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Fractal-COSYSMO Systems Engineering Cost Estimation for Complex Projects

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Dedicated to my Parents, Shweta and Vaishnavi

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by

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THESIS

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ABSTRACT

By

MANISH SHIVRAM KHADTARE

Cost estimation for engineering projects has advanced beyond the bottom-up counting of costs and the top-down use of analogies. Parametric cost estimation is now employed to estimate volatile software costs, as well as the cost of Systems Engineering (SE) effort in engineering projects. The Constructive Systems Engineering Cost Model (COSYSMO) estimates the number of Person-Months (PM) necessary to complete systems engineering projects by using project size and cost parameters. On re-examining the nature of systems engineering and the structures of complex projects that must be built in an orderly fashion on a variety of scales, it is apparent that a parametric formula that employs fractal dimensionality principles should be well suited for their cost estimation. This thesis therefore develops the connections between fractal dimensionality and cost estimation with COSYSMO. The result is Fractal-COSYSMO, a novel cost estimating formulation that can be used to determine the cost of developing systems that show complexity on a broad scale.

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1.1, Introduction: Effort Estimation for Engineering

Engineering projects can fail for many reasons. Besides performance failures and technical risks, sources of failure include:

- Cost overruns, and,
- Schedule overruns.

Tools and processes for estimating costs and schedules have been extensively developed.

The technical organization of an engineering project can be accomplished with Systems Engineering (SE). A method for specifically estimating SE cost and time has been developed, as the Constructive Systems Engineering Cost Model (COSYSMO) (R. Valerdi 2003) which takes engineering project inputs, and outputs (SE) Person Months (PM), a type of cost and schedule measure.

This thesis re-tailors COSYSMO for complex projects with fractal qualities, allowing more accurate cost estimation of a variety of measures, including project size, cost, schedule, or person months.

Parametric cost estimation models:

Parametric cost estimation methods are the most sophisticated and most difficult to develop. Parametric models generate cost estimates based on mathematical relationships between independent variables (i.e., requirements) and dependent variables (i.e., effort). The inputs characterize the nature of the work to be done, plus the environmental conditions under which the work will be performed and delivered. The definition of the mathematical relationships

between the independent and dependent variables is the heart of parametric modeling. These relationships are commonly referred to as Cost Estimating Relationships (CERs) and they are usually based upon statistical analyses of large amounts of data.

Advantages of parametric cost model:

- Fast
- Easy to use

Disadvantages of parametric cost model:

- Difficult and time consuming to develop
- Require a lot of clean, complete, and uncorrelated data to be properly validated.

1.2, Development of COSYSMO

"In 2000, USC Affiliates identified accurate Systems Engineering cost estimation as their most pressing need. In response, USC formed a task team to develop the model using volunteer experts from commercial firms (Motorola, Rational, Xerox, etc.), aerospace companies (Lockheed, Raytheon, TRW, etc.) and government organizations (DOD, FAA, NASA, etc.) who followed the process … . The first step involved investigating related literature on how existing cost models addressed Systems Engineering costing. What we found was that most of the modeling work done to date relied on heuristics or rules of thumb to cost Systems Engineering activities. Most of the firms we interviewed during this fact-finding period did bottoms-up costing that relied on engineers to provide task level estimates based upon their experience. These estimates were validated based on past experience and then summed to form the top-level estimates. Little was done to

factor into these estimates the synergy, dynamics, or confusion that occurs on large projects as teams are formed and organizations energized to do the work." (R. Valerdi 2003)

Constructive Systems Engineering Cost Model (COSYSMO):

- Estimates the number of person-months necessary to complete systems engineering projects.
- COSYSMO uses project size and cost estimates which are entered into a parametric formula.

As a parametric model, COSYSMO contains its own Cost Estimating Relationships (CER) and is structured in a way to accommodate the current systems engineering standards and processes.

COSYSMO Derivation History:

COSYSMO has gone through three major iterations. This section describes each of these spirals and the properties of the models.

- Strawman COSYSMO: The first version of COSYSMO contained a list of 16 systems engineering cost drivers. The model was referred to as the "strawman" version because it provided a skeleton for the model with limited content. The factors identified were ranked. Each parameter was determined to have a high, medium, or low influence level on systems engineering cost. The most influential application factor was *requirements understanding* and the most influential team factor was *personnel experience*.

- COSYSMO – IP: The second major version of COSYSMO included refined definitions and a revised set of cost drivers. At this stage, a general form for the model was proposed

containing three different types of parameters: additive, multiplicative, and exponential. This form of the model is the current COSYSMO form:

$$PM_{NS} = A * (SIZE)^{E} * \prod_{i=1}^{n} EM_{i} \tag{1}$$

Where,

PM = person months

A = calibration factor

Size = measure(s) of functional size of a system that has an additive effect on systems engineering effort

E = scale factor(s) having an exponential or nonlinear effect on systems engineering effort

EM = effort multipliers that influence systems engineering effort.

