2001). Misconceptions about probability can not only lead to faulty reasoning in everyday uses of probability, but also in the fields that depend upon determining the likelihood of uncertain events such as medicine, criminal justice, and finance.

The importance of being able to reason correctly about probability was also recognized by the National Council of Teachers of Mathematics (2000) in their Principles and Standards for Mathematics Instruction, which entails that students be able to use
statistical methods such as collecting, organizing, and displaying data and be capable of reasoning about probability and drawing inferences. However, many students fail to overcome their misconceptions about probability even after formal instruction. As Shaughnessy (1977) states, “Anyone who has recently had the opportunity to teach elementary probability and statistics to college students is probably aware of some misconceptions that these students possess prior to, and possibly after, a formal course in probability” (p. 295). Not only can probability misconceptions impede one’s ability to accurately determine the probability of unknown events, it can also lead to errors in statistical reasoning.

The purpose of this study was to examine the effectiveness of including prediction and classroom voting teaching methods in a probability lesson used to assist a group of high school precalculus students with one particular type of probability misconception—their overreliance on the representativeness heuristic.

1.1 Research Questions

The research questions examined in this study were:

(a) In what ways does the use of prediction and classroom voting via clickers contribute to students recognizing and overcoming their misconceptions about representativeness?

(b) To what extent can student probability misconceptions about representativeness be improved using prediction items and/or classroom voting with clickers?
1.2 Methodology

This research study investigated the ways in which the use of prediction and classroom voting via clickers in teaching probability concepts contributed to students overcoming their overreliance on the representativeness heuristic. In the Spring 2009 semester, the researcher taught a probability lesson related to the representativeness heuristic to three high school precalculus sections, with approximately 25 students within each section. Each precalculus section was taught as follows: one section was taught the lesson without the use of prediction and classroom voting, one section was taught the lesson using only the prediction teaching method, and the last section was taught the lesson using both prediction and classroom voting using clickers. To determine the ways in which the use of prediction and classroom voting via clickers contributed to students overcoming their misconceptions about representativeness, the following data was gathered: pre-tests and post-tests which assessed students’ use of the representativeness heuristic, interviews with students from each class section, observational data, and student-completed surveys.

1.3 Limitations and Delimitations

The students selected for this study were enrolled in the precalculus classes of two high school mathematics teachers that agreed to take part in this study. Unfortunately, due to the constraints of the research proposal and timeframe for the study, students could not be randomly assigned to the comparison prediction-only, and prediction-and-voting groups. Since the students were chosen because of their availability to the researcher and were not randomly assigned to the comparison and treatment groups, these students cannot be
assumed to be a representative sample of the population of high school students. Therefore, definitive generalizations about this study’s results cannot be made. It is also not possible to determine if the results of this study would have been different if a more representative sample of high school students were used. However, insights about how students demonstrate and interact intellectually with the representativeness heuristic can be taken from this study.

All three precalculus class sections used in this study were from the same high school and were classified under the same student registration criteria. It was important to limit the amount of variation among the three student groups so that comparisons could be made between the three precalculus sections. Unfortunately, there was no one precalculus teacher available at the time of this study that had three precalculus class sections. However, one precalculus teacher who agreed to take part in the study had two class sections which were randomly assigned as the comparison group and prediction-and-voting group, while another precalculus teacher’s class took part in the study as the prediction-only group. This study would have been further strengthened if all three class sections had the same precalculus teacher. Overall, variation among the three class sections was reduced by purposefully choosing the classes so that they resided in the same high school and were open to be taken by the same group of students.

This research study was completed using students in high school precalculus classes since probability is often taught during this course and was featured in the school district’s textbook for this grade-level, which were necessary features in order to obtain school district approval for this study. However, the results of this study might have been
different if a different high school grade level was chosen—such as Algebra I, Algebra II, Geometry, or Calculus—in which probability concepts are not usually taught.
2.1 Types of Probability Misconceptions

While probability misconceptions occur in a variety of different problem situations and contexts, researchers have classified four of the more common misconceptions about probability as: the overuse/overreliance of the availability heuristic, the application of the equiprobability bias, the employment of the outcome approach, and the overuse/overreliance of the representativeness heuristic (Fischbein & Schnarch, 1997; Kahneman & Tversky, 1972; Konold, 1995; Lecoutre, 1985; Lecoutre, 1992; Morsanyi, Primi, Chiesi, & Handley, 2009; Shaughnessy, 1977; Shaughnessy, 1992; Tversky & Kahneman 1974). The first type of probability misconception, the availability heuristic, is when one estimates the likelihood of events based on how easy it is to call to mind an instance of the event (Shaughnessy, 1992). While the availability heuristic can assist in estimating the probability of some situations, the complete reliance on the availability heuristic to determine the probabilities of events is considered a misconception. Another type of probability misconception, the equiprobability bias is described by Lecoutre (1985) as the tendency for people to think of random events as “equiprobable” by nature, and judge outcomes that occur with different probabilities—such as probability of the following outcomes (when rolling two dice simultaneously): getting a five on one die and a six on the other, or getting sixes on both dice—to be equally likely (Lecoutre, 1985; Lecoutre, 1992; Morsanyi et al., 2009). In the third type of probability misconception, students use the outcome approach when they perceive each trial on an experiment as a separate, individual phenomenon, and believe their task is to correctly decide for certain
what the next outcome of the experiment will be rather than to estimate what is likely to occur (Konold, 1995). Finally, the representativeness heuristic involves determining the likelihood of events based on how well an outcome represents some aspect of its parent population (Shaughnessy, 1992). The representativeness heuristic is valid for some probability and statistics situations, such as extending the characteristics of a large random sample to that of the entire population from which it was drawn (Shaughnessy, 1992). However, relying on the representativeness heuristic for all or many probability and statistics situations is considered a misconception.

2.2 The Representativeness Heuristic

The main focus of this study is to investigate the probability misconception that falls under the representativeness heuristic. As Kahneman and Tversky (1972) explain

A person who follows [the representativeness] heuristic evaluates the probability of an uncertain event, or a sample, by the degree to which it is: (i) similar in essential properties to its parent population; and (ii) reflects the salient features of the process by which it is generated. Our theory is that, in many situations, an event A is judged more probable than an event B whenever A appears more representative than B.

(p. 431)

For example, students who utilize the representativeness heuristic believe that in a series of six tosses of a fair coin, where H represents a result of heads and T represents the result of tails, the sequence HTTHTH is more likely to occur than the sequence HHHHTT. This reasoning is due to the fact that the first sequence, HTTHTH, appears to be more representative of the 50-50 distribution of heads and tails than the sequence
HHHHTT. However, the probability of each sequence occurring in a series of six coin tosses is the same. Kahneman and Tversky (1972) go on to state that, “To be representative, it is not sufficient that an uncertain event be similar to its parent population. The event should also reflect the properties of the uncertain process by which it is generated, that is, it should appear random” (p.434). Therefore, for the coin toss problem, a student who uses the representativeness heuristic would consider the sequence HTTHTH to be more likely than the sequence TTTHHH even though both sequences share a 50-50 distribution of heads and tails. The selection of the sequence HTTHTH is more likely because it appears more random than the sequence TTTHHH. However, once again the probability of each sequence occurring in a series of six coin tosses is the same. The representativeness heuristic is also demonstrated in what is called the gambler’s fallacy, which is the belief that after a long sequence of one outcome, the other outcome is more likely to occur (Kahneman & Tversky, 1972; Shaughnessy 1992). For instance, after several outcomes of heads when tossing a fair coin, tails would be considered to be more likely to come up as the next outcome. The gambler’s fallacy can be attributed to the belief that a representative sample of a population is one in which the characteristics of the parent population are represented not only globally in the entire sample, but also locally in each of its parts (Kahneman & Tversky, 1972). Therefore, in a series of six coin tosses, a student possessing the belief of local representativeness would expect the outcome to contain 3 heads and 3 tails. This belief of local representativeness is also called the belief in the law of small numbers (Kahneman & Tversky, 1972). Kahneman and Tversky (1972) noted that unlike the law of large numbers that ensures that very large samples will represent the probability and similar distributions of the
population from which they are drawn, the law of small numbers is the erroneous belief
that the law of large numbers applies to small samples of the population as well.

Unfortunately, the representativeness heuristic is not limited to naïve students.
Researchers Kahneman and Tversky (1971, 1974) found that even psychologists who had
substantial training in probability and statistics demonstrated misconceptions about
probability, including representativeness. If fact, many studies show that instruction in
probability and statistics is not necessarily sufficient for students to overcome their
misconceptions about probability (Hirsch & O’Donnell, 2001; Kahneman & Tversky,
1972; Shaughnessy, 1977). Therefore, the task of mathematics and statistics educators
becomes determining what teaching methods help students to effectively overcome and
correct their misconceptions about probability. Since the 1970s, numerous educational
research studies have been conducted in order to find an effective way to teach
probability so that students can overcome their misconceptions.

2.3 Teaching Methods

Research has shown that not all forms of instruction in probability and statistics assist
students in overcoming their probability misconceptions. Instead, researchers believe
that the “conventional lecture approach to the teaching of elementary probability and
statistics may not be the best way to overcome students’ misconceptions about
probability” (Shaughnessy, 1977, p.298). The work of Konold (1995), Shaughnessy
(1977), and Tversky & Kahneman (1971, 1974) particularly show that misconceptions
about probability, such as representativeness, appear to be psychological in nature and are
resistant to elimination during typical classroom instruction. Therefore, in order to help students overcome and correct their misconceptions of probability it is not sufficient to simply teach the underlying concepts of probability. Instead, instruction should be structured in order to make students aware of and effectively confront their misconceptions regarding probability.

As Fischbein and Schnarch (1997) suggested, it is not enough to simply present a probability problem and its correct solution—one must analyze the structure of the corresponding misconceptions. If instruction can lead students to actively experience the conflicts between their reasoning and conceptions regarding probability, and the correct reasoning of probability situations, then students can learn to analyze the causes of their misconceptions, and consequently they may be able to overcome them and attain a genuine probabilistic way of thinking (Fischbein & Schnarch, 1997).

In Hirsch and O’Donnell’s (2001) study involving various instructional interventions structured to help students overcome the representativeness heuristic, they aimed for participants to confront their misconceptions and experience cognitive conflict. In the Hirsch and O’Donnell (2001) study, cognitive conflict referred to the resulting conflict between participants’ misconceptions and correct probabilistic reasoning when participants were presented with correct probabilistic reasoning. The Hirsch and O’Donnell (2001) study suggests that if students are taught using instructional methods that create cognitive conflict and assist them in correctly resolving the conflict of their misconceptions with correct probabilistic reasoning, then students may be able to overcome their probability misconceptions. Research suggests that instructional methods that create cognitive conflict involve asking students to make predictions about certain

2.4 The Use of Prediction in Teaching

Lim (2008) suggests that prediction is not simply foretelling an outcome prior to observing the outcome of an event; prediction is conceiving an expectation of a certain result before performing the detailed operations associated with the result. In terms of mathematics and statistics education, prediction means stating one’s expectation for the answer to a problem or the outcome of an experiment prior to solving the problem or completing the experiment. Asking students to predict a result therefore frees them from the need for precision and accuracy of answers and instead makes the focus of the activity their conceptions, beliefs, intuitions, prior knowledge, and misconceptions about the topic at hand (Kim & Kasmer, 2007; Lim, Kim, Cordero, Buendía, & Kasmer, 2007). According to Fischbein and Grossman (1997), asking students to predict a result can assist students to overcome their misconceptions and improve their understanding of the underlying topic.

Encouraging the learner to guess intuitively, one creates a challenging situation. Another way of achieving this is through facing the student with a conflict between a personal guess and a mathematically accepted solution. Such a conflict may stimulate the interest of the learner and may help him or her to overcome his or her intuitive obstacles. Moreover, this understanding may contribute to the understanding of the mechanisms that shape the answer. (pp. 43-45)

Therefore, asking students to predict a mathematical or statistical result can aid in revealing students’ misconceptions; by comparing student misconceptions with correct mathematical reasoning, one can experience cognitive conflict.
Several research studies have shown that when students are asked first to make predictions about probability events and then compare their predictions with actual probability results, thereby creating cognitive conflict for the students, the students seem to learn more accurately the probability concepts (Garfield & Ben-Zvi, 2007; Hirsch & O’Donnell, 2001; Shaughnessy 1977). In fact, delMas and Bart (1987) conjectured that unless students record their predictions and then compare their predictions to correct probabilistic results, students will look for evidence to confirm their misconceptions. One way in which students can record their predictions is through classroom voting.

2.5 The Use of Classroom Voting in Teaching

Cline, Zullo, and Parker (2007b) define classroom voting as an instructional technique in which the instructor poses a multiple-choice or true/false question to the class, waits for a brief period of consideration, then has students vote for an answer, either by holding up colored index cards (a = red, b = blue, etc.) or by using a computerized personal response system such as a hand-held clicker. The term clicker generally refers to the technology in which each student uses a remote-like device to enter a response/vote that is received via an infrared receiver attached to a computer. The essential feature of the clicker technology is that classes of any size can use clickers to vote, and then both students and teachers can immediately view a bar graph showing the distribution of the responses (Draper, Cargill, & Cutts, 2002). The displayed bar graph shows the percentage of the class that voted for each answer choice, not a detailed breakdown of who voted for which answer choice. As a result, clickers can give students a safe voting environment in which
they have the opportunity to vote honestly without the peer-pressure to answer correctly. Classroom voting via clickers can be very useful because it makes students develop and record their prediction about the correct response to a question, provides immediate feedback to the teacher and students about the students’ responses, and supplies an answer distribution that can be used to develop meaningful classroom discussions about the question’s topic (Cline, 2006).

While some applications of clickers entail using them strictly as a method of assessment, research suggests that the most useful application of clickers is for initiating discussions among students (Cline, 2006; Draper et al., 2002). Questions for clicker lessons should be structured so that no answer-choice receives a majority of the votes; instead, two to four different answer choices should have similar voting percentages so that students can discuss their rationale for voting for each answer-choice (Cline, Zullo, & Parker, 2007a). As Cline et al. (2007a) explained, it is usually in the discussion of the answer choices that students uncover the appropriate logic of the problem and are able to obtain the correct answer to the problem without the instructor’s input. Cline (2006) also suggests that the most effective questions for classroom voting are designed to elicit common errors and misconceptions because “knowing where many students will go wrong, the teacher can use voting to confront and deal with their mistakes from the outset” (p.102).
Chapter 3

Methodology

This research study investigates how to help students overcome their probability misconceptions with respect to the representativeness heuristic. Research has shown that contrasting students’ conceptions regarding the representativeness heuristic with correct probabilistic thinking and reasoning can assist students in recognizing and overcoming their misconceptions. Consequently, this study was designed to investigate the ways in which prediction and classroom voting using clickers contributed to students effectively recognizing and overcoming their overuse or overreliance on the representativeness heuristic.

The study was designed to answer the following questions:

(a) In what ways does the use of prediction and classroom voting via clickers contribute to students recognizing and overcoming their misconceptions about representativeness?

(b) To what extent can student probability misconceptions about representativeness be improved using prediction items and classroom voting with clickers?

3.1 Participants

This study was conducted at a Texas public high school near the United States-Mexico border. The school’s student population was predominantly Hispanic (89%) and fifty-three percent of the students were considered economically disadvantaged (see Table 1).
Table 1: School Demographics (Texas Education Agency, 2008)

<table>
<thead>
<tr>
<th>Ethnicity</th>
<th>School Population Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hispanic</td>
<td>89 %</td>
</tr>
<tr>
<td>White</td>
<td>8 %</td>
</tr>
<tr>
<td>Black</td>
<td>2 %</td>
</tr>
<tr>
<td>Asian or Pacific Islander</td>
<td>&lt; 1 %</td>
</tr>
<tr>
<td>Native American</td>
<td>&lt; 1 %</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student Subgroups</th>
<th>School Population Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economically Disadvantaged</td>
<td>53 %</td>
</tr>
<tr>
<td>Special Education</td>
<td>8 %</td>
</tr>
<tr>
<td>Gifted/Talented Students</td>
<td>10 %</td>
</tr>
<tr>
<td>Limited English Proficient</td>
<td>7 %</td>
</tr>
</tbody>
</table>

The participants used in the study consisted of the students from three sections of precalculus classes from the high school. In total, the precalculus sections consisted of 76 students, with 3 students who declined to participate in the study. The students were aged from 16 to 18 years, depending upon whether they were taking the course as a high school junior or as a high school senior. The participants of the study were selected to be precalculus students because probability and statistics is often taught in high school precalculus courses and probability concepts were included in the school district’s precalculus textbooks. The classes that participated in this study had not received any instruction in probability or statistics in the classes prior to the start of the study.

One of each of the precalculus sections served as the following:

(a) A comparison group which consisted of 28 students and in which 3 students declined to participate in the study—leaving 25 students or 89.3% of the class that participated in the study.
(b) A prediction-only group in which the class was asked to predict probabilistic results. This precalculus section consisted of 23 students in which all students agreed to participate in the study.

(c) A prediction-and-voting group in which the class was asked to predict probabilistic results and vote for their prediction using clickers. This precalculus section consisted of 25 students in which all students agreed to participate in the study.

The comparison group, prediction-only group, and prediction-and-voting group were randomly assigned to the three available precalculus sections. The comparison group and prediction-and-voting group had the same precalculus teacher; the prediction-only group had a different precalculus teacher. The probability lessons used in this study were taught solely by the researcher to all three precalculus sections.

To determine if all three precalculus sections had equivalent knowledge of mathematics, the proportion of students that passed the 10th grade mathematics portion of the Texas Assessment of Knowledge and Skills (TAKS) Test was compared for each of the three sections. The proportion of students that passed or achieved commended performance of the 10th grade math portion of the TAKS Test for each of the three Pre-Calculus sections is shown in Table 2. All of the p-values for comparing any of the proportions listed in Table 2 failed to reach the 0.05 significance level. Therefore, the null hypothesis that any two proportions in each row are equivalent fails to be rejected and the proportions in each row can be considered statistically equivalent. The 10th grade mathematics TAKS test performance can therefore be considered equivalent for the three
class sections which suggests that all three precalculus sections had comparable levels of mathematics knowledge.

Table 2: 10th Grade TAKS Test Performance for Comparison and Treatment Groups

<table>
<thead>
<tr>
<th></th>
<th>Comparison Group</th>
<th>Prediction-Only Group</th>
<th>Prediction-and-Voting Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion of Students that Passed the 10th grade Math Portion of TAKS Test</td>
<td>20 out of 22</td>
<td>23 out of 23</td>
<td>23 out of 24</td>
</tr>
<tr>
<td>Proportion of Students that got Commended Performance on the 10th grade Math Portion of TAKS Test</td>
<td>4 out of 22</td>
<td>6 out of 23</td>
<td>2 out of 24</td>
</tr>
</tbody>
</table>

* p-value < 0.05

3.2 Research Design

To determine the ways in which inclusion of prediction and classroom voting using clickers into a probability lesson assisted students in recognizing and overcoming their overreliance on the representativeness heuristic, the researcher completed the following: introduction of the study to students, a probability review lesson, administration of a pre-test, a Representativeness Heuristic lesson, administration of two post-tests, and student interviews (see Figure 1).

As shown in Figure 1, the study had a mixed method research design in which quantitative data collection and analysis was the dominant research method. However, qualitative data in the form of lesson video recordings and student interviews were completed in order to support and interpret the results obtained by the quantitative data analysis.
3.2.1 Probability Review Lesson

The first probability lesson served as a general review of probability concepts taught in elementary and middle school grade levels. This review lesson consisted of a PowerPoint Presentation of the following probability concepts: sample space, event, outcome, complement, theoretical (classical) probability, experimental (relative frequency)
probability, independence, and tree diagrams (see Appendix A to view the slides for this presentation). Since the three precalculus sections were taught by two different precalculus teachers, the students in the three sections might not have had the same exposure and knowledge of the basic concepts of probability. The review lesson served to provide all students with a similar knowledge of probability before beginning the study. All three class sections were taught the same review lesson. The review lesson was taught by the researcher in one 90-minute class period a week before the administration of the study’s pre-test.

3.2.2 Representativeness Heuristic Lesson

The second probability lesson was presented by the researcher during one 90 minute class period one week after the administration of the pre-test. The lesson consisted of a PowerPoint Presentation of five multiple choice questions (which were screened for face validity by two college professors who have taught probability) that were structured so that students using the representativeness heuristic would select certain answer choice(s) and students using correct probabilistic reasoning would select a different answer choice (see Appendix B to view the slides for this presentation).

The prediction-and-voting precalculus section was presented the PowerPoint presentation in the following manner:

1. One of the multiple choice questions was posted and the students were asked to mentally predict the correct answer choice based on what they knew about probability and what they thought was the correct answer. (Students were allowed to employ any
resources in making their prediction, such as using calculators or solving the problem using pencil and paper methods.)

2. Students were asked to write down their reasoning on a supplied student response sheet (shown in Appendix C) and send in their vote using their clickers. Time taken for students to work out the individual questions and send in their vote varied depending on students’ ability to complete the question in a timely manner. This type of distribution, as shown research on classroom voting (Cline et al., 2007a), makes students curious about the correct answer and leads to richer student-to-student and whole-class discussions. When all or a large majority of the students selected an answer choice(s) indicating reliance on the representativeness heuristic then this type of situation, according to Cline et al. (2007a), will lead to richer whole-class discussion because students will be intrigued as to why their answer(s) is incorrect.

3. After all students voted using their clickers, the answer-choice distribution was shown to the class and students were given three to four minutes to discuss their reasoning for selecting their answer within their student group of three to four students. Student groups were selected to consist of at least three students in order to group enough students together so that multiple rationales and viewpoints could be discussed. However, the group size was limited to four students so that the student groups could effectively include each member’s rationale within the three to four minute time limit. The small-group discussions were intended to foster cognitive conflict so that the students could become aware of their misconceptions regarding representativeness and incorporate correct probabilistic reasoning into their conceptions about probability.
4. Once students had discussed their answer choices within their small-groups, the students were asked to volunteer to share their reasoning for selecting their initial answer choices with the whole class.

5. Lastly, slides from the PowerPoint Presentation (shown in Appendix B) were presented that used probability concepts from the probability review lesson to explain why each answer choices was correct and/or incorrect for the particular multiple-choice question being discussed.

A second precalculus class section served as the prediction-only group and was presented with the exact same PowerPoint Presentation (shown in Appendix B) lesson but were asked only to predict the correct answer and record their reasoning on the student response sheet (shown in Appendix C) before completing the student group discussion of the answer choices. The students were not asked to vote for their answer-choice and clickers were not provided to the students. The students discussed the multiple choice questions in their student groups and as a class.

The third precalculus section served as the comparison group for the study and was presented with the lesson without being asked to make predictions or vote for their answer choices. The students were presented with the exact same PowerPoint Presentation (shown in Appendix B) and discussed the multiple choice questions in their student groups and as a class. However, each student group was asked to fill out a student response sheet (shown in Appendix D) instead of each student filling out an individual student response sheet (shown in Appendix C). Students were asked to fill out the student response sheet as a group—instead of individually—so that students would
discuss and reason with their group members and thereby missed the opportunity to predict the answers on their own.

The three precalculus sections were randomly selected for their assignment as the comparison group (being taught the lesson without the prediction and classroom voting teaching methods), the prediction-only group (being taught the lesson only using the prediction teaching method), and the prediction-and-voting group (being taught the lesson using both prediction and classroom voting method). It turned out that the prediction-and-voting group and comparison group were taught by the same precalculus teacher—while the prediction-only group was taught by a different precalculus teacher. The fact that the comparison and prediction-and-voting groups had the same teacher is useful because it means that the two groups had very similar mathematical teaching backgrounds when the study was completed.

3.2.3 Representativeness Heuristic Test Design

The test questions used for the pre-test and post-tests used in this study are shown in Appendix E. The test questions consisted of a two-part multiple-choice format (taken from Hirsch & O’Donnell’s test used in their 2001 study regarding representativeness) in which students first selected their answer choice to a question, and then on the second part selected the answer-choice that most closely explained their reasoning for their answer selection—a sample test question is shown below.
A fair coin is tossed, and it lands heads up. The coin is tossed a second time. What is the probability that the second toss is also a head?

a. 1/2  
b. 1/3  
c. 1/4  
d. Slightly less than 1/2  
e. Slightly more than 1/2

Which of the following best describes the reason for your answer to the preceding question?

a. The chance of getting heads or tails on any one toss is always 1/2.  
b. The second toss is less likely to be a head because the first toss was a head.  
c. There are three possible outcomes when you toss a coin twice. Getting two heads is only one of them.  
d. There are four possible outcomes when you toss a coin twice. Getting two heads is only one of them.  
e. Other ________________________________

Each multiple-choice question involved a probability problem that could be answered using the representativeness heuristic, or by using correct probabilistic reasoning.

The test questions were based on the following five types of questions:

(a) determining the student’s ability to understand the independence of probabilistic events (questions coded A1, A2, A3, and A4)

(b) determining the student’s use of the “equally likely” response, also known as equiprobability bias (Lecoutre, 1985; Lecoutre, 1992), in which students answered that all probabilistic events were equally likely when the events actually had different probabilities of occurring (questions coded B1, B2, B3, and B4)

(c) determining the student’s use of the gambler’s fallacy for the representativeness heuristic (questions coded C1, C2, C3, and C4)

(d) determining the student’s use of the representativeness heuristic for problems involving flipping a fair coin (questions coded D1, D2, D3, and D4) and
(e) determining the student’s use of the representativeness heuristic for problems involving rolling a fair die (questions coded E1, E2, E3, and E4).

Test questions A1, A2, C1, C2, D1, D2, D3, D4, E1, E2, E3, and E4 were taken from Hirsch and O’Donnell’s (2001) study regarding creating a valid and consistent test for students’ use of the representativeness heuristic. The researcher obtained written permission from the creator of the representativeness heuristic test, Dr. Linda Hirsch, before beginning the study.

Questions A3 and A4 were created by the researcher based on Dr. Hirsch’s representativeness heuristic test in order to have enough questions to create two ten-question versions of the representativeness heuristic test to use in the study.

Questions B1, B2, B3, and B4 were created by the researcher in order to determine students’ use of the “equally likely” learned response. Due to the structure of the test questions, the researcher wanted to evaluate if or when students tended to answer that all probabilistic events were equally likely when the events listed actually had different probabilities of occurring. The action of answering that all probabilistic events are equally likely when it is inappropriate for a given problem situation was considered a learned response because students more frequently demonstrated the use of this action during the course of the study (i.e. after the pre-test). Since no other form of probability instruction was occurring, aside from the study itself, students were learning to respond to probability questions by selecting the “equally likely” answer choice due to some aspect of the study. Answering that probability questions are “equally likely” when it is inappropriate to do so has also been categorized in the probability and statistics literature as use of the equiprobability bias, in which one thinks of random events as
“equiprobable” by nature and judges outcomes that occur with different probabilities to be equally likely (Lecoutre, 1985; Lecoutre, 1992; Morsanyi et al., 2009).

### 3.3 Data Collection

To assess student learning and improvement of their use of the representativeness heuristic—assessed by the reduction of instances in which they relied on the representativeness heuristic to answer probability questions—the students each completed one pre-test and two post-tests. It is important to note that due to absence on the testing dates—not all students completed the pre-test and both post-tests. In the comparison group from the 25 students that agreed to be in the study, six students had not taken one or more of the tests—making 19 students that had completed all of the pre-test and post-tests. Of the 23 students in the prediction-only class, three students had not taken at least one of the tests—leaving 20 students with complete test data. In the prediction-and-voting class, five of the 25 students in the prediction-and-voting class did not take all of the pre-test and post-tests—leaving 20 students with complete test data.

#### 3.3.1 Representativeness Heuristic Pre-Test

Within the week prior to the Representativeness Heuristic lesson, the pre-test was administered to all of the precalculus sections. The pre-test consisted of ten questions, two questions from each of the five question type groups, taken from the possible list of twenty test questions (shown in Appendix E). The specific test items selected for the pre-test are shown in Table 3. The order in listing the types of questions on the Pre-test was
used so that similar questions were not placed close together within the test. By having the same types of test questions spaced in different areas of the Pre-test the researcher hoped to have students face each question with close examination instead of answering similar questions in the same manner without closely examining the question. The purpose of the pre-test was to determine which students overused the representativeness heuristic before the representativeness heuristic lesson.

Table 3: Test Items Selected for Use in the Pre-test

<table>
<thead>
<tr>
<th>Question #</th>
<th>Pre-Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A1</td>
</tr>
<tr>
<td>2</td>
<td>D1</td>
</tr>
<tr>
<td>3</td>
<td>B1</td>
</tr>
<tr>
<td>4</td>
<td>E1</td>
</tr>
<tr>
<td>5</td>
<td>C1</td>
</tr>
<tr>
<td>6</td>
<td>A2</td>
</tr>
<tr>
<td>7</td>
<td>D2</td>
</tr>
<tr>
<td>8</td>
<td>B2</td>
</tr>
<tr>
<td>9</td>
<td>E2</td>
</tr>
<tr>
<td>10</td>
<td>C2</td>
</tr>
</tbody>
</table>

3.3.2 Representativeness Heuristic Post-Tests

Two post-tests were given to all precalculus sections, at one-week and six-week intervals after the Representativeness Heuristic lesson was presented. The first post-test consisted of the ten questions, with two questions from each of the five questioning strategy groups, that were not selected for use in the pre-test. The second post-test served as a retention test and consisted of the same questions that were used in the pre-test. Since six weeks—including a two-week spring break vacation period from school—had elapsed since the administration of the pre-test, test familiarity was believed to not affect the student responses to the second post-test. The specific test items selected for the post-tests are shown in Table 4.
The first post-test served to determine which students were able to improve their overreliance on the representativeness heuristic in terms of the reduction of instances in which their answers to the tests demonstrated the use of the representativeness heuristic. The types of questions used in Post-test 1 were placed in the reverse order of those in Pre-test. The second post-test had identical questions to those used in Pre-test. The last post-test provided information about which students were able to maintain any improved use of the of the representativeness heuristic over time.

### Table 4: Test Items Selected for Use in the Post-tests

<table>
<thead>
<tr>
<th>Question #</th>
<th>Post-Test 1</th>
<th>Post-Test 2 (same items as Pre-Test)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C4</td>
<td>A1</td>
</tr>
<tr>
<td>2</td>
<td>E4</td>
<td>D1</td>
</tr>
<tr>
<td>3</td>
<td>B4</td>
<td>B1</td>
</tr>
<tr>
<td>4</td>
<td>D4</td>
<td>E1</td>
</tr>
<tr>
<td>5</td>
<td>A4</td>
<td>C1</td>
</tr>
<tr>
<td>6</td>
<td>C3</td>
<td>A2</td>
</tr>
<tr>
<td>7</td>
<td>E3</td>
<td>D2</td>
</tr>
<tr>
<td>8</td>
<td>B3</td>
<td>B2</td>
</tr>
<tr>
<td>9</td>
<td>D3</td>
<td>E2</td>
</tr>
<tr>
<td>10</td>
<td>A3</td>
<td>C2</td>
</tr>
</tbody>
</table>

### 3.3.3 Observational Data

In order to determine the ways in which the use of prediction and classroom voting might have contributed to students’ overcoming of their misconceptions about representativeness, observational data was gathered by video-taping each of the Representativeness Heuristic Lessons presentations for the three precalculus sections. The videotapes were viewed by the researcher to determine any differences that be observed between the comparison, prediction-only, and prediction-and-voting groups in the following areas:
1. Researcher’s presentation of the Representativeness Heuristic Lesson,
2. Researcher’s interaction with student groups during the lesson,
3. Students’ answers to the five representativeness questions presented during the lesson,
4. Students’ level of interaction during the representativeness heuristic lesson, and
5. Students’ learning displayed during the lesson.

The videotapes were also viewed by a collaborating teacher that had not previously seen the videotapes or was present for the Representativeness Heuristic Lessons. The collaborating teacher examined the videotapes for the same five criteria listed above. The researcher and the collaborating teacher compared their results to the five criteria and consolidated their results anywhere differences occurred in their conclusions.

### 3.3.4 Representativeness Heuristic Lesson Survey

A survey (shown in Appendix F) was administered at the end of the Representativeness Heuristic Lesson for each of the three precalculus sections to ascertain students' beliefs about whether they found the lesson to be engaging and interesting. To determine if the eight survey items actually individually measured students' perceived engagement and interest in the lesson, a factor analysis of the survey items was completed using the survey responses from all three precalculus sections. The statistical program Minitab was used to generate the scree plot of the eigenvalues for the survey items as shown in Figure 2. The scree plot shows that only factor one, with \( \lambda = 2.6908 \), and factor two, with \( \lambda = \)
1.25407, had eigenvalues greater than one. Since factor one and factor two have
eigenvalues greater than one, they can be considered as distinct factors in the survey
items. Unrotated factor loadings were calculated for the eight survey items as shown in
Table 5, where factor loadings with values greater than 0.40 (italicized in Table 5) were
considered to indicate that the survey item belonged to that factor. Factor one consists of
survey items 1, 2, 3, 6, 7, and 8 while factor two consists of survey items 4 and 5.
Closer examination of the survey items revealed that items that fall under factor one
(items 1, 2, 3, 6, 7, and 8) represent the student’s interest and/or engagement in learning
the probability topics presented in the Representativeness Heuristic Lesson, whereas
survey items that fall under factor two (items 4 and 5) represent the student’s
participation in class discussion during the lesson. The survey can therefore be
considered an adequate measurement of student engagement/interest and participation
during the Representativeness Heuristic Lesson.

Figure 2: Scree Plot of the Eigenvalues for the Survey Items
Table 5: Unrotated Factor Loadings for Factors 1 and 2

<table>
<thead>
<tr>
<th>Survey Item</th>
<th>Factor 1 Loading</th>
<th>Factor 2 Loading</th>
<th>Communality</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.572</td>
<td>0.076</td>
<td>0.334</td>
</tr>
<tr>
<td>2</td>
<td>0.429</td>
<td>0.344</td>
<td>0.303</td>
</tr>
<tr>
<td>3</td>
<td>0.667</td>
<td>0.112</td>
<td>0.458</td>
</tr>
<tr>
<td>4</td>
<td>0.344</td>
<td>0.689</td>
<td>0.594</td>
</tr>
<tr>
<td>5</td>
<td>0.151</td>
<td>-0.796</td>
<td>0.656</td>
</tr>
<tr>
<td>6</td>
<td>0.770</td>
<td>0.078</td>
<td>0.599</td>
</tr>
<tr>
<td>7</td>
<td>0.760</td>
<td>0.055</td>
<td>0.581</td>
</tr>
<tr>
<td>8</td>
<td>0.653</td>
<td>-0.016</td>
<td>0.426</td>
</tr>
<tr>
<td>Variance</td>
<td>2.6961</td>
<td>1.2541</td>
<td>3.9501</td>
</tr>
<tr>
<td>Proportion Variance</td>
<td>0.337</td>
<td>0.157</td>
<td>0.494</td>
</tr>
</tbody>
</table>

3.3.5 Student Interviews

Finally, the researcher completed semi-structured interviews with one student from each of the precalculus sections that demonstrated the most reduction in their overreliance on the representativeness heuristic between their pre-test and the two post-tests. The semi-structured interview questions (listed in Appendix G) consisted of two parts. The first part of the interview (question 0 in Appendix G) consisted of inquiring about the student’s reasoning behind selecting their answers on the pre-test and second post-test. The second post-test was selected since it had questions identical to the pre-test, which allowed students to more effectively compare their answers on the pre-test and post-test 2. All students were asked about their reasoning for selecting their answers in questions 2, 3, 4, 5, 7, and 8 on the pre-test and second post-test, which had identical question items. The second part of the interview (questions 1 through 8 in Appendix G) involved asking students to rate their reasoning abilities throughout the study and to describe the
aspects of the Probability Review Lesson and Representativeness Heuristic Lesson that contributed toward their improvement in probabilistic reasoning. The interviews were video-taped, transcribed, and coded by the researcher.
Chapter 4

Results

4.1 Representativeness Heuristics Tests

The Representativeness Heuristic Tests, shown in Appendix E, each had ten questions in which each question consisted of a two-part, multiple-choice format. Students first selected their answer choice to a test question, and then on the second part selected the answer-choice that most closely explained their reasoning for their answer selection. The second multiple-choice section for each question provided a reasoning option of “e. other” in which students could write in their reasoning for selecting their answer to the test question if their reasoning was not already described in the other answer choices. Each multiple-choice question involved probability questions that could be answered using the representativeness heuristic or by using correct probabilistic reasoning. A rubric (shown in Appendix H) was made prior to the administration of the tests in which each test question was scored in the following manner:

1. Students that selected the correct answer choice in the first part of the question and provided correct probabilistic reasoning in the second part of the question were scored as having answered the question correctly.

2. Students that selected an incorrect answer choice in the first part of the question and provided incorrect probabilistic reasoning in the second part of the question were scored as having answered the question wrong. (It is important to note, that student’s reasoning that lead to the selection of “wrong” answer choices may not simply be the incorrect reasoning of the student but might indeed be the use of a different type of heuristic or misconception by the student.)
3. Students that selected the answer choice indicating use of the representativeness heuristic in the first part of the question and provided probabilistic reasoning supporting the representativeness heuristic in the second part of the question were scored as having answered the question as using the representativeness heuristic (RH).

4. Students that selected the correct answer choice in the first part of the question but provided probabilistic reasoning supporting the representativeness heuristic in the second part of the question were also scored as having answered the question as using the representativeness heuristic (RH).