"The general rationale for whether a factor is additive, exponential, or multiplicative comes from the following criteria:" (Boehm, Valerdi et al. 2005)

- A factor is additive if it has a local effect on the included entity. For example, another source instruction, function point entity, requirement, module, interface, operational scenario, or algorithm to a system has mostly local additive effects. From the additive standpoint, the impact of adding a new item would be inversely proportional to its current size.
- A factor is multiplicative if it has a global effect across the overall system. For example, adding another level of service requirement, development site, or incompatible customer has mostly global multiplicative effects.

- A factor that is exponential has both a global effect and an emergent effect for larger systems. If the effect of the factor is more influential as a function of size because of the amount of rework due to architecture, risk resolution, team compatibility then it is treated as an exponential factor.

COSYSMO: The current version, referred to simply as COSYSMO, has a broader scope representative of the extensive participation from industrial affiliates and INCOSE (international council of systems engineering).The current operational form of the COSYSMO model is given as.

$$PM_{NS} = A * (SIZE)^{E} * \prod_{i=1}^{n} EM_{i} \qquad (2)$$

Where:

PM_{NS} = effort in Person Months (Nominal Schedule)

A = calibration constant derived from historical project data

Size = determined by computing the weighted sum of the four size drivers

E = represents economy/diseconomy of scale; default is 1.0

n = number of cost drivers (14)

EM_i = Effort Multiplier for the *i*th cost driver. Nominal is 1.0.

The Effort Multipliers are:

- Requirements Understanding
- Architecture Understanding
- Level of Service Requirements

- Migration Complexity
- Technology Risk
- Documentation
- # and diversity of installations/platforms
- # of recursive levels in the design
- Stakeholder team cohesion
- Personnel/team capability
- Personnel experience/continuity
- Process capability
- Multi-site coordination
- Tool Support

COSYSMO is designed to estimate the number of person months as a function of a system's functional size with considerations of diseconomies of scale. Namely, larger systems will require proportionally more systems engineering effort than smaller systems. That is, larger systems require a larger number of systems engineering person months to complete. The four metrics selected as reliable systems engineering size drivers are:

Number of systems requirements,

Number of major interfaces,

Number of critical algorithms, and

Number of Operational Scenarios.

The weighted sum of these drivers represents a system's functional size from the systems engineering standpoint and is represented in the following CER:

$$PM_{NS} = \sum_{k} W_e \phi_e + W_n \phi_n + W_d \phi_d \tag{3}$$

Where:

$k =$ *INTF, REQ, ALG,*

[*INTF: Number of Interfaces, REQ: Number of Requirements, ALG: Number of Algorithms.*]

$W =$ weight

$e =$ easy, $n =$ nominal, $d =$ difficult,

$\phi =$ driver count

The CER shown in Equation 3 is a representation of the relationship between functional size and systems engineering effort. The effect of each size driver on the number of systems engineering person months is determined by its corresponding weight factor. Different systems engineering efforts may exhibit different levels of productivity which must be represented in COSYSMO. An exponential factor, *E*, is added to the CER and is represented in Equation 4:

$$PM_{NS} = \left\{ \sum_{\mathrm{k}} W_{\mathrm{e}} \phi_e + W_n \phi_n + W_d \phi_d \right\}^E \tag{4}$$

In the case of small projects the exponent, *E*, could be equal to or less than 1.0. This would represent an economy of scale which is generally very difficult to achieve in large people intensive projects. Most large projects would exhibit diseconomies of scale and as such would employ a value greater than 1.0 for *E*. Systems development activities may have different diseconomies of scale because of two main reasons: growth of interpersonal communications overhead and growth of large-system integration overhead.

$$PM_{NS} = A \cdot (\Sigma w_e \phi_e + w_n \phi_n + w_d \phi_d)^E \cdot \prod_{j=1}^{14} EM_j \tag{5}$$

1.3, Objective and Use of COSYSMO

COSYSMO, as the name suggests, is a model that can be helpful in determining the cost of Systems Engineering projects. User objectives include the ability to make the following:

- "Investment decisions. Analysts calculating a return-on-investment need an estimate of the systems engineering cost or a life cycle effort expenditure profile.
- Budget planning. Managers need tools to help them allocate project resources.
- Tradeoffs. Decisions often need to be made between cost, schedule, and performance.
- Risk management. Sensitivity analyses allow for consideration of unavoidable uncertainties influenced by the given parameters for systems engineering cost estimation.
- Strategy planning. Setting mixed investment strategies to improve an organization's systems engineering capability via reuse, tools, process maturity, or other initiatives.
- Process improvement measurement. Investment in training and initiatives often need to be measured."(Boehm 2005)

The main aspects of these Objectives are to provide a model which is.

- Accurate
- Tailor able
- Simple
- Well-defined
- Constructive
- Pragmatic

COSYSMO has been used in a wide variety of engineering companies, including Lockheed Martin, Raytheon, Northrop Grumman, SAIC, General Dynamics, and BAE Systems.

1.4, Variations of COSYSMO (COSYSMO-Risk)

Managing a project well is crucial, due to the fact that many important risks can set the project up for failure. The identification of risks is often dampened by the tendency of humans to be overly optimistic about their ability to perform on schedule and on budget. This is especially true in the early stages of project planning when stakeholder requirements are not well articulated.

COSYSMO-Risk uses the three-point values to estimate the (non-parametric) probability distributions that effectively incorporate the uncertainty of risk into COSYSMO. For example, the technical team may input the scope risk by estimating the 5%, the 50%, and the 95% values for the following scope characteristics:

- Number of system Requirements,
- Number of system Interfaces,
- Number of system-specific Algorithms, and
- Number of Operational Scenarios.

Additionally, the technical team may estimate the performance risk by estimating the best, nominal, and worst case values for the performance characteristics, or effort multipliers. These characteristics are representative of the most influential systems engineering cost drivers as defined by industry experts and validated by historical data (Valerdi 2005)

Project risks may arise by adopting novel technology. Some work has been done to show the negative effects of technologies over a long time horizon and frameworks have been developed to show how products with short life cycles can affect the overall project risk (Smith and Bahill 2007) . Hence the cost estimation and the schedule risk both should be mitigated at

the early phases of the project and during the course of the project to develop values for EACs (Estimates at Completion).

CHAPTER II, Fractals

2.1, Introduction: Conceptual Foundations

Benoit Mandelbrot, the pioneer of classifying geometry, first coined the term “fractal” in 1975 from the Latin word *fractus*, which means broken. According to Webster's Dictionary, a fractal is defined as being "derived from the Latin *fractus* meaning broken, uneven; any of various extremely irregular curves or shape that repeat themselves at any scale on which they are examined." That is, a fractal is a rough geometric shape that can be split into parts, each of which is a reduced- size copy of whole. Fractal geometries have found an intricate place in science as a representation of some of the unique geometrical features occurring in nature. Many naturally occurring objects, such as trees, coastlines or clouds, are now considered to have fractal properties; and most of the interest in this topic comes from the different attempts that were made in order to produce the natural phenomena using computer graphics.

Fractals often have the following features:

- Finite structure at arbitrarily small scales.
- Self-similarity at all scales
- Defined by a simple and recursive definition (A definition which is used to define an object in terms of itself.)

One of the essential properties is ‘self-similarity,’ which refers to an infinite nesting structure on all scales. Fractals are considered to be infinitely complex because they appear similar at all levels of magnification. Theoretical fractal objects are infinitesimally sub-divisible in each subset. However small the set, it contains no less details then the complete set. These concepts of self-similarity and infinitesimal sub-divisibility are important, since fractal methods can be used

to analyze any ‘system,’ whether natural or artificial, that decomposes into parts in a self-similar fashion.

Problems defining dimensions:

The complexity of defining dimensions can be summarized in the following visualization depicted in Figure 1, of a microscopic fly flying towards a piece of paper (Garg 1990.). The fly starts out very far from the object in Figure 1(a), and thus it appears as a zero dimensional speck. As the fly gets closer, in Figure 1(b), the speck begins to elongate into a one dimensional line. Upon flying over the line, in Figure 1(c), the fly sees that it is actually a two dimensional plane, flying even closer in Figure 1(d). The fly sees that the plane has a depth to it as well, forming a three dimensional prism. Thereafter, by flying closer still, the fly sees only a two dimensional plane. Finally the fly flies into the piece of paper, seeing a two dimensional network of fibers.

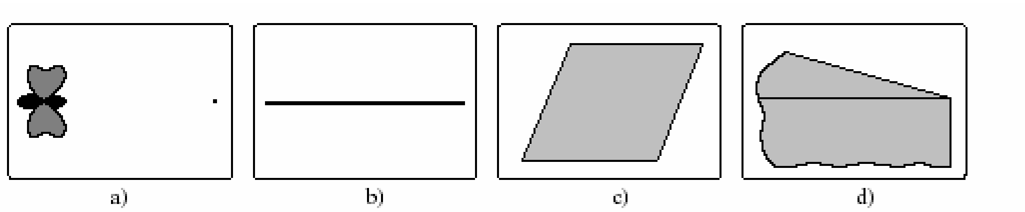


Figure 1: A fly flying towards a piece of paper from very far away revels the problem of defining dimensions

BROWNIAN MOTION: (Perrin, Wiener)

In Brownian motion, a particle advances ‘forward’ in one dimension but actually travels through the whole of the volume in which it is housed. “Brownian motion is topologically of dimension 1. However, being practically plane filling, it is fractals of dimension 2. The discrepancy

between these two values … qualify Brownian motion as being a fractal" (Mandelbrot 1977). Brownian motion has a space-filling property.