5. Students that selected an answer choice in the first part of the question and an answer choice in the second part of the question that was not listed together in the test rubric (see Appendix H) were scored as having provided a inconclusive or “not a matched” response. Students’ selections that fell under this scoring usually occurred because students selected contradictory answer selections for the two multiple-choice parts. For an example of this, if a student selected answer choice C for part one and answer choice C for part two of the following question:

A bag has 9 pieces of fruit: 3 apples, 3 pears, and 3 oranges. Four pieces of fruit are picked, one at a time. Each time a piece of fruit is picked, the type of fruit is recorded, and it is then put back in the bag. If the first 3 pieces of fruit were apples, what is the fourth piece MOST LIKELY to be?

a. A pear
b. An apple
c. An orange
d. An orange or a pear are both equally likely and more likely than an apple.
e. An apple, orange, or pear are all equally likely.
Which of the following best describes the reason for your answer to the preceding question?

a. This piece of fruit is just as likely as any other.

b. The apples seem to be lucky.

c. The picks are independent, so each fruit has an equally likely chance of being picked.

d. The fourth piece of fruit won't be an apple because too many have already been picked.

e. Other ______________________________

(Question 9 in Appendix E)

The first answer choice selection of “c. An orange” indicates an incorrect answer and the second answer choice of “c. The picks are independent, so each fruit has an equally likely chance of being picked” indicates correct probabilistic reasoning. The student could have selected these answer choices for any of the following reasons:

1. The student had inadvertently selected answer choices for either part that did not represent their true knowledge (i.e. accidentally selecting answer-choice C for part one when the student had intended to select answer-choice E).

2. The student misinterpreted or misread the question statement leading to their answer-choice selections.

3. The student had intentionally selected random answer choices in the hopes of getting the question correct out of pure coincidence.

Without more knowledge of the students’ answering procedures, one cannot infer the students’ knowledge and reasoning underlying their answer-choice selections. Therefore, students’ answers that did not occur on the test rubric were marked as inconclusive or “not a matched” response, meaning that no conclusions could be drawn from the students’ answers to the test question. It is important to note that inconclusive answer choice selections were not included in the statistical analysis of the test data. Only correct, representativeness heuristic (RH), and wrong responses were examined in the statistical analyses of students’ test data. On average, students selected inconclusive
answer choices only 7.6% of the time on any of their tests. The lowest occurrence of students providing inconclusive answer choice selections was on the comparison group’s post-test 1 in which approximately 2% of students’ test data was inconclusive. The highest occurrence of students supplying inconclusive answer choices was the prediction-only group on post-test 2 with 13% of their answer choice selections being inconclusive.

The question items on the pre-test and post-tests falling under the following question group types:

A: determining the student’s ability to understand the independence of probabilistic events (questions coded A1, A2, A3, and A4)

C: determining student’s use of the gambler’s fallacy for the representativeness heuristic (questions coded C1, C2, C3, and C4)

D: determining student’s use of the representativeness heuristic for problems involving flipping a fair coin (questions coded D1, D2, D3, and D4) and

E: determining student’s use of the representativeness heuristic for problems involving rolling a fair die (questions coded E1, E2, E3, and E4)

were graded according to the test rubric (see Appendix H). The number of correct, incorrect, representativeness heuristic (RH), and no match answers were summed for each student on their pre-test, post-test 1, and post-test 2. The term “overall correct answers” will be used to refer to a student’s summed number of correct responses to question types A, C, D, and E. Likewise, the term “overall incorrect answers” will be used to refer to a student’s summed number of incorrect responses to question types A, C, D, and E. Finally, the term “overall RH answers” will be used to refer to a student’s
summed number of answer responses indicating use of the representativeness heuristic for question types A, C, D, and E.

The question items falling under question group type B, determining student’s use of the “equally likely” (or equiprobability bias) response in which students answered that all probabilistic events were equally likely when the events actually had different probabilities of occurring (questions coded B1, B2, B3, and B4), were not included in each student’s overall test score. The questions of type B were excluded from the number of correct, incorrect, representativeness heuristic (RH), and no match answers that were summed for each student because they dealt with the equiprobability bias and not with student’s use of the representativeness heuristic.

4.1.1 Overreliance of Representativeness Heuristic between Comparison and Treatment Groups on Pre-Test

To determine if the comparison, prediction-only, and prediction-and-voting groups had the same level of overreliance on the representativeness heuristic at the beginning of the study, a One-way Analysis of Variance (ANOVA) test was completed using the statistical software package Minitab and the students’ pre-test data. The mean number of overall correct, incorrect, and representativeness heuristic (RH) responses were compared between the three precalculus sections. The results of the One-way ANOVA are summarized in Table 6 shown below. The mean number of correct responses on the pre-test for the prediction-and-voting group ($\bar{x}_{p \& v} = 5.333$) and comparison group ($\bar{x}_c = 5.238$) can be considered statistically equivalent. However the mean number of correct responses on the pre-test for the prediction-only group ($\bar{x}_{p \text{ only}} = 3.1913$) is so much
lower than the mean number of correct responses for the prediction-and-voting and comparison groups that it must be considered statistically different. Therefore, the prediction-only group had a statistically lower knowledge level of probability with respect to correct probabilistic reasoning compared to the prediction-and-voting and comparison groups.

Table 6: One-way ANOVA Results for Comparing Entering Probabilistic Knowledge between Comparison and Treatment Groups

<table>
<thead>
<tr>
<th></th>
<th>Prediction-and-Voting</th>
<th>Prediction-Only</th>
<th>Comparison</th>
<th>ANOVA result</th>
</tr>
</thead>
<tbody>
<tr>
<td># Correct Responses</td>
<td>$\bar{x}_{pv} = 5.333$</td>
<td>$\bar{x}_{p,\text{only}} = 3.191$</td>
<td>$\bar{x}_{c} = 5.238$</td>
<td>$p = 0.041$</td>
</tr>
<tr>
<td></td>
<td>$s_{pv} = 2.140$</td>
<td>$s_{p,\text{only}} = 2.130$</td>
<td>$s_c = 1.921$</td>
<td></td>
</tr>
<tr>
<td># Correct Responses</td>
<td>$\bar{x}_{pv} = 5.333$</td>
<td>$\bar{x}_{c} = 5.238$</td>
<td>$s_{pv} = 2.140$</td>
<td>$s_c = 1.921$</td>
</tr>
<tr>
<td># RH Responses</td>
<td>$\bar{x}_{pv} = 1.167$</td>
<td>$\bar{x}_{p,\text{only}} = 1.565$</td>
<td>$\bar{x}_{c} = 1.476$</td>
<td>$p = 0.592$</td>
</tr>
<tr>
<td></td>
<td>$s_{pv} = 1.435$</td>
<td>$s_{p,\text{only}} = 1.409$</td>
<td>$s_c = 1.327$</td>
<td></td>
</tr>
<tr>
<td># Wrong Responses</td>
<td>$\bar{x}_{pv} = 0.708$</td>
<td>$\bar{x}_{p,\text{only}} = 1.087$</td>
<td>$\bar{x}_{c} = 0.619$</td>
<td>$p = 0.194$</td>
</tr>
<tr>
<td></td>
<td>$s_{pv} = 0.908$</td>
<td>$s_{p,\text{only}} = 0.996$</td>
<td>$s_c = 0.805$</td>
<td></td>
</tr>
</tbody>
</table>

$\bar{x}$: mean
$s$: standard deviation
$p$: p-value

The mean number of representativeness heuristic (RH) and wrong responses on the pre-test for the prediction-and-voting group, prediction-only group, and comparison group are close enough that they are considered statistically equivalent. Therefore, the
prediction-and-voting and comparison groups can be judged as having the same knowledge of probability with respect to correct probabilistic reasoning and all three groups can be judged as having comparable levels of dependence upon the use of the representativeness heuristic.

4.1.2 Overall Improvement of Students’ Post-Test Scores

To establish how the comparison, prediction-only, and prediction-and-voting groups performed in terms of answering more test questions correctly and fewer questions using the representativeness heuristic, the mean number of correct and representativeness heuristic test questions were calculated for questions of types A, C, D, and E. The questions of type B were excluded when determining a student’s overall test score because questions in group B were structured to determine student’s use of the “equally likely” response—not their use of the representativeness heuristic. The graphical representation of the performance of the comparison and treatment groups for the mean number of correct responses given—ranging from zero questions correct to all eight questions correct—on the pre- and post-tests is shown in Figure 3. Similarly, Figure 4 is the graphical representation of the performance of the three precalculus sections for the mean number of questions that were answered to indicate the use of the representativeness heuristic—ranging from zero questions to all eight questions answered using the representativeness heuristic—for the pre- and two post-tests.

It is important to note that the mean values presented in figures 3 and 4 only include the students that had taken the pre-test, the first post-test, and the second post-test. If a student was absent when one or more of the tests were administered their test
data for any of the other tests were not used in calculating the mean values. The removal of students with incomplete test data was completed so that the mean values for the pre-test, post-test 1, and post-test 2 would consistently represent the same students. In the comparison group from the 25 students that agreed to be in the study, six students had not taken one or more of the tests—making 19 students (76% of the participating students) that had completed all of the pre-test and post-tests. Of the 23 students in the prediction-only class, three students had not taken at least one of the tests—leaving 20 students (87% of the class) with complete test data. Finally, five of the 25 students in the prediction-and-voting class did not take all of the tests—leaving 20 students (80% of the class) with complete test data. To guarantee that including all available student test data in calculating the mean values did not affect the overall trends of the graphical representations, additional graphical representations were also completed include all available student test data. However, the graphs for the mean values including all available student test data demonstrated the same overall patterns and trends that were already present in the graphs for which the mean values were calculated with removing students with incomplete test data.

As shown in Figure 3, the comparison group, prediction-only group, and prediction-and-voting group were all able to show improvement in the mean number of correct responses given from the pre-test to post-test 1. Yet, the comparison group demonstrated the largest improvement in the mean number of correct responses from the pre-test to post-test 1. All three groups, to a certain degree, were able to maintain these improvements up to six weeks after the representativeness lesson when post-test 2 was administered. Although the comparison group showed the greatest improvement from
the pre-test to post-test 1, the comparison group also performed the worse among the three groups in terms of maintaining the mean number of correct responses from post-test 1 to post-test 2.

Figure 3: Comparison and Treatment Groups Mean Number of Correct Responses for All Test Questions

In Figure 4 the comparison group, prediction-only group, and prediction-and-voting group were all able to show improvement in reducing the mean number of representativeness heuristic responses given from the pre-test to post-test 1. However, the comparison group still demonstrated the largest reduction in the mean number of representativeness heuristic responses from the pre-test to post-test 1. In addition, all three groups were able to somewhat maintain any reduction in the use of the representativeness heuristic until post-test 2 was administered. Although the comparison group showed the greatest reduction in the use of the representativeness heuristic from
the pre-test to post-test 1, the comparison group also performed the worse among the three groups in terms of maintaining the mean number of representativeness heuristic from post-test 1 to post-test 2.

![Figure 4: Comparison and Treatment Groups Mean Number of Representativeness Heuristic Responses for All Test Questions](image)

To determine if the comparison, prediction-only, and prediction-and-voting groups demonstrated a statistically significant improvement in their overreliance on the representativeness heuristic, paired t-tests were completed on students’ pre-test and post-test scores. The summary of the paired t-test results are shown in Table 7.

The comparison group demonstrated a statistically significant improvement in the number of correct responses given from the pre-test to both post-tests and a highly statistically significant reduction in the number of representativeness heuristic (RH) responses from the pre-test to both post-tests. This means that the comparison group demonstrated a significant shift from the use of the representativeness heuristic to the use
of correct probabilistic reasoning and the students were able to maintain this improvement in their probabilistic reasoning up to six weeks after the Representativeness Heuristic lesson.
The prediction-only group showed a statistically significant improvement in the number of correct responses given from the pre-test to the first post-test, but did not demonstrate a statistically significant improvement in the number of correct responses given from the pre-test to the second post-test. The prediction-only group also demonstrated a statistically significant reduction in the number of representativeness heuristic (RH) responses from the pre-test to the first post-test, but again did not have a statistically significant reduction in the number of representativeness heuristic (RH) responses from the pre-test to the second post-test. The students in the prediction-only group were therefore able to demonstrate a significant improvement in their probabilistic reasoning and use of the representativeness heuristic between the pre-test and first post-test, but were unable to maintain these improvements until the time the second post-test was administered.

The prediction-and-voting group showed a statistically significant improvement in the number of correct responses given from the pre-test to both the first and second post-tests and a statistically significant reduction in the number of representativeness heuristic (RH) responses from the pre-test to the first post-test. The p-value for the reduction in the number of representativeness heuristic (RH) responses from the pre-test to the second post-test barely missed the 0.05 statistically significance cutoff with a value of 0.056. Overall, the prediction-and-voting group students displayed a significant shift from their use of the representativeness heuristic to the use of correct probabilistic reasoning and, for the most part, were able to maintain this improvement in their probabilistic reasoning up to six weeks after the Representativeness Heuristic lesson.
All three precalculus sections demonstrated a significant shift from the use of the representativeness heuristic to the use of correct probabilistic reasoning between the pre-test and first post-test. While students in all three precalculus sections also showed a shift from their use of the representativeness heuristic to the use of correct probabilistic reasoning between the pre-test and second post-test, only the comparison group had strongly statistically significant p-values. The comparison, prediction-only, and prediction-and-voting groups can be considered to have reduced their overreliance on the representativeness heuristic and, to a certain extent, maintained these improvements up to six weeks after the Representativeness Heuristic Lesson. However, in order to determine which precalculus section showed the most overall improvement in the use of the representativeness heuristic between the pre-test and post-tests, effect sizes must be examined.

4.1.3 Effect Size for Student Overall Achievement on Post-Tests between Comparison and Treatment Groups

The effect size (Cohen’s d) value for the differences in student achievement between the comparison and treatment groups in terms of the gain in the total number of correct and representativeness heuristic (RH) responses given from the students’ pre-tests to post-tests were calculated. The effect sizes are shown in Table 8. The negative values of the effect sizes for the number of correct responses on the post-tests indicate that the comparison group, on average, demonstrated more of an improvement in the number of correct responses from their pre-test to post-tests than the prediction-and-voting and prediction-only groups. The positive values of the effect sizes for the number of
representativeness heuristic (RH) responses on the post-tests indicate that, on average, the prediction-and-voting and prediction-only groups’ improvement in their use of the representativeness heuristic from their pre-test to post-tests was below the improvements demonstrated by the comparison group.

Any effect size value greater than or less than 0.3 was judged as having practical (or real-world) significance. The effect sizes for the prediction-and-voting group versus the comparison group for students’ gain in the number of correct and representativeness responses provided from the pre-test to post-test 1 can all be judged as having practical significance. Therefore, the prediction-and-voting group demonstrated a significantly lower gain in average achievement from the pre-test to first post-test than the comparison group for practical, classroom teaching purposes.

<table>
<thead>
<tr>
<th>Table 8: Effect Sizes for Student Achievement Gain from Pre-Tests to Post-Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Correct Responses</strong></td>
</tr>
<tr>
<td>Prediction-and-Voting Group versus Comparison Group</td>
</tr>
<tr>
<td>Prediction-Only Group versus Comparison Group</td>
</tr>
<tr>
<td><strong>Number of Representativeness Heuristic Responses</strong></td>
</tr>
<tr>
<td>Prediction-and-Voting Group versus Comparison Group</td>
</tr>
<tr>
<td>Prediction-Only Group versus Comparison Group</td>
</tr>
</tbody>
</table>

The effect sizes for the prediction-only group versus the comparison group are not significant for students’ gain in the number of correct responses provided on either of the post-tests. However, the average student gain from the pre-test to second post-test for the
number of representativeness heuristic responses given by prediction-only students was larger than the gain in representativeness responses provided by the comparison students.

While the effect sizes provide an overall description of how the students in the treatment groups performed from the pre- to post-tests with respect the comparison group, a more thorough analysis of the pre-test and post-test data must be completed in order to take into account any variables, such as student attendance or student demonstration of probability knowledge on the pre-test, that might have influenced student achievement on the post-tests. To incorporate this more thorough analysis of the test data, a multiple regression analysis was completed on the students’ post-test scores.

4.1.4 Regression Analysis of Post-Test Scores

To determine students’ improvement in terms of an increase in the total number of correct responses and a decrease in the total number of representativeness heuristic (RH) responses from the pre-test to the post-tests, four regression models were used. The four regression models used the following data as the dependent variable:

1. The total number of correct responses on the first post-test,
2. The total number of representativeness heuristic (RH) responses on post-test 1,
3. The total number of correct responses on the second post-test, and
4. The total number of representativeness heuristic (RH) responses on post-test 2.

The independent variables considered to have influence over the success of a student’s post-test score were included in all regression models as follows:

1. If the student was in the prediction-and-voting class,
2. If the student was in the prediction-only class,
3. If the student attended the Probability Review Lesson,

4. If the student attended the Representativeness Heuristic Lesson, and

5. The student’s number of correct responses on the Pre-Test.

To incorporate the independent variables of the student belonging to one of the three precalculus sections and the student’s attendance of the lessons, the data was dummy coded (see Table 9). For example, to indicate that a student was in the prediction-only group and attended only the review lesson, the student’s coding was assigned as follows:

- Prediction-and-Voting = 0
- Prediction-Only = 1
- Attended_Representativeness_Heuristic_Lesson = 0
- Attended_Review_Lesson = 1

<table>
<thead>
<tr>
<th>Student’s Class Coding</th>
<th>Prediction-and-Voting</th>
<th>Prediction-Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction-and-Voting Class</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Prediction-Only Class</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Comparison Class</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student’s Attendance Coding</th>
<th>Attended_Representativeness_Heuristic_Lesson</th>
<th>Attended_Review_Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student was Present</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Student was Absent</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Once all the students’ test data were appropriately coded, all four regression models were completed using the Minitab statistical software package. The summary of the regression results for all four regression models can be found in Appendix I. However, for the purposes of reviewing the regression model results we will only examine the regression...
models that had statistically significant coefficients for the Prediction-and-Voting and Prediction-Only independent variables.

The only regression model that had slope coefficients significantly different from zero for the Prediction-and-Voting and Prediction-Only variables was the regression for the number of correct responses given on the first post-test. The regression equation, coefficients, standard error values, t-values, and p-values for the number of students’ correct responses on the first post-test are shown in Table 10.

Table 10: Regression Output for the Number of Correct Responses on Post-Test 1

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coefficient</th>
<th>Standard Error of Coefficient</th>
<th>T-Value</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>5.039**</td>
<td>1.385</td>
<td>3.64</td>
<td>0.001</td>
</tr>
<tr>
<td>Prediction-and-Voting</td>
<td>-0.8901</td>
<td>0.4960</td>
<td>-1.79</td>
<td>0.078</td>
</tr>
<tr>
<td>Prediction-Only</td>
<td>-1.0450*</td>
<td>0.5049</td>
<td>-2.07</td>
<td>0.043</td>
</tr>
<tr>
<td>Attended_Review_Lesson</td>
<td>-0.7196</td>
<td>0.9275</td>
<td>-0.78</td>
<td>0.441</td>
</tr>
<tr>
<td>Attended_Representativeness_Heuristic_Lesson</td>
<td>1.0998</td>
<td>0.9325</td>
<td>1.18</td>
<td>0.243</td>
</tr>
<tr>
<td>Pre-Test_Correct</td>
<td>0.33911**</td>
<td>0.09559</td>
<td>3.55</td>
<td>0.001</td>
</tr>
</tbody>
</table>

* = coefficient has p-value < 0.05
** = coefficient has p-value < 0.001

The Prediction-and-Voting slope coefficient was not significantly different from zero; therefore, the prediction-and-voting group, on average, did not get any more questions correct on the first post-test than the comparison group. However, the coefficient for the Prediction-Only variable can be judged as being statistically different from zero. This means that a student’s inclusion into the prediction-only group reduced
the student’s number of correct responses on the first post-test by 1.0405 responses compared to the comparison group. Therefore, the comparison group demonstrated a greater improvement in the number of correct responses from the pre-test to the first post-test when compared to the prediction-only group.

The other regression models (see Appendix I) did not have statistically significant p-values for coefficients of the Prediction-and-Voting and Prediction-Only variables. This means that the coefficients are not statistically different from zero and, therefore, the prediction-and-voting and prediction-only groups did not do better than the comparison group in terms of the number of correct responses given on the second post-test and the number of representativeness heuristic (RH) responses given on both post-tests.

In all four regression models, the treatment groups did not demonstrate a greater overall decrease in their overreliance on the representativeness heuristic than the comparison group. In fact, the comparison group, on average, answered one more questions correct on the first post-test than the prediction-only group. While the regression analysis provides an overall picture of how the comparison, prediction-only, and prediction-and-voting groups performed, it fails to include details regarding how the groups performed for each of the following question group types:

A: the independence of probabilistic events (question group A),
B: the use of the “equally likely”/equiprobability bias responses (question group B)
C: the gambler’s fallacy for the representativeness heuristic (question group C),
D: the representativeness heuristic for problems involving flipping a fair coin (question group D), and
E: the representativeness heuristic for problems involving rolling a fair die (question group E).

To determine how each group performed on the pre-test and post-tests with respect to the question groups A, B, C, D, and E, further analysis needed to be completed.

4.1.5 Overall Achievement of Comparison and Treatment Groups on each Test Question Group

In order to gain a deeper understanding of how students in the prediction-only and prediction-and voting-groups performed on each test item type—A, B, C, D, and E—relative to the comparison group the students pre-test and post-tests responses were evaluated using logistic regression, paired t-test, and effect size analysis methods. The results of these analysis methods are listed in Appendices J, K, and L and are summarized in tables 11 and 12 to follow. A more detailed discussion of the Representativeness Heuristic test analysis results for each individual test question group is presented in the following five sections.
Table 1: Results of Test Question Group Analysis for the Number of Correct Responses Provided by Students

<table>
<thead>
<tr>
<th>Pre-Test to Post-Test 1</th>
<th>Pre-Test to Post-Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction-and-Voting Group</td>
<td>Prediction-Only Group</td>
</tr>
<tr>
<td><strong>Test Item Group</strong></td>
<td><strong>Test Item Group</strong></td>
</tr>
<tr>
<td>A (independence of events)</td>
<td>-PTT (p = 0.005)</td>
</tr>
<tr>
<td>B (equiprobability bias/ equally likely response)</td>
<td>+LRA (p = 0.012)</td>
</tr>
<tr>
<td></td>
<td>+PTT (p = 0.001)</td>
</tr>
<tr>
<td>C (gambler’s fallacy)</td>
<td>+PTT (p = 0.000)</td>
</tr>
<tr>
<td>D (rep. heuristic with coin)</td>
<td>-LRA (p = 0.018)</td>
</tr>
<tr>
<td>E (rep. heuristic with dice)</td>
<td>+PTT (p = 0.000)</td>
</tr>
</tbody>
</table>

- **LRA** = Logistic Regression Analysis had a statistically significant (p < 0.05) **negative** coefficient, meaning that the student group provided significantly fewer correct responses to the test questions compared to the comparison group.
- **PTT** = Paired T-Test showed a statistically significant (p < 0.05) **positive** coefficient, meaning that the student group provided significantly more correct responses to the test questions compared to the comparison group.
- **PTT** = Paired T-Test showed a statistically significant (p < 0.05) **decrease** in the number of correct responses students provided on the post-test from the number of correct responses provided on the pre-test.
- **PTT** = Paired T-Test showed a statistically significant (p < 0.05) **increase** in the number of correct responses students provided on the post-test from the number of correct responses provided on the pre-test.
- **ES** = Effect Size was **negative**, meaning that the treatment group provided a significantly smaller (|Cohen’s d| > 0.3) gain in correct responses from the pre-test to post-test compared to the comparison group for the practical purposes of the study.
- **ES** = Effect Size was **positive**, meaning that the treatment group provided a significantly larger (|Cohen’s d| > 0.3) gain in correct responses from the pre-test to post-test compared to the comparison group for the practical purposes of the study.
Table 12: Results of Test Question Group Analysis for the Number of Representativeness Heuristic and Equiprobability Bias/“Equally Likely” Learned Responses Provided by Students

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Test to Post-Test 1</td>
<td>Pre-Test to Post-Test 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-ES (d = -0.337)</td>
<td>-LRA (p = 0.018)</td>
<td></td>
<td>A (independence of events)</td>
<td>-LRA (p = 0.003)</td>
<td>-ES (d = -0.450)</td>
<td></td>
</tr>
<tr>
<td>+ES (d = 0.310)</td>
<td></td>
<td></td>
<td>B (equiprobability bias/“equally likely response)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+ES (d = 0.315)</td>
<td>+ES (d = 0.660)</td>
<td>-PTT (p = 0.002)</td>
<td>D (rep. heuristic with coin)</td>
<td>+PTT (p = 0.003)</td>
<td>+ES (d = 0.400)</td>
<td>+ES (d = 0.549)</td>
</tr>
<tr>
<td>-PTT (p = 0.012)</td>
<td>-PTT (p = 0.031)</td>
<td>-PTT (p = 0.025)</td>
<td>E (rep. heuristic with dice)</td>
<td>+PTT (p = 0.003)</td>
<td>-PTT (p = 0.003)</td>
<td>-PTT (p = 0.003)</td>
</tr>
</tbody>
</table>

- LRA = Logistic Regression Analysis had a statistically significant \( p < 0.05 \) negative coefficient, meaning that the student group provided significantly fewer Representativeness Heuristic (RH) responses (or equiprobability bias/“equally likely” learned responses for test question group B) to the test questions compared to the comparison group.

+ LRA = Logistic Regression Analysis had a statistically significant \( p < 0.05 \) positive coefficient, meaning that the student group provided significantly more RH (or “equally likely”) responses to the test questions compared to the comparison group.

- PTT = Paired T-Test showed a statistically significant \( p < 0.05 \) decrease in the number of Representativeness Heuristic (RH) responses (or equiprobability bias/“equally likely” learned responses for test question group B) to the test questions from the number of RH (or “equally likely”) responses provided on the pre-test.

+ PTT = Paired T-Test showed a statistically significant \( p < 0.05 \) increase in the number of RH responses (or “equally likely” responses) to the test questions from the number of RH (or “equally likely”) responses provided on the pre-test.

- ES = Effect Size was negative, meaning that the treatment group provided a significantly smaller (\( |Cohen's d| < 0.3 \)) gain in their use of the Representativeness Heuristic (RH) (or equiprobability bias/“equally likely” learned bias responses for test question group B) from the pre-test to post-test compared to the comparison group for the practical purposes of the study.

+ ES = Effect Size was positive, meaning that the treatment group provided a significantly larger (\( |Cohen's d| > 0.3 \)) gain in their use of the RH (or “equally likely” LR) from the pre-test to post-test compared to the comparison group for the practical purposes of the study.

Test Item Group:
- A (independence of events)
- B (equiprobability bias/“equally likely response)
- C (gambler’s fallacy)
- D (rep. heuristic with coin)
- E (rep. heuristic with dice)
4.1.5 (A) Achievement of Comparison and Treatment Groups on Test Question Group A

Test questions within group A were used to determine each student’s ability to understand the independence of probabilistic events and were coded A1, A2, A3, and A4 (see Appendix E). To establish how the comparison, prediction-only, and prediction-and-voting groups performed in terms of answering more test questions of type A correctly and fewer using the representativeness heuristic, the mean number of correct and representativeness heuristic test questions were calculated out of the two questions of type A given on each of the tests. The graphical representation of the performance of the comparison and treatment groups for the mean number of correct responses given—ranging from zero questions correct to two questions correct on the pre- and post-tests for question group A is shown in Figure 5. While Figure 6 is the graphical representation of the performance of the three precalculus sections for the mean number of group A questions that were answered to indicate the use of the representativeness heuristic—ranging from zero questions to two questions answered using the representativeness heuristic for the pre-test and two post-tests.

It is important to note that the mean values presented in figures 5 through 14 in this section, only use test data for students that had taken the pre-test, the first post-test, and the second post-test. The removal of students with incomplete test data was completed so that the mean values for all three tests would consistently represent the same students, thereby avoiding attrition bias. For the comparison group, six of the 25 students had not taken one or more of the tests—leaving 19 students (76% of the participants in the comparison group) that had completed all pre- and post-tests. Of the 23 students in the
prediction-only class, three students had not taken at least one of the tests—leaving 20 students (87% of the class) with complete test data. Finally, five of the 25 students in the prediction-and-voting class did not take all of the tests—leaving 20 students (80% of the class) with complete test data.

![Graph](image)

Figure 5: Comparison and Treatment Groups Mean Number of Correct Responses for Test Question Group A

Interestingly, none of the three precalculus sections demonstrated an improvement in the mean number of correct responses from the pre-test to the first post-test. In fact, the prediction-only group and prediction-and-voting group both demonstrated a decrease in the number of correct responses from the pre-test to post-test 1. However, the results of the paired t-test between each group’s mean number of correct responses given on the pre-test and post-test 1 (see Appendix K) show that only the prediction-and-voting group had a statistically significant decrease in the number of correct responses given from the pre-test to post-test 1. In addition, the effect sizes indicate that the prediction-and-voting group performed significantly worse than the comparison group in students’ average gain
in the number of correct responses provided for type A questions from the pre-test to first post-test (see appendix L). With the data collected for this study, it is unclear as to why the prediction-and-voting group showed a significant decrease in the number of correct responses for group A test questions.

At six weeks after the representativeness lesson, when the second post-test was administered, none of the three precalculus sections improved past their pre-test score for the mean number of correct responses. While the prediction-only group and prediction-and-voting group both decreased in the number of correct responses given from the pre-test to the first post-test, the logistic regression for test question group A (see Appendix J) failed to have coefficient different from zero and the effect sizes for group A (see Appendix L) also failed to reach the threshold for practical significance. These results indicate that the prediction-only group and prediction-and-voting group did not perform significantly worse (statistically or practically) than the comparison group performed for question group A on post-test 1 and post-test 2.

As shown in Figure 6, all three precalculus sections demonstrated an increase in the mean number of responses indicating use of the representativeness heuristic from the pre-test to post-test 1. However, the results of the paired t-test between each group’s mean number of representativeness heuristic responses given on the pre-test and post-test 1 (see Appendix K) show that none of the three groups demonstrated a statistically significant increase in the number of representativeness heuristic responses given from the pre-test to post-test 1.
At the time of the administration of post-test 2, only the prediction-and-voting group had demonstrated an mean number of representativeness heuristic responses less than or equal to their pre-test score for the mean number of representativeness heuristic responses. The logistic regression for test question group A (see Appendix J) failed to have coefficient different from zero. However, the effect sizes for group A (see Appendix L) did reach practical significance threshold which indicates that the prediction-and-voting group performed significantly better, for practically purposes, than the comparison group for reducing the number of representativeness heuristic responses given for question group A from the pre-test to post-test 1.
4.1.5 (B) Achievement of Comparison and Treatment Groups on Test Question

Group B

Test questions within group B were used to determine each student’s use of the equiprobability bias/“equally likely” learned response in which students would answer that all probabilistic events were equally likely when the events actually had different probabilities of occurring. The action of answering that all probabilistic events are equally likely when it is inappropriate for a given problem situation was considered a learned response because students more frequently demonstrated the use of this action during the course of the study (i.e. after the pre-test). Since no other form of probability instruction was occurring, aside from the study itself, students were learning to respond to probability questions by selecting the “equally likely” answer choice due to some aspect of the study.

However, it is also important to note that the judgment of outcomes with different probabilities to be equally likely has been characterized as the equiprobability bias in probability literature (Lecoutre, 1985; Lecoutre, 1992; Morsanyi et al., 2009). Lecoutre (1985) described the equiprobability bias as the tendency for people to think of random events as “equiprobable” by nature—a conclusion he came to when he observed that many interview subjects believed that the chances of getting a five and a six, versus two sixes when simultaneously throwing two dice, were equally likely (Lecoutre, 1985; Lecoutre, 1992). What makes this study different from Lecoutre’s 1985 and 1992 studies is that students in general did not already possess the tendency to inappropriately answer “equally likely” to probability problems. Instead, students in this study tended to answer the group B questions correctly on the pre-test but then selected the “equally likely”
answer choice on their post-tests. Therefore, some aspect of this study motivated students to adopt the use of the “equally likely” response. In Lecoutre’s 1985 and 1992 studies, no instructional intervention was employed. Instead, Lecoutre aimed to observe the probabilistic reasoning that participants already possessed through a series of interviews. Therefore, students’ shift to the use of the “equally likely” response on probabilistic scenarios could be a result of test taking strategies they developed due to this study or could be the result of their adoption of Lecoutre’s equiprobability bias. Hence, the questions that are categorized as group B test questions will represent the equiprobability bias/“equally likely” learned response test questions. (For other researchers’ conjectures for why students might adopt the equiprobability bias, see Chapter 5: Conclusions)

Questions in group B were coded B1, B2, B3, and B4 (see Appendix E). To establish how the comparison, prediction-only, and prediction-and-voting groups performed in terms of answering more test questions of type B correctly and fewer using the equiprobability bias/“equally likely” learned response, the mean number of correct and learned response questions were calculated out of the two questions of type B given on each of the tests. The graphical representation of the performance of the comparison and treatment groups for the mean number of correct responses given—ranging from zero questions correct to two questions correct on the pre-test and post-tests for question group B is shown in Figure 7. Figure 8 is the graphical representation of the performance of the three precalculus sections for the mean number of group B questions that were answered to indicate the use of the equiprobability bias/equally likely learned response—ranging from zero questions to two questions answered using the equally likely response for the
pre-test and two post-tests. As with test questions in group A, incomplete student test
data was removed when creating the graphical representations for group B test questions
so that the mean values for the pre-test, post-test 1, and post-test 2 consistently represent
the same students. In the comparison group 19 of the 25 students (76%) were included in
test data, 20 out of the 23 students (87%) in the prediction-only class were included, and
20 out of the 25 students (80%) in the prediction-and-voting class had their test data used
for the graphs.

Figure 7: Comparison and Treatment Groups Mean Number of Correct Responses for
Test Question Group B

As shown in figure 7, the prediction-only and prediction-and-voting groups
demonstrated an increase in the mean number of correct responses to group B questions
from the pre-test to the first post-test. Interestingly, the comparison group provided fewer
correct responses for questions of type B on post-test 1 and post-test 2 than on the pre-
test. The logistic regression the number of correct responses on post-test 1 questions (see
Appendix J) had coefficient statistically greater than zero for the prediction-and-voting group—which indicates that the prediction-and-voting group performed statistically better than the comparison group in terms of providing more correct answers to questions of type B on post-test 1.

At six weeks after the representativeness lesson, when the second post-test was administered, only the prediction-and-voting group was able to still maintain any improvements in the mean number of correct responses given from the pre-test to the first post-test. In contrast, the prediction-only group and comparison group both provided fewer correct responses to questions of group B on post-test 2 than on the pre-test. However, the results of the paired t-test between each group’s mean number of correct responses given on the pre-test and each post-test (see Appendix K) show that none of the three precalculus classes had a statistically significant increase or decrease in the number of correct responses given from the pre-test to post-test 1 and from the pre-test to post-test 2. Finally, the effect sizes for the number of correct responses provided for question group B (see Appendix L) reveal that the prediction-and-voting group provided a significant greater gain in the number of correct answers (for practical purposes) from the pre-test to post-test 1 and post-test 2 than the comparison group. These results indicate that the prediction-and-voting group performed statistically significantly better than the comparison group for type B test questions on post-test 1 and practically significantly better than the comparison group in terms of gains from the pre-test to post-test 1 and post-test 2.

As shown in Figure 8, all of the three precalculus sections demonstrated an increase in the mean number of responses indicating use of the equiprobability bias/
“equally likely” learned response from the pre-test to post-test 1 and from the pre-test to post-test 2. However, the results of the paired t-test between each group’s mean number of equally likely responses given on the pre-test and each post-test (see Appendix K) show that none of the three groups demonstrated a statistically significant increase in the number of equally likely responses given from the pre-test to each post-test.

Figure 8: Comparison and Treatment Groups Mean Number of Equiprobability Bias/“Equally Likely” Learned Responses for Test Question Group B

Although all classes demonstrated an increase in the number of equiprobability bias/“equally likely” learned responses provided on the post-tests, the comparison group showed the largest increase in the number of equally likely responses given on post-test 1 and post-test 2. In contrast, the prediction-and-voting group provided significant (both statistically and practically) fewer equally likely responses than the comparison group as shown by the results of the logistic regression coefficients (see Appendix J) and the effect
sizes (see Appendix L). The prediction-only group also provided fewer equally likely responses than the comparison group but failed to meet the criteria for practical and statistical significance (in the effect sizes and the logistic regression analysis, respectively).

4.1.5 (C) Achievement of Comparison and Treatment Groups on Test Question

Group C

Test questions within group C were used to determine each student’s use of the gambler’s fallacy for the representativeness heuristic and were coded C1, C2, C3, and C4 (see Appendix E). To establish how the comparison, prediction-only, and prediction-and-voting groups performed in terms of answering more test questions of type C correctly and fewer using the representativeness heuristic, the mean number of correct and representativeness heuristic test questions were calculated out of the two questions of type C given on each of the tests. The graphical representation of the performance of the comparison and treatment groups for the mean number of correct responses given—ranging from zero questions correct to two questions correct on the pre-test and post-tests for question group C is shown in Figure 9.

All three precalculus sections demonstrated an increase in the mean number of correct responses from the pre-test to the post-test 1; however, the comparison group had the greatest increase among all three precalculus sections which was closely followed by the prediction-only group. The results of the paired t-test between each group’s mean number of correct responses given on the pre-test and post-test 1 (see Appendix K) show that the comparison group barely missed statistical significance with a p-value of 0.056
but the prediction-only group did have a statistically significant increase in the number of correct responses given from the pre-test to post-test 1.

Figure 9: Comparison and Treatment Groups Mean Number of Correct Responses for Test Question Group C

For the second post-test, only the comparison group were able to maintain improvements in their number of correct responses for question types C past their pre-test scores. While both the prediction-only group and prediction-and-voting group decreased in the number of correct responses given from the post-test 1 to post-test 2, the prediction-only group alone performed statistically worse than the comparison group for post-test 2 based on the logistic regression for test question group C (see Appendix J). The effect sizes (see Appendix L) also show that prediction-only group demonstrated a significantly smaller gain in correct responses (for practical purposes) for questions of type C from the pre-test to post-test 2. Therefore, the prediction-only group and prediction-and-voting group both had a lower mean performance than the comparison
group in terms of providing correct responses to questions of type C. Yet, only the prediction-only group performed statistically or practically worse than the comparison group for either of the post-tests.