The examples above show that there is a need for a geometry that handles these situations better than Euclidean geometry. Euclidean structures have whole number dimensions, such as one dimensional line or a two dimensional plane. Benoit Mandelbrot first defined the term "Fractal" as including fractal dimensions in 1975 in order to handle geometries with dimensions that do not fall neatly into a whole number category. In fractals never is a point where the fundamental whole-number building blocks are found. This is because the building blocks themselves have the same form as the original object with infinite complexity in each one. An example of this in nature can be seen in a fern, shown in Figure 2.

Figure 2: A fern is a common example of geometry in nature that is easily modeled using fractal geometry

The entire fern has the same structure as each branch. If the individual branches were zoomed in upon, it is quite conceivable to imagine this as a completely separate fern with branches of its

own. Self-similarity of fractals can occur under some change in scale either strictly or statistically. Strictly self-similar fractals do not change their appearance significantly when viewed under a microscope of arbitrary magnifying power, whereas for statistically self-similar fractals, a smaller portion of the fractal is seemingly but not exactly similar to the original fractal itself. Fractal objects by definition contain infinite detail, containing the same degree of detail in each part as is contained in entire object, no matter how many times a section
is enlarged.

Fractals before Mandelbrot:

There were several types of fractals objects that were mainly devised in the pre-Mandelbrot era, in order to gain insight as to their true structures.

Cantor:

The Theory of Sets was developed during the late nineteenth century. Mathematicians delighted in producing sets with strange properties, many of them now recognized to be fractal in nature. One of these sets was developed by George Cantor (1845-1918) (Lauwerier 1987). Its construction is relatively simple and is illustrated in Figure 3.

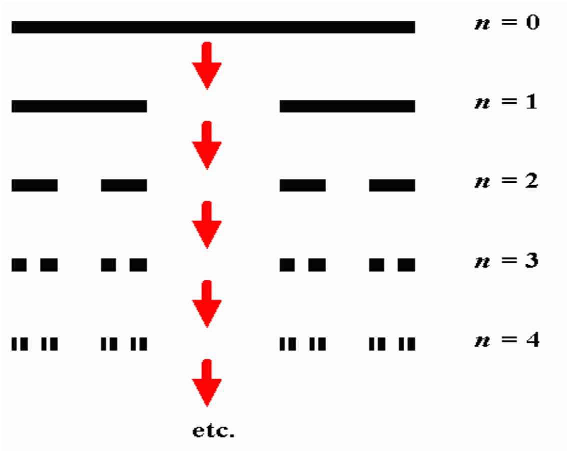


Figure 3: Generation of Cantor set

Begin the construction process with all real numbers in the interval [0, 1] of the real number line. Extract the interval [1/3, 2/3] which constitutes the central third of the original interval, leaving the two closed intervals [0, 1/3] and [2/3, 1]. Continue this process at each stage, extracting the central third of any interval that remains. If this process is continued *ad infinitum*, the remaining points rest on the edges of the teeth of the 'comb' as illustrated. This may not look particularly remarkable at first, but the set has some unusual properties. An infinite series corresponding to the lengths of the extracted sections can be given in a simple geometric progression.

$$\left[1+\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+\ldots\right]/3=1 \tag{6}$$

As at first stage, an interval of length 1/3 is cut out; in subsequent stages twice as many intervals, each 1/3 the length of the previous cut out intervals, are extracted. The points remaining in the cantor set, although infinite in number, are crammed into a total length of magnitude zero. Such points are disconnected. There is some unfilled space between any pair of points in the set, no matter how close these points may be. The set is said to form a 'dust'.

“Numbers in general can be represented with respect to any number base. As the cantor set construction involves repeated division by 3, it is informative to represent these numbers to the base 3. A typical value in the interval [0, 1] could be represented in this form as a sequence of the digits 0, 1, 2, for example:

$$0.221120210_{(Base3)}$$

As the middle third part of each existing interval has been discarded, the value illustrated in the above example could not belong to the Cantor set, since the digit 1 cannot occur in the base 3 representation of a value in the cantor set. Only the digit 0 and 2 can occur. For example, the following number is in the Cantor set:

$$0.220020200_{(Base3)}$$

The digits 0 and 2 can be used to trace a path down the cantor comb to the required value, 2 indicating that the right 'tooth' is taken, 0 indicating the left 'tooth'. Hence, we can say that there is a one-to-one correspondence between numbers in the cantor set and the binary numbers in the range [0, 1]”.(A.J. Crilly 1991) Much of Cantor’s work was involved with separating the paradoxes of the concept of infinity. As(Boyer 1968) indicates: “A system S is said to be infinite when it is similar to a proper part of itself.”

Van Koch curve:

The curve generated by Helge Van Koch [1904] is typical of the type of curve generated in that era, and is one of the classical fractal objects. The Van Koch curve is shown in Figure 4.

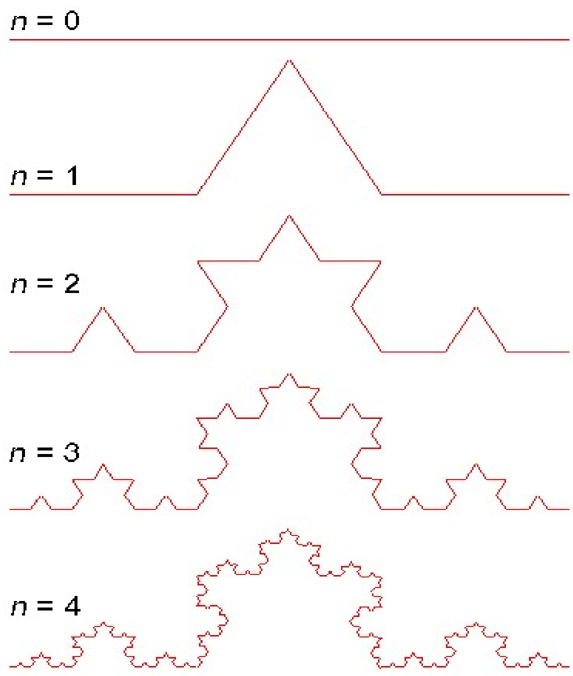


Figure 4: Four stages in the generation of a Koch curve

The curve is constructed from a line segment, which we can take, without loss of generality, to be of unit length. Then the central third of the line is extracted and replaced by two lines of length 1/3. This process is continued, the central third of the line segment being replaced at each stage by two lines of length one third that of the segment. The main aspect for generating a Koch curve is that, the projection of the replacement is always on the same side of the curve. You can easily see the change as the number of stage increases. At each stage, the total length of the curve is multiplied by 4/3. Also, we know that the repeated multiplication of unity by 4/3 creates a number too large to store in any calculator. In fact, the length of the completed Koch curve is infinite.

“In the generation process of the Koch curve, the middle third part of the line is removed and is replaced by a triangle which we can call the triangular Koch curve. Now consider the area between the Koch curve and the original line. As one triangle is added, its area is equal to:

$$A = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{\sqrt{3}}{6}\right) = \frac{\sqrt{3}}{36} \tag{7}$$

The formula used is a basic geometry formula which is 'half times base times height' for the area of triangle. The actual value of A is immaterial for this development. At the next generation stage, with n=2, four triangles are added, their size being linearly scaled down by a factor of 1/3. Thus, the additional area added by one triangle is $(1/3)^2$ = (1/9) the original area added. At each stage, four times as many new triangles are added to the total area, but the area of each triangle added is one-ninth of those added at the previous stage. Thus, the increase in area at any stage is four- ninths the area added at the previous stage. This leads to a geometric progression for the total area

$$A = \left(1 + \left(\frac{4}{9}\right) + \left(\frac{4}{9}\right)^2 + \left(\frac{4}{9}\right)^3 + ...\right) \tag{8}$$

This has a finite sum as:

$$\frac{9A}{5} = \frac{\sqrt{3}}{20} \tag{9}$$

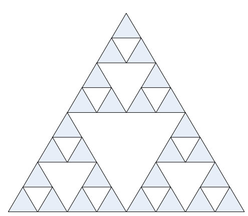
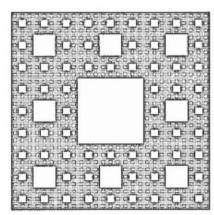
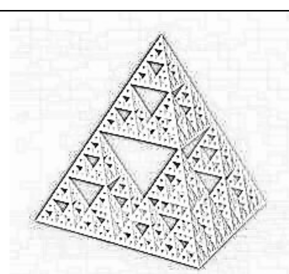
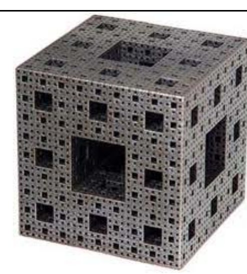
Thus, we have a curve of infinite length which encloses a finite area. From this theory we can say that that the curve does not have a well-defined slope at any point. Hence, the curve is nowhere differentiable. Also it contains infinite number of self-similar images. Any cut down sub-image will also be of infinite length as the whole curve itself is infinite in length. Hence we

come to the conclusion that for any two points on the curve, no matter how 'close' they are, the curve between them is of infinite length. (A.J. Crilly 1991)

Sierpinski:

Warclaw Sierpinski [1882 - 1969] gave his name to a number of fractal objects, like the Sierpinski arrowhead (or triangle, or gasket), the Sierpinski carpet, which is based in two-dimensional space, and the Sierpinski tetrahedron and sponge, based in three-dimensional space, as shown in Table 1.