![Figure 10: Comparison and Treatment Groups Mean Number of Representativeness Heuristic Responses for Test Question Group C](image)

Figure 10 is the graphical representation of the performance of the three precalculus sections for the mean number of group C questions that were answered in ways that indicate the use of the representativeness heuristic—ranging from zero questions to two questions answered using the representativeness heuristic for the pre-test and two post-tests. As shown in Figure 10, the comparison group and prediction-and-voting group demonstrated an decrease in the mean number of responses indicating use of the representativeness heuristic from the pre-test to post-test 1. Similarly, the comparison group and prediction-and-voting group were able to maintain the reduction in representativeness responses until post-test 2. The prediction-only group, on the other
hand, showed an overall increase in the number of representativeness heuristic responses from the pre-test to post-test 2.

The results of the paired t-test between each group’s mean number of representativeness heuristic responses given on the pre-test and the post-tests (see Appendix K) show that none of the three groups demonstrated a statistically significant decrease or increase in the number of representativeness heuristic responses given from the pre-test to post-test 1 and from the pre-test to post-test 2. The logistic regression analysis (see Appendix J) for test question group C both failed to have p-values to indicate statistical significance. However, for the average gain in the number of question group C representativeness responses given from the pre-test to second post-test was significant for the prediction-only group’s effect sizes (see Appendix L). Therefore, the prediction-only group performed significantly worse (for practical purposes) than the comparison group in terms of providing fewer representativeness heuristic responses to group C test questions.

4.1.5 (D) Achievement of Comparison and Treatment Groups on Test Question Group D

Test questions within group D were used to determine each student’s use of the representativeness heuristic for problems involving flipping a fair coin and were coded D1, D2, D3, and D4 (see Appendix E). Using the same procedure as the previous question groups, the mean number of correct and representativeness heuristic test questions were calculated out of the two questions of type D given on each of the tests. Figure 11 is the graphical representation of the performance of the comparison and
treatment groups for the mean number of correct responses given—ranging from zero questions correct to two questions correct on the pre- and post-tests for question group D. Also like the previous question groups, the removal of students with incomplete test data was completed so that the mean values for the tests would consistently represent the same students.

Figure 11: Comparison and Treatment Groups Mean Number of Correct Responses for Test Question Group D

All of the three precalculus sections demonstrated an improvement in the mean number of correct responses from the pre-test to the first post-test. The comparison group demonstrated the greatest increase of correct responses of type D from the pre-test to post-test 1 with the prediction-and-voting group then prediction-only group ranking second and third in the increase of correct responses for group D. Analysis of the paired t-tests between each group’s mean number of correct responses given on the pre-test and post-test 1 (see Appendix K) show that all three precalculus sections demonstrated a
statistically significant increase in the number of correct responses given from the pre-test to post-test 1 for questions of type D.

At six weeks after the representativeness lesson, when the second post-test was administered, all three precalculus sections were able to maintain their increase in correct responses to type D questions well above their pre-test scores. The paired t-tests (see Appendix K) revealed that the comparison and prediction-and-voting groups were able to maintain their statistically significant increase in the number of correct responses from the pre-test to post-test 2; whereas, the prediction-only group was unable to maintain the statistically significant increase in correct responses of questions within group D for post-test 2. While the prediction-only group and prediction-and-voting group performed below the comparison group in the number of correct responses given for questions of type D on both post-tests, the logistic regression for test question group D (see Appendix J) reveals that only the prediction-only group performed statistically significantly below the comparison group for post-test 2. The effect sizes for questions in group D (see Appendix L) also reveal that only the prediction-only group performed significantly below the comparison group for the average gain in correct responses from the pre-test to post-test 2.

Figure 12 is the graphical representation of the performance of the three precalculus sections for the mean number of group D questions that were answered to indicate the use of the representativeness heuristic—ranging from zero questions to two questions answered using the representativeness heuristic for the pre-test and two post-tests. Within Figure 12, one can see that all three precalculus sections demonstrated a decrease in the mean number of representativeness heuristic responses from the pre-test
to the first post-test. The comparison group had the greatest decrease in the number of representativeness heuristic responses of type D from the pre-test to post-test 1 with the prediction-and-voting group then prediction-only group ranking second and third.

Analysis of the paired t-test between each group’s mean number of representativeness heuristic responses given on the pre-test and post-test 1 (see Appendix K) show that the comparison and prediction-and-voting groups demonstrated a statistically significant decrease in the number of representativeness heuristic responses given from the pre-test to post-test 1 for question of type D. Whereas, the prediction-only group failed to show a statistically significant decrease in the number of representativeness heuristic responses for questions of type D from the pre-test to the first post-test.

![Figure 12: Comparison and Treatment Groups Mean Number of Representativeness Heuristic Responses for Test Question Group D](image)

At the time of the administration of post-test 2, all three precalculus sections were able to maintain their decrease in representativeness heuristic responses to type D questions below their pre-test scores. However, the paired t-tests (see Appendix K)
revealed that only the comparison groups was able to maintain their statistically test to post-test 2. The prediction-only group and the prediction-and-voting group did have a statistically significant decrease in the number of representativeness heuristic responses of questions within group D for post-test 2.

The prediction-only group and prediction-and-voting group performed below the comparison group in terms of providing fewer representativeness heuristic responses or questions of type D on both post-tests, the logistic regression for test question group D (see Appendix J) show that neither the prediction-only group nor the prediction-and-voting group performed significantly below the comparison group for post-test 1 and post-test 2. However, the effect sizes (see Appendix L) reveal that the prediction-and-voting and prediction-only groups performed significantly worse than the comparison group (for practical purposes) in terms of reducing the number of representativeness responses provided from the pre-test to both post-tests.

4.1.5 (E) Achievement of Comparison and Treatment Groups on Test Question Group E

Group E test questions were coded E1, E2, E3, and E4 (see Appendix E) and were used to determine each student’s use of the representativeness heuristic for problems involving rolling a fair die. Again, the mean number of correct and representativeness heuristic test questions were calculated out of the two questions of type E given on each of the tests. Figure 13 is the graphical representation of the performance of the comparison and treatment groups for the mean number of correct responses given—ranging from zero questions correct to two questions correct on the pre-test and post-tests for question group
Students with incomplete test data were removed when creating the graphical representations so that the mean values for the tests would consistently represent the same students.

As shown in Figure 13, all three of the precalculus sections demonstrated an increase in the mean number of correct responses to questions of type E from the pre-test to the first post-test. The prediction-only group demonstrated the greatest increase in the number of correct responses to type E questions from the pre-test to post-test 1. The comparison group ranked second and the prediction-and-voting group ranked third in terms of the increase of correct responses for questions in group E. When the second post-test was administered, all three precalculus sections were able to maintain their increase in correct responses to type E questions well above their pre-test scores. Analysis of the paired t-tests between each group’s mean number of correct responses given on the pre-test and each post-test (see Appendix K) show that all three precalculus sections demonstrated a statistically significant increase in the number of correct responses given from the pre-test to post-test 1 and from the pre-test to post-test 2 for questions of type D.

While the prediction-only group and prediction-and-voting group overall performed below the comparison group in the number of correct responses given for questions of type E on both post-tests, the logistic regression for test question group E (see Appendix J) reveals that only the prediction-only group performed statistically significantly below the comparison group for post-test 1 and post-test 2. The effect sizes for questions in group E (see Appendix L) reveal that the prediction-only group
performed significantly below (or practical purposes) the comparison group for students’
gain in the number of correct response provided from the pre-test to both post-tests.

Figure 13: Comparison and Treatment Groups Mean Number of Correct Responses for
Test Question Group E

Figure 14 is the graphical representation of the performance of the three
precalculus sections for the mean number of group E questions that were answered to
indicate the use of the representativeness heuristic—ranging from zero questions to two
questions answered using the representativeness heuristic for the pre-test and two post-
tests.

As shown in Figure 14, all three precalculus sections demonstrated an decrease in
the mean number of representativeness heuristic responses to questions of type E from
the pre-test to the first post-test. The prediction-only group demonstrated the greatest
decrease in the number of representativeness heuristic responses to type E questions from
the pre-test to post-test 1. While, the prediction-and-voting group ranked second and the
comparison group ranked third in terms decreasing the number of representativeness heuristic responses provided for questions in group E. For the second post-test, all three precalculus sections were able to maintain their decrease in the number of representativeness heuristic responses to type E questions below their pre-test scores. Analysis of the paired t-tests (see Appendix K) show that the comparison and prediction-only groups demonstrated a statistically significant decrease in the number of representativeness heuristic responses given from the pre-test to post-test 1 and the pre-test to post-test 2 for questions of type E. Whereas, the prediction-and-voting group only showed a statistically significant decrease in the number of representativeness heuristic responses for questions of type E from the pre-test to the post-test 1.

Figure 14: Comparison and Treatment Groups Mean Number of Representativeness Heuristic Responses for Test Question Group E

The prediction-only group and prediction-and-voting group performed below the comparison group in terms of providing fewer representativeness heuristic responses or
questions of type E on both post-tests; however, the logistic regression for test question

group E (see Appendix J) reveals that the prediction-only group and prediction-and-
voting group did not perform statistically significantly different from the comparison
group. However, the effect sizes for type E questions (see Appendix L) failed to reach
the Cohen’s d cut-off for significance.

4.2 Analysis of Representativeness Heuristic Lesson Video Recordings

In order to determine the ways in which the use of prediction and classroom voting might
have contributed to students’ overcoming their misconceptions about representativeness,
observational data was gathered by video-taping each of the Representativeness Heuristic
Lesson presentations for each of the three precalculus sections. The videotapes were
viewed by the researcher and a collaborating teacher that had not previously seen the
videotapes or been present for the Representativeness Heuristic Lessons. The
collaborating teacher was chosen based on their availability to view all of the video-taped
lessons, which totaled to approximately 4 hours of video. The video-taped lessons were
analyzed to determine any observable differences between the comparison, prediction-
only, and prediction-and-voting groups in the following areas:

1. Researcher’s presentation of the Representativeness Heuristic Lesson,

2. Researcher’s interaction with student groups during the lesson,

3. Students’ answers to the five representativeness questions presented
during the lesson,

4. Students’ level of interaction during the representativeness heuristic
lesson, and
5. Students’ learning displayed during the lesson.

After individual viewing of the videotapes, the researcher and the collaborating teacher compared their observations for the five criteria and reconciled their results anywhere differences occurred in their conclusions. The detailed results for the video-taped lesson analysis are presented in the following five subsections.

4.2.1 Presentation of the Representativeness Heuristic Lesson

The Representativeness Heuristic Lesson for the comparison and treatment groups was presented by the researcher using the same PowerPoint Presentation of five multiple choice questions, structured in such a way as to reveal students’ use of the representativeness heuristic and promote students’ use of correct probabilistic reasoning (see Appendix B to view the slides for this presentation). Due to the researcher’s use of the same PowerPoint Presentation during each lesson presentation, the visual content of the lesson was identical for all three precalculus classes.

The only differences in the presentation of the lesson to the three precalculus classes occurred due to the design of the comparison and treatment groups. For instance, when a representativeness question was presented to the comparison group, the class was asked to immediately discuss the question in their student groups. In contrast, students in the prediction-only class were asked to individually predict the correct answer to the question and record their prediction on a supplied handout before discussing the questions within their student groups. Finally, students in the prediction-and-voting group was asked to individually predict the correct answer choice, record their prediction on a supplied handout, and vote for their prediction using clickers before discussing the
question in their student groups. Aside from these procedural lesson instruction variations that were built into the research design of the study, the researcher and collaborating teacher did not observe any differences in how the representativeness heuristic lesson was presented to the three precalculus classes.

4.2.2 Researcher’s Interaction with Student Groups

During the viewing of the video-taped lessons, the researcher and collaborating teacher watched for any differences in how the researcher interacted with students from the comparison, prediction-only, and prediction-and-voting classes. Throughout all three classes, the researcher interacted with students in the following ways:

1. When students were given individual time to work on the questions presented during the lesson, the researcher would walk around the room to encourage on-task behavior but would not talk to any students.

2. During the group discussion portions of the lesson, the researcher would circulate around the class and talk to each of the student groups. The researcher asked each group for their answer choice selection and their reasoning for selecting the answer choice. When groups did not agree on the same answer choice, the researcher would ask for students’ reasoning for each answer choice and would suggest that the students find some way to “come to a group consensus” or “convince one another that their answer choice selection is correct.” While talking to the student groups, the researcher would not indicate to students what the correct reasoning for the problems was and would not reveal which answer choice was correct. By not providing any
instruction regarding the problem, the researcher hoped to not interfere with the naturally-occurring dynamics of the student group discussions.

3. Once all students groups had sufficient time to discuss the question and come up with a group answer choice, the researcher led a whole-class discussion in which students explained their reasoning for selecting each answer choice. If no students volunteered to discuss an answer choice for a question, then the researcher would move on to the next answer choice. Once all answer choices for a question were discussed, the researcher presented (via the PowerPoint Presentation, see Appendix B) the mathematical work behind finding the probabilities of the answer choices and would then reveal the correct answer for each question.

The collaborating teacher and the researcher did not observe any differences between the comparison, prediction-only, and prediction-and-voting groups in terms of how the researcher interacted with students during the lesson. In all of the precalculus classes, the researcher addressed all students in the three ways listed previously. Therefore, the researcher’s interaction with students during the lesson should not have affected the results attained by the lesson given to the comparison and treatment groups.

4.2.3 Students’ Answers to Representativeness Heuristic Questions during the Lesson

During the representativeness heuristic lesson, all students were asked to record their answers to the five questions presented in the lesson. Students in the comparison group recorded their answers to the questions as groups on a student-group handout (see Appendix D). However, the prediction-only and prediction-and-voting group students
independently recorded their answers to the questions on an individual-student handout (see Appendix C). The answer choices selected by students in the comparison, prediction-only, and prediction-and-voting group are listed in Table 13 for all five questions presented in the lesson.

To determine if the proportion of student responses to the lesson questions were statistically different for any of the answer choices, all class proportions of students’ answer choices to each question from Table 13 were analyzed using two-proportion comparison analysis (see Appendix P for full results). Student answer choices for questions 1, 2, and 4 (the questions involving students’ use of the representativeness heuristic) all failed to reach the 0.05 p-value significance level required for any two proportions to be judged statistically different. Therefore, all student answer choice responses for questions 1, 2, and 4 can be judged as statistically equivalent for all three precalculus classes. However, for questions 3 and 5 (the questions involving students’ use of the equiprobability bias/“equally likely” learned response) the comparison group provided answers that were different from the answers provided by the prediction-only and prediction-and-voting groups.

In question 3, the students were asked to determine which of the following was most likely to occur when flipping a coin two times: getting two heads, two tails, or one head and one tail. The correct answer choice was “B. getting one head and one tail” with probability of $\frac{1}{2}$, since the probability of getting two heads and two tails are both $\frac{1}{4}$. An answer choice selection of “D. all of the above outcomes are equally likely” might indicate that the student was not using correct probabilistic reasoning and was relying on the equiprobability bias/“equally likely” learned response. The comparison group
Table 13: Students’ Answers to Representativeness Heuristic Questions during the Lesson

<table>
<thead>
<tr>
<th>Question</th>
<th># of student responses</th>
<th>Answer Choice</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>0</td>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>(100%)</td>
</tr>
<tr>
<td>Q2</td>
<td>0</td>
<td>A</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>(40%)</td>
</tr>
<tr>
<td>Q3</td>
<td>0</td>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(0%)</td>
</tr>
<tr>
<td>Q4</td>
<td>0</td>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(0%)</td>
</tr>
<tr>
<td>Q5</td>
<td>0</td>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(0%)</td>
</tr>
</tbody>
</table>

Note: Computed answer to question RH = Representativeness heuristic response to question EQ = “Equally likely” responses to question

students all selected answer choice D on their group-completed handouts, whereas 16 students (73%) from the prediction-only class and 15 students (60%) from the prediction-
and-voting class selected answer choice D on their individual-completed handouts. The proportion of students in the prediction-only and prediction-and-voting groups answering D to question 3 was statistically equivalent. However, the comparison group provided significantly more equiprobability bias/“equally likely” learned responses (answer choice D) than either of the treatment groups ($p < 0.01$). Similarly, when examining the number of correct student responses to question 3 (answer choice B) the comparison group provided significantly fewer correct responses than either of the treatment groups ($p < 0.01$). The prediction-only and prediction-and-voting groups provided statistically equivalent number of correct responses to question 3. These results reveal that the comparison group was relying more heavily on the equiprobability bias/“equally likely” learned response than the prediction-only and prediction-and-voting groups for question 3 during the representativeness heuristic lesson.

In question 5, the students were asked to determine which of the following was most likely to occur when rolling two fair dice: getting a sum of 3, 5, 6, or 9. The correct answer choice was “C. Sum = 6” with probability of $5/36$, the highest probability that occurred in all of the answer choices. An answer choice selection of “E. all of the above outcomes are equally likely” would indicate that the student was not using correct probabilistic reasoning and was relying on the equiprobability bias/“equally likely” learned response. The comparison group students all selected answer choice C on their group-completed handouts, whereas 16 students (73%) from the prediction-only class and 20 students (80%) from the prediction-and-voting class selected answer choice C on their individual-completed handouts. The proportion of students in the prediction-only class and the proportion of students in the prediction-and-voting class were statistically
equivalent for students’ responses of C for question 5. However, the comparison group provided significantly more correct responses (answer choice C) than either of the treatment groups \((p < 0.02)\). Similarly, when examining the number of equiprobability bias/ “equally likely” learned student responses to question 5 (answer choice E) the comparison group provided significantly fewer equiprobability bias/ “equally likely” learned responses than the prediction only group \((p < 0.02)\). However, the comparison group barely missed the 0.05 statistical significance cutoff point for achieving significantly fewer equiprobability bias/ “equally likely” learned responses than the prediction-and-voting group \((p = 0.065)\). The prediction-only and prediction-and-voting groups provided statistically equivalent number of “equally likely” responses to question 5. These results show that the comparison group was providing more correct responses than both the prediction-only and prediction-and-voting groups for question 5 during the representativeness heuristic lesson.

The analysis of student answer choice selection for questions 3 and 5 reveal that the comparison group initially provided more equiprobability bias/ “equally likely” learned responses for question 3 than the prediction-only and prediction-and-voting groups. However, for question 5, the comparison group students transitioned to providing more correct answer choices than either of the treatment groups. After examining the video-taped lessons and student replies on the handouts, the causes for the comparison group’s shift from using the equiprobability bias/ “equally likely” learned responses on question 3 to using correct probabilistic reasoning on question 5 cannot be determined. If the student completed handouts were individually completed by students in the comparison group so that their reasoning was explicitly explained for each
question, then some explanation(s) for the students’ shift in responses for questions 3 and 5 might have been revealed. However, since the comparison group filled out the lesson’s handout as student groups, the reasoning used for each student cannot be determined.

Examining Table 13 for each group’s answer choice selections for the five lesson questions, it is interesting to note that within each class, the students transitioned from providing representativeness heuristic responses to question 1 (approximately 60% of class responses) to providing more correct responses on questions 2 and 4 (80% to 100% of class responses). Similarly, students within each class transitioned from providing “equally likely” responses on question 3 (60% to 100% of class responses) to providing more correct responses to question 5 (70% to 100% of class responses). The shifts in students’ answer choices during the lesson reveals that students within all of the classes were able to transition from relying on the representativeness heuristic to using correct probabilistic reasoning during the lesson. Likewise, students in all three precalculus classes shifted from using the equiprobability bias/ “equally likely” learned response to correctly applying probabilistic reasoning to solve probability problems during the lesson.

4.2.4 Students’ Level of Interaction during the Representativeness Heuristic Lesson

The researcher and collaborating teacher identified three types of student interaction during the viewing of the video-taped representativeness lesson:

1. Interaction among students during the student-group discussions of the answer choices,

2. Students’ interaction with the researcher during the student-group discussions, and
3. Students’ interaction with the researcher during the presentation of the correct answer choice.

During the representativeness heuristic lesson, each class was video-taped via a video-camera placed at the back of each of the classrooms. This allowed the video-camera to capture an overall view of the classroom environment while the lesson was occurring (see Figures 15 through 17). While viewing the video-taped lessons, the researcher and the collaborating teacher noticed that the positioning of the video-camera at the back of the classroom did not capture what was occurring within many of the student group discussions. In fact, the video-camera was able to capture student dialogue only for the student group located closest to the camera. Therefore, only the student groups located closest to the video-camera were observed for student interaction during the lesson.

Within all three classes, the student groups appeared to engage in similar group discussions. The students would reveal the answer choice that they considered to be correct, the students would then explain their reasoning for selecting the answer choice (sometimes writing out any mathematical work they had used), and the students would determine a group answer choice. After a group answer choice was selected, students would remain quiet or would often engage in other conversations involving their outside interests until the whole-class discussion began. Due to the limitation in audio recorded by the video camera, the researcher and collaborating teacher could not identify any differences in student to student interaction among the comparison and treatment groups.
Figure 15: Comparison Class Layout for Video Recording

Figure 16: Prediction-Only Class Layout for Video Recording
As described earlier (section 4.2.2), during the group discussion portions of the lesson, the researcher would walk among the class and talked to each of the student groups. The researcher would ask each group for their answer choice selection and their reasoning for selecting the answer choice. In each of the classes, the students interacted with the researcher in similar ways. If the student group agreed on the selection of an answer choice, the researcher would simply ask for the groups’ reasoning behind their answer choice selection before moving on to the next student group. When groups did not agree on one answer choice, the researcher would ask for individual students’ reasoning for each answer choice and would suggest that the students find some way to “come to a group consensus” or “convince one another that their answer choice selection is correct.” With this suggestion, the researcher would move to a different student group, so that the undecided group could discuss the question and answer choices further.
Occasionally, the students would ask the researcher a question regarding the problem or would seek verification that they had selected the correct answer choice. However, the researcher would not instruct any students in the correct reasoning to use and would not reveal which answer choice was correct. By not providing any instruction regarding the problem, the researcher hoped to not provide any additional instruction to any one of the classes, which could affect the results of the lesson and the study. Within the videotapes of the representativeness lesson, the researcher and collaborating teacher did not observe any additional student-researcher interactions during the group discussions for any of the three precalculus classes. Therefore, the student-researcher interactions during the student-group discussions can be considered the same for the comparison and treatment groups.

The researcher and collaborating teacher did observe differences in how the students in the three precalculus classes interacted with the researcher during the presentation of the correct answer choice. The prediction-and-voting group’s students remained silent during the researcher’s presentation of the probability work that led to the correct answer choice for each of the five lesson questions. In contrast, the prediction-only group’s students often talked within their groups during the researcher’s presentation. In fact, the researcher had to ask the prediction-only class to be quiet on two separate occasions during the video of the lesson. The comparison group also talked within their student groups during the researcher’s presentation of the correct answer choice. However, the comparison group was not as disruptive as the prediction-only group and the researcher did not have to ask the comparison class to be quiet during the presentation. Thus, the prediction-and-voting group was the most composed and
attentive class during the researcher’s presentation of the correct answer choice for the five lesson questions.

### 4.2.5 Students’ Learning Displayed during the Representativeness Heuristic Lesson

Based on students’ replies on the lesson handouts (see Table 13 in section 4.2.3), students in all three precalculus classes transitioned from providing representativeness heuristic responses and equiprobability bias/“equally likely” learned responses to providing more correct responses to the questions by the end of the lesson. Thus, the students in the comparison and treatment groups displayed that they had learned to correctly apply probabilistic reasoning to solve probability problems during the lesson.

While observing the video-taped lessons, the researcher and collaborating teacher noticed that the prediction-and-voting group seemed to have more knowledge of probability than the comparison and prediction-only classes. During the whole-class discussion of the answer choices, the prediction-and-voting students usually gave reasoning that involved the use of probability theory, such as “the probability of that outcome is ½ and the other answer choices have probabilities of ¼.” Whereas the comparison and prediction-only students often provided reasons that were based on expectation and personal experience, such as “you don’t usually get two heads or two tails in a row.” The level at which the students used probability theory in their reasoning for selecting answer choices led the researcher and collaborating teacher to believe that the prediction-and-voting students had a better understanding and/or deeper knowledge of probability than the students in the comparison and prediction-only classes. However, when the student answer choice selections were compared for the three precalculus
classes (see Table 13), the prediction-and-voting group only significantly outperformed the comparison group for the number of correct answers to question 3 \( (p < 0.001) \). In all other cases, the prediction-and-voting group did not perform significantly better than the comparison or prediction-only classes.

4.3 Analysis of Representativeness Heuristic Lesson Survey

In order to determine any differences in students’ self-rated engagement and interest during the Representativeness Heuristic Lesson, the eight survey items were examined using a two sample t-test and a Mann-Whitney comparison of students’ median survey scores. The survey items were categorized into two groups: factor one group consisted of survey items 1, 2, 3, 6, 7, and 8, and represents students’ interest and/or engagement in learning the probability topics presented in the Representativeness Heuristic Lesson. Survey items categorized under factor two, (items 4 and 5), represent students’ participation in class discussion during the lesson. For the two sample t-test, the mean class rating for each factor was compared among the comparison, prediction-only, and prediction-and-voting groups using the statistical program Minitab. The median class rating for each factor was compared among the comparison, prediction-only, and prediction-and-voting groups using the Mann-Whitney Test for equality of medians in the statistical program Minitab. The mean and median student survey ratings are summarized in Table 14 shown below. For complete Mann-Whitney and two sample t-test results for the analysis of the survey items, see Appendix N and O respectively.

Students’ ratings of survey items in factor one ranged from 6 to 30, with a low score indicating minimal student interest and/or engagement during the
Representativeness Heuristic Lesson and a high score indicating strong student interest and/or engagement in learning the probability topics presented in the lesson.

Alternatively, students’ ratings of survey items in factor two could range from 2 to 10, with a low score indicating little to no student participation in class discussion during the lesson and a high score indicating frequent student participation during the Representativeness Heuristic Lesson.

### Table 14: Mean and Median Group Rating for Factor 1 and Factor 2 Survey Items

<table>
<thead>
<tr>
<th></th>
<th>Factor 1: Survey Items 1, 2, 3, 6, 7, and 8 (Range: 6 to 30)</th>
<th>Factor 2: Survey Items 4 and 5 (Range: 2 to 10)</th>
</tr>
</thead>
</table>
| Comparison Group     | Mean = 22.08  
Median = 22  
N = 20 | Mean = 8.84  
Median = 9  
N = 20 |
| Prediction-Only Group| Mean = 21.27  
Median = 21.5  
N = 22 | Mean = 8.86  
Median = 9.5  
N = 22 |
| Prediction-and-Voting Group | Mean = 21.75  
Median = 23  
N = 25 | Mean = 9.00  
Median = 9  
N = 25 |

All Mann-Whitney Tests for equality of medians and two sample t-tests for equality of means for factor one and factor two failed to reach the 0.05 level of significance. Therefore, students’ ratings for factor one survey items can be judged as statistically equivalent among the comparison, prediction-only, and prediction-and-voting groups. Similarly, the students’ ratings for factor two survey items are statistically equivalent between the comparison and treatment groups. Therefore, there were no discernable differences between students’ survey ratings in the comparison, prediction-only, and prediction-and-voting groups.

While the three groups did not significantly differ in students’ survey ratings for factors one and two, it is important to note that all groups had mean and median ratings...
that were above the neutral rating cut-off for each factor (factor one neutral rating cut-off was at 18 and factor two rating cut-off was at 6). Thus, all students reported participating in class discussion and feeling interested and/or engaged in their learning of probability during the Representativeness Heuristic Lesson.

4.4 Analysis of Student Interviews

The semi-structured interviews were transcribed by the researcher and color-coded, based on the following categories:

1. Students’ probabilistic reasoning abilities before the lesson presentations (italicized and bolded text with the footnote of 1 in the interview transcript shown in Appendix M),

2. Students’ probabilistic reasoning abilities after the first lesson presentation (italicized and bolded text with the footnote of 2 in the interview transcript),

3. Students’ probabilistic reasoning abilities after both lesson presentations (italicized and bolded text with the footnote of 3 in the interview transcript),

4. Students’ current probabilistic reasoning abilities (italicized and bolded text with the footnote of 4 in the interview transcript),

5. Students’ comments about the first lesson presentation (italicized and bolded text with the footnote of 5 in the interview transcript),

6. Students’ comments about the second lesson presentation (italicized and bolded text with the footnote of 6 in the interview transcript),

7. Students’ general comments about the lesson presentations (italicized and bolded text with the footnote of 7 in the interview transcript), and
8. Students’ comments regarding previous probability instruction (italicized and bolded text with the footnote of 8 in the interview transcript shown in Appendix M).

Using the above eight categories and the color-coded in the interview transcript (shown in Appendix M), the researcher coded and analyzed the students’ interview comments and the results are summarized in Table 15, Table 16, and Table 17 that follow.

A total of three students were interviewed during this study, one student from each of the comparison, prediction-only, and prediction-and-voting groups. Students had to meet the following criteria before they were interviewed:

   a. Student demonstrated use of the representativeness heuristic on their pre-test,

   b. Student demonstrated improvement in their use of the representativeness heuristic from their pre-test to first post-test, and

   c. Student consented to be interviewed for the study.

The students’ use and improvement over the representativeness heuristic was important criteria since the interview questions (see Appendix G) involved asking students about the aspects of the lesson(s) that they believed improving their probabilistic reasoning abilities.

It is important to note that multiple students met the first two criteria listed above. However, the students with the greatest improvement in their use of the representativeness heuristic within the comparison, prediction-only, and prediction-and-voting classes were asked to give student interviews. All of the students with the best improvement their use of the representativeness heuristic consented to be interviewed. If any of the students had not consented to being interviewed, the next student with the
highest improvement in their use of the representativeness heuristic would have been asked to be interviewed.

4.4.1 Students’ Probability Reasoning

The results of the researcher’s analysis of the coded interviews for students' comments regarding their probability reasoning are summarized in Table 15 to follow.

Students were asked in the interview to rate their probability reasoning abilities before the lesson presentations, after the first lesson presentation, and after both lessons. The students’ self-reported rating and their reasoning for giving each rating are listed in Table 15. The comparison and prediction-and-voting students reported that their reasoning regarding probability had improving over the course of the study. The prediction-only student, however, did not report any improvement in her probability reasoning abilities.

When asked about their reasoning regarding questions on the Pre-Test, all three students’ comments demonstrated the use of the representativeness heuristic belief that a list of probabilistic outcomes should appear random and that an outcome with repeated values is less-likely to occur than outcomes with no repeated values. The comparison group and prediction-only group students’ interview comments also demonstrated the representativeness heuristic belief that any list of outcomes should model the characteristics of the parent population such as a listing of the outcome of 10 coin tosses should consist of 5 heads and 5 tails. The prediction-and-voting student also mentioned initially holding the erroneous belief that rolling the number six on a die was less likely than rolling a smaller number. Based on the prediction-and-voting student’s remarks in
Table 15: Students' Comments During Interview Regarding their Probability Reasoning

<table>
<thead>
<tr>
<th>Student's Probability Reasoning</th>
<th>Comparison Student</th>
<th>Prediction-Only Student</th>
<th>Prediction-and-Voting Student</th>
</tr>
</thead>
</table>
| **Before Lessons** | Presented Reliance on Representativeness Heuristic:  
1. Characteristics of parent population  
(50-50 distribution of H and T)  
2. Randomness of sample  
(repeated values are less likely to occur)  
Reasoning Rating: *Average (just above average)* because student was unable to show any proof of work - "just reasoning" | Presented Reliance on Representativeness Heuristic:  
1. Characteristics of parent population  
(50-50 distribution of H and T)  
2. Randomness of sample  
(repeated values are less likely to occur)  
Reasoning Rating: *Below Average* because student was unable to show any proof of work - "just reasoning" | Presented Reliance on Representativeness Heuristic:  
1. Randomness of sample  
(repeated values are less likely to occur)  
Other Misconception: rolling sixes are less likely than rolling smaller number on a die  
Reasoning Rating: *Below Average* because student could only base probabilistic reasoning on their experiences |
| **After First Lesson** | Reasoning Rating: *Above Average* because student had learned how to use the "tree chart" for working out probability problems | Reasoning Rating: *Below Average* because student was unsure about how to apply probabilistic rules and methods or how to tell if two events were independent | Reasoning Rating: *Average because student learned how to order and organize probabilistic events |
| **After Both Lessons** | Student showed the following improvements: 1. Student remarked on their use of multiplying the probabilities of independent events. 2. Student remarked on their use of tree charts and tables to organize probabilistic events  
Reasoning Rating: *Above Average* because student felt they had improved but still had some difficulty in solving some probability problems | Student showed the following problem behavior: 1. Student was unable to determine when two events are independent 2. Student would answer "equally likely" when they believe the problem involves independent events but student does not mathematically verify independence of events.  
Reasoning Rating: *Below Average* because student was unsure about how to apply probabilistic rules and methods or how to determine if two events were independent | Student showed the following improvements: 1. Student remarked on their use of multiplying the probabilities of independent events. 2. Student remarked on their use of tree charts to organize probabilistic events  
Reasoning Rating: *Average (a little above average)* because student had practiced organizing probabilistic events and determining their probabilities |
| **Current Reasoning Shown During the Interview** | Student seemed unsure about when to answer "equally likely" to some probability questions. Student would answer "equally likely" but seemed unsure as to why the problem had "equally likely" outcomes. | Student believes that all independent events are "equally likely".  
Student is unsure about how to determine when two events were independent.  
Student did not appear to "work out" any answers, but relied entirely on their reasoning. | Student seemed to genuinely understand that some events will have same probability regardless of outcome (one face of a die is 1/6th probability regardless of number on the face of the die).  
Student demonstrated use of probabilistic rules to work out probability problems. However, the student mistakenly added probabilities when he should have multiplied.  
Student did not show any proof during the interview that he did not fall for "equally likely" learned response. |

The belief that rolling a six occurred less frequently than rolling lower valued numbers, such as a two or three. Determining the probability of events based on one's previous experiences is characterized as the use of the availability heuristic, in which one estimates the likelihood of events based on how easy it is to call to mind an instance of
the event (Shaughnessy, 1992). Therefore, it is possible that the prediction-and-voting student used both the representativeness and availability heuristics in determining the probability of events before the lesson presentations.

When questioned about their reasoning regarding questions on Post-Test 2, which was administered six weeks after the administration of the Pre-Test, the comparison group and prediction-and-voting group students’ interview comments indicated the appropriate use of probabilistic tools, such as using tree diagrams to organize probabilistic outcomes and multiplying the probabilities of independent events. However, the comparison student demonstrated that although he had selected the answer choice for “equally likely” outcomes, he was unsure of why the probabilistic outcomes were “equally likely.” The prediction-only student repeatedly stated that she considered any probabilistic event that was independent to have equally likely outcomes, regardless of the type of outcome. The prediction-only student also admitted to having difficulty in determining if probabilistic events were independent, even though her answers to the majority of the Post-Test 2 questions were “equally likely.” From their responses during the interview, the comparison group and prediction-only students demonstrated the equiprobability bias/“equally likely” learned response, in which one judges probabilistic events to be equally likely even when the events do not have the same probability of occurring. While the prediction-and-voting student comments showed an improvement in his probability reasoning, such as no longer relying on the availability heuristic, the student did not provide any statements in the interview that would indicate that he did develop the equiprobability bias/“equally likely” learned response.
Overall, the comparison student and prediction-and-voting student showed the most use of correct probability reasoning during their interviews. However, it is important to note that the prediction-and-voting student appeared to use more correct probability reasoning than the comparison student at the time when the pre-test was administered. The prediction-only student showed the least use of correct probability reasoning during the interview and repeatedly commented on how she answered “equally likely” to test questions without doing any scratch work and without being able to identify any definite probabilistic reasoning to justify her answers to probability test questions. All students' interview comments indicated that they would answer questions on the tests without completely working out the problem, showing the students’ tendency to rely on their reasoning to answer probabilistic questions instead of using probabilistic rules and/or procedures to work out the probabilities involved in the problems to determine the correct answer choice.

Since the interviews consisted of only one student per the comparison, prediction-only, and prediction-and-voting groups, generalizations from the individual student interviews to each group cannot be made. However, the interviews can provide some insight into the probability reasoning of students in each of the groups.

4.4.2 Students’ Comments About the Lessons

The results of the researcher's analysis of the coded interviews for students' comments regarding the probability lessons are summarized in Table 16 below. All three students that were interviewed remembered that the first lesson involved learning how to organize
the outcomes and determine the probability of events, and that the second lesson involved practicing the concepts learned in the first lesson using student groups.

The prediction-only student stated that the second lesson was “more interactive” (Appendix M, line 1705) since it involved the class discussing their predictions of several probability problems in groups of three to four students. The prediction-and-voting student recalled the use of clickers during the second lesson and stated that using the clickers made the lesson “more interactive, [because] you were actually able to get into the lesson” (Appendix M, line 2594). Overall, all of the students described that the two lessons assisted them in recalling or learning probability concepts.