Table 1: Table showing Sierpinski fractal objects in 2-D and 3-D space

	Sierpinski triangle	Sierpinski carpet
Fractal object in 2-D space		
	Sierpinski tetrahedron	Sierpinski sponge
Fractal object in 3- D space		

Construction process:

"To construct a Sierpinski triangle, extract from an original triangle the inverted half-scale copy of itself formed by joining the mid points of the three sides. Three half-scale triangles now remain so one fourth of the area of the original triangle has been removed. The process is now repeated for each triangle remaining in the object. At the second stage, one- fourth of the area of all three triangles is removed, each of which is one-fourth of the area of the previous. At third stage, there are nine triangles removed, of area $\left(\frac{1}{4}\right)^3$ of the original triangle's area. If that original area is set to A, the area removed by this process gives another geometric progression which sums to A:

$$A\left[\frac{1}{4}+3\left(\frac{1}{4}\right)^2+3^2\left(\frac{1}{4}\right)^3+3^3\left(\frac{1}{4}\right)^4+.....\right]=\frac{A}{4}\left[1+\frac{3}{4}+\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3+....\right]=A \tag{10}$$

In the Cantor set building process, where we extracted a region of the same size as the whole of the original space, the Sierpinski triangle building process leaves a set of points, but no area. The points exist in a zero magnitude area, separated, and forming a dust. In practice, the Sierpinski triangle set can only be drawn to a given number of subdivisions, as shown in Figure 5". (A.J. Crilly 1991)

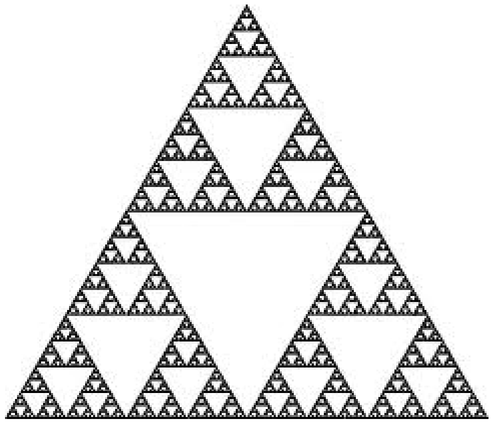


Figure 5: A Sierpinski triangle after five subdivisions

All the other Sierpinski objects, such as the Sierpinski square, tetrahedron and the Sierpinski carpet, can be constructed in a similar way, although their shapes are different.

Deterministic and Random fractals:

Fractals have two major variations:

1. Deterministic fractals
2. Random fractals

Deterministic fractals:

Those fractals that are composed of several scaled down and rotated copies of them-selves, such as the Koch curve. Julia set also falls in same category. The whole set can be obtained by applying a non-linear iterated map to an arbitrarily small section of the original object. Thus, the structure of a Julia set is contained in any smaller fraction of the set. So, Julia sets are also called algebraic fractals.

Algebraic fractals:

Algebraic fractals form the biggest class of fractals. They are created by using nonlinear processes in n-dimensional spaces.

Geometric fractals:

The fractals of this class are visual. In the two-dimensional case, they are made of a broken line (or of a surface in three-dimensional case) called *the generator*. Each of the segments which form the broken line is replaced by the broken line generator at the corresponding scale for any step of the algorithm. The process can be repeated infinitely, creating a geometrical fractal. Koch curve is an example of Geometric fractal

Both algebraic and geometric fractals are termed "deterministic" fractals. Since the generation requires use of a particular rule which is repeated recursively over and over again, they exhibit the property of strict self-similarity. For example Julia sets.

Random fractals:

Random Fractals include those fractals which have an additional element of randomness allowing for simulation of natural phenomenon, so they exhibit properties of statistical self-similarity.

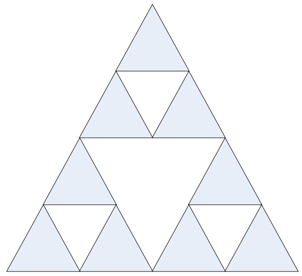
Stochastic fractals:

The stochastic fractals are obtained in the case of an iterative process with accidental parameters. Stochastic fractal processes can create replicas of natural objects. Two dimensional stochastic fractals, for example, are used for recreating the surface of the sea, or for relief modeling.