Table 16: Students’ Comments During Interview Regarding the Probability Lessons

<table>
<thead>
<tr>
<th>Student Comments About the Lessons</th>
<th>Comparison Student</th>
<th>Prediction-Only Student</th>
<th>Prediction-and-Voting Student</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Lesson</strong></td>
<td>Student remembered:</td>
<td>Student remembered:</td>
<td>Student remembered:</td>
</tr>
<tr>
<td>1. Use of predicting some</td>
<td>1. Dice Simulation</td>
<td>1. Presentation of how</td>
<td>1. The use of student groups</td>
</tr>
<tr>
<td>probabilities</td>
<td>2. Presentation of</td>
<td>to organize probabilistic</td>
<td>and having to come up with</td>
</tr>
<tr>
<td></td>
<td>“tools” like the</td>
<td>events</td>
<td>group answer to probability</td>
</tr>
<tr>
<td></td>
<td>tree chart to</td>
<td></td>
<td>questions</td>
</tr>
<tr>
<td></td>
<td>organize</td>
<td></td>
<td>2. The main feature of the</td>
</tr>
<tr>
<td></td>
<td>probabilistic</td>
<td></td>
<td>lesson was to practice</td>
</tr>
<tr>
<td></td>
<td>events</td>
<td></td>
<td>probability concepts learned</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>in the first lesson.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3. Student called the lesson</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>“more interactive” than the</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>first lesson.</td>
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<tr>
<td><strong>Second Lesson</strong></td>
<td>Student remembered:</td>
<td>Student remembered:</td>
<td>Student remembered:</td>
</tr>
<tr>
<td>1. The use of student groups</td>
<td>1. The use of student</td>
<td>1. The use of student</td>
<td>1. The use of student groups</td>
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<tr>
<td></td>
<td>groups</td>
<td>groups and having come</td>
<td>groups to discuss answer to</td>
</tr>
<tr>
<td></td>
<td></td>
<td>up with group answer</td>
<td>probability questions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>to probability questions</td>
<td>2. The use of clickers during</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>the lesson.</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>3. The main feature of the</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>lesson was to practice</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>probability concepts learned</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>in the first lesson.</td>
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<td></td>
<td></td>
<td></td>
<td>4. Student said that the</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>clickers made the lesson</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>“more engaging,” “helpful,”</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>and “fun.”</td>
</tr>
<tr>
<td><strong>Comments in General About the</strong></td>
<td>Student emphasized</td>
<td>Student emphasized</td>
<td>Student emphasized</td>
</tr>
<tr>
<td><strong>Lessons</strong></td>
<td>that the use of</td>
<td>that the use of the</td>
<td>that the use of the</td>
</tr>
<tr>
<td></td>
<td>the tree chart to</td>
<td>tree chart to organize</td>
<td>tree chart to organize</td>
</tr>
<tr>
<td></td>
<td>organize</td>
<td>probabilistic events</td>
<td>probabilistic events</td>
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<tr>
<td></td>
<td>organize</td>
<td>was one of the most</td>
<td>was one of the most</td>
</tr>
<tr>
<td></td>
<td>probabilistic</td>
<td>helpful aspects of the</td>
<td>helpful aspects of the</td>
</tr>
<tr>
<td></td>
<td>events was one of</td>
<td>lessons since it helped</td>
<td>lessons.</td>
</tr>
<tr>
<td></td>
<td>the most helpful</td>
<td>her “visualize” the problem</td>
<td></td>
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<tr>
<td></td>
<td>aspects of the</td>
<td>at hand.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>lessons.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.4.3 Students’ Previous Instruction in Probability

The results of the researcher's analysis of the coded interviews for students' comments regarding previous probability instruction are summarized in Table 17 below. The comparison group and prediction-only students both recalled having received previous
instruction in probability in elementary and middle school. The comparison student also stated that he had received some probability instruction in high school in determining the probability of genetic traits in his biology class. However, the prediction-and-voting student did not recall having received any previous instruction in probability, even though his elementary and middle school education took place in the United States.

Again, it is important to note that the semi-structured student interviews consisted of only one student per the comparison, prediction-only, and prediction-and-voting groups and therefore generalizations from the individual student interviews to each group cannot be made. Therefore, the students’ reasoning abilities, beliefs about the lessons, and previous instruction in probability cannot be considered indicative of all of the students in the comparison group, prediction-only group, and prediction-and-voting group. The student interviews do, however, provide the invaluable perspective of individual students’ views of the probability lesson presentations and any changes in their individual reasoning abilities throughout the research intervention.

Table 17: Students’ Comments During Interview Regarding Previous Probability Instruction

<table>
<thead>
<tr>
<th>Previous Probability Instruction</th>
<th>Comparison Student</th>
<th>Prediction-Only Student</th>
<th>Prediction-and-Voting Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student remembered doing short probability lessons in elementary school and middle school.</td>
<td>Student remembered doing short probability lessons in elementary school and middle school.</td>
<td>Student did not remember receiving any previous probability instruction.</td>
<td></td>
</tr>
<tr>
<td>Student stated that he used the “grid method” of determining the probability of hereditary traits in his biology class.</td>
<td>Student did not remember being taught the “rules” of probability in her prior probability teachings.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.4.4 Students’ Reasoning Abilities shown on their Tests and During their Interviews

Based on their answers on the pre-test and post-tests (results summarized in Table 18) and their comments during the interview, the comparison and prediction-only
students demonstrated the greatest improvement in reforming their answers from the representativeness heuristic to correct probability reasoning. However, the comparison and prediction-only students also repeatedly answered that probabilistic events were “equally likely” when the probability of the events were in fact not equivalent. In contrast, the prediction-and-voting student did not show any shift from correct probability reasoning to the inappropriate use of the “equally likely” response. However, the prediction-and-voting student also demonstrated the least use of the representativeness heuristic at the administration of the pre-test and therefore the improvement in his use of the Representativeness heuristic was minimal.

Table 18: Summarized Results of Interview Students’ Pre-test and Post-tests

<table>
<thead>
<tr>
<th>Number of test questions were student moved from using:</th>
<th>Comparison Student</th>
<th>Prediction-Only Student</th>
<th>Prediction-and-Voting Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Representativeness Heuristic to using correct probability reasoning</td>
<td>2 (Q2 and Q4 on Pre-test &amp; Post-test 2)</td>
<td>3 (Q2, Q4, and Q9 on Pre-test and Post-test 2)</td>
<td>1 (Q4 on Pre-test and Post-test 2)</td>
</tr>
<tr>
<td>Correct probability reasoning to answering equiprobability bias/ “equally likely” learned response</td>
<td>1 (Q3 on Pre-test &amp; Post-test 2)</td>
<td>1 (Q3 on Post-test 1 and Q8 on Post-test 2)</td>
<td>0</td>
</tr>
</tbody>
</table>

Therefore, it is unclear if the use of prediction and/or classroom voting in the Representativeness Heuristic Lesson assisted the interviewed students in replacing their overreliance on the Representativeness Heuristic with the use of correct probability reasoning.

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Chapter 5

Conclusions

During the course of this study, all three precalculus sections demonstrated a statistically significant ($p < 0.05$ in paired t-tests) shift from their use of the representativeness heuristic to the use of correct probabilistic reasoning between the pre-test and first post-test. The comparison, prediction-only, and prediction-and-voting groups can be considered to have reduced their overreliance on the representativeness heuristic and, to a certain extent (resulting in students answering on average up to one more question correctly on the second post-test), maintained this improvement up to six weeks after the Representativeness Heuristic Lesson. This finding is consistent with the results of the Morsanyi et al. (2009) study which revealed that “teaching students about the representativeness heuristic successfully decreased the number of representativeness-based responses in the case of the British psychology students” (p.219).

The analysis of students’ use of the equiprobability bias/ “equally likely” learned response (type B questions) on the pre-test and post-tests reveals that the prediction-and-voting group provided significantly fewer learned responses than the comparison group (statistically and for practical purposes), as shown by the results of the logistic regression coefficients and the effect sizes. The prediction-only group also provided fewer learned responses than the comparison group but failed to meet the criteria for practical and statistical significance in the effect sizes and the logistic regression analysis. These results would indicate that the comparison group, while out-performing the treatment groups on the representativeness heuristic test questions, failed to adopt correct probabilistic reasoning and instead adopted the equiprobability bias/ “equally likely”
learned response, in which students answer that all probabilistic events are equally likely regardless of the type of probability question being asked.

While the equiprobability bias/“equally likely” learned response would provide students with correct answers to the representativeness heuristic test questions, this response would be inappropriate and incorrect for other probability questions, such as type B test questions; unfortunately, only two of the ten questions on the tests were equiprobability bias/“equally likely” learned response (type B) questions. As so few “equally likely” response (type B) questions were on the tests, there is not enough evidence to state conclusively that the use of prediction and classroom voting via clickers assisted students in learning correct probabilistic reasoning, without the adoption of the equiprobability bias/“equally likely” learned response.

While it could not be conclusively shown with the research data that was collected during this study, the researcher believes that the use of prediction and classroom voting made the probability lesson more interactive for students and increased students’ personal accountability for their learning. By making students more focused on their answers and current conceptions of the probability topics, the prediction-and-voting group might have had a better opportunity to root out their false preconceptions.

The pre- and post-tests used in this study only examined students’ use of the representativeness heuristic and the equiprobability bias/“equally likely” learned response. Therefore, the researcher cannot determine if the prediction-and-voting groups were able to more effectively remove their false preconceptions regarding probability than the comparison or prediction-only groups. However, these results do offer the motivation for further research into the use of prediction and classroom voting via
clickers and its impact on the depth of student learning and understanding of mathematical and statistical concepts.

The comparison group’s shift toward answering “equally likely” to probability problems on the post-tests supports the Morsanyi et al. (2009) study results that “learning about a certain type of heuristic does not affect the use of another heuristic (e.g., students who learnt to resist the representativeness heuristic continued to give erroneous equiprobability responses)” (p. 219). In addition, all three precalculus classes showed an overall increase in the number of “equally likely” responses to type B questions after the representativeness lesson—this is similar to the findings of researchers in the field of probability and statistics who found that subjects’ use of the equiprobability bias increases with statistics education (Batanero, Serrano, & Garfield, 1996; Lecoutre, 1985; Lecoutre, 1992; Morsanyi et al., 2009). Researchers believe that this counterintuitive increase in subject’s use of the equiprobability bias, with an increase in subject’s statistical knowledge, can be due to the following:

1. “If they viewed the outcome of each flip as independent of the outcomes of all prior flips, they could deduce that all sequences are equally likely without computing the probability” (Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993, p. 396).

2. Subjects believe that “equally likely does not mean that the sequences have the same numeric probability of occurrence, but that there is no basis for making a prediction of what will happen” (Konold et al., 1993, p. 399).
3. “It is possible that students with a poor understanding of the concept of randomness use it to justify their initial intuitions about the equiprobability of different outcomes” (Morsanyi et al., 2009, p. 216).

Overall, neither of the treatment groups out-performed the comparison group in terms of replacing their use of the representativeness heuristic with correct probabilistic reasoning. However, this does not mean that the use of prediction and classroom voting hinders one’s efforts of reducing students’ overreliance on the representativeness heuristic. All three precalculus sections showed significant improvement in their use of the representativeness heuristic after the Representativeness Heuristic Lesson. Therefore, teaching to address students’ overuse of the representativeness heuristic did assist the precalculus students in learning correct probabilistic reasoning.

Due to time constraints, it was not possible for the researcher to interview multiple students from each of the comparison, prediction-only, and prediction-and-voting groups, the semi-structured interviews with one student from each group do provide some insight into: the students’ reasoning abilities throughout the research intervention, their observations and opinions regarding the probability lessons, and their previous instruction in probability. The three students’ interviews all indicated that they held one or more of the representativeness heuristic beliefs at the time that they took the pre-test, and while all three students stated that their probabilistic reasoning abilities had improved after the lessons, only the comparison and prediction-and-voting students’ comments indicated an understanding of probabilistic rules and concepts.

The prediction-only student stated repeatedly that she was unsure of how to determine if two events were independent and seemed to struggle in being able to explain
or apply probabilistic concepts taught in the two lessons. Both the prediction-only and comparison students provided equiprobability bias/ “equally likely” learned responses in their interview response and in answering probability test questions. However, the prediction-and-voting student did not make any statements in the interview nor answered any questions on his tests that would indicate that he did not also develop the equiprobability bias/ “equally likely” learned response. While the results from the interviews cannot be generalized to represent all students in the comparison, prediction-only, and prediction-and-voting groups, the interviews did provide the insight that all of the interviewed students began with an over-reliance on the representativeness heuristic and that the two probability lessons were not enough to provide each of the students with a solid understanding of probability concepts and correct probabilistic reasoning.

The comparison group used in this study used student group discussion since it was the most “generic” representation of the classroom structure used in the prediction-only and prediction-and-voting groups. However, most traditional classroom practices involve students working independently and not in student groups. Therefore, the inclusion of a fourth classroom group of students completing the representativeness heuristic lesson independently would have provided a second comparison group that more closely modeled the traditional classroom structure. It is the researcher’s belief that this “traditional comparison group” would most likely have shown even smaller gains in students’ adoption of correct probabilistic reasoning and a continued use of the representativeness heuristic. It is the researcher’s hope that further studies will be completed using the traditional comparison group so that the effectiveness of the representativeness lesson can be explored using individual student work.

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The students selected for this study cannot be assumed to be a representative sample of the population of high school students and, therefore, definitive generalizations about this study’s results cannot be made. Due to students’ absences when portions of the study’s data collection and intervention were completed, some students were not present for the Representativeness Heuristic Lesson and/or for the administration of one of the Representativeness Heuristic Tests. These missing data values could have an effect on the conclusions made from this study’s data. Although the regression analysis and paired t-test took into account the missing data values and deleted incomplete test data from the analyses, a complete set of test data would have strengthened the confidence that the conclusions drawn from the test data accurately represent what occurred during the study.

As mentioned previously, the prediction-only group had a different classroom teacher than the prediction-and-voting and comparison group’s teacher. Students in all three groups had equivalent knowledge of probability based on the two proportion tests of their mathematics TAKS scores. However, the prediction-only group provided significantly fewer correct responses on their pre-test than the comparison and prediction-and-voting groups (based on one-way ANOVA). Therefore, it is not surprising that the prediction-only group demonstrated the least improvement over the representativeness heuristic out of the three groups used in the study.

It must also be noted that the pre-test and second post-test used for this study were identical, while, the post-test 1 used different test questions that assessed the same concepts as the pre-test and post-test 2. More specifically, the questions in post-test 1 differed from the questions in the pre-test/post-test 2 by the following:
a. The order of the questions in post-test 1 was the reverse order of the types of questions presented in the pre-test/post-test 2.

b. The context of the question was changed slightly (i.e. by asking for the probability of getting the result of heads in the pre-test/post-test 2 and the probability of getting the result of tails on post-test 1, asking for the most likely result on the pre-test/post-test 2 and asking for the least likely result on post-test 1, using a sequence of 8 die rolls on pre-test/post-test 2 and 6 die rolls on post-test 1, etcetera).

For more detailed explanation regarding the differences between the pre-test and post-tests see sections 3.3.1 and 3.3.2 or refer to Appendix E to view the questions used on the tests. This testing structure was used so that test results would not include a testing familiarity effect. However, since not all tests were identical, students’ achievement on post-test 1 might not accurately reflect the knowledge of probability that was shown on the pre-test or second post-test.

One major limitation to the observational data collected for this study was the use of only one camera to capture student interaction during the representativeness heuristic lesson. As stated in section 3.3.3, the level of student interaction within their student groups could not be effectively captured on the video recording of the lesson. The inclusion of an additional video camera or audio recorders in each student group would have greatly improved the researcher’s ability to observe students’ interaction within their student groups during the lesson. However, due to the researcher’s lack of resources, additional video and audio equipment could not be obtained for this study.

The analysis of test data was completed for this study included: regression analysis, logistic regression analysis, paired t-tests, and effect sizes. However, a repeated
measures design (3 groups × 3 tests × 2 test items) might have also been used to analyze the test data collected for the study. Due to time constraints, this repeated measures test item analysis could not be completed. The addition of the repeated measure test analysis would have provided an additional look into the test data and could have possibly strengthened the results provided by the other test data analyses.

While the addition of prediction and classroom voting teaching methods did not appear to have assisted in students’ recognizing and overcoming their misconceptions about representativeness, this does not mean that prediction and classroom voting would not assist in students overcoming other mathematical or statistical misconceptions—such as the outcome approach or the availability heuristic. More research about using these teaching methods must be completed before researchers and teachers can determine where prediction and classroom voting are the most useful tools in assisting student learning of mathematical and statistical concepts.
References


Appendix A: PowerPoint Presentation Slides for the Probability Review Lesson

What are the odds?

What are the odds?
- How many times would you expect to get two 6s when rolling a pair of dice for:
  - 10 rolls?
  - 100 rolls?
  - 200 rolls?

A Review of Probability

What is Probability?
- The study of chance associated with the occurrence of events
- Types of Probability:
  - Classical (Theoretical)
  - Relative Frequency (Experimental)

Definitions
- Sample Space - the list of all possible outcomes from a probability experiment
  - Example:
    - Days of the week:
      - 5 = { Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday }
- Each individual item in the list is called a Single Event or Outcome
- An Event is any combination of Single Events

Probability Notation
- Probability of an Event Occurring
  - \( P(\text{Event}) = \frac{\text{number of desired outcomes}}{\text{total number of possible outcomes}} \)
  - Example:
    - \( P(\text{Head}) = \frac{1 \text{ Head}}{2 \text{ Sides of Coin}} \)

Fundamentals of Probability
- What is wrong with the following statements?
  - The probability of rain today is .10%
  - The probability of rain tomorrow is 120%
  - The probability of rain or no rain this week is 50%
- \( 0.0 \leq P(\text{Event}) \leq 1.0 \)
- \( P(\text{Sample Space}) = 1.0 \)
Finding Relative Frequency (Experimental) Probability

- What is the probability of rolling two 6s for a pair of dice for:
  - 10 rolls?
  - 100 rolls?
  - 200 rolls?

Finding Classical (Theoretical) Probability

- What is the theoretical probability of rolling two 6s for a pair of dice?

\[
P(6,6) = \frac{1}{36}
\]

- 36 outcomes

\[
P(6,6) = \frac{1}{36} \approx 0.0278
\]

Finding the Complement

- What is the theoretical probability of NOT rolling two 6s for a pair of dice?

\[
P(\text{not } 6,6) = 1 - \frac{1}{36} = \frac{35}{36}
\]

Rules of Probability

- When rolling one die:
  - What is the sample space?

Sample Space: \{1, 2, 3, 4, 5, 6\}

- \(P(\text{even#}) = \frac{3}{6} = \frac{1}{2}\)
- \(P(1, 2, \text{or }3) = \frac{3}{6} = \frac{1}{2}\)
- \(P(\text{even# or 1, 2, 3}) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{5}{6}\)

Independence

- Events are “independent” if knowledge that one event has happened does not affect the probability of the other event happening.

- Events A and B are independent if:

\[
P(A \text{ and } B) = P(A) \cdot P(B)
\]
Independence

- Example: Is the probability of getting a “3” on one roll of a die independent of getting a “5” or “6” on the roll of a second die?
  
  Will be independent if:
  
  \[ P(A \text{ and } B) = P(A) \cdot P(B) \]

Independence

- Example: Is the probability of getting a “3” on one roll of a die independent of getting a “5” or “6” on the roll of a second die?
  
  \[ P(A \text{ and } B) = P(A) \cdot P(B) \]

Independence

\[
\begin{array}{c|cccc}
  & 1 & 2 & 3 & 4 \\
\hline
1 & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} \\
2 & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} \\
3 & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} \\
4 & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} \\
5 & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} \\
6 & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} \\
\end{array}
\]

- Getting a “3” on one roll of a die is independent of getting a “5” or “6” on the roll of a second die.

Rules of Probability

- \[ P(A \text{ and } B) = P(A) \cdot P(B) \], if events A and B are independent
  
  - Why?
  
  When events A and B are independent:
  
  \[ P(A \text{ and } B) = P(A) \cdot P(B) \]

Let’s Do It!

- What is the probability of getting all heads when flipping 3 coins?

- What is the sample space?
Let’s Do It!

What is the probability of getting two heads and one tail when flipping 3 coins?
Appendix B: PowerPoint Presentation Slides for the Representativeness

Heuristic Lesson

Question 1:
A fair die is rolled four times and the results are recorded in the order that they appear.

For each roll, the die lands with the number 1, 2, 3, 4, 5, or 6 facing up.

The following outcome is the LEAST LIKELY to occur:

A. 1 2 3 4
B. 5 5 6 6
C. 3 1 4 6
D. 3 1 6 4
E. All of the above sequences are equally likely

Q: Does the order of the sequence of die rolls matter?
A: Yes, the order of the sequence is the order in which the die was rolled.

This means, 3 1 4 6 ≠ 3 1 6 4

Q: Is one roll of the die independent from the other rolls?
A: Will be independent if

\[ P(A \text{ and } B) = P(A) \cdot P(B) \]

P(3 on 1st roll and 11 on 2nd roll) = \( \frac{1}{36} \)

\[
\begin{array}{c|cccc}
\text{Number on 1st Die} & 1 & 2 & 3 & 4 \\
\hline
\text{Number on 2nd Die} & (1,1) & (1,2) & (1,3) & (1,4) \\
& (2,1) & (2,2) & (2,3) & (2,4) \\
& (3,1) & (3,2) & (3,3) & (3,4) \\
& (4,1) & (4,2) & (4,3) & (4,4) \\
& (5,1) & (5,2) & (5,3) & (5,4) \\
& (6,1) & (6,2) & (6,3) & (6,4) \\
\end{array}
\]

P(3 on 1st roll) = \( \frac{1}{6} \)
P(11 on 2nd roll) = \( \frac{1}{6} \)
P(3 on 1st roll and 11 on 2nd roll) = \( \frac{1}{36} \)
P(3 on 1st roll or 11 on 2nd roll) = \( \frac{5}{36} \)

Therefore, the rolls of the die are independent.

P(3 on 1st roll) \cdot P(11 on 2nd roll) = \( \frac{1 \cdot 1 \cdot 1}{6 \cdot 6 \cdot 36} \)
Q. What are the probabilities for each of the following sequence of die rolls?

\[
\begin{align*}
P(3\,1) &= \frac{1\times1\times1}{6 \times 6 \times 6} = \frac{1}{216} \\
P(5\,5) &= \frac{1\times1\times1}{6 \times 6 \times 6} = \frac{1}{216} \\
P(3\,1\,4) &= \frac{1\times1\times1}{6 \times 6 \times 6} = \frac{1}{216} \\
P(5\,5\,5) &= \frac{1\times1\times1}{6 \times 6 \times 6} = \frac{1}{216}
\end{align*}
\]

A. 123 4
B. 555 5
C. 314 6
D. 318 4
E. All of the above sequences are equally likely

Question 2:
Assume the probability of a birth being a baby boy is \(\frac{1}{2}\).

Which of the following sequences, where B represents having a boy and G represents having a girl, is MOST LIKELY to occur for having six children?

A. B B B B G B
B. B G G B G B
C. B B B G G G
D. All of the above sequences are equally likely

Q: Does the order of the sequence of the births matter?

A: Yes, the order of the sequence is the order in which the children are born.

This means, B G G B G B ≠ B B B G G G.

Q: Is the gender of the birth of one child independent from the gender of the birth of another child?

A: Will be independent if:

\[P(A \text{ and } B) = P(A) \times P(B)\]
Q: What are the probabilities for each sequence of births?

P(A and B) = P(A) \cdot P(B)

P(boy for 1st child and boy for 2nd child) = \frac{1}{4}
P(boy for 1st child) = \frac{1}{2}
P(boy for 2nd child) = \frac{1}{2}

P(boy for 1st child) \cdot P(boy for 2nd child) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}

The gender of the birth of one child is independent from the gender of the birth of another child.

A.

P(BBBBGBB) = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{256}

P(BGGGBGB) = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{256}

P(BBBBGGG) = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{256}

A. BBBBGBB 2 2 2 2 2 2 64
B. BBBGGBB
C. BBBBBGG
D. All of the above sequences are equally likely

Question 3:

Mary has flipped a coin two times. Which outcome is MOST LIKELY to occur?

A. Getting two heads
B. Getting one head and one tail
C. Getting two tails
D. All of the above outcomes are equally likely

Q: What is the sample space for flipping a coin two times?

A:

```
    H T
   / \ / \ 
  H  T  H  T
 / \ / \ / \ 
T  T  T  T  T
```

Q: What are the probabilities for flipping a coin two times?

A:

```
    H T
   / \ / \ 
  H  T  H  T
 / \ / \ / \ 
T  T  T  T  T
```

Outcome: Probability
HH: \(\frac{1}{2}\) \(\frac{1}{2}\) = \(\frac{1}{4}\)
HT: \(\frac{1}{2}\) \(\frac{1}{2}\) = \(\frac{1}{4}\)
TH: \(\frac{1}{2}\) \(\frac{1}{2}\) = \(\frac{1}{4}\)
TT: \(\frac{1}{2}\) \(\frac{1}{2}\) = \(\frac{1}{4}\)
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**Question 4:**
The spinner shown below has three equal areas labeled A, B, and C.

![Spinner Diagram]

The probability of the spinner landing on A and then A again is _____.
A. less likely than the spinner landing on A and then B.
B. equally likely as the spinner landing on A and then B.
C. more likely than the spinner landing on A and then B.

**Q:** What is the sample space for spinning the spinner two times?

**A:**

**Q:** What are the probabilities for spinning the spinner two times?

**A:**

**Question 5:**
Two fair dice are rolled and the resulting numbers facing upwards on the dice is summed.

Which of the following sums is the MOST LIKELY to occur:

A. Sum = 3
B. Sum = 5
C. Sum = 6
D. Sum = 9
E. All of the above sums are equally likely

**A:**

![Dice Table]

**Q:** What is the sample space for summing the two dice?

**A:**

![Dice Sum Table]
Q: What is the probability for the sums?
A:

\[ P(\text{Sum} = 3) = \frac{2}{36} = \frac{1}{18} \]
\[ P(\text{Sum} = 5) = \frac{4}{36} = \frac{1}{9} \]
\[ P(\text{Sum} = 6) = \frac{5}{36} \]
\[ P(\text{Sum} = 9) = \frac{4}{36} = \frac{1}{9} \]
A. Sum = 3  \quad 30  9
B. Sum = 5
C. Sum = 6
D. Sum = 9
E. All of the above sums are equally likely

This Lesson is Complete
Thank you!
Appendix C: Student Response Sheet used during Representativeness

Heuristic Lesson

Name: _______________________     Group Number: _______     Date: __________

Write your prediction for the following questions and write how you arrived at your prediction.

1. A fair die is rolled four times and the results are recorded in the order that they appear. For each roll, the die lands with the number 1, 2, 3, 4, 5, or 6 facing up.

   The following outcome is the LEAST LIKELY to occur:
   A.   1 2 3 4
   B.   5 5 5 5
   C.   3 1 4 6
   D.   3 1 6 4
   E.   All of the above sequences are equally likely

   I predicted ________ because …

2. Assume the probability of a birth being a baby boy is ½. Which of the following sequences, where B represents having a boy and G represents have a girl, is MOST LIKELY to occur for having six children?

   A.   B B B B G B
   B.   B G G B G B
   C.   B B B G G G
   D.   All of the above sequences are equally likely

   I predicted ________ because …
Write your prediction for the following questions and write how you arrived at your prediction.

3. Mary has flipped a coin two times.

Which outcome is MOST LIKELY to occur?

A. Getting two heads
B. Getting one head and one tail
C. Getting two tails
D. All of the above outcomes are equally likely

I predicted _______ because …

4. The spinner shown on the right has three equal areas labeled A, B, and C.

The probability of the spinner landing on A and then A again is __________ the spinner landing on A and then B:

A. less likely than
B. equally likely as
C. more likely than

I predicted _______ because …

5. Two fair dice are rolled and the resulting numbers facing upwards on the dice is summed.

Which of the following sums is the MOST LIKELY to occur:

A. Sum = 3
B. Sum = 5
C. Sum = 6
D. Sum = 9
E. All of the above sums are equally likely

I predicted _______ because …
Appendix D: Student Group Response Sheet used during Representativeness

Heuristic Lesson

Group Member Names: ________________________________     Date: ____________

________________________________

Write your group’s answer for the following questions and write how you arrived at your answer.

1. A fair die is rolled four times and the results are recorded in the order that they appear. For each roll, the die lands with the number 1, 2, 3, 4, 5, or 6 facing up.

   The following outcome is the LEAST LIKELY to occur:

   A.   1 2 3 4
   B.   5 5 5 5
   C.   3 1 4 6
   D.   3 1 6 4
   E.   All of the above sequences are equally likely

   We answered _______ because …

2. Assume the probability of a birth being a baby boy is ½. Which of the following sequences, where B represents having a boy and G represents have a girl, is MOST LIKELY to occur for having six children?

   A.   B B B B G B
   B.   B G G B G B
   C.   B B B G G G
   D.   All of the above sequences are equally likely

   We answered _______ because …
Write your group’s answer for the following questions and write how you arrived at your answer.

3. Mary has flipped a coin two times.

Which outcome is MOST LIKELY to occur?

A. Getting two heads
B. Getting one head and one tail
C. Getting two tails
D. All of the above outcomes are equally likely

We answered ________ because …

4. The spinner shown on the right has three equal areas labeled A, B, and C.

The probability of the spinner landing on A and then A again is __________ the spinner landing on A and then B:

A. less likely than
B. equally likely as
C. more likely than

We answered ________ because …

5. Two fair dice are rolled and the resulting numbers facing upwards on the dice is summed.

Which of the following sums is the MOST LIKELY to occur:

A. Sum = 3
B. Sum = 5
C. Sum = 6
D. Sum = 9
E. All of the above sums are equally likely

We answered ________ because …
Appendix E: Pre-Test and Post-Test Questions

1. A fair coin is tossed, and it lands heads up. The coin is tossed a second time. What is the probability that the second toss is also a head?
   a. 1/2  
   b. 1/3  
   c. 1/4  
   d. Slightly less than 1/2  
   e. Slightly more than 1/2  
   
Which of the following best describes the reason for your answer to the preceding question?
   d. The chance of getting heads or tails on any one toss is always 1/2.  
   e. The second toss is less likely to be a head because the first toss was a head.  
   f. There are three possible outcomes when you toss a coin twice. Getting two heads is only one of them.  
   g. There are four possible outcomes when you toss a coin twice. Getting two heads is only one of them.  
   h. Other ________________________________

2. The first roll of a fair die results in a 3. The die is rolled a second time. What is the chance that the second roll also results in a 3?
   a. 1/36  
   b. 1/5  
   c. 1/6  
   d. Slightly less than 1/6  
   e. Slightly more than 1/6  
   
Which of the following best describes the reason for your answer to the preceding question?
   a. There are thirty-six possible outcomes when you roll a die twice. Getting two 3's is only one of them.  
   b. The second toss is less likely to be a 3 because the first toss was a 3.  
   c. The chance of getting a 3 on any one roll is always 1/6.  
   d. Any of the other five numbers is more likely than a 3.  
   e. Other ________________________________
3. A fair coin is tossed, and its outcome is a tail. The coin is tossed a second time. What is the probability that the second toss is also a tail?

a. 1/4  
b. 1/3  
c. 1/2  
d. Slightly less than 1/2  
e. Slightly more than 1/2

Which of the following best describes the reason for your answer to the preceding question?

a. The chance of getting heads or tails on any one toss is always 1/2.  
b. The second toss is less likely to be a tail because the first toss was a tail.  
c. There are three possible outcomes when you toss a coin twice. Getting two tails is only one of them.  
d. There are four possible outcomes when you toss a coin twice. Getting two tails is only one of them.  
e. Other ________________________________

4. The first roll of a fair die results in a 5. The die is rolled a second time. What is the chance that the second roll also results in a 5?

a. 1/5  
b. 1/6  
c. 1/36  
d. Slightly less than 1/6  
e. Slightly more than 1/6

Which of the following best describes the reason for your answer to the preceding question?

a. There are thirty-six possible outcomes when you roll a die twice. Getting two 5's is only one of them.  
b. The second toss is less likely to be a 5 because the first toss was a 5.  
c. The chance of getting a 5 on any one roll is always 1/6.  
d. Any of the other five numbers is more likely than a 5.  
e. Other ________________________________
5. When three coins are tossed, which of the following outcomes is LEAST LIKELY to occur?

a. Three heads
b. Two heads and one tail
c. One head and two tails
d. All the above outcomes are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. The outcome of having exactly three heads is more likely than having exactly two heads, which is more likely than having exactly one head.
b. Since coin tossing is a random event, every outcome has exactly the same probability of occurring.
c. You are much more likely to get a mixture of heads and tails than all heads or all tails.
d. There are four possible outcomes and each outcome has a probability of 1/4.
e. Other ______________________________

6. When two fair die are rolled, which of the following outcomes is MOST LIKELY to occur?

a. Both die show an odd number
b. Both die show an even number
c. One die shows an odd number and one die shows an even number
d. All the above outcomes are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. If the first die is an odd number then it is more likely that the second die is an even number, and vice versa.
b. Since die rolling is a random event, every outcome has exactly the same probability of occurring.
c. You are much more likely to get a mixture of odd and even numbers than both even or both odd.
d. There are three possible outcomes and each outcome has a probability of 1/3.
e. Other ______________________________
7. When three coins are tossed, which of the following outcomes is MOST LIKELY to occur?
   a. Three heads
   b. Three tails
   c. One head and two tails
   d. All the above outcomes are equally likely.

Which of the following best describes the reason for your answer to the preceding question?
   a. The outcome of having exactly three tails is more likely than having exactly one tail.
   b. You are much more likely to get a mixture of heads and tails than all heads or all tails.
   c. Since coin tossing is a random event, every outcome has exactly the same probability of occurring.
   d. The outcome of having exactly three heads is more likely than having exactly two heads.
   e. Other ________________________________

8. When three fair die are rolled, which of the following outcomes is MOST LIKELY to occur?
   a. All three die show an odd number
   b. All three die show an even number
   c. Two die show an odd number and one die shows an even number
   d. All the above outcomes are equally likely.

Which of the following best describes the reason for your answer to the preceding question?
   a. There are four possible outcomes and each outcome has a probability of 1/4.
   b. You are much more likely to get a mixture of odds and evens than all odds or all evens.
   c. Since die rolling is a random event, every outcome has exactly the same probability of occurring.
   d. If the first two die have shown an odd number then it is more likely that the third die will show an odd number.
   e. Other ________________________________
9. A bag has 9 pieces of fruit: 3 apples, 3 pears, and 3 oranges. Four pieces of fruit are picked, one at a time. Each time a piece of fruit is picked, the type of fruit is recorded, and it is then put back in the bag. If the first 3 pieces of fruit were apples, what is the fourth piece MOST LIKELY to be?
   a. A pear  
   b. An apple  
   c. An orange  
   d. An orange or a pear are both equally likely and more likely than an apple.  
   e. An apple, orange, or pear are all equally likely.

Which of the following best describes the reason for your answer to the preceding question?
   a. This piece of fruit is just as likely as any other.  
   b. The apples seem to be lucky.  
   c. The picks are independent, so each fruit has an equally likely chance of being picked.  
   d. The fourth piece of fruit won't be an apple because too many have already been picked.  
   e. Other ________________________________

10. A box contains 6 balls: 2 are red, 2 are white, and 2 are blue. Four balls are picked at random, one at a time. Each time a ball is picked, the color is recorded, and the ball is put back in the box. If the first 3 balls are red, what color is the fourth ball MOST LIKELY to be?
   a. Red  
   b. White  
   c. Blue  
   d. Blue and white are equally likely and more likely than red.  
   e. Red, blue, and white are all equally likely.

Which of the following best describes the reason for your answer to the preceding question?
   a. The fourth ball should not be red because too many red ones have already been picked.  
   b. The picks are independent, so every color has an equally likely chance of being picked.  
   c. Red seems to be lucky.  
   d. This color is just as likely as any other color.  
   e. Other ________________________________
11. A Halloween basket has 9 candy bars: 3 Mars bars, 3 Kit-kat bars, and 3 Snickers bars. Four candy bars are picked, one at a time. Each time a candy bar is randomly picked, the type of bar is recorded, and it is then put back in the basket. If the first 3 candy bars were Mars bars, what is the fourth piece MOST LIKELY to be?

a. A Mars bar  

b. A Kit-kat bar  

c. A Snickers bar  

d. A Snickers bar or a Kit-kat bar are both equally likely and more likely than an Mars bar.  

e. All three types of bar are equally likely.  

Which of the following best describes the reason for your answer to the preceding question?

a. The Mars bars seem to be lucky.  

b. One type of candy bar is just as likely as any other type.  

c. The picks are independent, so each bar has an equally likely chance of being picked.  

d. The fourth candy bar won't be a Mars bar because too many have already been picked.  

e. Other __________________________  

12. A bag contains 6 marbles: 2 are blue, 2 are green, and 2 are yellow. Four marbles are picked at random, one at a time. Each time a marble is picked, the color is recorded, and the marble is put back in the bag. If the first 3 marbles are blue, what color is the fourth marble MOST LIKELY to be?

a. Blue  

b. Green  

c. Yellow  

d. Yellow and green are equally likely and more likely than blue.  

e. Blue, yellow, and green are all equally likely.  

Which of the following best describes the reason for your answer to the preceding question?

a. The fourth marble should not be blue because too many blue ones have already been picked.  

b. The picks are independent, so every color has an equally likely chance of being picked.  

c. Blue seems to be lucky.  

d. This color is just as likely as any other color.  

e. Other __________________________  

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13. If a fair coin is tossed five times, which of the following sequence of heads (H) and tails (T), if any, is MOST LIKELY to occur?

a. H T H T T
b. T H H H H
c. H T H T H
d. Sequences (a) and (c) are equally likely.
e. All of the above sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. Every sequence of five tosses has exactly the same probability of occurring.
b. Since tossing a coin is random, the coin should not alternate between heads and tails.
c. Since tossing a coin is random, you should not get a long string of head or tails.
d. There ought to be roughly the same number of tails as heads.
e. Other ________________________________

14. If a fair coin is tossed twelve times, which of the following sequences of heads (H) and tails (T), if any, is LEAST LIKELY to occur?

a. H T H T H T H T H T H T
b. H H T H T H H T T H H
c. T T H T H T H T H T H

d. H H H H H H T T T T
e. All sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. There ought to be roughly the same number of tails as heads.
b. Since tossing a coin is random, you should not get a long string of head or tails.
c. Every sequence of twelve tosses has exactly the same probability of occurring.
d. Since tossing a coin is random, the coin should not alternate between heads and tails.
e. Other ________________________________
15. If a fair coin is tossed eight times, which of the following sequences of heads (H) and tails (T), if any, is MOST LIKELY to occur?

   a.  T T H H H T T
   b.  H H H H H H H
   c.  H T H T H T H
   d.  H H T H T H H
   e.  All sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

   a.  Every sequence of eight tosses has exactly the same probability of occurring.
   b.  Since tossing a coin is random, the coin should not alternate between heads and tails.
   c.  There ought to be roughly the same number of tails as heads.
   d.  Since tossing a coin is random, you should not get a long string of head or tails.
   e.  Other ________________________________

16. If a fair coin is tossed six times, which of the following sequences of heads (H) and tails (T), if any, is LEAST LIKELY to occur?

   a.  H T H T H T
   b.  T T H H T H
   c.  H H H H T T
   d.  H T H H T H
   e.  All sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

   a.  Since tossing a coin is random, you should not get a long string of head or tails.
   b.  Every sequence of six tosses has exactly the same probability of occurring.
   c.  There ought to be roughly the same number of tails as heads.
   d.  Since tossing a coin is random, the coin should not alternate between heads and tails.
   e.  Other ________________________________
17. If a fair die is rolled four times, which of the following sequences of results, if any, is LEAST LIKELY to occur?

a. 6 4 3 5
b. 5 6 2 6
c. 2 3 4 5
d. 2 1 4 3
e. All sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. Since rolling a die is a random event, a result like that is very unlikely.
b. You are much more likely to get a mixture of different numbers than an ordered sequence.
c. All sequences of rolls have exactly the same probability of occurring.
d. You are much more likely to get a mixture of different numbers than numbers that repeat.
e. Other ________________________________

18. If a fair die is rolled eight times, which of the following sequences of results, if any, is MOST LIKELY to occur?

a. 5 6 2 6 3 5 4 2
b. 2 1 4 3 1 5 4 6
c. 6 4 3 2 4 1 5 6
d. 2 3 4 5 6 1 2 3
e. All sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. You are much more likely to get a mixture of different numbers than an ordered sequence.
b. Since rolling a die is a random event, a result like that is very likely.
c. You are much more likely to get a mixture of different numbers than numbers that repeat.
d. All sequences of rolls have exactly the same probability of occurring.
e. Other ________________________________
19. If a fair die is rolled eight times, which of the following sequences of results, if any, is LEAST LIKELY to occur?

a. 2 3 4 5 6 1 2 3  

b. 6 4 3 2 4 1 5 6  

c. 5 6 2 6 3 5 4 2  

d. 2 1 4 3 1 5 4 6  

e. All sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. You are much more likely to get a mixture of different numbers than an ordered sequence.

b. Since rolling a die is a random event, a result like that is very unlikely.

c. All sequences of rolls have exactly the same probability of occurring.

d. You are much more likely to get a mixture of different numbers than numbers that repeat.

e. Other ________________________________

20. If a fair die is rolled six times, which of the following sequences of results, if any, is MOST LIKELY to occur?

a. 5 6 2 6 4 3  

b. 2 1 4 3 2 4  

c. 6 4 3 2 5 1  

d. 1 2 3 4 5 6  

e. All sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. All sequences of rolls have exactly the same probability of occurring.

b. You are much more likely to get a mixture of different numbers than numbers that repeat.

c. Since rolling a die is a random event, a result like that is very likely.

d. You are much more likely to get a mixture of different numbers than an ordered sequence.

e. Other ________________________________
Appendix F: Representativeness Heuristic Lesson Survey

Name: ___________________________             Date: ____________

**Probability Lesson Survey**

Please circle the number that that shows how much you agree or disagree with each of the following statements:

Strongly Disagree = 1
Somewhat Disagree = 2
Unsure/ Neither Agree or Disagree = 3
Somewhat Agree = 4
Strongly Agree = 5

<table>
<thead>
<tr>
<th></th>
<th>Strongly Disagree</th>
<th>Somewhat Disagree</th>
<th>Unsure/ Neither Agree or Disagree</th>
<th>Somewhat Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. During the probability lesson I felt bored.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2. The lesson made me think very little about probability concepts.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3. The lesson has made me more interested in learning about probability.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4. During the lesson, I talked about the probability concepts with my classmates.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5. I did not actively participate in the lesson.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>6. The lesson presentation captured my interest.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>7. The lesson did not make probability concepts interesting to me.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>8. During the lesson presentation I felt engaged into the lesson.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
Appendix G: Semi-structured Student Interview Questions

0. Before we begin with the main part of the interview, can I ask you a few questions to better understand your reasoning that underlie your answers in the tests?

[ I will have selected certain questions from the student’s pre-tests and post-tests in which they selected the representativeness heuristic answer choice for the pre-test but the correct answer choice for the post-test. ]

You chose ____ (A, B, C, or D) in the pre-test but ____ (A, B, C, or D) in the post-test. Can you help me understand what you were thinking back then?

Possible Paraphrasing:
- You chose ____ (A, B, C, or D) in this problem but you chose ____ (A, B, C, or D). What were you thinking when you chose those answers?

1. What does having “good reasoning ability for probability” mean to you?

Possible Paraphrasing:
- What qualities does a person with “good reasoning ability for probability” have?

2. Before the probability lesson presentations, how would you rate your reasoning abilities for probability?

[Possible student responses: Poor/Bad, Average/Fair, or Excellent/Good]

Sub-Questions:

2.1 Why do you say your reasoning ability is [Student Response to Question 2]?

2.2 What factors do you think contributed to your reasoning for probability being [Student Response to Question 2]?

Possible Paraphrasing:
- What aspects do you think contributed to your reasoning for probability being [Student Response to Question 2]?
- What things do you think made your reasoning for probability [Student Response to Question 2]?

2.3 Have you been taught probability before?

If student replies yes, I will ask the following follow-up questions:

In what class(es)? When did you take this class?

Possible Paraphrasing:
- What type of previous instruction or teaching in probability have you received before the probability lesson presentations?
- When was the last time that you learned or practiced probability?
Take a moment to remember as much as you can about the second lesson on probability that you had with me.

[Give student time to recollect or ask any questions]

Are you ready?

Now, I would like for you to take a moment to remember as much as you can about the first lesson on probability that you had with me.

[Give student time to recollect or ask any questions]

How would you rate your reasoning ability for probability after the first lesson but before the second lesson?

[Possible student responses: Poor/Bad, Average/Fair, or Excellent/Good]

Sub-Questions:

3.1 What factors do you think contributed to your reasoning for probability becoming [Student Response to Question 3]?

Possible Paraphrasing:

• What things do you think contributed to your reasoning for probability becoming [Student Response to Question 3]?
• What things do you think made your reasoning for probability [Student Response to Question 3]?

4 How would you rate your reasoning abilities for probability after the second lesson?

[Possible student responses: Poor/Bad, Average/Fair, or Excellent/Good]

Possible Paraphrasing:

• After both lessons that I taught, how would you rate your reasoning abilities for probability?

5 Now I would like you to compare the first lesson you had with me and the second lesson you had with me.

[Give student time to recollect or ask any questions]

Can you name a few differences between the two lessons?

6 Do you think your reasoning for probability has increased or remained the same because of these two probability lessons?
If student replies increased, I will ask the following sub-questions:

6.1 In your opinion, which lesson made a greater contribution to an increase in your reasoning ability for probability?

6.2 Why?

Possible Paraphrasing:
• What aspects of the first/second lesson do you think contributed to an increase in your reasoning ability for probability?

Possible list of aspects:
• Use of student groups
• Discussing concepts with other students during the lesson
• Use of technology
• Use of diagrams
• Etc.

7 What is the most important thing that you learned from the probability lesson presentations?

Possible Paraphrasing:
• What is something you learned form the probability lesson presentations?

8 What is one thing that you are still confused about regarding probability?
## Appendix H: Representativeness Heuristic Test Rubric

<table>
<thead>
<tr>
<th>Question</th>
<th>Correct</th>
<th>Wrong</th>
<th>RH / “Equally Likely” Response</th>
<th>RH (or “Equally Likely”) based on other reasoning</th>
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<tbody>
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<td>1 (A1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Part 1</td>
<td>A</td>
<td>B, C, E</td>
<td>A, D</td>
<td>D</td>
</tr>
<tr>
<td>Part 2</td>
<td>A</td>
<td>C, D</td>
<td>B</td>
<td>E</td>
</tr>
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<td>2 (A2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Part 1</td>
<td>C</td>
<td>A, E</td>
<td>B, D</td>
<td>B, D</td>
</tr>
<tr>
<td>Part 2</td>
<td>C</td>
<td>A</td>
<td>B, D</td>
<td>E</td>
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<tr>
<td>3 (A3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Part 1</td>
<td>C</td>
<td>B, A, E</td>
<td>A, D</td>
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<tr>
<td>Part 1</td>
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<td>C, E</td>
<td>A, D</td>
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<td>B, D</td>
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<td>A, B</td>
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<td>D</td>
<td>D</td>
</tr>
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<td>C, A</td>
<td>E</td>
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<td>D</td>
<td>D</td>
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<td>D</td>
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</tr>
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<td>10 (C2)</td>
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<td>D</td>
<td>D</td>
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<tr>
<td>Part 2</td>
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<td>C</td>
<td>B, A</td>
<td>D</td>
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137
<table>
<thead>
<tr>
<th>Question</th>
<th>Correct</th>
<th>Wrong</th>
<th>RH / “Equally Likely” Response</th>
<th>RH (or “Equally Likely”) based on other reasoning</th>
</tr>
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<tbody>
<tr>
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<td>C, D</td>
<td>A</td>
<td>E</td>
</tr>
<tr>
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<td>B</td>
<td>A, C, D</td>
<td>E</td>
</tr>
<tr>
<td>14 (D2)</td>
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<td>B, C, D</td>
<td>A</td>
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<tr>
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<td>A, B, D</td>
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<td>A, D</td>
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<tr>
<td>16 (D4)</td>
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<td>A, C</td>
<td>A, C</td>
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<tr>
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<td>A, C, D</td>
<td>A, C, D</td>
<td>E</td>
</tr>
<tr>
<td>17 (E1)</td>
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<td>A, D</td>
<td>B, C</td>
<td>B, C</td>
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<tr>
<td>Part 2</td>
<td>C</td>
<td>A</td>
<td>B, D</td>
<td>E</td>
</tr>
<tr>
<td>18 (E2)</td>
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<td>B, C</td>
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<td>Part 2</td>
<td>D</td>
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<td>A, C</td>
<td>E</td>
</tr>
<tr>
<td>19 (E3)</td>
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<td>A, C</td>
<td>A, C</td>
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<td>B</td>
<td>A, D</td>
<td>E</td>
</tr>
<tr>
<td>20 (E4)</td>
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<td>A, C</td>
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<tr>
<td>Part 2</td>
<td>A</td>
<td>C</td>
<td>B, D</td>
<td>E</td>
</tr>
</tbody>
</table>

A Group: Probability of Independent Events  
B Group: "Equally Likely" Learned Response (LR)  
C Group: Gamblers Fallacy  
D Group: Most Likely/Least Likely Coins  
E: Group: Most Likely/Least Likely Dice
Appendix I: Regression Analysis Results for Post-Test Scores

Regression for Number of Correct Responses on Post-Test 1

Regression Equation:

\[ \text{Post-Test1\_Correct} = 5.04 - 0.890(\text{Prediction-and-Voting}) - 1.05(\text{Prediction-Only}) - 0.720(\text{Attended\_Review\_Lesson}) + 1.10(\text{Attended\_Representativeness\_Heuristic\_Lesson}) + 0.339(\text{Pre-Test\_Correct}) \]

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coefficient</th>
<th>Standard Error of Coefficient</th>
<th>T-Value</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>5.039**</td>
<td>1.385</td>
<td>3.64</td>
<td>0.001</td>
</tr>
<tr>
<td>Prediction-and-Voting</td>
<td>-0.8901</td>
<td>0.4960</td>
<td>-1.79</td>
<td>0.078</td>
</tr>
<tr>
<td>Prediction-Only</td>
<td>-1.0450*</td>
<td>0.5049</td>
<td>-2.07</td>
<td>0.043</td>
</tr>
<tr>
<td>Attended_Review_Lesson</td>
<td>-0.7196</td>
<td>0.9275</td>
<td>-0.78</td>
<td>0.441</td>
</tr>
<tr>
<td>Attended_Representativeness_Heuristic_Lesson</td>
<td>1.0998</td>
<td>0.9325</td>
<td>1.18</td>
<td>0.243</td>
</tr>
<tr>
<td>Pre-Test_Correct</td>
<td>0.33911**</td>
<td>0.09559</td>
<td>3.55</td>
<td>0.001</td>
</tr>
</tbody>
</table>

* = coefficient has p-value < 0.05  
** = coefficient has p-value < 0.001

Analysis of Variance

<table>
<thead>
<tr>
<th></th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
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<td>54.230</td>
<td>10.846</td>
<td>4.55</td>
<td>0.001</td>
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<td>Residual Error</td>
<td>58</td>
<td>138.207</td>
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<tr>
<td>Total</td>
<td>63</td>
<td>192.438</td>
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R-Squared  28.2%  
R-Squared (adjusted)  22.0%
Regression for Number of Representativeness Heuristic (RH) Responses on Post-Test 1

Regression Equation:

Post-Test1_RH = -0.138 + 0.249(Prediction-and-Voting) + 0.500(Prediction-Only) + 0.431(Attended_Review_Lesson) – 0.172(Attended_Representativeness_Heuristic_ Lesson) + 0.155(Pre-Test_Correct)

<table>
<thead>
<tr>
<th>Predictor</th>
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<th>P-Value</th>
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<tbody>
<tr>
<td>Constant</td>
<td>-0.1383</td>
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<td>-0.17</td>
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<tr>
<td>Prediction-and-Voting</td>
<td>0.2487</td>
<td>0.3034</td>
<td>0.82</td>
<td>0.416</td>
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<tr>
<td>Prediction-Only</td>
<td>0.4998</td>
<td>0.2983</td>
<td>1.68</td>
<td>0.099</td>
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<tr>
<td>Attended_Review_Lesson</td>
<td>0.4312</td>
<td>0.5683</td>
<td>0.76</td>
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<tr>
<td>Attended_Representativeness_Heuristic_Lesson</td>
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<td>0.5692</td>
<td>-0.30</td>
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<tr>
<td>Pre-Test_RH</td>
<td>1.55538</td>
<td>0.08591</td>
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<td>0.0076</td>
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* = coefficient has p-value < 0.05
** = coefficient has p-value < 0.001

Analysis of Variance

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<th>P-Value</th>
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<td>Regression</td>
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<td>5.9406</td>
<td>1.1881</td>
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<td>51.4969</td>
<td>0.8879</td>
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<tr>
<td>Total</td>
<td>63</td>
<td>57.4375</td>
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<td></td>
<td></td>
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</table>

R-Squared: 10.3%
R-Squared (adjusted): 2.6%
Regression for Number of Correct Responses on Post-Test 2

Regression Equation:

\[
\text{Post-Test2\_Correct} = 5.65 - 0.108(\text{Prediction-and-Voting}) - 1.04(\text{Prediction-Only}) - 1.10(\text{Attended\_Review\_Lesson}) + 0.034(\text{Attended\_Representativeness\_Heuristic\_Lesson}) + 0.345(\text{Pre-Test\_Correct})
\]

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<th>P-Value</th>
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<td>Prediction-and-Voting</td>
<td>-0.1081</td>
<td>0.5500</td>
<td>-0.20</td>
<td>0.845</td>
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<tr>
<td>Prediction-Only</td>
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<td>0.5525</td>
<td>-1.89</td>
<td>0.064</td>
</tr>
<tr>
<td>Attended_Review_Lesson</td>
<td>-1.098</td>
<td>1.232</td>
<td>-0.89</td>
<td>0.376</td>
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<tr>
<td>Attended_Representativeness_Heuristic_Lesson</td>
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<td>0.8303</td>
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<tr>
<td>Pre-Test_Correct</td>
<td>0.3447*</td>
<td>0.1063</td>
<td>3.24</td>
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* = coefficient has p-value < 0.05
** = coefficient has p-value < 0.001

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<th>MS</th>
<th>F</th>
<th>P-Value</th>
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<td>164.005</td>
<td>2.877</td>
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<tr>
<td>Total</td>
<td>62</td>
<td>224.603</td>
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</table>

R-Squared | 27.0%
R-Squared (adjusted) | 20.6%
Regression for Number of Representativeness Heuristic RH Responses on Post-Test 2

Regression Equation:

\[ \text{Post-Test2\_RH} = -0.185 + 0.115(\text{Prediction-and-Voting}) + 0.586(\text{Prediction-Only}) + 0.416(\text{Attended\_Review\_Lesson}) - 0.025(\text{Attended\_Representativeness\_Heuristic\_Lesson}) + 0.180(\text{Pre-Test\_RH}) \]

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<th>P-Value</th>
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<tr>
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<td>-0.20</td>
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<tr>
<td>Prediction-and-Voting</td>
<td>0.1150</td>
<td>0.3482</td>
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<td>0.742</td>
</tr>
<tr>
<td>Prediction-Only</td>
<td>0.5863</td>
<td>0.3408</td>
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<td>Attended_Review_Lesson</td>
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<td>0.7788</td>
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<td>0.595</td>
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<td>0.5234</td>
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<td>Pre-Test_RH</td>
<td>0.17975</td>
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* = coefficient has p-value < 0.05
** = coefficient has p-value < 0.001

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R-Squared: 11.2%
R-Squared (adjusted): 3.4%
### Appendix J: Logistic Regression Analysis Results for Each Test Question Group

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<th>Test Question Group</th>
<th>Number of Correct Responses to Post-Test 1</th>
<th>Number of Correct Responses to Post-Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prediction-and-Voting Class</td>
<td>Prediction-Only Class</td>
</tr>
<tr>
<td></td>
<td>Coef = -0.105</td>
<td>Coef = -1.70</td>
</tr>
<tr>
<td></td>
<td>Z = -0.15</td>
<td>Z = -1.70</td>
</tr>
<tr>
<td></td>
<td>Odds = 0.90</td>
<td>Odds = 0.34</td>
</tr>
<tr>
<td></td>
<td>p = 0.881</td>
<td>p = 0.090</td>
</tr>
<tr>
<td></td>
<td>Coef = -0.458</td>
<td>Coef = -0.845</td>
</tr>
<tr>
<td></td>
<td>Z = -0.90</td>
<td>Z = -1.68</td>
</tr>
<tr>
<td></td>
<td>Odds = 0.63</td>
<td>Odds = 0.43</td>
</tr>
<tr>
<td></td>
<td>p = 0.366</td>
<td>p = 0.094</td>
</tr>
<tr>
<td></td>
<td>Coef = -1.010</td>
<td>Coef = -2.537</td>
</tr>
<tr>
<td></td>
<td>Z = -0.86</td>
<td>Z = -2.36</td>
</tr>
<tr>
<td></td>
<td>Odds = 0.36</td>
<td>Odds = 0.08</td>
</tr>
<tr>
<td></td>
<td>p = 0.390</td>
<td>p = 0.018*</td>
</tr>
<tr>
<td></td>
<td>Coef = -0.105</td>
<td>Coef = -1.70</td>
</tr>
<tr>
<td></td>
<td>Z = -0.15</td>
<td>Z = -1.70</td>
</tr>
<tr>
<td></td>
<td>Odds = 0.90</td>
<td>Odds = 0.34</td>
</tr>
<tr>
<td></td>
<td>p = 0.881</td>
<td>p = 0.090</td>
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<tr>
<td></td>
<td>Coef = -0.458</td>
<td>Coef = -0.845</td>
</tr>
<tr>
<td></td>
<td>Z = -0.90</td>
<td>Z = -1.68</td>
</tr>
<tr>
<td></td>
<td>Odds = 0.63</td>
<td>Odds = 0.43</td>
</tr>
<tr>
<td></td>
<td>p = 0.366</td>
<td>p = 0.094</td>
</tr>
<tr>
<td></td>
<td>Coef = -1.010</td>
<td>Coef = -2.537</td>
</tr>
<tr>
<td></td>
<td>Z = -0.86</td>
<td>Z = -2.36</td>
</tr>
<tr>
<td></td>
<td>Odds = 0.36</td>
<td>Odds = 0.08</td>
</tr>
<tr>
<td></td>
<td>p = 0.390</td>
<td>p = 0.018*</td>
</tr>
<tr>
<td></td>
<td>Coef = -0.105</td>
<td>Coef = -1.70</td>
</tr>
<tr>
<td></td>
<td>Z = -0.15</td>
<td>Z = -1.70</td>
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<tr>
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<td>Odds = 0.90</td>
<td>Odds = 0.34</td>
</tr>
<tr>
<td></td>
<td>p = 0.881</td>
<td>p = 0.090</td>
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<tr>
<td></td>
<td>Coef = -0.458</td>
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<td>Z = -0.90</td>
<td>Z = -1.68</td>
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<td>Odds = 0.63</td>
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<tr>
<td></td>
<td>p = 0.366</td>
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<tr>
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<td>Coef = -1.010</td>
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<td>Z = -0.86</td>
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<td></td>
<td>Odds = 0.36</td>
<td>Odds = 0.08</td>
</tr>
<tr>
<td></td>
<td>p = 0.390</td>
<td>p = 0.018*</td>
</tr>
</tbody>
</table>

Coef = logistic regression equation coefficient  
Z = z-score  
Odds = odds ratio  
p = p-value

**Note:**
The independent variables in all logistic regression models are:
1. If the student was in the prediction-and-voting class
2. If the student was in the prediction-only class

The coding for students attendance in one of the treatment classes, listed above, was the same as the regression in section 4.1.4.

The coding for the number of questions correct was:
0 if the student answered neither of the two questions per test question group correct
1 if the student answered one of the two questions per test question group correct
2 if the student answered both of the two questions per test question group correct
<table>
<thead>
<tr>
<th>Test Question Group</th>
<th>Number of RH Responses to Post-Test 1</th>
<th>Number of RH Responses to Post-Test 2</th>
</tr>
</thead>
<tbody>
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<td>Prediction-Only Class</td>
</tr>
<tr>
<td></td>
<td>Coef = -0.131</td>
<td>Coef = !</td>
</tr>
<tr>
<td></td>
<td>Z = -0.09</td>
<td>Z = !</td>
</tr>
<tr>
<td></td>
<td>Odds = 0.88</td>
<td>Odds = !</td>
</tr>
<tr>
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<td>p = 0.927</td>
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<tr>
<td></td>
<td>Coef = 0.717</td>
<td>Coef = 0.896</td>
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<tr>
<td></td>
<td>Z = 0.58</td>
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<tr>
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<td>Odds = 2.05</td>
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</tr>
<tr>
<td></td>
<td>p = 0.564</td>
<td>p = 0.471</td>
</tr>
<tr>
<td></td>
<td>Coef = !</td>
<td>Coef = !</td>
</tr>
<tr>
<td></td>
<td>Z = !</td>
<td>Z = !</td>
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<td></td>
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<td>Odds = !</td>
</tr>
<tr>
<td></td>
<td>p = !</td>
<td>p = !</td>
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<td></td>
<td>Coef = 0.896</td>
<td>Coef = 1.034</td>
</tr>
<tr>
<td></td>
<td>Z = 0.72</td>
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<tr>
<td></td>
<td>p = 0.471</td>
<td>p = 0.299</td>
</tr>
</tbody>
</table>

|                     | Coef = !                              | Coef = !                              |
|                     | Z = !                                | Z = !                                |
|                     | Odds = !                              | Odds = !                              |
|                     | p = !                                | p = !                                |
|                     | Coef = 1.034                         | Coef = 1.386                         |
|                     | Z = 1.04                             | Z = 1.64                             |
|                     | Odds = 2.53                          | Odds = 4.00                          |
|                     | p = 0.299                            | p = 0.101                            |

**Note:**

The independent variables in all logistic regression models are:

1. If the student was in the prediction-and-voting class
2. If the student was in the prediction-only class

The coding for students attendance in one of the treatment classes, listed above, was the same as the regression in section 4.1.4.

The coding for the number of questions correct was:

0 if the student answered neither of the two questions per test question group indicating use of the representativeness heuristic

1 if the student answered one of the two questions per test question group indicating use of the representativeness heuristic

2 if the student answered both of the two questions per test question group indicating use of the representativeness heuristic
Appendix K: Paired T-Test Results for Comparison and Treatment Groups on Each Test Question Group

<table>
<thead>
<tr>
<th># Correct Responses</th>
<th>Pre-Test</th>
<th>Post-Test 1</th>
<th>Pre-Test</th>
<th>Post-Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Question Group A</strong></td>
<td>$\bar{x}_{\text{pre-test}} = 1.474$</td>
<td>$\bar{x}_{\text{post-1}} = 1.474$</td>
<td>$P = 1.000$</td>
<td>$\bar{x}_{\text{pre-test}} = 1.286$</td>
</tr>
<tr>
<td>$s_{\text{pre-test}} = 0.772$</td>
<td>$s_{\text{post-1}} = 0.841$</td>
<td>$P = 0.358$</td>
<td>$s_{\text{post-2}} = 0.784$</td>
<td></td>
</tr>
<tr>
<td>$N = 19$</td>
<td>$N = 19$</td>
<td></td>
<td>$N = 21$</td>
<td></td>
</tr>
<tr>
<td><strong>Question Group B</strong></td>
<td>$\bar{x}_{\text{pre-test}} = 0.526$</td>
<td>$\bar{x}_{\text{post-1}} = 0.368$</td>
<td>$P = 0.420$</td>
<td>$\bar{x}_{\text{pre-test}} = 0.476$</td>
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<tr>
<td>$s_{\text{pre-test}} = 0.513$</td>
<td>$s_{\text{post-1}} = 0.761$</td>
<td>$P = 0.171$</td>
<td>$s_{\text{post-2}} = 0.512$</td>
<td></td>
</tr>
<tr>
<td>$N = 19$</td>
<td>$N = 19$</td>
<td></td>
<td>$N = 21$</td>
<td></td>
</tr>
<tr>
<td><strong>Question Group C</strong></td>
<td>$\bar{x}_{\text{pre-test}} = 1.579$</td>
<td>$\bar{x}_{\text{post-1}} = 1.842$</td>
<td>$P = 0.056$</td>
<td>$\bar{x}_{\text{pre-test}} = 1.619$</td>
</tr>
<tr>
<td>$s_{\text{pre-test}} = 0.769$</td>
<td>$s_{\text{post-1}} = 0.501$</td>
<td>$P = 0.428$</td>
<td>$s_{\text{post-2}} = 0.740$</td>
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</tr>
<tr>
<td>$N = 19$</td>
<td>$N = 19$</td>
<td></td>
<td>$N = 21$</td>
<td></td>
</tr>
<tr>
<td><strong>Question Group D</strong></td>
<td>$\bar{x}_{\text{pre-test}} = 0.842$</td>
<td>$\bar{x}_{\text{post-1}} = 1.737$</td>
<td>$P = 0.002^{**}$</td>
<td>$\bar{x}_{\text{pre-test}} = 0.810$</td>
</tr>
<tr>
<td>$s_{\text{pre-test}} = 0.834$</td>
<td>$s_{\text{post-1}} = 0.562$</td>
<td>$P = 0.001^{**}$</td>
<td>$s_{\text{post-2}} = 0.814$</td>
<td></td>
</tr>
<tr>
<td>$N = 19$</td>
<td>$N = 19$</td>
<td></td>
<td>$N = 21$</td>
<td></td>
</tr>
<tr>
<td><strong>Question Group E</strong></td>
<td>$\bar{x}_{\text{pre-test}} = 1.263$</td>
<td>$\bar{x}_{\text{post-1}} = 2.000$</td>
<td>$P = 0.002^{**}$</td>
<td>$\bar{x}_{\text{pre-test}} = 1.333$</td>
</tr>
<tr>
<td>$s_{\text{pre-test}} = 0.872$</td>
<td>$s_{\text{post-1}} = 0.000$</td>
<td>$P = 0.025^{*}$</td>
<td>$s_{\text{post-2}} = 0.856$</td>
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<tr>
<td>$N = 19$</td>
<td>$N = 19$</td>
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<td>$N = 21$</td>
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</table>

<table>
<thead>
<tr>
<th># Misconception Responses</th>
<th>Pre-Test</th>
<th>Post-Test 1</th>
<th>Pre-Test</th>
<th>Post-Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Question Group A</strong></td>
<td>$\bar{x}_{\text{pre-test}} = 0.000$</td>
<td>$\bar{x}_{\text{post-1}} = 0.0526$</td>
<td>$P = 0.331$</td>
<td>$\bar{x}_{\text{pre-test}} = 0.000$</td>
</tr>
<tr>
<td>$s_{\text{pre-test}} = 0.000$</td>
<td>$s_{\text{post-1}} = 0.2294$</td>
<td>$P = 0.329$</td>
<td>$s_{\text{post-2}} = 0.048$</td>
<td></td>
</tr>
<tr>
<td>$N = 19$</td>
<td>$N = 19$</td>
<td></td>
<td>$N = 21$</td>
<td></td>
</tr>
<tr>
<td><strong>Question Group B</strong></td>
<td>$\bar{x}_{\text{pre-test}} = 1.158$</td>
<td>$\bar{x}_{\text{post-1}} = 1.579$</td>
<td>$P = 0.088$</td>
<td>$\bar{x}_{\text{pre-test}} = 1.238$</td>
</tr>
<tr>
<td>$s_{\text{pre-test}} = 0.602$</td>
<td>$s_{\text{post-1}} = 0.769$</td>
<td>$P = 0.186$</td>
<td>$s_{\text{post-2}} = 0.625$</td>
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</tr>
<tr>
<td>$N = 19$</td>
<td>$N = 19$</td>
<td></td>
<td>$N = 21$</td>
<td></td>
</tr>
<tr>
<td><strong>Question Group C</strong></td>
<td>$\bar{x}_{\text{pre-test}} = 0.263$</td>
<td>$\bar{x}_{\text{post-1}} = 0.158$</td>
<td>$P = 0.331$</td>
<td>$\bar{x}_{\text{pre-test}} = 0.238$</td>
</tr>
<tr>
<td>$s_{\text{pre-test}} = 0.653$</td>
<td>$s_{\text{post-1}} = 0.501$</td>
<td>$P = 0.186$</td>
<td>$s_{\text{post-2}} = 0.625$</td>
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</tr>
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<td>$N = 19$</td>
<td>$N = 19$</td>
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<td>$N = 21$</td>
<td></td>
</tr>
<tr>
<td><strong>Question Group D</strong></td>
<td>$\bar{x}_{\text{pre-test}} = 0.737$</td>
<td>$\bar{x}_{\text{post-1}} = 0.158$</td>
<td>$P = 0.012^{**}$</td>
<td>$\bar{x}_{\text{pre-test}} = 0.810$</td>
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<tr>
<td>$s_{\text{pre-test}} = 0.733$</td>
<td>$s_{\text{post-1}} = 0.375$</td>
<td>$P = 0.002^{**}$</td>
<td>$s_{\text{post-2}} = 0.750$</td>
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<tr>
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<td>$N = 19$</td>
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<td>$N = 21$</td>
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</tr>
<tr>
<td><strong>Question Group E</strong></td>
<td>$\bar{x}_{\text{pre-test}} = 0.474$</td>
<td>$\bar{x}_{\text{post-1}} = 0.000$</td>
<td>$P = 0.003^{*}$</td>
<td>$\bar{x}_{\text{pre-test}} = 0.429$</td>
</tr>
<tr>
<td>$s_{\text{pre-test}} = 0.612$</td>
<td>$s_{\text{post-1}} = 0.000$</td>
<td>$P = 0.031^{*}$</td>
<td>$s_{\text{post-2}} = 0.598$</td>
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</table>

Note: The possible number of correct responses ranged from 0 to 2 and the possible number of misconception responses ranged from 0 to 2.
### Prediction-Only Group Paired T-Test Results

<table>
<thead>
<tr>
<th># Correct Responses</th>
<th>Pre-Test</th>
<th>Post-Test 1</th>
<th>Pre-Test</th>
<th>Post-Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question Group A</td>
<td>$\bar{x}_{\text{pre-test}} = 1.227$</td>
<td>$\bar{x}_{\text{post-test}} = 1.227$</td>
<td>$\bar{x}_{\text{pre-test}} = 1.238$</td>
<td>$\bar{x}_{\text{post-test}} = 1.238$</td>
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<td>$s_{\text{pre-test}} = 0.752$</td>
<td>$s_{\text{post-test}} = 0.869$</td>
<td>$s_{\text{pre-test}} = 0.768$</td>
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<td>N = 21</td>
</tr>
<tr>
<td>Question Group B</td>
<td>$\bar{x}_{\text{pre-test}} = 0.318$</td>
<td>$\bar{x}_{\text{post-test}} = 0.409$</td>
<td>$\bar{x}_{\text{pre-test}} = 0.238$</td>
<td>$\bar{x}_{\text{post-test}} = 0.190$</td>
</tr>
<tr>
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<td>$s_{\text{pre-test}} = 0.477$</td>
<td>$s_{\text{post-test}} = 0.734$</td>
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<td>N = 21</td>
</tr>
<tr>
<td>Question Group C</td>
<td>$\bar{x}_{\text{pre-test}} = 1.273$</td>
<td>$\bar{x}_{\text{post-test}} = 1.545$</td>
<td>$\bar{x}_{\text{pre-test}} = 1.429$</td>
<td>$\bar{x}_{\text{post-test}} = 1.286$</td>
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<tr>
<td>Question Group D</td>
<td>$\bar{x}_{\text{pre-test}} = 0.727$</td>
<td>$\bar{x}_{\text{post-test}} = 1.318$</td>
<td>$\bar{x}_{\text{pre-test}} = 0.810$</td>
<td>$\bar{x}_{\text{post-test}} = 1.143$</td>
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<td>$s_{\text{pre-test}} = 0.883$</td>
<td>$s_{\text{post-test}} = 0.894$</td>
<td>$s_{\text{pre-test}} = 0.928$</td>
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<td>N = 21</td>
</tr>
<tr>
<td>Question Group E</td>
<td>$\bar{x}_{\text{pre-test}} = 0.545$</td>
<td>$\bar{x}_{\text{post-test}} = 1.545$</td>
<td>$\bar{x}_{\text{pre-test}} = 0.619$</td>
<td>$\bar{x}_{\text{post-test}} = 1.333$</td>
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<td>$s_{\text{pre-test}} = 0.596$</td>
<td>$s_{\text{post-test}} = 0.739$</td>
<td>$s_{\text{pre-test}} = 0.669$</td>
<td>$s_{\text{post-test}} = 0.730$</td>
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</table>

<table>
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<tr>
<th># Misconception Responses</th>
<th>Pre-Test</th>
<th>Post-Test 1</th>
<th>Pre-Test</th>
<th>Post-Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question Group A</td>
<td>$\bar{x}_{\text{pre-test}} = 0.046$</td>
<td>$\bar{x}_{\text{post-test}} = 0.091$</td>
<td>$\bar{x}_{\text{pre-test}} = 0.048$</td>
<td>$\bar{x}_{\text{post-test}} = 0.095$</td>
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<tr>
<td></td>
<td>$s_{\text{pre-test}} = 0.213$</td>
<td>$s_{\text{post-test}} = 0.294$</td>
<td>$s_{\text{pre-test}} = 0.218$</td>
<td>$s_{\text{post-test}} = 0.301$</td>
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<td>N = 21</td>
<td>N = 21</td>
<td>N = 21</td>
<td>N = 21</td>
</tr>
<tr>
<td>Question Group B</td>
<td>$\bar{x}_{\text{pre-test}} = 1.136$</td>
<td>$\bar{x}_{\text{post-test}} = 1.273$</td>
<td>$\bar{x}_{\text{pre-test}} = 1.190$</td>
<td>$\bar{x}_{\text{post-test}} = 1.333$</td>
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<tr>
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<td>$s_{\text{pre-test}} = 0.710$</td>
<td>$s_{\text{post-test}} = 0.827$</td>
<td>$s_{\text{pre-test}} = 0.680$</td>
<td>$s_{\text{post-test}} = 0.856$</td>
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<td>N = 22</td>
<td>N = 22</td>
<td>N = 21</td>
<td>N = 21</td>
</tr>
<tr>
<td>Question Group C</td>
<td>$\bar{x}_{\text{pre-test}} = 0.227$</td>
<td>$\bar{x}_{\text{post-test}} = 0.136$</td>
<td>$\bar{x}_{\text{pre-test}} = 0.143$</td>
<td>$\bar{x}_{\text{post-test}} = 0.190$</td>
</tr>
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<td>$s_{\text{pre-test}} = 0.612$</td>
<td>$s_{\text{post-test}} = 0.468$</td>
<td>$s_{\text{pre-test}} = 0.478$</td>
<td>$s_{\text{post-test}} = 0.512$</td>
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<td>N = 22</td>
<td>N = 22</td>
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<td>N = 21</td>
</tr>
<tr>
<td>Question Group D</td>
<td>$\bar{x}_{\text{pre-test}} = 0.500$</td>
<td>$\bar{x}_{\text{post-test}} = 0.409$</td>
<td>$\bar{x}_{\text{pre-test}} = 0.524$</td>
<td>$\bar{x}_{\text{post-test}} = 0.476$</td>
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<td>$s_{\text{pre-test}} = 0.602$</td>
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<td>N = 22</td>
<td>N = 22</td>
<td>N = 21</td>
<td>N = 21</td>
</tr>
<tr>
<td>Question Group E</td>
<td>$\bar{x}_{\text{pre-test}} = 0.846$</td>
<td>$\bar{x}_{\text{post-test}} = 0.227$</td>
<td>$\bar{x}_{\text{pre-test}} = 0.810$</td>
<td>$\bar{x}_{\text{post-test}} = 0.286$</td>
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<tr>
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<td>$s_{\text{pre-test}} = 0.774$</td>
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<td>$s_{\text{post-test}} = 0.463$</td>
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<td>N = 22</td>
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<td>N = 21</td>
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Note: The possible number of correct responses ranged from 0 to 2 and the possible number of misconception responses ranged from 0 to 2.
## Prediction-and-Voting Group Paired T-Test Results

<table>
<thead>
<tr>
<th># Correct Responses</th>
<th>Pre-Test</th>
<th>Post-Test 1</th>
<th>Pre-Test</th>
<th>Post-Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Question Group A</strong></td>
<td>x _ pre-test = 1.625 \text{,} s _ pre-test = 0.647 \text{,} N = 24</td>
<td>x _ post _ test = 1.208 \text{,} s _ post _ test = 0.833 \text{,} N = 24</td>
<td>x _ pre-test = 1.619 \text{,} s _ pre-test = 0.669 \text{,} N = 21</td>
<td>x _ post _ test = 1.429 \text{,} s _ post _ test = 0.811 \text{,} N = 21</td>
</tr>
<tr>
<td><strong>Question Group B</strong></td>
<td>x _ pre-test = 0.542 \text{,} s _ pre-test = 0.509 \text{,} N = 24</td>
<td>x _ post _ test = 0.833 \text{,} s _ post _ test = 0.868 \text{,} N = 24</td>
<td>x _ pre-test = 0.524 \text{,} s _ pre-test = 0.512 \text{,} N = 21</td>
<td>x _ post _ test = 0.571 \text{,} s _ post _ test = 0.507 \text{,} N = 21</td>
</tr>
<tr>
<td><strong>Question Group C</strong></td>
<td>x _ pre-test = 1.625 \text{,} s _ pre-test = 0.711 \text{,} N = 24</td>
<td>x _ post _ test = 1.875 \text{,} s _ post _ test = 0.448 \text{,} N = 24</td>
<td>x _ pre-test = 1.619 \text{,} s _ pre-test = 0.740 \text{,} N = 21</td>
<td>x _ post _ test = 1.667 \text{,} s _ post _ test = 0.577 \text{,} N = 21</td>
</tr>
<tr>
<td><strong>Question Group D</strong></td>
<td>x _ pre-test = 0.833 \text{,} s _ pre-test = 0.868 \text{,} N = 24</td>
<td>x _ post _ test = 1.542 \text{,} s _ post _ test = 0.721 \text{,} N = 24</td>
<td>x _ pre-test = 0.857 \text{,} s _ pre-test = 0.910 \text{,} N = 21</td>
<td>x _ post _ test = 1.571 \text{,} s _ post _ test = 0.746 \text{,} N = 21</td>
</tr>
<tr>
<td><strong>Question Group E</strong></td>
<td>x _ pre-test = 1.250 \text{,} s _ pre-test = 0.794 \text{,} N = 24</td>
<td>x _ post _ test = 1.875 \text{,} s _ post _ test = 0.338 \text{,} N = 24</td>
<td>x _ pre-test = 1.286 \text{,} s _ pre-test = 0.784 \text{,} N = 21</td>
<td>x _ post _ test = 1.714 \text{,} s _ post _ test = 0.561 \text{,} N = 21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th># Misconception Responses</th>
<th>Pre-Test</th>
<th>Post-Test 1</th>
<th>Pre-Test</th>
<th>Post-Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Question Group A</strong></td>
<td>x _ pre-test = 1.619 \text{,} s _ pre-test = 0.669 \text{,} N = 24</td>
<td>x _ post _ test = 1.429 \text{,} s _ post _ test = 0.811 \text{,} N = 21</td>
<td>x _ pre-test = 1.619 \text{,} s _ pre-test = 0.669 \text{,} N = 21</td>
<td>x _ post _ test = 1.429 \text{,} s _ post _ test = 0.811 \text{,} N = 21</td>
</tr>
<tr>
<td><strong>Question Group B</strong></td>
<td>x _ pre-test = 0.524 \text{,} s _ pre-test = 0.512 \text{,} N = 24</td>
<td>x _ post _ test = 0.571 \text{,} s _ post _ test = 0.507 \text{,} N = 21</td>
<td>x _ pre-test = 1.048 \text{,} s _ pre-test = 0.669 \text{,} N = 21</td>
<td>x _ post _ test = 1.095 \text{,} s _ post _ test = 0.700 \text{,} N = 21</td>
</tr>
<tr>
<td><strong>Question Group C</strong></td>
<td>x _ pre-test = 1.619 \text{,} s _ pre-test = 0.740 \text{,} N = 24</td>
<td>x _ post _ test = 1.667 \text{,} s _ post _ test = 0.577 \text{,} N = 24</td>
<td>x _ pre-test = 0.143 \text{,} s _ pre-test = 0.359 \text{,} N = 21</td>
<td>x _ post _ test = 0.301 \text{,} s _ post _ test = 0.666 \text{,} N = 21</td>
</tr>
<tr>
<td><strong>Question Group D</strong></td>
<td>x _ pre-test = 0.857 \text{,} s _ pre-test = 0.910 \text{,} N = 24</td>
<td>x _ post _ test = 1.571 \text{,} s _ post _ test = 0.746 \text{,} N = 24</td>
<td>x _ pre-test = 0.571 \text{,} s _ pre-test = 0.746 \text{,} N = 21</td>
<td>x _ post _ test = 0.238 \text{,} s _ post _ test = 0.436 \text{,} N = 21</td>
</tr>
<tr>
<td><strong>Question Group E</strong></td>
<td>x _ pre-test = 1.286 \text{,} s _ pre-test = 0.784 \text{,} N = 24</td>
<td>x _ post _ test = 1.714 \text{,} s _ post _ test = 0.561 \text{,} N = 24</td>
<td>x _ pre-test = 0.524 \text{,} s _ pre-test = 0.680 \text{,} N = 21</td>
<td>x _ post _ test = 0.190 \text{,} s _ post _ test = 0.512 \text{,} N = 21</td>
</tr>
</tbody>
</table>

Note: The possible number of correct responses ranged from 0 to 2 and the possible number of misconception responses ranged from 0 to 2.