Some of the more complicated sets we have discussed above do not sit naturally in such restricted spaces. Consider the Cantor set; it consists of a set of disjoint individual values so it can be thought of as having dimension zero. Yet it has a one-to-one correspondence with the set of values on the real line in the interval [0, 1], which is fairly evidently a one-dimensional set. The Koch curve may appear to be one-dimensional according to the argument above. Yet points along the curve cannot be labeled in the normal way by distances from a fixed point, as the distance between any two points on the Koch curve is infinite. The Koch curve does not fill the two-dimensional space in which it exists, so it cannot be two-dimensional. Yet it seems to be more than one-dimensional. Similar, problems occur when considering the Sierpinski gasket and fractal objects which exist in three-dimensional space, such as Sierpinski sponge and tetrahedron.

Taking all these things into account there seems to be a need for a fractal dimension to describe the structure of such constructions; this is provided by the 'fractal dimension', or the 'Hausdorff dimension'(Voss 1988) which is another definition of fractals as defined earlier.

A simple approach can be made by, considering the way in which dimension can be defined for some normal figures, for example a square or a triangle. If we look at Figure 6, we can say that the shapes are properly two-dimensional. We shall discuss about the self-similar property of fractal by considering its background that is, the fractal object can be split into parts, each of which is the reduced size copy of whole. Squares and triangles can be subdivided into parts each of which is the reduced size copy of whole, shown in Figure (9). The square and triangle below can be sub divided into four copies at 1/2 scale, nine copies at 1/3 scale, 16 copies at 1/4 scale.



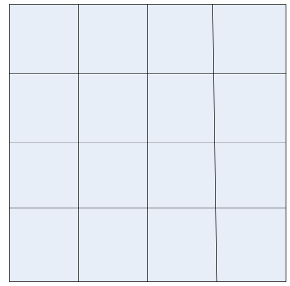


Figure 6: Triangle and Square sub-divided in parts to show the fractal dimensionality

In each case, if the number of copies in N and the scale factor is f we have the relationship

$$N = \left(\frac{1}{f}\right)^2 \tag{11}$$

As with the other figures the cube is also broken down into smaller cubes of 1/4 the size of the original.

It takes 64 of these smaller cubes to create the original cube.

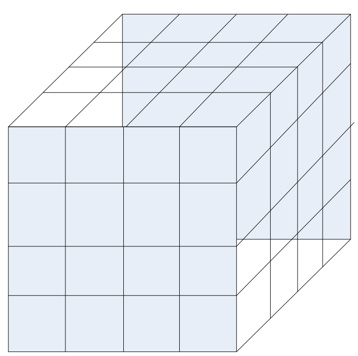


Figure 7: A cube sub-divided in parts to show the fractal dimensionality

Here the relation between N and f becomes

$$N = \left(\frac{1}{f}\right)^{3} \tag{12}$$

The power of $\left(\frac{1}{f}\right)$ indicates the dimension of the object. We can generalize this form. If an object can be subdivided into N copies of itself at scale f, then its dimension is the value D which satisfies.

$$N = \left(\frac{1}{f}\right)^{D} \tag{13}$$

D can be obtained by applying the logarithm operation on Equation (13) which gives, (A.J. Crilly 1991)

$$D = \frac{\log(N)}{\log\left(\frac{1}{f}\right)} \tag{14}$$

We will consider Equation (14) to calculate the fractal dimension of the fractal objects as defined earlier. For example, the Cantor set, contains two copies of itself at $1/3$ scale, thus its dimension is

$$D = \frac{\log(2)}{\log(3)} = 0.6309 \tag{15}$$

To four decimal places, this is a non-integer dimension between 0 and 1, which is exactly what we want a fractal dimension between 0 and 1. Let us check the fractal dimension for the Koch curve we have four copies at $1/3$ scale. Thus

$$D = \frac{\log(4)}{\log(3)} = 1.2619 \tag{16}$$

To four decimal places, this is a non-integer dimension between 1 and 2. The Sierpinski triangle contains three copies at $1/2$ scale, so has dimension

$$D = \frac{\log(3)}{\log(2)} = 1.5850 \tag{17}$$

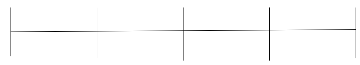
to four decimal places. Extending beyond two dimensions, the Sierpinski sponge has 20 self-similar copies at $1/3$ scale, giving dimension of

$$D = \frac{\log(20)}{\log(3)} = 2.7268 \tag{18}$$

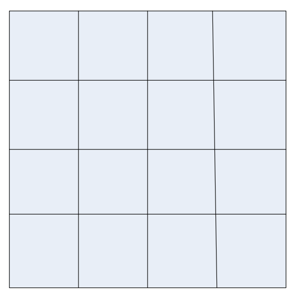
To four decimal places, being a three dimensional fractal as expected it gives the value between 2 and 3. This method works very well for objects which contain exact sub copies of themselves; but not very many objects are so well behaved, even if they have exact integer dimensions. Hausdorff's definition overcomes this problem. He uses the concept of a 'neighborhood', which for our purposes can be considered as a small region of regular shape centered on a particular point. In normal one-dimensional space, a neighborhood is a short line segment, in two dimensions it is a small circle.