\( \bar{x} \): mean
\( s \): standard deviation
\( p \): p-value
** = The two-tailed p-value met the 0.05 statistical significance level
** = The two-tailed p-value met the 0.01 statistical significance level
N: sample size (number of test included in the Paired T-test analysis)
Appendix L: Effect Sizes for Student Achievement Gain on Each Test Question Group

<table>
<thead>
<tr>
<th>Question Group</th>
<th>Number of Correct Responses</th>
<th>Effect Size (Cohen’s d)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-Test to Post-Test #1</td>
<td>Pre-Test to Post-Test #2</td>
</tr>
<tr>
<td><strong>Question Group A</strong></td>
<td>Prediction-and-Voting Group versus Comparison Group</td>
<td>-0.470*</td>
</tr>
<tr>
<td></td>
<td>Prediction-Only Group verses Comparison Group</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>Question Group B</strong></td>
<td>Prediction-and-Voting Group versus Comparison Group</td>
<td>0.526*</td>
</tr>
<tr>
<td></td>
<td>Prediction-Only Group verses Comparison Group</td>
<td>0.300</td>
</tr>
<tr>
<td><strong>Question Group C</strong></td>
<td>Prediction-and-Voting Group versus Comparison Group</td>
<td>-0.017</td>
</tr>
<tr>
<td></td>
<td>Prediction-Only Group verses Comparison Group</td>
<td>0.018</td>
</tr>
<tr>
<td><strong>Question Group D</strong></td>
<td>Prediction-and-Voting Group versus Comparison Group</td>
<td>-0.202</td>
</tr>
<tr>
<td></td>
<td>Prediction-Only Group verses Comparison Group</td>
<td>-0.290</td>
</tr>
<tr>
<td><strong>Question Group E</strong></td>
<td>Prediction-and-Voting Group versus Comparison Group</td>
<td>-0.146</td>
</tr>
<tr>
<td></td>
<td>Prediction-Only Group verses Comparison Group</td>
<td>-0.320*</td>
</tr>
</tbody>
</table>

*: \( |\text{Cohen’s d}| > 0.300\)

Note: The effect sizes were calculated from the gain if the mean scores of the Paired T-Tests shown in Appendix K.
<table>
<thead>
<tr>
<th>Question Group</th>
<th>Number of RH Responses</th>
<th>Pre-Test to Post-Test #1</th>
<th>Pre-Test to Post-Test #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group A</td>
<td>Prediction-and-Voting Group versus Comparison Group</td>
<td>-0.052</td>
<td>-0.337*</td>
</tr>
<tr>
<td></td>
<td>Prediction-Only Group versus Comparison Group</td>
<td>-0.023</td>
<td>0.000</td>
</tr>
<tr>
<td>Group B</td>
<td>Prediction-and-Voting Group versus Comparison Group</td>
<td>-0.450*</td>
<td>-0.245</td>
</tr>
<tr>
<td></td>
<td>Prediction-Only Group versus Comparison Group</td>
<td>-0.299</td>
<td>-0.148</td>
</tr>
<tr>
<td>Group C</td>
<td>Prediction-and-Voting Group versus Comparison Group</td>
<td>0.140</td>
<td>0.199</td>
</tr>
<tr>
<td></td>
<td>Prediction-Only Group versus Comparison Group</td>
<td>0.024</td>
<td>0.310*</td>
</tr>
<tr>
<td>Group D</td>
<td>Prediction-and-Voting Group versus Comparison Group</td>
<td>0.400*</td>
<td>0.315*</td>
</tr>
<tr>
<td></td>
<td>Prediction-Only Group versus Comparison Group</td>
<td>0.549*</td>
<td>0.660*</td>
</tr>
<tr>
<td>Group E</td>
<td>Prediction-and-Voting Group versus Comparison Group</td>
<td>0.098</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Prediction-Only Group versus Comparison Group</td>
<td>-0.213</td>
<td>-0.250</td>
</tr>
</tbody>
</table>

*: | Cohen’s d | > 0.300

Note: The effect sizes were calculated from the gain if the mean scores of the Paired T-Tests shown in Appendix K.
Appendix M: Coded Transcript of Semi-Structured Student Interviews

NOTE:

All transcriptions were completed using the Transana Version 2.20 Program.

All transcriptions were modeled after the Jeffersonian Transcription Notation as described by G. Jefferson, *Transcription Notation*, in J. Atkinson and J. Heritage (eds), *Structures of Social Interaction*, New York: Cambridge University Press, 1984.

Some notation used in the transcriptions below is as follows:

<table>
<thead>
<tr>
<th>(pause .)</th>
<th>A pause in speech lasting approximately two to five seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>( text )</td>
<td>Speech which is unclear or in doubt in the transcript</td>
</tr>
<tr>
<td>(( italic text ))</td>
<td>Annotation of non-verbal activity</td>
</tr>
<tr>
<td>…</td>
<td>A break in one’s speech or an interruption of one’s speech by another</td>
</tr>
<tr>
<td>Ω&lt; number &gt;</td>
<td>Time codes that were inserted to aid in the transcription of the interviews</td>
</tr>
<tr>
<td>{ text }</td>
<td>Test questions that are being discussed in the interview but are not part of the interview speech itself.</td>
</tr>
<tr>
<td>1</td>
<td>Coding used for student’s probability reasoning before the lessons</td>
</tr>
<tr>
<td>2</td>
<td>Coding used for student’s probability reasoning after the first lesson</td>
</tr>
<tr>
<td>3</td>
<td>Coding used for student’s probability reasoning after both lessons</td>
</tr>
<tr>
<td>4</td>
<td>Coding used for student’s current probability reasoning shown during the interview</td>
</tr>
<tr>
<td>5</td>
<td>Coding used for student’s comments about the first lesson</td>
</tr>
<tr>
<td>6</td>
<td>Coding used for student’s comments about the second lesson</td>
</tr>
<tr>
<td>7</td>
<td>Coding used for student’s comments in general about the lessons</td>
</tr>
<tr>
<td>8</td>
<td>Coding used for student’s comments about previous probability instruction</td>
</tr>
</tbody>
</table>

Comparison Class Student Interview:

Total Time for Interview: 32 minutes

NOTE: This interview occurred before school two days after the student being interviewed took the Post-test 2. The interview took place in a tutoring room located next to the school’s copy room. Throughout the interview there was occasional noise that came from the copy room next door.

((Interviewer is sitting on camera to the left, but the desk in front of the interviewer is in the camera's left bottom corner and shows the test papers and notes the interviewer has. S1, a student from the Comparison Class, is sitting opposite the interviewer at the right side of the camera's view. The student and the blank desk in front of the student are visible to the camera. A tape recorder is placed between the interviewer and student S1.))
Okay. So before I get into like the main part of the questions. Um, I want to ask you just a few questions to get a better understanding of your reasoning, um for some of your answers on your test.

S1: Alrighty.

I: So, here is a copy of your Pre-test and the Post-tests you (have taken). So if you want to look over this you can. ([Interviewer lays Pre-test and Post-test #1 in front of the student and S1 begins to look over the papers])

S1: What do the numbers mean?

I: Oh.

S1: You circled a number.

I: I'll ask certain things (about those).

S1: Alright.

I: So are you ready? ((Student nods yes)) Okay. Um, so for your Pre-test for question number two and if you can also turn to number nine on your Post-test. It's on the last page. It's the one in green.

{ Pre-test Question #2:
If a fair coin is tossed five times, which of the following sequence of heads (H) and tails (T), if any, is MOST LIKELY to occur?

f. H T H T T
g. T H H H H
h. H T H T H
i. Sequences (a) and (c) are equally likely.
j. All of the above sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

f. Every sequence of five tosses has exactly the same probability of occurring.
g. Since tossing a coin is random, the coin should not alternate between heads and tails.
h. Since tossing a coin is random, you should not get a long string of head or tails.
i. There ought to be roughly the same number of tails as heads.
j. Other ________________________________

Post-test 1 Question #9:
If a fair coin is tossed eight times, which of the following sequences of heads (H) and tails (T), if any, is MOST LIKELY to occur?

f. T T H H H H T T
Which of the following best describes the reason for your answer to the preceding question?

f. Every sequence of eight tosses has exactly the same probability of occurring.
g. Since tossing a coin is random, the coin should not alternate between heads and tails.
h. There ought to be roughly the same number of tails as heads.
i. Since tossing a coin is random, you should not get a long string of head or tails.
j. Other ________________________________

S1: (Um-huh).

I: Okay so, um why did you choose, um, D and (pause .) D for your answer choice that the sequences A and C were equally likely, um, because they had to be roughly the same number of heads and tails on your Pre-test. But then on number nine its the same sort of question right? It deals with the most likely for the heads for the coin.

S1: Uh-huh.

I: On the Post-test you said that all sequences were equally likely because each sequence of eight tosses has exactly the same probability. So what kind of what sort of reasoning lead you to select the answers you did or uh, the first part?

S1: Um, I just (pause .) I don't know why. When I saw like the equal number and I thought like a 50-50 chance of getting a head or tails. Well that's what I thought, and then I, I kind of realized. Like on this one ((points to Post-test 1)) I kind of, by then I knew that like all sequences were equally likely because its 50-50 regardless.

I: Okay. So, you said on the first one the 50-50 chance for heads or tails made you select A and C but what about, I mean you said the same thing for...

S1: A and C?

I: ...like, what was it about those sequences that made them seem more, uh, more likely than say B?

S1: On that test?


S1: Well that's not the same question.
S1: Like that one (points to question two on Pre-test)) there's an answer choice that says that A and B are equally likely.

{ Pre-test Question #2:
If a fair coin is tossed five times, which of the following sequence of heads (H) and tails (T), if any, is MOST LIKELY to occur?

a. H T H T T
b. T H H H H
d. Sequences (a) and (c) are equally likely.
e. All of the above sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?
a. Every sequence of five tosses has exactly the same probability of occurring.
b. Since tossing a coin is random, the coin should not alternate between heads and tails.
c. Since tossing a coin is random, you should not get a long string of head or tails.
d. There ought to be roughly the same number of tails as heads.
e. Other ____________________________ }

S1: Oh, um, I just. I don't know, just the fact that like they. I can't. Well I can't see it because this one's (points to question nine on Post-test I)) not the same.

{ Post-test 1 Question #9:
If a fair coin is tossed eight times, which of the following sequences of heads (H) and tails (T), if any, is MOST LIKELY to occur?

a. T T H H H T T
d. H H T H T H T H
e. All sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?
a. Every sequence of eight tosses has exactly the same probability of occurring.
b. Since tossing a coin is random, the coin should not alternate between heads and tails.
c. There ought to be roughly the same number of tails as heads.
d. Since tossing a coin is random, you should not get a long string of head or tails.
e. Other ____________________________ }
I: Okay. Um, well something that might help too. (Interviewer brings out S1's Post-test 2) Is this right here, because the Post-test number one is slightly different but. (Student looks at Post-test 2) So that's the test I gave you this previous week. This is the last test right?

S1: Right.

I: And so basically the only difference between that and the Pre-test is I labeled one the Pre-test so they have the exact same questions. So go ahead and look at number two on that one. Maybe that will help.

{ Post-test 2 Question #2:
If a fair coin is tossed five times, which of the following sequence of heads (H) and tails (T), if any, is MOST LIKELY to occur?

a. H T H T T
b. T H H H H
c. H T H T H
d. Sequences (a) and (c) are equally likely.
e. All of the above sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. Every sequence of five tosses has exactly the same probability of occurring.
b. Since tossing a coin is random, the coin should not alternate between heads and tails.
c. Since tossing a coin is random, you should not get a long string of head or tails.
d. There ought to be roughly the same number of tails as heads.
e. Other ________________________________  }

S1: (Student examines questions number two on Post-test 2) Yeah I saw that both of them had three heads and three tails...

I: Oh, okay.

S1: ...and I thought that it was just like, that those two were the most likely to occur because there should be like, like a (pause .) no that's not why it should. I should not have circled that. I was, I was thinking that there should be roughly, like the same amount of heads as tails because its half and half.1

I: Okay and was that your reasoning for both, or just one of the tests?

S1: Um, not just that one (points to Pre-test) because this one I didn't put A and C. I put that they were equally likely.

I: Okay, and what made you think that all of the sequences were equally likely?
S1: Um, because they're like (pause .) because you have to toss it each time and you
have a 50-50 chance of getting heads or tails and since you're tossing it the same
amount of times, like five times each, it's the same probability for each sequence. 3
I: Okay. Um, so is it easier to look at the test number two or test number. Well, this one
( (Interviewer holds up Post-test 1) ) because we can look at either one.
S1: This one's...
I: The second one.
S1: ...is (more similar), yeah.
<304893>I: Okay. So, um, then look at. The next one I had a question about was
question number four for both tests. ( (Interviewer and S1 turn to question four on the
Pre-test and question four on the Post-test 2) )
{ Pre-test and Post-test 2 Question #4:
If a fair die is rolled four times, which of the following sequences of results, if any, is LEAST
LIKELY to occur?
f. 6 4 3 5
g. 5 6 2 6
h. 2 3 4 5
i. 2 1 4 3
j. All sequences are equally likely.
Which of the following best describes the reason for your answer to the preceding question?
f. Since rolling a die is a random event, a result like that is very unlikely.
g. You are much less likely to get a mixture of different numbers than an ordered sequence.
h. All sequences of rolls have exactly the same probability of occurring.
i. You are much less likely to get a mixture of different numbers than numbers that repeat.
j. Other ________________________________ }
I: So (pause.) for the first test, your Pre-test uh, you selected that uh, sequence B was
least likely because you are much more likely to get a mixture of different numbers
than numbers that repeat. So what sort of reasoning uh, led you to select these answers?
S1: (pause.) Oh because I saw that the two sixes, and I thought like, I don't know I just
thought that um, the chance of getting two sixes in one sequence is uh, less likely. 1 But
then again like, the whole thing with the, you know each roll being individual and like
having one six chance of getting that like. On this one I changed it to all sequences are
equally likely too. 3
I: Okay. So on your Post-test you changed it to equally likely (answer) right?
S1: Um-huh.

I: So initially, when you first did the test you said that the two sixes is what lead you to say that it was least likely, right?

S1: Yeah.

I: Um, was there (pause.) anything, any reason why the other sequences didn't seem uh, or seemed more likely (for you)?

S1: Because they didn't (pause.) because um, they didn't have any numbers repeating.

I: Okay. So the repetition made it less likely, right? ((S1 nods yes)) Right, um okay. So the next one I just had a question about was (pause.) yeah, number five, for both of them. ((Interviewer and S1 look at question five on the Pre-test)) So on the first one ((Interviewer points to question five on Pre-test)) you said uh, its dealing with the bag of fruit with nine pieces of fruit and you first selected D. That an orange or a pear are both equally likely and more likely than an apple and then your reasoning was that there are no more apples left. So, can you kind-of explain what your reasoning was there?

{ Pre-test and Post-test 2 Question #5:

A bag has 9 pieces of fruit: 3 apples, 3 pears, and 3 oranges. Four pieces of fruit are picked, one at a time. Each time a piece of fruit is picked, the type of fruit is recorded, and it is then put back in the bag. If the first 3 pieces of fruit were apples, what is the fourth piece MOST LIKELY to be?

f. A pear
g. An apple
h. An orange
i. An orange or a pear are both equally likely and more likely than an apple.
j. An apple, orange, or pear are all equally likely.

Which of the following best describes the reason for your answer to the preceding question?

f. This piece of fruit is just as likely as any other.
g. The apples seem to be lucky.
h. The picks are independent, so each fruit has an equally likely chance of being picked.
i. The fourth piece of fruit won't be an apple because too many have already been picked.
j. Other ________________________________  }

S1: I didn't, I forgot. Well, I mean, I guess I didn't read the "and put back in the bag" ((S1 points to the line in question five on the Pre-test that say "and put back in the bag").)

I: Okay.

S1: Yeah that wasn't, my mistake. I guess I I don't know why I didn't read that.
I: So you thought um...
S1: That they...
I: ...like they had taken out all three apples and there was nothing else left?
S1: Yeah.
I: Okay. So, thinking back to then, if you known that the apples were replaced would that change your answer?
S1: Yeah, I would have.
I: What would you answer?
S1: Um, that all of them are equally likely.
I: Okay. And on your Post-test you put that they were equally likely.
S1: Yeah, because I had read that part.

I: Okay. Alright, um, and then the next one is number seven. ((Interviewer reads from question seven on the Pre-test)) So on the first one you put ((Interviewer reads from question seven on the Pre-test)) um, its a (heads and) coins question but its done with least likely and you put that uh, sequence uh, D was the least likely because there would be roughly the same number of heads and tails.

If a fair coin is tossed twelve times, which of the following sequences of heads (H) and tails (T), if any, is LEAST LIKELY to occur?

f. H T H T H T H T H T H T T

g. H H T H T T H T H T H

h. T T H T T H T H T H T T

i. H H H H H H H T T T T

j. All sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

f. There ought to be roughly the same number of tails as heads.

g. Since tossing a coin is random, you should not get a long string of head or tails.

h. Every sequence of twelve tosses has exactly the same probability of occurring.

i. Since tossing a coin is random, the coin should not alternate between heads and tails.

j. Other ___________________________}
S1: Yeah the same reasoning that I had for the first one. Like the first question, ((S1 student turns Pre-test to page that has question two)) like on number two.

I: Okay. So you were expecting uh, what exactly for this sequence?

S1: Uh, for there to be like. (pause.) ((S1 flips between pages of the Post-test to look at questions two and seven)) That's weird. Like the, the answer underneath it I put the right answer. Oh no, this is the other test.

I: Yeah.

S1: Oh um, I just saw that long string of heads and I thought that was pretty unlikely. 1

I: Okay.

S1: And then, but I changed it because I still had that. Uh, like each coin toss having a fifty-fifty chance thing. 3

I<527854>: Um, and then the last one I wanted to do was question number three ((Interviewer turns to question three on the Pre-test and question three on the Post-test 2)) so I (we go back) just a little. ((S1 turns to question three on the Pre-test and question three on the Post-test 2)) So on the first test you said um, ((Interviewer reads from question three on the Pre-test)) its a question with the three coins and we were asking which one was least likely. Three heads, one heads and one tail, one head and two tails uh, or they were all equally likely. So first you said that um, answer choice A, three heads, was the least likely because the outcome of having um, equally three heads was less likely than having exactly two heads which was less, less likely than have, having exactly one head. So what sort-of uh, reasoning or uh, work lead you to that?

{ Pre-test and Post-test 2 Question #3:
   When three coins are tossed, which of the following outcomes is LEAST LIKELY to occur?
   
   e. Three heads
   f. Two heads and one tail
   g. One head and two tails
   h. All the above outcomes are equally likely.

   Which of the following best describes the reason for your answer to the preceding question?
   
   f. The outcome of having exactly three heads is less likely than having exactly two heads,
   which is less likely than having exactly one head.
   g. Since coin tossing is a random event, every outcome has exactly the same probability of occurring.
   h. You are much less likely to get a mixture of heads and tails than all heads or all tails.
   i. There are four possible outcomes and each outcome has a probability of 1/4.
   j. Other ________________________________  }
S1: Well I just, I thought that it was uh, um less likely to have all three heads and the reason. I've, I should have put "other" on that one and like filled it out because like its not really that I thought it was less likely than having "(S1 reads from question three on Pre-test) "exactly two heads which is less likely than having exactly one head." But just because like its, I thought it was less likely to get three heads in a row. And um, I think that. Hold on, ((S1 looks at question number three on the Pre-test and sighs)) if there was a, to me if there was a, a answer that said B and C are like. Oh, "least likely" ((S1 pauses reads through the question and in a low voice reads "equally likely")) I don't know now, I am still conflicted with that one, because I would assume they are all equally likely because I tried to think of that, that little tree thing, that we did and uh. ((S1 motions like drawing a probability tree in the air and pauses)) You know, I am still, I am fighting with this right now. ((S1 smiles and looks away))

I: So initially, did you use like any of the tree diagrams or anything like that for your answers?

S1: No. I just thought getting three heads in a row would...

I: Okay, so you just, on that one you just thought of you know, if you had to flip a coin three times you wouldn't expect to get three heads, right?

S1: Right.

I: Okay, so if for instance say, there was an answer choice with with three heads it was three tails would you still say it was least likely?

S1: (pause .) Um, before the tree thing?

I: Yeah. So think back to when you first took the, the test.

S1: Yeah, I would say it was least likely.

I: Okay. So what would you. Say if you didn't have to think about least likely or anything like that, what would you expect to get, for getting, for flipping three coins?

S1: Either B or C.

I: So you would expect to get, um, not just all heads or all tails but a mixture right?

S1: Right.

I: Okay. Um, so now looking at your last test ((Interviewer reads from question three on the Post-test)) you put that all of the above outcomes are equally likely um, because,
tossing a coin is a random event and every outcome has the same probability. So what sort of reasoning led you to that?

S1: Um, because (pause .) _because every time you uh, you toss a coin its fifty-fifty_. That and um I was just thinking uh, since you're tossing three times like. With like, with the _tree thing in minds its, ([S1 gestures with hand like he was drawing a probability tree]) they're going to be equally likely_.

I: Okay, so you did some work and that lead you to say that it was equally likely?

S1: Yeah, well I tried to. _I don't know what I was doing there, but whatever it was it sort of helped me to come to that conclusion_.

I: Okay. Um (pause. ) Alright, and then the last one was question number eight for both of them ((Interviewer and S1 turn to question eight on the Pre-test and question eight on the Post-test 2)) So there you answered the same way for both of them.

You said that um, ((Interviewer reads from question eight on the Pre-test)) when two fair die are rolled that the most likely outcome was uh, all of the above. So getting uh, both die to show odd um, or both of them to show even, or that one shows an odd number and one shows an even number. And you said it was because rolling a die was a random event and every outcome has uh, exactly the same probability of occurring. So for both of those, what's your, your reasoning for answering (that way)?

[S1 reads question eight to himself] They're equally likely just in, just because of uh. (_S1 pauses and looks at question eight_) _They are all equally likely because, I don't know they all have a chance, I mean, they all have the same chance of occurring_. (_S1 rubs his hand and leans onto the table_) I don't know
how to say that. *I just, I know that they are all equally likely.* 4 ((S1 continues rubbing hand and looks at the questions eight while raising his eyebrows))

I: So did you do like any work in your head? Trying to find out the chances for each or did you, did you just uh, have like that intuition that it has to be equally likely?

S1: *Just that it has to be equally likely, because there's, you know there's three odd numbers and three even numbers on a die and, um you're rolling it twice so like, they're equally, like you're equally likely to get um, even or odd on one die so its going to be the same for the other one too.* 4 I know like, its hard to explain. ((Student looks down at paper and then squints at Interviewer))

I: Okay. (pause .) So do you feel like there is any sort of um, I guess intuition or like a knee-jerk reaction to think of like probability of events as being all equally likely? (pause .) Or is it just within, like the certain cases that you looked at?

S1: It dep. Um, (pause .) well with all these, like the one's I said were equally likely its because they're. There's uh, basically a fifty-fifty chance of um, each event. Like with the coin it was uh, one to two and then with the, the die it was three to six which is, you know, one to two. 4

I: Uh-huh.

S1: So I, I saw that and that made me believe that, the one's like that were (pretty) equally likely to happen.

I: Okay, so if it was a situation that, say maybe you had uh, an unfair coin or an unfair die where the probability wasn't equally one-half and one-half. Say, you had like the probability of getting a head was um, three-fourths and a tail was one-fourth. Would that change your, your rea... How you look at the different problems?

S1: Yeah.

I: Okay. Would you feel that you would still answer that it was equally likely? Or do you think that you would be more um, I guess hesitant to say that?

S1: I'd would be more hesitant to say it, I would have like uh, sort of, I'd have to tr. Um it, well it wouldn't be likely because. Well it wouldn't be as equally likely because one has a three-fourths chance of happening and one has a one-fourth chance of happening.

I: Okay.

S1: And, so yeah that would change my answer.
I: Okay. (Alright.) So that was basically the, the first part of what I wanted to ask you.

S1: Okay.

I: So thank you ((Interviewer laughs)) for that, I know that it is tough looking over all of this stuff. Okay. ((Interviewer collects the test papers in front of S1)) Um, so for the second part I am just going to ask you a couple questions and if something is unclear just let me know, okay?

S1: Okay.

I: Um so first, what does having good reasoning ability for probability mean to you?

S1: Uh, I think (pause .) it means like, like if you're actually going to work out, like if you're actually fi… Uh, use probability to see what your chances are of getting a certain outcome you have a way better chance of um, being accurate. If, if you're able to reason well. You, you can be more accurate with your uh, ((S1 pauses while the door in interview room is closed loudly)) estimations.

I: Okay. So I guess just so that I understand like um, somebody that has good reasoning ability is able to I guess to make accurate estimates or uh, predictions about certain events, right?

S1: Right.

I: Okay. Um, so, before the probability lesson presentations, how would you rate your reasoning abilities for probability?

S1: Um. Well to me I thought I was reasoning pretty well, but it helped to have to tools like that tree chart. That helped me a lot. So, um I didn't know as much as I do now (that's). ((S1 stops talking when the door in interview room is closed loudly))

I: So if you had to put, at some sort of rating like, um like say um poor, you know below average, average, above average, or excellent like how would you rate your reasoning ability before all the presentations that I did?

S1: Um, maybe average, average or just above average.

I: Okay. Um so why would you say your reasoning abilities is average? Or was average, rather?

S1: Well because looking back at it now like some of my answers were kind of (pause .) like dumb. Like that, that saying that three heads is less likely than two heads and one tail. Just uh, you know like, they were not really big mistakes but they were, I don't
know. I would say that, um, they were average because I, one I hadn't done it in a long
time and two because I didn't, I didn't really have anything to uh um, back up my
answers other than just that flat-out reasoning. But yeah, like I really appreciated that
tree chart.

I: Okay. So (to understand) I guess you didn't feel like you had maybe some of the skills
that you have now.

S1: Yeah.

I: I think you mainly answered this question, but uh, you know what other factors do you
think might have contributed to your reasoning for probability um, you know being at
that average state?

S1: What other, what do you mean?

I: Because I think you, you sort of explained kind of that question right now. You talked
about maybe the, the skills that you had and that you felt you couldn't back up your
reasoning. So was there anything else that you think um, might have contributed to your
reasoning, feeling that your reasoning was average? Or is that basically...

S1: That's basically it.

I: ...(Basically?)

S1: Yeah.

S<1238592>I: Um, so my next question is um, have you been or were you taught
probability before?

S1: Yes.

I: Okay. Um, in what classes?

S1: ((S1 sighs)) Um, I think I was introduced to it in the third grade. ((Interviewer
laughs softly)) (That's), no one like, none of my teachers ever really like uh, spent a lot
of time on it but we'd go over it for like a day or two.

I: Okay.

S1: Just, I guess a refresher.

I: And like what type of, or what classes did they do that refresher in?

S1: What do you mean what classes?
I: Like was it, did you do it in Algebra One...

S1: Oh no, like I hadn't, we hadn't really messed with probability since middle school.

I: Middle school?

S1: Yeah.

I: So about seventh, eighth grade?

S1: Like, like eighth grade math.

I: Okay. So since then had you gotten, like anymore refresher instruction with it?

S1: No, because um, because since middle school I hadn't had a class that really needed probability to do anything.

I: Okay. Alright. Um, so now I want you go ahead and take a moment and remember as much as you can about the second lesson that I did. And, (if you'll) take your time and if you have any questions (you can go ahead and ask them).

S1: I second lesson.

I: Uh-huh.

S1: Were we in groups for that one?

I: You were in groups for both of them. But the first lesson uh, was the one that I did kind of like a general review.

S1: Yeah.

I: And then the second one uh, was when I did the more specific questions and you guys were in groups and kind of talked through, I presented like those five questions?

S1: Uh-huh.

I: And you guys talked through them in groups.

S1: Yeah, I remember that. Yeah, uh I just remember arguing with Paul. And uh, (S1 sighs and smiles) I don't remember any, uh the exact questions I just remember there was some about a coin. There was some about uh, dice and then there was some about balls, I think and then I just remember Mr. Guevara was sitting right behind me. ((Interviewer laughs softly)) That's all I remember really.
I: Okay. Um so, now um, take a moment and remember as much as you can about the first lesson.

S1: ((S1 rubs their head)) (Yeah.) I think you had one of the first slides on that PowerPoint had like a dice on it, some red dice. Huh, and then that's when you were, that's when you showed us that tree things ((S1 smiles)) and it was where. Yeah you were using heads and tails for that tree chart, the tree example and then um. There was one where people were getting a bunch of crazy like, predictions and of course Paul ended up being right somehow. ((Interviewer laughs softly)) So, um yeah that what I remember is that like we all put out answers on the board and some people were way off.

I: Okay. Um so how would you rate your reasoning ability for probability after the first lesson, but before the second lesson? So in between those two.

S1: Definite, well above average now but not excellent because I am still like, I am still like, not struggling but wrestling with some of the questions like we saw.

I: Okay. So, is that, would you say its above average now or was it above average like after that first lesson?

S1: Oh, after that first lesson?

I: Uh-huh.

S1: Um, above average because you introduced us to that tree chart...

I: Okay.

S1: ...and that was like my favorite tool.

I: ((Interviewer laughs softly)) Okay. Um, so what factors do you think uh, contributed to your reasoning for probability becoming above average?

S1: Can you say that again?

I: Okay. So like what sort of factors or things do you think contributed to your reasoning for probability uh, going from just average to above average?

S1: Uh, well a refresher lesson for sure because I hadn't seen it in a while and then um. I knew there were some other ways other than tree chart that you showed us but I can't remember. I can't remember what they were called...

I: Uh.
S1: ...*there was the grid one*.  

I: Oh, doing a chart?  

S1: Yeah.  

I: Okay.  

S1: ((S1 clears throat)) So just the, the *like ways of checking my answers*.  

I: Okay. Um, so how would you rate your reasoning abilities for probability like after the second lesson. So kind of where you are right now.  

S1: Yeah, *better*. Like oh rate it, like above aver...  

I: Uh-huh.  

S1: *definitely above average but like I said not excellent*.  

I: So would you say that it went up even more after the first lesson? Or did it kind of stay around the same?  

S1: (Um), after, after the first lesson or the second lesson?  

I: So you said after the first lesson you felt it was above average right?  

S1: Right.  

I: So do you think it went up any more after the second lesson? Or did it stay around the same?  

S1: Um, I think *it stayed around the same*.  

I: Okay. Um, so now I want you to compare, the first lesson that you had with the second lesson that you had with me.  

S1: Um, I think the first lesson, the *first lesson sort of gave us uh, some tools to use when it came to reasoning with the problems* and the *second lesson uh, like let us exercise those like we were able to like put them to use*.  

I: Okay. Uh can you name I guess anymore differences between the two lessons?  

S1: Um, (pause .) I can't.
I: No? Okay.
S1: No.
I: Um...
S1: But the first lesson was like, like I said just, some ways to help you figure out probability questions and then the second one was just like, seeing if you could like put them to use.

I: Okay. Um, so do you think your reasoning ability for probability has increased? Or remained the same because of the two probability lessons?
S1: Its increased.
I: Increased?
S1: Yeah.
I: Okay. Uh, so in your opinion which lesson made a greater contribution to an increase in your reasoning for probability?
S1: Lesson one.
I: Lesson one? Okay, and why?
S1: Because again the introduction to those, those tools that we could use to help ah, reason with those problems.
I: Okay. And of those tools did you, have you ever been taught those before? Or?
S1: The, the grid one? I think we use that in Biology for uh, for uh, I forget what they are called. Its like seeing, seeing your probability of um, like getting brown eyes as opposed to blue and you know what I mean?
I: Uh-huh, the genetic trait squares, right?
S1: Yeah.
I: Um, but in terms of anything else that I taught you did you remember learning that before? Or was it completely new?
S1: It was basically new. Yeah.
I: Okay. Um, so what was the most important thing um, that you learned from the probability lesson presentations?

S1: Um, ((S1 pauses and sighs)) the most important lesson? Not to, not to just assume that um, because you have like say for the, the tossing the coin three times like not to just assume that because you have three heads that its less likely to occur than two heads and one tail. 

I: Okay. So maybe um, is that, is that the same thing as saying like um, maybe not to depend upon like whatever your initial reasoning was? Or am I, misunderstanding that?

S1: I don't know. I think, I think its just to, to um, ((S1 pauses and moves in chair)) like use what we learned, like the um, the tree chart and that grid. Uh, use them to make sure that you're right. Like, so I guess, I would, like before I, I circled my answer I would sort of test it. (pause .) Yeah. ((S1 lets out a breath))

I: Okay. I would just, I would put those to use.

I: So would you, like if you were looking at a question would you initially like read it through and then uh, select an answer that you think might be right and then check it? Or would you check it and then answer?

S1: Um, check it and then answer. But before it was what I thought was right. 

I: Okay. So the lessons helped you in I guess, thinking through your answers more clearly?

S1: Right.

I: Okay. And what is one thing that you still feel confused about regarding probability?

S1: Um (pause .) I don't know that one. ((S1 sighs)) Its not that I feel confused about it I guess just, uh, I need to, I need to keep practicing it and maybe I'll, then I'll probably uh, feel a little more comfortable with, with my answers.

I: Okay. So you just feel like, you need maybe a little bit more practice to be more sure in, in all of your thinking, (right)?

S1: Yeah.

I: Um, I think that's about it. I mean is there anything else that you wanted to add? Or?

S1: Well I enjoyed it.
I: You enjoyed it?

S1: Yeah, it was, it was a nice break away from sine and cosine.

I: All the normal Pre-Calculus stuff.

S1: Yeah.

I: Okay. Um, do you think you know what I taught you is going to be helpful at some point?

S1: Yeah.

I: Yeah?

S1: Yeah, like if um, if I ever, if I ever run into it again or I need to use it some real-life situation its going to help.

I: Okay. Alright that was about it.

S1: Alright.

((Video ends))

Prediction-Only Class Student Interview:

Total Time for Interview: 33 minutes

NOTE: This interview occurred after school on the day the student being interviewed took the Post-test 2. The interview took place in a tutoring room located next to the school’s copy room. Throughout the interview there was occasional noise that came from the copy room next door.

((Interviewer is sitting on camera to the left. S2, a student from the Prediction-Only Class, is sitting opposite the interviewer in the right side of the camera’s view. The student and the blank desk in front of the student are visible to the camera. The student’s bag and books are on the table in front of them and a tape recorder is placed between the interviewer and S2. The student S2 begins the interview with a hand placed on their face in front of their mouth.))

I: So um, before we begin like the main part of the interview I just want to ask you a few questions to kind of get a better understanding of your reasoning um, that underlie some of the answers on your test. Okay? ((S2 nods yes but still has had on their mouth))
So um, here is a copy of the first test you took the Pre-test (Interviewer places Pre-test and Post-test 2 on the desk in front of S2, the student re-adjusts their position in their seat and removes their hand from their mouth to take the tests and look at them) and then the test that you took today.

S2: Uh-huh.

I: And so the first question I wanted to look at you with was...

S2: Oh weird! (S2 begins flipping pages for both tests to compare the test questions and the Interviewer laughs)

I: Yeah they were, they were basically the exact same items I just changed the title. So...

S2: Also I keep getting the same ((S2 laughs)) ones wrong. I did do very badly!

I: No.

S2: Yeah...

I: No, not really. ((S2 puts both hands on their face in front of their mouth and begins looking at question two on the Pre-test)) Okay for the first one was number two. And so on the Pre-test you put, it was the one involving the coin and asking which one was most likely.

{ Pre-test and Post-test 2 Question #2:
If a fair coin is tossed five times, which of the following sequence of heads (H) and tails (T), if any, is MOST LIKELY to occur?

a. H T H T T
b. T H H H H
c. H T H T H
d. Sequences (a) and (c) are equally likely.
e. All of the above sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. Every sequence of five tosses has exactly the same probability of occurring.
b. Since tossing a coin is random, the coin should not alternate between heads and tails.
c. Since tossing a coin is random, you should not get a long string of head or tails.
d. There ought to be roughly the same number of tails as heads.
e. Other ________________________________ }

S2: Uh-huh.

I: And at first you said that the sequences A and C were equally likely because there ought to be roughly the same number of heads and tails.
S2: Uh-huh.

I: So what kind of, what reasoning made you answer um, D?

S2: Um, the first one I just, (S2 removes hands from mouth and gestures while they are speaking)) it just seemed, there was no legit reasoning, it just seemed. Just I don't know, it just seemed seemed like it would be more probable for them to be an equal number of heads and tails.

I: What do you mean by like an equal number?

S2: Just three heads and three tails, out of, I mean wait is it ((S2 looks at question two on the Pre-test)), oh its five. Well not a equal number, but roughly the ((S2 laughs)) same amount. I don't know I guess I can't explain it any better than that.

I: Okay. (S2 returns hands to mouth and looks at the Pre-test) So I guess, for B did that one seem like the least likely of all of them?

S2: Uh-huh.

I: And, I mean what made it seem least likely?

S2: (S2 removes hands from face and places right hand under the table) There were, just the way the heads out-numbered the tails. There seemed, it just seemed disproportionate. I don't know...

I: Okay.

S2: ...Just by looking at it.

I: Okay. And then on the last test you, (S2 puts left hand on mouth and reads question two on Post-test 2) it was the same question but you said that all of the above sequences were equally likely because every sequence of five tosses has exactly the same probability. So what sort of reasoning led you to answer that they were all equally likely?

S2: Just because each flip is independent, of the other flips. ((S2 removes left hand from mouth and gestures with it while talking)) I mean its still a fifty percent chance. But then, I guess now that I look at it its, ((S2 looks at question two on Post-test 2)) I don't know. (S2 cocks head while looking at the Post-test 2)) I don't know what I was thinking. ((S2 laughs)) Um, this isn't helping. Um, (pause .) it just seemed, like each toss would be independent from the one before.
I: Okay. (S2 places left hand on face next to their mouth) So did you, in doing the answer did you just kind of work it out in your head? Or did you do any sort of um, scratch work or anything that lead you to it?

S2: Um, no.

I: You just...

S2: (S2 takes left hand from face and gestures with it while talking) I just kind of, I, I can't explain I'm sorry. (S2 laughs)

I: Okay. Um, so the next one I wanted to look at was number four (S2 turns to question four in the Pre-test and Post-test 2 and places both hands on their mouth) yeah its on the next page (for question four). Okay, so on the first test uh, this one is involving the die now and where its asking which one is least likely. And one the first test you put that sequence five-six-two-six was the least likely because you are much more likely get a mixture of different numbers than numbers that repeat.

{ Pre-test and Post-test 2 Question #4:
If a fair die is rolled four times, which of the following sequences of results, if any, is LEAST LIKELY to occur?

a. 6 4 3 5
b. 5 6 2 6
c. 2 3 4 5
d. 2 1 4 3
e. All sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. Since rolling a die is a random event, a result like that is very unlikely.
b. You are much less likely to get a mixture of different numbers than an ordered sequence.
c. All sequences of rolls have exactly the same probability of occurring.
d. You are much less likely to get a mixture of different numbers than numbers that repeat.
e. Other ____________________________

S2: Uh-huh.

I: So, what sort of reasoning uh, did you use to pick answer B?

S2: Just that none of the other ones had um, (S2 removes their hands from their face and gestures with them while they are talking) yeah none of the other ones had any repeats of numbers and at the time I just thought that it would make more sense to have, to have completely different numbers, if there are six of them. I...(S2 laughs and places both of hands on their mouth)
I: Okay. Um, and then on the, the last test you answered that they were all equally likely because all the sequences of rolls all have the exact same probability of occurring. So what sort of reasoning did you use for that one?

S2: I, just figured they'd be independent? I don't know. Um, (pause.) I guess yeah, I didn't, I didn't do the little chart thing or anything.

I: So, did um, having the rolls be independent (in any way) influence your reasoning? Because you selected C, right? That all of them have the same probability.

S2: (S2 places both hands back on their face next to their mouth) Uh-huh.

I: So how did you decide that out of all the reasons that it was answer choice C? (S2 places both hands below the table)

S2: Uh, I, I can't remember I don't know, I just don't know what I was thinking at the time. But um, I guess I just figured (pause.) they all had exactly the same probability of occurring. I don't, I'm sorry.

I: No, its okay.

S2: (I am not helping here).

I: Alright. So, lets see, next one is number seven so its on the next thing. (S2 turns to question seven on the Pre-test and Post-test 2 and then puts both hands on their mouth) Okay. So, for the first test its um, the coin example again but this time it is dealing with least likely and there's twelve tosses. So you selected answer choice D was the least likely because since tossing the coin is random you should get, you should not get a long string of heads or tails. So what sort of reasoning lead you to answer choice D?

{ Pre-test and Post-test 2 Question #7:
If a fair coin is tossed twelve times, which of the following sequences of heads (H) and tails (T), if any, is LEAST LIKELY to occur?

a. H T H T H T H T H T H T H
b. H H T H T H H T H T H

c. T T H H T H T H T H

d. H H H H H H T T T T

e. All sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. There ought to be roughly the same number of tails as heads.

b. Since tossing a coin is random, you should not get a long string of head or tails.}
c. Every sequence of twelve tosses has exactly the same probability of occurring.
d. Since tossing a coin is random, the coin should not alternate between heads and tails.
e. Other ____________________________ }

S2: ((S2 removes hands from face and gestures with them while they talk)) *It just seemed that they were so I mean, disproportionate so that there were eight H's and only four T's that I don't know, it seemed that they should be more equal.* 

I: So I guess you were not expecting to get, so uh, let's see how (should I put this). So in, even if say you had gotten um, six heads and six tails.

S2: Uh-huh.

I: In that order, so six heads all at once and then six tails would that still be considered as something that is least likely? Or would that...

S2: ((S2 removes hands from their face)) Oh, no it was, it was just, just the numbers for me at the time. I think that it was just the numbers. *It didn't have anything to do with the order.* 

I: So if you had seen, *if there was like an answer that had the six heads and the six tails would still say that D was the least likely?* 

S2: Yeah. You mean in that order right?

I: Yeah.

S2: Yeah.

I: Okay, and um, then for answer choice seven on the last test you put they were all equally likely because they have the same probability of occurring. So, what sort of reasoning did you use for answering that question?

S2: Um, ((S2 uses hands to gesture as they speak)) *I guess again thinking each, each flip was independent of the previous, coin toss.* 

I: So there I guess on that one did the, the fact that one sequences, sequence D had more heads did that affect, how you figured out the probability? Or did it not matter anymore?

S2: ((S2 takes hands from under the table and gestures with them while they talk)) It didn't matter any more, *I didn't really look at them much just "oh their independent"* 

((S2 mimics circle answer choice on the test)) *equally likely.*
I: Okay. Um, the next one is number nine so its on the last page.((S2 turns to number nine in the Pre-test and Post-test 2)) So on the first test um,((S2 puts both hands on face, below the mouth and reads over question nine on the Pre-test)) its the fair die being rolled eight times and its for the most likely and you put that the most likely sequence was uh, answer choice B because you're more likely to get a mixture of different numbers than numbers that repeat. So what sort um, of reasoning or logic went into that question?

Pre-test and Post-test 2 Question #9:
If a fair die is rolled eight times, which of the following sequences of results, if any, is MOST LIKELY to occur?

f. 5 6 2 6 3 5 4 2
g. 2 1 4 3 1 5 4 6
h. 6 4 3 2 4 1 5 6
i. 2 3 4 5 6 1 2 3
j. All sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

f. You are much more likely to get a mixture of different numbers than an ordered sequence.
g. Since rolling a die is a random event, a result like that is very likely.
h. You are much more likely to get a mixture of different numbers than numbers that repeat.
i. All sequences of rolls have exactly the same probability of occurring.
j. Other ________________

S2: Um, just that the, I mean C did D have? ((S2 uses hands to gesture while they talk)) Yeah A, C, and D just all had (pause .) doubles. You know what I mean? Like weren't, it didn't seem likely that you would get doubles. You know what I mean? Like weren't, one, two three, four, five. Not in that order you know, but you should fill up all your numbers first and then have repeats and since there was only eight. I thought, I don't know? No B has repeats too, I don't know what I was thinking. ((S2 laughs)) (pause .) Maybe I guessed? I don't know.

I: So I guess just...

S2: Even now looking back I was just like, why did I pick that? I don't know.

I: ((Interviewer laughs softly)) Okay. So I guess on that one it just initially appeared?

S2: It just looked right. In my head at the time? I don't know. ((S2 places left hand on face next to mouth))

I: Okay. And on the last one it was that they were all equally likely um, because they all have the same probability. So was there, what sort of reasoning did you use for answering that question?
S2: Um, (pause .) ((S2 takes left hand from face and uses it to gesture while talking)) I guess I just figured that, I guess this time since they all had doubles I figured that it didn't matter. ((S2 laughs and places left hand back on face)) I don't know. 3 ((S2 puts both hands in air and then puts them under the table)) Um, yeah that's, that's it I guess.

I: Okay. So do, you feel, that maybe, ((S2 takes hands from under table)) do you feel you are more like that. Uh, do you feel that you, answer things are equally likely most of the time? 4

S2: Yeah. Especially, ((S2 laughs)) especially with the last one.

I: Okay. Um, let's see and then, I am kind of back-tracking, but the next one I wanted to ask you about was question number five its on the second page. ((S2 turns to question five on the Pre-test and Post-test 2 and reads over the question)) Okay so this one is slightly different, its the one involving um, drawing fruit from a bag.

{ Pre-test and Post-test 2 Question #5:
A bag has 9 pieces of fruit: 3 apples, 3 pears, and 3 oranges. Four pieces of fruit are picked, one at a time. Each time a piece of fruit is picked, the type of fruit is recorded, and it is then put back in the bag. If the first 3 pieces of fruit were apples, what is the fourth piece MOST LIKELY to be?

a. A pear
b. An apple
c. An orange
d. An orange or a pear are both equally likely and more likely than an apple.
e. An apple, orange, or pear are all equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. This piece of fruit is just as likely as any other.
b. The apples seem to be lucky.
c. The picks are independent, so each fruit has an equally likely chance of being picked.
d. The fourth piece of fruit won't be an apple because too many have already been picked.
e. Other ________________________________ }

S2: Uh-huh.

I: And its asking um, that "Barry drew three pieces of fruit which were apples" and they want to know if the fourth, what the fourth piece is most likely going to be. And on the first test you put, ((sound of Interviewer turning papers)) on the first test you put answer choice E that the apple, orange, or pear were all equally likely because the picks were, were independent so the fruit had an equal likely chance of being picked. So what sort of reasoning did you use uh, in answering that question?
S2: Um, (pause .) I just figured at that point since, since they were put back into the bag they were, it was totally independent. Those actually are independent, I think. Um, because there was no, it wasn't, it wasn't like a sequence, like the sequence didn't matter it was just that one mattered. So that one, those actually were independent I think. I think, I don't know. But, yeah, just because the sequence of it didn't have anything to do. Well? I don't know. Um, yeah they were all, they were an equal number of all of them and I figured the pick was equal.

I: Okay. So I know you're saying a lot of the reasoning is because its independent right?

S2: Yeah.

I: So, I guess the question I have is what criteria or what um, how do you decide if something is independent?

S2: Um, (pause .) I think. I don't know. I just kind of pick um, for five it that actually kind of made sense but for some of the other ones I don't know why I thought they were independent. Um, (pause .) I'm sorry. I don't know.

I: Okay and on that one the, the Post-test you answered the same thing so...

S2: Yeah. That one was just my own, I was in a rush or something, I don't know.

I: Okay, so I guess on that one...

S2: That one was just my own, I was in a rush or something, I don't know.

I: Okay.
S2: I'm sorry.

I: No, it's alright. Um, and then on, the last test its the same question um, but here you answered D that all of them were equally likely because there are four possible outcomes and each outcome has a probability of one-fourth.

S2: I just thought like, *I just thought to the little like tails heads thing. ((S2 motions like drawing a probability tree))*. I mean, *it can be heads, heads, heads, or head. Well there's two. It can be heads, heads or heads, tails you know? And I figured each one. Like all, there are four possible outcomes in each one has a probability of one-fourth.* You know, just, I am picturing the chart but I can't verbalize it very well. ((S2 laughs)) But, yeah.

I: And so from there, *I guess (visualizing) the chart did that help you in selecting that they were equally likely?* 4

S2: Uh-huh.

I: Okay. Um, okay. On this one I just wanted, this was your, the test that was in between the Post-test one.

S2: Uh-huh.

I: ((Interviewer hands S2 a copy of the Post-test 1 turned to question three)) And on that one I just wanted to ask you something about the same question, number three.

{ Post-test 1 Question #3:
  When three fair die are rolled, which of the following outcomes is MOST LIKELY to occur?
  
  e. All three die show an odd number
  f. All three die show an even number
  g. Two die show an odd number and one die shows an even number
  h. All the above outcomes are equally likely.

  Which of the following best describes the reason for your answer to the preceding question?
  
  f. There are four possible outcomes and each outcome has a probability of 1/4.
  g. You are much more likely to get a mixture of odds and evens than all odds or all evens.
  h. Since die rolling is a random event, every outcome has exactly the same probability of occurring.
  i. If the first two die have shown an odd number then it is more likely that the third die will show an odd number.
  j. Other ____________________________

S2: Okay. ((S2 reads question three on the Post-test 1))
I: So here, it was the same question but you said that there was um, am I on? No I am not.

It was question number eight, sorry. So its on the, third page. ((S2 turns to question eight in the Post-test 1)) Okay, so this is the, the same coin question but here you selected one head and two tails...

{ Post-test 1 Question #8:
When three coins are tossed, which of the following outcomes is MOST LIKELY to occur?

e. Three heads
f. Three tails
g. One head and two tails
h. All the above outcomes are equally likely.

Which of the following best describes the reason for your answer to the preceding question?
f. The outcome of having exactly three tails is more likely than having exactly one tail.
g. You are much more likely to get a mixture of heads and tails than all heads or all tails.
h. Since coin tossing is a random event, every outcome has exactly the same probability of occurring.
i. The outcome of having exactly three heads is more likely than having exactly two heads.
j. Other __________________________________________  }

S2: Uh-huh.

I: ...because you are much more likely to get a mixture of heads and tails than then all heads or all tails. So what sort of reasoning did you use there?

S2: I just figured they should be different. At the time ((S2 laughs)) I don't, I don't know. I just figured that it, would be more likely to get one of each than to have all one or the other. I mean to have more of a balance, I mean its not going to be balanced with three, but. 1

I: So on that one did you, like use the, the chart or any...

S2: No, I just looked at it and wrote something down. ((S2 laughs))

I: Okay. And then, let me see. ((sound of interviewer turning pages)) Okay, and then for the Pre-tests and the last one its question number eight. ((S2 turns the Pre-test and Post-test 2 to question number eight)) Which is on the third page. And then on the test I just gave you its question number three. ((S2 flips to question three on the Post-test 1 and begins reading through each of the test questions)) So we'll go through in a more sequential order here. So this one, its involving two die and its asking which one is most likely so on the very first test you put um, ((Interviewer whispers to herself)) on the very first test you put that all of the above outcomes were equally likely since rolling a die is a random event every outcome has the exactly the same probability of occurring. So I guess, what reasoning did you use there?
Pre-test and Post-test 2 Question #8:
When two fair die are rolled, which of the following outcomes is MOST LIKELY to occur?
a. Both die show an odd number
b. Both die show an even number
c. One die shows an odd number and one die shows an even number
d. All the above outcomes are equally likely.

Which of the following best describes the reason for your answer to the preceding question?
a. If the first die is an odd number then it is more likely that the second die is an even number, and vice versa.
b. Since die rolling is a random event, every outcome has exactly the same probability of occurring.
c. You are much more likely to get a mixture of odd and even numbers than both even or both odd.
d. There are three possible outcomes and each outcome has a probability of 1/3.
e. Other ________________________________

Post-test 1 Question #3:
When three fair die are rolled, which of the following outcomes is MOST LIKELY to occur?
a. All three die show an odd number
b. All three die show an even number
c. Two die show an odd number and one die shows an even number
d. All the above outcomes are equally likely.

Which of the following best describes the reason for your answer to the preceding question?
a. There are four possible outcomes and each outcome has a probability of 1/4.
b. You are much more likely to get a mixture of odds and evens than all odds or all evens.
c. Since die rolling is a random event, every outcome has exactly the same probability of occurring.
d. If the first two die have shown an odd number then it is more likely that the third die will show an odd number.
e. Other ________________________________

S2: Um, (pause.) I guess I just thought (pause.) that it would be equal for all of them? I don't. Because there's an equal number of odd and even, well I guess I thought that didn't matter. ((S2 laughs)) Um, I, I guess I just figured it was completely random so, it didn't matter what odds and evens, didn't factor in at all? I don't know.
I: Okay. And on that one did you work it out in your head? Or did you do any sort of scratch-work?
S2: Ummm...((S2 shakes head no))
I: No?
S2: Nothing.

I: Okay. And then so on the second question it was the same question but here it was number three. (S2 looks at question three on the Post-test 1) And it says um, when a fair die is rolled the most likely you put uh, that the two die would show an odd number and one die would show. Sorry, one die would show an odd number and one die would an even number and it said, and you put that you were much more likely to get a mixture of odds and evens than all odds or all evens.

S2: Uh-huh. I, I guess at the time (S2 laughs) I thought that it was just likely because there were three. I guess I then thought then that there were three of each so it made sense to have a mixture. Between odds and evens.

I: Okay. And did you just do that one mentally as well? (S2 nods yes) Okay. And let's see (pause.) and then on the very last test you I guess you answered, almost the same as before but you put that they were all equally likely because there are three possible outcomes and each outcome has a probability of one-third.

S2: Um (pause.) I guess I just, maybe there was a chart? I don't remember, know this was just today but huh, I guess I just thought. I don't know what I thought. I'm sorry I can't remember.

I: That's okay. Okay, so that basically all I wanted to do for this part. I just kind of wanted to go through and, and see you know kind of what your reasoning was for answering the questions. (S2 hands the tests to the Interviewer) Thank-you.

S2: Uh-huh.

I: Okay. So for the next part, I just going to ask you some questions and you know if I, if the question doesn't make sense or you want me to repeat it just let me know and um, feel free to take your time, as much time as you need. Okay?

S2: Uh-huh.

I: Okay. Um so first uh, what does having good reasoning ability for probability mean to you?

S2: Um, I guess, (pause.) I guess it would mean just being able to see proportions and stuff. Being able to, well no, I don't know. I guess it would just be (pause.) yeah I guess just being able to see proportions and just kind of generalize things I guess. I don't know if that makes any sense. Just to kind of (pause.) I don't know. (S2 laughs) I'm sorry...

I: (It's okay).
S2: ...Okay, um (pause .) or I guess it means to be able to (pause .) differentiate between (pause .) things that (pause .) I don't know. ((S2 laughs)) I'm sorry (it's just)... 

I: No, it's okay. You can take as much time as you need. I know these questions are kind of hard (and stuff). 

S2: I was just like, well I don't know. What do it think is good? Um, (pause .) I guess just being able to, to look at something and know maybe, I don't. I'm sorry can you repeat it? 

I: Okay. Um, so like what, what type of qualities do you think a person that has good reasoning for probability would have? 

S2: Oh! Oh, my bad. Um... 

I: There's a couple different ways you can see it. 

S2: Okay, I think they, would be able to (pause .) I think they would be able to um, kind of classify things really well? Like, somebody who could, like you know the charts but if they were like kind of already in your head if you could kind of already picture that without the charts. You know what I mean? If you could kind of figure that out just naturally, I guess that's, that's the type of person. Someone who could classify things well and figure things out quickly, just like that. I, I don't know if that makes sense, but yeah. 

I: Okay. So maybe, just so I kind of, I guess understand uh what you mean, so would it be maybe somebody um, that like understood, like the rules for probability really well and were able to work them out... 

S2: Yeah. Somebody that... 

I: ...(Did I get that right)? 

S2: ...Yeah, somebody that, (pause .) somebody that has that kind of, that reasoning just engrained in them, I guess. I don't know how to... 

I: Okay, that's good. Um, so before the probability lesson presentations that I did, how would you rate your reasoning abilities for probability? 

S2: Like zero. ((S2 and Interviewer laugh)) Um, not very, well. I mean, I was like okay in third grade but I've never, I've never had to really to deal with it since then. So I guess kind of hard to gauge because, I never really had to, use it. 

I: Okay. So would you say um, I guess if you had to rate it like would you say it was like poor or um, below average, average, above average, excellent...
S2: Probably like, below average...

I: Below average?

S2: Because yeah.

I: Okay. And so why would um, why would you say your reasoning was below average at that point?

I: Because, (pause.) I don't know. Just because I mean from looking at the Pre-test kind of, I don't know. I don't know how much of it was me actually thinking and trying to figure it out. But now just when I look back at it just doesn't click, like it doesn't make sense, you know? I don't know.

I: Okay. So um, do you feel like maybe you used like, instead of trying to work it out, maybe you used...

S2: Yeah I just looked at and thought...

I: ...your...

S2: ...Oh! ((S2 motions like circling an answer choice on the test)) That looks...

I: ...that instinct or something?

S2: ...things look equal, yeah.

I: Okay. Um (pause.) and this might be uh, a similar question but you know, what type of factors do you think might have contributed to your reasoning um, being below average at that time? I know you mentioned uh, that you hadn't seen it for a while, right?

S2: Yeah.

I: Was there anything else?

S2: Yeah, but just that I hadn't seen it for a while and that um, there was never, there was never any. Before your lessons whether like um, there was never any rules for anything. It was kind of like, since again, since like there were no rules that even though you weren't using it, you just remembered the rules of, there, there were no rules. I mean I'm sure there were but you can't, you know? Like you never, the rules weren't displayed in that, that concrete way I guess when, when I was younger and its like, to me at least. So there were no set rules to use and I hadn't seen those in so long.

I: Okay. Um, so some of this is repetitive but I have to ask it so, uh, have you been taught probability before?
S2: Uh-huh.

I: Okay and then in what classes?

S2: **Just in elementary school. I seriously don't think I have seen it since like fourth or fifth grade.**

I: Fourth or fifth grade?

S2: Yeah.

I: Okay. Um...

S2: **No I have, in middle school.** Spinney, ((S2 uses hand to gesture in a circular motion)) spinney things like on game boards...

I: Oh, okay.

S2: ...there were those. All over the place in middle school. ((Interviewer laughs softly)) I forgot about those but they were there, yes.

I: Okay. So did they do just kind of like review stuff? Or did they...

S2: **No, they just did review stuff.**

I: Okay.

S2: Maybe we had, **I think maybe in, in one day in eighth grade. I think my math teacher taught probability, I think one day.**

✉<1465231>I: Okay. Um, so right now I want you to go ahead and take a moment and just remember as much as you can about the second lesson on probability that you had with me.

S2: Okay. ((S2 pauses, looks around, and then motions towards the Interviewer)) Okay.

I: Yeah? Okay. ((S2 laughs)) Um, so now I want to also take a moment and remember as much as you can about the very first lesson.

S2: Okay. ((S2 pauses, looks around, and then nods at the Interviewer))

I: Yeah? ((S2 laughs))

S2: Yeah, its like...
I: So, how would you rate your reasoning ability for probability after the first lesson, but before the second?

S2: Um, I don't know if there's much of a change really. I mean I, I think what, what the lessons did do and from the first lesson, it showed, it kind of taught how to, how to do things one you knew what they were. I mean like how to treat, how to treat probability once you knew what they were. But they didn't really, like whether it wasn't independent or whether, I mean whether you should use your little chart or like, whether it was just. Like the, the, you remember the rolls and it would, ((S2 gestures like images popping up in the air)) on, on the screen. You know what I mean? The dice would roll and...

I: Uh-huh.

S2: ...and then you would see the, the numbers on the screen. Like, they taught like that kind of thinking and like how to, how to use, like how to figure out once you knew which type the probability problem was. But I still don't think I knew when, when to use, you know what I mean? Like when to use what type of thinking.5

I: Okay.

S2: I guess, if that makes sense.

I: And by what type of thinking do you mean like um, like the experimental probability verses theoretical? Or...

S2: Um, just what type of thinking is in, just how to, how to think of the problem. I mean, do I think of (pause .) the (pause .) the thing as independent or as. I mean once, once you know its independent, you know all of the outcomes are equally likely and then once you. I mean if you, if its a sequence then you should do like with the dice, the one out of thirty-six. You know what I mean? Like I know how to use that but I didn't know when to use, when to use which one,2 you know?

I: Okay. So after that first lesson would you say, was it still? You said there wasn't much change but would you still say that it was below average? Or...

S2: I, I guess so.

I: Yeah? And you know what type of factors, or what factors so you think uh, contributed to your reasoning staying the same?

S2: Just that, that I still didn't know when to differentiate like, I, I didn't know how to pick which method to use, I didn't know when it was independent and when it wasn't.2
I: Okay. Um, so now how would you rate your reasoning abilities after the second lesson?

S2: Um they're still probably about the same ((S2 laughs and Interviewer laughs softly)) because I still, you know? I mean it helped in that I never thought of anything like that as, as the one out of thirty-six and there are thirty-six possibilities and. Or maybe I had in, again in third grade or fifth grade or whenever. But um, so *I know how to use that type of thinking but I still don't know when something is independent and when it is not.* I mean I, maybe I do and I just, didn't on the test because I was kind of huh ((S2 gestures like writing on the test)) but yeah.

I: Okay. So you'd say that...

S2: I mean maybe, *I think it has improved a little bit, I think maybe average now* because. 

I: Okay.

S2: (I liked them).

I: ((Loud noises of scraping and doors closing come into the room from the copy room next door)) Um, so now I am gonna, just go ahead and I guess compare um, the first lesson you had with me and the second one. And just go ahead and think about the um, the comparison between the two.

S2: You mean?

I: ((Noises from copy room continue)) Or just take your time to think about it I guess.

S2: Okay.

I: And just let me know when you are ready.

S2: Okay. ((S2 laughs))

I: Okay. So can you name a few differences between the two lessons? ((Noises from copy room stop))

S2: Um, the first one everything was kind of (pause .) I, I think in the first one I'm, maybe I am getting them confused. But in the first one, *the first one was where all the dice, the dice were visually rolling* ((S2 gestures dice rolling and images popping up in the air)) and you could see everything on the screen. *The second one was just kind of like recalling that but without actually displaying it.* I think, was it, is that true? Or...
S2: ...like did we do it again, I can't remember.

I: For your class, the first one we did do the, the little simulation from the computer program and I think we started talking about some of the, it was kind of like a review of concepts and things like that and we worked out, like maybe a couple of problems. On the second lesson, that's when you guys were really like doing more of the group work stuff.

S2: Uh-huh.

I: And I like posted five different questions and we like talked them all through...

S2: Yeah and the group had to agree and give a communal, answer...

I: Uh-huh.

S2: ...communal answer ((S2 gestures to make quotation marks and S2 and the Interviewer laugh)) yeah, the group answer.

I: Yeah.

S2: Yeah.

I: Okay, so I mean, was there, I guess now that we kind of, I guess we both remember better. ((Interviewer laughs))

S2: Yeah, sorry.

I: No, its okay. So I mean what other, you know what sort of differences were there for the two lessons for you?

S2: The second one was a lot more interactive,  and the first one was just more of a review. I think, or a basic stating like overall this is what we are doing, and just an introduction.

I: Okay. Um, (pause .) was there any other differences maybe in like uh, what I expected from you or how I presented it, was there any differences?

S2: Uh, you just expected us to participate more and I guess that's it.

I: Okay. Um, so do you think your reasoning for probability has increased or remained the same because of the two probability lessons?

S2: Um I, I think its increased a little bit.
I: Okay. Um, so in your opinion which lesson made a greater contribution to an increase in your reasoning?

S2: Um, I think at the time, like during the second lesson, I could understand things really well and while it was in front of me I was fine with it. So I think during the second lesson, that one made the greatest impact like during the lesson. But the first, I like, I still remember things from the first one kind of the basic things stuck with me better. But at that time, the second one made a more drastic improvement, I think.

I: So overall, would it be the first or the second?

S2: Um, I guess the first.

I: Okay. So um, you know what, I guess why was the first lesson the one that made the most impact?

S2: I remember it better, I think. Not, obviously not that well ((S2 laughs)) but I remember it a little better.

I: And was there anything in particular about that lesson that, I guess um, helped make it more memorable? Or...

S2: No, I don't know. I, that was probably on my part, I was just probably more there, ((S2 uses hands to gesture from head forward into the air)) I guess. Even though the second one was more interactive, I could, I can't, I can't even remember the, the problems or the things we did and with the other one I can remember a little bit better.

I: Okay. And so what was the most important thing that you learned from the probability presentations?

S: Hum (pause.) I guess the, I guess the heads and tails type charts or the one out of thirty-six type thing. You know the, you know the tree thing...

I: Uh-huh.

S2: ...that thing. That helps visualize, I guess.

I: Okay. And what is one thing that you still feel confused about regarding probability?

S2: That I can't tell, like sometimes I can and sometimes I can't tell the difference between when its independent. I mean, now looking back I am am like oh well this should have been. But I guess during the test I just was like, I, I didn't know what to
I: Okay. Um, well that about all of the questions that I have for you.

S2: Okay.

I: I mean was there anything else that you wanted to add?

S2: No.

I: Or any questions?

S2: No.

I: No? Okay, well do you think the lessons were helpful, hopefully? ((Interviewer laughs softly))

S2: I think they were, yeah. ((S2 smiles and laughs))

I: Okay. Um, okay. I'll shut everything down.

((Video ends))

**Prediction-and-Voting Class Student Interview:**

Total Time for Interview: 30 minutes

NOTE: This interview occurred after school in a classroom of one of the participating teacher's classrooms approximately one week after the student being interviewed took the Post-test 2.

((Interviewer is sitting on camera to the left, but the desk in front of the interviewer is in the camera's left bottom corner and shows the test papers and notes the interviewer has. S3, a student from the Prediction-and-Voting Class, is sitting opposite the interviewer in the right side of the camera's view. The student and the blank desk in front of the student are visible to the camera. A tape recorder is placed between the interviewer and student S3.))

I: Okay. So kind of like what I said before, so um, the first part that we are going to do is I am going to show you your test and then kind of ask you to explain your reasoning or what you were thinking when you answered. Um...

S3: Okay.
I: ...So, let's see. (Interviewer hands S3 copies of their Pre-test and Post-test 2)) Okay, so that's a copy of the first test you took, the Pre-test and then this is a copy of the very last test that you took last week?

S3: Okay. ((S3 begins looking over both tests))

I: And essentially there were exactly the same test, ((Interviewer laughs softly)) the exact same questions I just um, gave you guys one at the beginning and one at the end. Um, so the first one I kind of wanted to look at with you was question number two, for both of them.

{ Pre-test and Post-test 2 Question #2:
If a fair coin is tossed five times, which of the following sequence of heads (H) and tails (T), if any, is MOST LIKELY to occur?

a. H T H T T  
b. T H H H H  
c. H T H T H  
d. Sequences (a) and (c) are equally likely.  
e. All of the above sequences are equally likely. 
}

Which of the following best describes the reason for your answer to the preceding question?

a. Every sequence of five tosses has exactly the same probability of occurring.  
b. Since tossing a coin is random, the coin should not alternate between heads and tails.  
c. Since tossing a coin is random, you should not get a long string of head or tails.  
d. There ought to be roughly the same number of tails as heads.  
e. Other ________________________________  }

S3: Uh-huh. ((S3 turns their attention to looking over question number two on the Pre-test and Post-test 2))

I: And that was the problem involving um the coin were you flip it five times and figuring out which sequence was most likely. And on the first test you answered that sequence B was the most likely because every sequence of five tosses has exactly the same probability of occurring.

S3: Uh-huh.

I: So, what sort of uh, reasoning or thinking were you doing when you answered that question?

S3: Um, with the, there's a fifty, uh yeah, fifty percent. Because if you have a fifty-fifty chance of each roll doesn't that mean has, has an equal chance of landing? I think I probably did the tree for that one.

I: Okay.
S3: Because that's also fifty-fifty.

I: Yeah so this test was before I gave you guys like the, it was after the first lesson but before the second one. So it might be hard to think back, that far...

S3: Yeah.

I: ...But, I mean if you can kind of, I know its hard, but if you can try to think maybe what you were, what sort of reasoning or whatever you used back then. Because I know probably by now, you know you might, your impressions might be different, but...

S3: Uh-huh.

I: ...So did anything kind of strike you as uh, as to why sequence B might be more likely than say A or C?

S3: I am not quite sure. (pause .) I am not very sure how I picked B on this one.

I: Okay, that's fine it was a long time ago. ((Interviewer laughs softly))

S3: Yeah.

I: And then on the very last test, for the same question.

S3: Uh-huh.

I: You had put that all of the above sequences were equally likely and you picked the same reason that every sequence of five tosses has exactly the same probability of occurring. So what sort of reasoning did you use there?

S3: Well like um, on where it says A right here ((S3 points to reason A for question two on Post-test 2)) where each of them has the same probability then if, uh, there what, same possibility of it landing each roll so it would probably, it would be all equal. It be what is it? Two, four, six, eight, ten. Right? Uh, for a one tenth probability of it landing like that. 4

I: Okay. So on that question did you kind of work it out in your head like you did right now or did you um...

S3: Yeah kind of like what I did right now. Each one of them has a fifty percent chance so it should all be equally likely. 4

I: Okay. Okay, so the next question I want to look at was number four. Its on the second page ((S3 and Interviewer turn to question four on the Pre-test and Post-test...)}
2) So on the very first test um, \((\text{noise of people outside the classroom becomes audible})\) its the question involving the die and this time its least likely and you had picked uh, the answer choice B was least likely because you are much more likely to get a mixture uh of different numbers than numbers that repeat. So what sort of uh, reasoning lead you to that answer?

{ Pre-test and Post-test 2 Question #4: 
If a fair die is rolled four times, which of the following sequences of results, if any, is LEAST LIKELY to occur? 

a. 6 4 3 5  
b. 5 6 2 6  
c. 2 3 4 5  
d. 2 1 4 3  
e. All sequences are equally likely. 

Which of the following best describes the reason for your answer to the preceding question? 

a. Since rolling a die is a random event, a result like that is very unlikely.  
b. You are much less likely to get a mixture of different numbers than an ordered sequence.  
c. All sequences of rolls have exactly the same probability of occurring.  
d. You are much less likely to get a mixture of different numbers than numbers that repeat.  
e. Other \[ \text{______________________________} \] 

S3: \((S3 \text{ pauses to look over question four on the Pre-test})\) Let me see if I can remember that one. (pause .) Okay, well because I saw that it was six-six so it might be that popped into my head that its really slim for it to be uh, for the number to come up twice within the, set of limit you have. 

I: Okay. So in terms of the other sequences um, I guess what excluded them from being least likely? 

S3: Well I just saw, for some weird reason six always seemed like a larger number uh, by room I guess you could say. Three seemed well, if you look at the paper three looks kind of small so what popped up right away was the six. 

I: Okay. So if, say for example sequence one has a six in it as well, so would choice A be you know, more likely or least likely than say C or D, just because it has that six? 

S3: Well what I meant by six is you know each of them, size is different right?... 

I: Uh-huh. 

S3: \(\text{...one has a slim, two is at a larger, six has actually a pretty larger number so I saw two of it. Two six, so I automatically popped uh, popped into my head.} \)
I: Okay. So is it just the number six itself or that is was twice...

S3: Well it was just the number six. Well, the six popped up twice.

I: Oh okay. ((S3’s cell phone rings and the student takes a moment to answer the phone call. S3’s dialogue while on the phone has not been transcribed in this interview. The following is what occurred in the interview after S3 was off the phone.)) So if for that, if everything else was the same but say instead of two sixes they had um, two ones or something would you still consider it as being least likely?

S3: That probably wouldn’t have popped into my head. The six just was right there and I saw (like)...

I: Okay. So the, there you’re thinking the larger the number the least likely its going to be to be rolled right? 

S3: Sort of yeah.

I: Okay. ((S3’s phone rings)) Um, go ahead that’s alright.

S3: ((S3 turns off phone)) That was just a text.

I: Okay.

S3: Cool. ((S3 puts phone away)) Yeah.

I: Okay. So then on the last test, its the same question but here you answered answer choice E that all sequences were equally likely ((S3 coughs softly)) because all sequence of rolls has the exact same probability of occurring. So what sort of reasoning did you use there?

S3: Same as the coin toss except its one-sixth. So each one of them has a same amount.

I: Okay.

S3: Its one-sixth for the first one, one-sixth for the second one, one-sixth for the third, and so on.

I: So in terms of like, and on the first one you said that, in terms of the value for the six that’s what you considered to be least likely. Did that reasoning or any of that change? Do you think, by the time you took the last test?
S3: Um, the time we had, a little bit because then I, I guess I was kind of rushing this one but right here when I was taking it the second time I saw that each number had the same chance of popping up.

I: So there you didn't, did you see a larger number as being, as having like a least likely chance of occurring on this (one)?

S3: No, it just had the same possibility.

¤<422992>I: Okay. Uh so the next one is number five so its right below.

{ Pre-test and Post-test 2 Question #5:
A bag has 9 pieces of fruit: 3 apples, 3 pears, and 3 oranges. Four pieces of fruit are picked, one at a time. Each time a piece of fruit is picked, the type of fruit is recorded, and it is then put back in the bag. If the first 3 pieces of fruit were apples, what is the fourth piece MOST LIKELY to be?

a. A pear
b. An apple
c. An orange
d. An orange or a pear are both equally likely and more likely than an apple.
e. An apple, orange, or pear are all equally likely.

Which of the following best describes the reason for your answer to the preceding question?
a. This piece of fruit is just as likely as any other.
b. The apples seem to be lucky.
c. The picks are independent, so each fruit has an equally likely chance of being picked.
d. The fourth piece of fruit won't be an apple because too many have already been picked.
e. Other ________________________________ }

S3: Uh-huh. ((S3 looks at question five on the Pre-test))

I: So on the first test, its the one involving the different types of fruit the three apples, pears, and oranges. And its asking um, if you have three pieces, if you had uh drawn three apples and it wanted to know if the fourth piece of fruit uh, would most likely be a pear, apple, or orange. And you put that an orange or a pear are both equally likely and more likely than an apple and then your reasoning was the apple would not be uh picked due to the fact that all of them are out, an orange and a pear have the same likelihood. So what sort of reasoning did you use?

S3: Uh, well on this one I thought that since, what was it, the pear was picked three times and there was only three pears uh, yeah I didn't read it right. I thought that they were left out instead of putting them back in.

I: Oh, okay. So when you were reading it you were I guess picturing that fruit being drawn out three times and then...
I: Okay. And then on the very last test ("S3 looks at question five on the Post-test 2") it's the same question but here you put that an apple, orange, or pear are equally likely because each piece of fruit is as likely as any other.

S3: Yes. And right here I realized that the fruit was being put back in so it had the same possibility (of being drawn out).

I: Okay. So do you think if, if you had read the question through the first time...

S3: If I had read it all and actually read it, I would have probably would have come up with the same thing.

I: The same answer?

S3: Yes ma'am.

I: Okay. Uh so the next one is number seven its on the next side. ("S3 and the interviewer turn to question seven on the Pre-test and Post-test 2")

{ Pre-test and Post-test 2 Question #7:
If a fair coin is tossed twelve times, which of the following sequences of heads (H) and tails (T), if any, is LEAST LIKELY to occur?

a. H T H T H T H T H T H T
b. H H T H T H H H T T H H
c. T T H T H T T H H T H
d. H H H H H H H T T T T
e. All sequences are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. There ought to be roughly the same number of tails as heads.
b. Since tossing a coin is random, you should not get a long string of head or tails.
c. Every sequence of twelve tosses has exactly the same probability of occurring.
d. Since tossing a coin is random, the coin should not alternate between heads and tails.
e. Other ___________________________
}

S3: Seven. Aw (I used a lot didn't I) ("Interviewer laughs softly")

I: And on that question its a coin one again but this time its a longer sequence, its twelve flips and its asking which one is least likely. And you put sequence D was the least likely because every sequence of twelve tosses has exactly the same probability of occurring.

So what sort of reasoning did you use there?
S3: Right there automatically I caught heads, heads, heads, heads, heads, tail, tail, tail. Automatically that came up to me and I selected that one on that. 

I: Okay, and what kind of, what was it that stood out to you for that sequence?

S3: That is was all in an order instead of being mashed up, I guess.

I: Okay, just because that one had like I guess a long sequence of heads and then tails...

S3: Yes ma’am.

I: ...It seemed, wrong? ((Interviewer laughs softly))

S3: Yeah kind of.

I: Okay. And then on the same question for the last test you put that they were all equally likely because each sequence of twelve tosses has exactly the same probability of occurring. So what sort of uh, reasoning happened there?

S3: Right here at first I looked at the, the first answer and then the second answer and I realized that it was still the same chance fifty, fifty each section so then I looked at C and D. Uh, B and C?

I: Uh-huh.

S3: And I realized it was still going to be fifty-fifty so it all has to be equally likely.

I: Okay. So on this one when you worked it through did you kind of do it in your head um, working it out or did you...

S3: Well what came up right away is coin fifty uh, half of uh, fifty percent heads fifty percent tail and realized no matter how many times you toss it, its still going to be the same chance, fifty-fifty.

I: Okay. So there did um, having like a long sting of heads or tails like change how you viewed the probability?

S3: Uh, no its should still be the same.

I: Okay. Uh, so for the last part um, I kind of back tracked a little bit but its number three which was on the second page. ((S3 and Interviewer turn to question three on the Pre-test and Post-test 2))

{ Pre-test and Post-test 2 Question #3:
When three coins are tossed, which of the following outcomes is LEAST LIKELY to occur?

a. Three heads
b. Two heads and one tail
c. One head and two tails
d. All the above outcomes are equally likely.

Which of the following best describes the reason for your answer to the preceding question?
a. The outcome of having exactly three heads is less likely than having exactly two heads, which is less likely than having exactly one head.
b. Since coin tossing is a random event, every outcome has exactly the same probability of occurring.
c. You are much less likely to get a mixture of heads and tails than all heads or all tails.
d. There are four possible outcomes and each outcome has a probability of 1/4.
e. Other ________________________________
question three on the Post-test 1)) So there um, its the same, oh no wait its not number three its number eight, sorry.

S3: Okay, I was like that's dice. ((Interviewer laughs softly and S3 turns to question eight on the Post-test 1))

{ Post-test 1 Question #8:
  When three coins are tossed, which of the following outcomes is MOST LIKELY to occur?
  a. Three heads
  b. Three tails
  c. One head and two tails
  d. All the above outcomes are equally likely.

Which of the following best describes the reason for your answer to the preceding question?
  a. The outcome of having exactly three tails is more likely than having exactly one tail.
  b. You are much more likely to get a mixture of heads and tails than all heads or all tails.
  c. Since coin tossing is a random event, every outcome has exactly the same probability of occurring.
  d. The outcome of having exactly three heads is more likely than having exactly two heads.
  e. Other ________________________________ }

I: On this one ((Interviewer holds up the paper for Post-test 1)) it was just in a different sequence.

S3: Okay.

I: Yeah, so there um, instead of answering A three heads you put C uh, that it would be one head and two tails and you put that you are much more likely to get a mixture of heads and tails then all heads or all tails. So what sort of reasoning did you use uh, for that one?

S3: This one, ((S3 pauses to look over the question)) this one I actually kind of looked through.

I: Uh-huh.

S3: Uh, ((S3 pauses to read through the question eight on the Post-test 1)) I guess I just little, looked at this a little harder.

I: Uh-huh.

S3: And I thought of that one instead of actually going of what I think. This one I kind of thought of a little bit more.

I: Okay. On this one I'm sorry I forgot to mention but...
S3: Yes.

I: ...So on, on the first test and the last test the question was asking for least likely, right?

S3: Uh-huh.

I: But on this middle one, its the exact same question but here its asking most likely. (((S3 says "most likely" at the same time as the Interviewer)) So I don't know, how did that change your answer, when you looked at that problem?

S3: (((S3 looks at question eight on the Post-test 1)) I didn't realize that.

I: Yeah I, I (forgot and) just saw it right now too, so. ((Interviewer laughs softly))

S3: What was your question again?

I: So on this middle test when you answered the same question but here we were looking at the most likely...

S3: Uh-huh.

I: ...Outcome. Um, so what sort of reasoning do you think lead to selecting that you got the one head and two tails?

S3: Okay. Well since I guess I saw that um, I guess I thought of the exact same thing on the previous test. Just by uh, um, occurrence that I flipped a coin.

I: Okay. So when you are flipping a coin three times what sort of expectations I guess were you looking for?

S3: Usually when you flip a coin three times its, you always get a mixture than, rather than a solid uh, sequence.

I: Okay. So on that one it was just kind of based off of real world impressions not really working out...

S3: Yes ma’am.

I: Okay. Alright and then the last one, so on the, this is the last question on the, its number eight (((S3 places Post-test 1 to the side)) its on the third page.

S3: Okay. (((S3 and Interviewer turn to question eight on the Pre-test and Post-test 2))

{ Pre-test and Post-test 2 Question #8:
When two fair die are rolled, which of the following outcomes is MOST LIKELY to occur?

a. Both die show an odd number
b. Both die show an even number
c. One die shows an odd number and one die shows an even number
d. All the above outcomes are equally likely.

Which of the following best describes the reason for your answer to the preceding question?

a. If the first die is an odd number then it is more likely that the second die is an even number, and vice versa.
b. Since die rolling is a random event, every outcome has exactly the same probability of occurring.
c. You are much more likely to get a mixture of odd and even numbers than both even or both odd.
d. There are three possible outcomes and each outcome has a probability of 1/3.
e. Other ________________________________

I: Okay. So on the very first test this one's asking uh, this time its with die and its asking which of the outcomes is most likely and you have a choice of getting um, all odds, all evens, or a mixture odd uh, odd number on one die and even on another die. And you answered that all of the outcomes were equally likely since rolling a die is a random event, every outcome has the exact same probability of occurring. So what sort of reasoning did you use in answering that?

S3: Uh, rolling the die is uh, it doesn't matter um, what kind of dice it is it still has the same chance of it being one of six either one through six.

I: Okay. So on that one did, I guess having an even or odd number effect how you saw it being...

S3: No I saw it as one-sixth of a chance of getting um, that number.

I: Okay. And was that kind of just your impression or did you do anything, like any scratch work or mental work that lead you to that answer?

S3: No I just thought of a die, a die has six sides there's one chance of it being on one side, one-sixth of a chance.

I: Okay. And I think that's about it the other questions for this you answered the same way pretty much.

S3: Yeah.

I: So um, there's not much more I can ask you about that. (Interviewer laughs softly and the Interviewer and S3 begin to clear the tests from the table. S3 hands the interviewer the copies of the tests.) Thank you.
S3: Uh-huh.

I: So this next part I am just going to ask you kind of some um, general questions about the lessons and maybe what you think about probability.

S3: Uh-huh.

I: And if the question is unclear or you need to take some time to answer it just feel free.

S3: Okay.

I: Um so the first question I have, is uh, what does having good reasoning ability for probability mean to you?

S3: Uh, good reasoning?

I: Uh-huh.

S3: Being able to see just from one point you have to actually look and see if there is another chance, how looking at it, see if you still get the same. You reason more with this, with just you think but actually you can work out the problem with.

I: Okay...

S3: I guess.

I: ...So I guess just so I understand what you’re saying. So does having good reasoning ability mean um, that like a person can look at what they think is um, is the answer in terms of probability and then also look at the, the work and the theory?

S3: Uh-huh.

I: Is that right?

S3: Yes ma’am.

I: Okay. So the next question is um, before the probability lesson presentations how would you rate your reasoning abilities for probability?

S3: Um, \textit{not that high}.

I: Not that high.

S3: Just, general knowledge.
I: Okay. So I guess um, ((Noise outside the classroom becomes audible)) if you had to rate it overall would you say it was like um, you know poor, below average, average, above average, excellent what sort of...

S3: Maybe a little bit below average.

I: A little bit below average? Okay. Um now why, why do you say your reasoning ability um, was below average at that point?

S3: At that point I only had one view of how probability was. I didn't have, there, after your, the lesson I learned that there was the tree and then you can actually mentally think of it out.

I: Okay. So what was uh, you said you had one view, so would, how would you classify your, your view of probability at that time? ((Noise outside the classroom continues))

S3: Basically I, your just experience. Probability of this happening would just effect my, would, at the experience that effect my... Answers.

I: So you kind of based all of the questions on what you had experienced or what you had expect to experience, is that right?

S3: Yes ma'am.

I: Okay. And um, what factors do you think contributed to your reasoning for probability being below average at that time?

S3: I only had the experience I didn't actually study probability. I didn't, just generally knew what probability was, it was just a chance of what you can get.

I: Okay. So you didn't feel like you had a lot of instruction or background in that yet?

S3: Yes ma’am.

I: Okay. Um, so I guess here is the next question. Have you been taught probability before?

S3: No ma’am, just with you... Just with me.
I: Okay. Um, and I guess did you go to school here? Um, like through elementary school and middle school or where did you...

S3: Uh for elementary I was at (school name has been censored to protect the privacy of the student) for middle I was at (school name has been censored to protect the privacy of the student) and then high school (school name has been censored to protect the privacy of the student)).

I: Okay. I was just wondering because it depends upon maybe sometimes where you go to school. Because probability might be taught at elementary or maybe middle school but I don't know. If some, some people actually go to like um, live across the border and then come here for high school so that might affect it. But so you went to school here and that's fine. Um, so go ahead and take a moment and remember as much as you can about the second lesson on probability that you had with me.

S3: Okay.

I: Alright. Um, and then now I want you to take another moment and just remember as much as you can about the first lesson.

S3: Ooh, that's a little bit harder. ((Interviewer laughs softly))

I: Yeah it was a long time ago. And if you want to ask any questions to help remember feel free.

S3: When was the first lesson?

I: Um, (Interviewer laughs softly) I think it was in February but I first came by and I gave you guys my whole spiel about the forms you need to sign and everything like that. And then the first lesson um, I kind of did the PowerPoint presentation.

S3: Uh-huh.

I: And it was kind of like a general review of probability and the different concepts that come up. And then um, you guys took the first test the next week. And then the week after that I came back and another lesson the last, the second lesson.

S3: That one was with the little clicker things, right?

I: Uh-huh. Yeah, exactly.

S3: Okay. I think I somewhat remember now.
I: Yeah? Okay. So how would you rate your reasoning ability for probability after the first lesson but before the second lesson?

S3: *I understood it a little bit more, it was still a little fuzzy but I knew somewhat how to organize the probability outcomes.*

I: Okay. So would you say it terms of your rating what, did it go like, do you think it remained below average or do you think it went up?

S3: Probably, *around average maybe.*

I: Okay. So what factors do you think contributed um, to your reasoning going from below average to average?

S3: *The fact that I actually, can put it in order what the probabilities would be, instead of just experience.*

I: Okay. So you felt like you could use maybe some of the stuff that you learned?

S3: Yes ma’am.

I: Okay. ((Sound of people screaming and talking outside the classroom becomes audible)) And how would you rate your reasoning abilities for probability after everything, after the second lesson?

S3: Uh, *I have a little bit more insights into it, I can mentally think of it and if I have a hard time with that I can draw up the tree and figure out what the possibilities would be from there.*

I: Okay. So you know after the first lesson you said it was about, your reasoning was about average. So do you think it stayed about average or did it increase or, I would hope that it didn't decrease but you never know. ((S3 laughs softly))

S3: Uh, *it increased a little more because by the second time I got more into since I figured out a little bit about it.*

I: Uh-huh.

S3: Um, *how to actually do the tree correctly it can get a little confusing after a while, after it reaches a certain number.*

I: Okay. So I guess that helped, *that second lesson helped cement I guess some of the, the ideas that you were using?*
I: Okay. And so now I want you to go ahead and compare the first lesson you had with the second lesson.

S3: What do you mean by compare?

I: Um, just whatever, struck you as being different between the two. Maybe how I presented it, what you felt like I expected from you, what I was teaching you, you know whatever. If there was, I mean I am going to ask you to name a few uh, differences anyway so if there's anything you felt was you know, different what was it. If there was anything you felt was the same um, what was it.

S3: Well one thing that was the same was that you reinforced what you taught from the first one.

I: Uh-huh.

S3: You uh, we actually able to go through it a little bit more than the first time. Because the first time we had to kind of rush it through. What was different was um, for me the second time when we were working in partners I was able to get insight from everyone else, how they did it so I had a little bit of insight how others do it so I can get, I guess a little bit of knowledge.

I: Okay. Um, was there anything else that um, stood out as being you know different than the first lesson?

S3: Was it more fun. ((Interviewer laughs softly))

I: More fun. Well its up to you, if it was, it was.

S3: Well yeah it was a little bit more fun because we were actually able to interact with what your lessons were. Figure out, and then after we answered you uh, I guess questioned, well not questioned but try getting how they reasoned out for that one was.

I: Okay. Um, so do you think your reasoning for probability overall has increased or remained the same because of the two probability lessons?

S3: Its increased a little bit, now that I have an insight of how probability works.

I: Okay. Um so in your opinion which lesson made a greater contribution to an increase in your reasoning?

S3: The second one.
I: The second lesson, and why? ((Interviewer laughs softly))

S3: Just because we were able to, you were able, people were able to say how they got it and you were able to explain I guess, you can write down, you were write down on the board how did this and how that was incorrect.  

I: Okay. So I, I guess kind of seeing what everybody thought and maybe know what answers were wrong and why.  

S3: Uh-huh.

I: Okay. Um, was there anything else that you think might have um, might have made the second less uh, might have made the second lesson have a greater impact?  

S3: Uh, (pause .) that's that main thing that actually helped.  

I: Okay. Um, so what is the most important thing um, that you learned from the probability lesson presentations?  

S3: That probability is really confusing. ((S3 and the Interviewer laugh softly)) Um, from the second lesson?  

I: Just overall.  

S3: Just overall, um, probability can change depending on uh, how many times you're gonna, how many items you are going to use uh, how long you are going to use it. Uh, probability has so many variables.  

I: Okay. ((Noise outside the classroom becomes audible)) And um what is one thing that you still feel confused about regarding probability?  

S3: Um, (pause .) I'm not quite sure. One thing that I am still confused about. ((Noise outside classroom continues and becomes louder))  

I: Or if there is nothing, that's okay too. ((Interviewer laughs softly)) That's fine.  

S3: Uh, yeah let's just skip that one for now.  

I: Okay. So you mentioned before um, that one of the elements that you noticed in the lesson was the clickers, right?  

S3: Yes ma’am.
I: Okay. ((A janitor enters the classroom and the Interviewer tells the janitor that it is okay to come in and get the take the classroom's trash. This discussion is not included in the transcription.)) Um, so do you believe that using um, the clickers affected how you learned during the lesson?

S3: A little bit, ((Noise outside classroom continues)) because I was able to put my answer up there and you were able to show whether it was wrong or right. 6

I: Okay. And in what ways do you think um, the clickers affected how you learned? ((Noise outside the classroom stops))

S3: It was more interactive, you were actually able to get into the lesson. 6

I: Okay. Um, so besides being interactive was there anything else that you felt might have affected um, or is there anything else that you think the clickers maybe helped or contributed to the lesson overall?

S3: Um, nothing else just that you were actually able to...

I: Okay.

S3: ... Continue with the lesson.

I: So do you think um, clickers were helpful, or unhelpful, or neither really helpful or unhelpful in assisting in your learning of probability?

S3: They were really helpful. You were able to see what people were thinking, why they got that answer, and how many people were actually on the same track as you. 6

I: Okay. So like when you were voting using the clickers did um, like me showing the distribution and seeing the answers, who answered what. Like how did that um, I guess affect your learning?

S3: Um, well when you gave a first answer you kind of think about it, Maybe it could have been THIS one, maybe it could have been THAT one, and usually someone would come up with that answer. So well you'll give out your answer, you can actually see what your second guess was. 6

I: Okay. So did uh, seeing how other people answered the question, did that make you more open to different possible choices or reasoning?

S3: Uh, yes ma’am.

I: Okay. Um, is there anything that you felt that the clickers might have not helped with. Is there anything that seemed unhelpful with them?
S3: I guess you can say just kind of the nerves and if you get like one, you're just that one vote, there just going up there and explaining why you did it.

I: Okay. Did you feel that, I mean because on, when it, when it puts it up it doesn't really show anybody's name it just shows the number that answered each question right?

S3: Right.

I: So did having it be anonymous help that, with that?

S3: Yes ma'am, because when you, who voted for what so if you didn't want to go up for that one you didn't have to but you knew that it was incorrect.

I: Okay. And um, so do you feel like the voting did it like, was it something that your group talked about or did it help bring any discussion?

S3: What do you mean by?

I: Um, because when we presented the lesson I don't know if you remember or not uh, but I think like I put a question up and then you guys thought about it independently and voted.

S3: Uh-huh.

I: And then you guys talked about it in like a group or with partners, right?

S3: Right.

I: So did, how, did voting and having that up, did that help when you were discussing it with other people in your group? Or was it just kind of like, just something extra?

S3: It was just something extra that you, you said what you thought, they said what you thought, and then you put it together and figured out what it was.

I: Okay.

S3: It was just, I guess you could saw an extra tool, just there.

I: Okay. Um, so that's basically all that I had. Um, is there anything else that you wanted to add or to comment on?

S3: Um, that'll be all.
I: No?

S3: No.

I: Okay.

((Video Ends))
Appendix N: Mann-Whitney Test Results for Equality of Median Survey Ratings

<table>
<thead>
<tr>
<th></th>
<th>Prediction-and-Voting Group</th>
<th>Prediction-Only Group</th>
<th>Comparison Group</th>
<th>Mann-Whitney Test Result</th>
</tr>
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<tbody>
<tr>
<td><strong>Factor 1:</strong> Survey Items 1, 2, 3, 6, 7, and 8</td>
<td>Median p &amp; v = 23 N = 25</td>
<td>Median p only = 21.5 N = 22</td>
<td>Median c = 22 N = 20</td>
<td>p = 0.382 Fail to Reject H₀: Median p &amp; v = Median p only = Median c</td>
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<td>Median p &amp; v = 23 N = 25</td>
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<td>Median p only = 21.5 N = 22</td>
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<td>p = 0.450</td>
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<td><strong>Factor 2:</strong> Survey Items 4 and 5</td>
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<td>Median p only = 9.5 N = 22</td>
<td></td>
<td></td>
<td>p = 0.900</td>
</tr>
<tr>
<td></td>
<td>Median c = 9 N = 20</td>
<td></td>
<td></td>
<td>p = 0.706</td>
</tr>
</tbody>
</table>

N: sample size (number of students whose survey data was used for the Mann-Whitney Test)

p: p-value
Appendix O: Two Sample T-Test Results for Survey Ratings

<table>
<thead>
<tr>
<th>Factor 1: Survey Items 1, 2, 3, 6, 7, and 8</th>
<th>Prediction-and-Voting Group</th>
<th>Prediction-Only Group</th>
<th>Comparison Group</th>
<th>T-Test Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
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<tr>
<td>$s_{p &amp; v} = 4.42$</td>
<td>$s_{p &amp; v} = 4.42$</td>
<td>$s_{p &amp; v} = 4.42$</td>
<td>$s_{p &amp; v} = 4.42$</td>
<td>$s_{p &amp; v} = 4.42$</td>
</tr>
<tr>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
</tr>
<tr>
<td>$s_{p &amp; v} = 4.42$</td>
<td>$s_{p &amp; v} = 4.42$</td>
<td>$s_{p &amp; v} = 4.42$</td>
<td>$s_{p &amp; v} = 4.42$</td>
<td>$s_{p &amp; v} = 4.42$</td>
</tr>
<tr>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
<td>$\bar{X}_{p &amp; v} = 22.08$</td>
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<tr>
<td>$s_{p &amp; v} = 4.42$</td>
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<td>$s_{p &amp; v} = 4.42$</td>
<td>$s_{p &amp; v} = 4.42$</td>
<td>$s_{p &amp; v} = 4.42$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Factor 2: Survey Items 4 and 5</th>
<th>Prediction-and-Voting Group</th>
<th>Prediction-Only Group</th>
<th>Comparison Group</th>
<th>T-Test Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{X}_{p &amp; v} = 8.84$</td>
<td>$\bar{X}_{p &amp; v} = 8.84$</td>
<td>$\bar{X}_{p &amp; v} = 8.84$</td>
<td>$\bar{X}_{p &amp; v} = 8.84$</td>
<td>$\bar{X}_{p &amp; v} = 8.84$</td>
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<tr>
<td>$s_{p &amp; v} = 1.75$</td>
<td>$s_{p &amp; v} = 1.75$</td>
<td>$s_{p &amp; v} = 1.75$</td>
<td>$s_{p &amp; v} = 1.75$</td>
<td>$s_{p &amp; v} = 1.75$</td>
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<tr>
<td>$\bar{X}_{p &amp; v} = 8.84$</td>
<td>$\bar{X}_{p &amp; v} = 8.84$</td>
<td>$\bar{X}_{p &amp; v} = 8.84$</td>
<td>$\bar{X}_{p &amp; v} = 8.84$</td>
<td>$\bar{X}_{p &amp; v} = 8.84$</td>
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<tr>
<td>$s_{p &amp; v} = 1.75$</td>
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<td>$s_{p &amp; v} = 1.75$</td>
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<td>$\bar{X}_{p &amp; v} = 8.84$</td>
<td>$\bar{X}_{p &amp; v} = 8.84$</td>
<td>$\bar{X}_{p &amp; v} = 8.84$</td>
<td>$\bar{X}_{p &amp; v} = 8.84$</td>
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<td>$s_{p &amp; v} = 1.75$</td>
<td>$s_{p &amp; v} = 1.75$</td>
</tr>
</tbody>
</table>

$\bar{X}$: sample mean  
$s$: standard deviation  
$N$: sample size (number of students whose survey data was used for the two sample t-test)  
$p$: p-value
### Appendix P: Two-Proportion Test Results for Students’ Answers to Representativeness Heuristic Questions during the Lesson

<table>
<thead>
<tr>
<th>Question 1</th>
<th>Answer Choice</th>
<th>Comparison Group</th>
<th>Prediction-Only Group</th>
<th>Prediction-and-Voting Group</th>
<th>Two-Proportion Test Result</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>0 out of 5 (0%)</td>
<td>1 out of 22 (5%)</td>
<td>1 out of 22 (5%)</td>
<td>p = 0.306</td>
<td>Fail to Reject H₀: p &amp; v = p only = c</td>
</tr>
<tr>
<td></td>
<td>0 out of 5 (0%)</td>
<td>1 out of 22 (5%)</td>
<td>0 out of 25 (0%)</td>
<td>p = 0.306</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 out of 5 (0%)</td>
<td>1 out of 22 (5%)</td>
<td>0 out of 25 (0%)</td>
<td>p = 1.000</td>
<td></td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>3 out of 5 (60%)</td>
<td>12 out of 22 (54%)</td>
<td>12 out of 22 (54%)</td>
<td>p = 0.823</td>
<td>Fail to Reject H₀: p &amp; v = p only = c</td>
</tr>
<tr>
<td></td>
<td>3 out of 5 (60%)</td>
<td>12 out of 22 (54%)</td>
<td>16 out of 25 (64%)</td>
<td>p = 0.509</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 out of 5 (60%)</td>
<td>16 out of 25 (64%)</td>
<td>16 out of 25 (64%)</td>
<td>p = 0.867</td>
<td></td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>0 out of 5 (0%)</td>
<td>0 out of 22 (0%)</td>
<td>0 out of 25 (0%)</td>
<td>p = 0.509</td>
<td>Fail to Reject H₀: p &amp; v = p only = c</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>0 out of 5 (0%)</td>
<td>0 out of 22 (0%)</td>
<td>0 out of 25 (0%)</td>
<td>p = 0.867</td>
<td>Fail to Reject H₀: p &amp; v = p only = c</td>
</tr>
<tr>
<td><strong>E</strong></td>
<td>2 out of 5 (40%)</td>
<td>9 out of 22 (41%)</td>
<td>9 out of 22 (41%)</td>
<td>p = 0.730</td>
<td>Fail to Reject H₀: p &amp; v = p only = c</td>
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<tr>
<td></td>
<td>9 out of 22 (41%)</td>
<td>9 out of 22 (41%)</td>
<td>9 out of 25 (36%)</td>
<td>p = 0.867</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9 out of 25 (36%)</td>
<td>9 out of 25 (36%)</td>
<td>p = 0.867</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Question 2</th>
<th>Answer Choice</th>
<th>Comparison Group</th>
<th>Prediction-Only Group</th>
<th>Prediction-and-Voting Group</th>
<th>Two-Proportion Test Result</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>0 out of 5 (0%)</td>
<td>1 out of 22 (5%)</td>
<td>1 out of 22 (5%)</td>
<td>p = 0.306</td>
<td>Fail to Reject H₀: p &amp; v = p only = c</td>
</tr>
<tr>
<td></td>
<td>0 out of 5 (0%)</td>
<td>1 out of 22 (5%)</td>
<td>1 out of 25 (4%)</td>
<td>p = 0.927</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 out of 5 (0%)</td>
<td>1 out of 25 (4%)</td>
<td>1 out of 25 (4%)</td>
<td>p = 0.306</td>
<td></td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>0 out of 5 (0%)</td>
<td>2 out of 22 (9%)</td>
<td>2 out of 22 (9%)</td>
<td>p = 0.138</td>
<td>Fail to Reject H₀: p &amp; v = p only = c</td>
</tr>
<tr>
<td></td>
<td>2 out of 22 (9%)</td>
<td>2 out of 22 (9%)</td>
<td>0 out of 25 (0%)</td>
<td>p = 0.138</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 out of 22 (9%)</td>
<td>2 out of 22 (9%)</td>
<td>0 out of 25 (0%)</td>
<td>p = 1.000</td>
<td></td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>0 out of 5 (0%)</td>
<td>0 out of 22 (0%)</td>
<td>0 out of 25 (0%)</td>
<td>p = 0.867</td>
<td>Fail to Reject H₀: p &amp; v = p only = c</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>5 out of 5 (100%)</td>
<td>19 out of 22 (86%)</td>
<td>19 out of 22 (86%)</td>
<td>p = 0.062</td>
<td>Fail to Reject H₀: p &amp; v = p only = c</td>
</tr>
<tr>
<td></td>
<td>5 out of 5 (100%)</td>
<td>19 out of 22 (86%)</td>
<td>24 out of 25 (96%)</td>
<td>p = 0.246</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 out of 5 (100%)</td>
<td>24 out of 25 (96%)</td>
<td>p = 0.307</td>
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*p: p-value*  
* *= p-value < 0.05
<table>
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<th>Answer Choice</th>
<th>Comparison Group</th>
<th>Prediction-Only Group</th>
<th>Prediction-and-Voting Group</th>
<th>Two-Proportion Test Result</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>0 out of 5 (0%)</td>
<td>0 out of 22 (0%)</td>
<td>0 out of 25 (0%)</td>
<td>p &amp; v = p only = c</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0 out of 5 (0%)</td>
<td>6 out of 22 (27%)</td>
<td></td>
<td>p = 0.004* Reject H₀: p &amp; v = c (In fact, p only &gt; c)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6 out of 22 (27%)</td>
<td>10 out of 25 (40%)</td>
<td>p = 0.351 Fail to Reject H₀: p &amp; v = p only</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0 out of 5 (0%)</td>
<td>0 out of 22 (0%)</td>
<td>0 out of 25 (0%)</td>
<td>p &amp; v = p only = c</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>5 out of 5 (100%)</td>
<td>16 out of 22 (73%)</td>
<td></td>
<td>p = 0.004* Reject H₀: p &amp; v = c (In fact, c &gt; p only)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>16 out of 22 (73%)</td>
<td>15 out of 25 (60%)</td>
<td>p = 0.351 Fail to Reject H₀: p &amp; v = p only</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 out of 5 (100%)</td>
<td>15 out of 25 (60%)</td>
<td></td>
<td>p = 0.000* Reject H₀: p &amp; v = c (In fact, c &gt; p &amp; v)</td>
</tr>
<tr>
<td>Question 4</td>
<td>A</td>
<td>1 out of 5 (20%)</td>
<td>0 out of 22 (0%)</td>
<td></td>
<td>p = 0.264 Fail to Reject H₀: p &amp; v = p only = c</td>
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<td>0 out of 22 (0%)</td>
<td>1 out of 25 (4%)</td>
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<td>p = 0.307</td>
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<tr>
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<td>1 out of 5 (20%)</td>
<td>1 out of 25 (4%)</td>
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<td>p = 0.382</td>
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<td>B</td>
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<td>21 out of 22 (95%)</td>
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<td>p = 0.402 Fail to Reject H₀: p &amp; v = p only = c</td>
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<td>21 out of 22 (95%)</td>
<td>24 out of 25 (96%)</td>
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<td>p = 0.927</td>
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<td>4 out of 5 (80%)</td>
<td>24 out of 25 (96%)</td>
<td></td>
<td>p = 0.382</td>
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<tr>
<td></td>
<td>C</td>
<td>0 out of 5 (0%)</td>
<td>1 out of 22 (5%)</td>
<td></td>
<td>p = 0.306 Fail to Reject H₀: p &amp; v = p only = c</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 out of 22 (5%)</td>
<td>0 out of 25 (0%)</td>
<td></td>
<td>p = 0.306</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0 out of 5 (0%)</td>
<td>0 out of 25 (0%)</td>
<td></td>
<td>p = 1.000</td>
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*p: p-value
* = p-value < 0.05
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<tr>
<th>Question 5</th>
<th>Answer Choice</th>
<th><strong>Comparison Group</strong></th>
<th><strong>Prediction-Only Group</strong></th>
<th><strong>Prediction-and-Voting Group</strong></th>
<th><strong>Two-Proportion Test Result</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>0 out of 5 (0%)</td>
<td>0 out of 22 (0%)</td>
<td>0 out of 25 (0%)</td>
<td><strong>p &amp; v = p only = c</strong></td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>0 out of 5 (0%)</td>
<td>0 out of 22 (0%)</td>
<td>0 out of 25 (0%)</td>
<td><strong>p &amp; v = p only = c</strong></td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>5 out of 5 (100%)</td>
<td>16 out of 22 (73%)</td>
<td></td>
<td><strong>p = 0.004</strong>&lt;sup&gt;*&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>Reject H₀: p &amp; v = c</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(In fact, c &gt; p only)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16 out of 22 (73%)</td>
<td>20 out of 25 (80%)</td>
<td><strong>p = 0.558</strong></td>
<td>Fail to Reject H₀: p &amp; v = p only</td>
</tr>
<tr>
<td>D</td>
<td>C</td>
<td>5 out of 5 (100%)</td>
<td>20 out of 25 (80%)</td>
<td><strong>p = 0.012</strong>&lt;sup&gt;*&lt;/sup&gt;</td>
<td><strong>Reject H₀: p &amp; v = c</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(In fact, c &gt; p &amp; v)</td>
</tr>
<tr>
<td>E</td>
<td>D</td>
<td>0 out of 5 (0%)</td>
<td>1 out of 22 (5%)</td>
<td><strong>p = 0.306</strong></td>
<td>Fail to Reject H₀: p &amp; v = p only = c</td>
</tr>
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<td></td>
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</tr>
<tr>
<td></td>
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<td>1 out of 22 (5%)</td>
<td>2 out of 25 (8%)</td>
<td><strong>p = 0.622</strong></td>
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<tr>
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<td>0 out of 5 (0%)</td>
<td>2 out of 25 (8%)</td>
<td><strong>p = 0.140</strong></td>
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</tr>
<tr>
<td></td>
<td>E</td>
<td>0 out of 5 (0%)</td>
<td>5 out of 22 (22%)</td>
<td><strong>p = 0.011</strong>&lt;sup&gt;*&lt;/sup&gt;</td>
<td><strong>Reject H₀: p &amp; v = c</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(In fact, p only &gt; c)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 out of 22 (22%)</td>
<td>3 out of 25 (12%)</td>
<td><strong>p = 0.332</strong></td>
<td>Fail to Reject H₀: p &amp; v = p only</td>
</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0 out of 5 (0%)</td>
<td>3 out of 25 (12%)</td>
<td><strong>p = 0.065</strong>&lt;sup&gt;*&lt;/sup&gt;</td>
<td><strong>Reject H₀: p &amp; v = c</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(In fact, p &amp; v &gt; c)</td>
</tr>
</tbody>
</table>

<p: p-value

* = p-value < 0.05
Curriculum Vita

Tami Dashley was born in El Paso, Texas, in April 1987 to Geraldine Rajski and Richard Dashley. Tami Dashley graduated from Eastwood High School, El Paso, Texas, in May 2005 and began attending The University of Texas at El Paso in the Fall 2005 semester. While pursuing her Bachelor of Science degree in Mathematics with a minor in secondary education, Tami was awarded The Texas Council of Teachers of Mathematics (TCTM) Specialist Scholarship. In the Spring 2007 semester, she was accepted as a Mathematics Scholar for The University of Texas at El Paso’s newly created Math and Science Teachers (MaST) Academy. After three semesters in the MaST Academy, in the Spring semester of 2008, Tami received The Outstanding MaST Academy Scholar Award. Upon graduating Summa Cum Laude with her undergraduate degree in the Spring of 2008, she was awarded The Outstanding Undergraduate Mathematics Student Award and the University Banner Bearer position for the graduation commencement ceremony. The following semester, in Fall of 2008, Tami began her first semester of graduate work at The University of Texas at El Paso. On January 14th, 2010, Tami presented her thesis research at the MAA/AMS Joint Mathematics Meetings in San Francisco, California. In Spring 2010 semester, Tami was awarded The Outstanding Graduate Mathematics Student Award and graduated Summa Cum Laude with her master’s degree. Tami has accepted a teaching position at the Transmountain Early College High School for the El Paso Independent School District beginning in the 2010/2011 academic school year.

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El Paso, TX 79936