Another definition of fractals is: “A fractal is by definition a set for which the Hausdorff-Besicovitch dimension strictly exceeds the topological dimension.” To understand the second definition we need to be able to understand the fractal dimension. So first we have to look at understanding how to calculate the dimension of an object. Below we have three different objects.

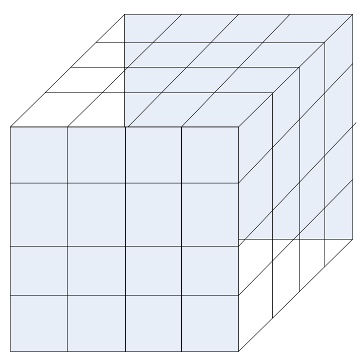
1. As we can see the line is broken into 4 smaller lines. Each of these lines is similar to the original line, but they are all 1/4 the scale. This is the idea of *self-similarity*.



2. The square below is also broken into smaller pieces. Each of which is 1/4th the scale of the original. In this case it takes 16 of the smaller pieces to create the original.



3. As with the others the cube is also broken down into smaller cubes of 1/4 the scale of the original. It takes 64 of these smaller cubes to create the original cube.



By looking at this we begin to see a pattern:

$4 \text{ (Line)} = 4^1$

$16 \text{ (Square)} = 4^2$

$64 \text{ (Cube)} = 4^3$

This let us to think about the Equation:

$$N = S^D \tag{19}$$

Where N is the number of small pieces that go into the larger one, S is the scale to which the smaller pieces compare to the larger one and D is the dimension. We now have the tools to be able to calculate the dimension. Just solve for D in the previous equation. When we do this we find that the

Dimension is:

$$D = \log N / \log S \tag{20}$$

This dimension is the Hausdorff-Besicovitch dimension.

In fractal dimensionality, the **fractal dimension**, *D*, is nothing but a statistical quantity that tell us of how completely a fractal appears to fill space, as one goes to finer and finer scales. The fractal dimension D is a basic parameter of a fractal, revealing important aspects of its structure, as it quantifies the degree of irregularity or fragmentation. It also indicates the level of complexity or the amount of details through scales and describes the distribution of the mass around a center (Sobreira Fabiano 2002). There are many definitions of fractal dimension. The most important theoretical fractal dimensions are the Rényi dimension, the Hausdorff dimension

and packing dimension. Although for some classical fractals all these dimensions do coincide, in general they are not equivalent. There are two main approaches to generate a fractal structure.

- Growing the fractal from a unit object.
- Creating subsequent divisions in the original structure like the Sierpinski triangle.

The Sierpinski triangle also called as the Sierpinski gasket was named after the great mathematician ‘Waclaw Sierpinski’ who described it in 1915. This is one of the basic examples of self -similar sets. It is a mathematically generated pattern that can be reproduced at any magnification or reduction. One Sierpinski triangle can be built into many different triangles within itself as shown in the figures.

Figure 8 shows the Sierpinski triangle for the zero iteration, Figure 9 shows the Sierpinski triangle for one iteration, Figure 10 shows the Sierpinski triangle for two iteration.

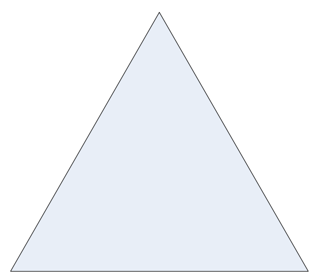


Figure 8: Sierpinski triangle for zero iteration

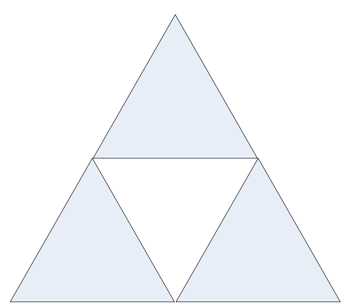


Figure 9: Sierpinski triangle for one iteration

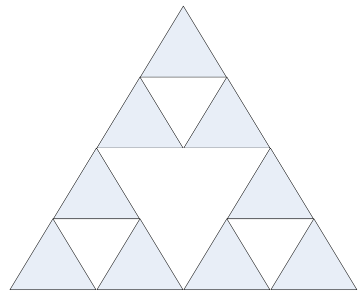


Figure 10: Sierpinski triangle for two iteration

2.2 Fractal Dimensionality Mathematics:

There are various techniques to develop and to produce fascinating fractal images. Two techniques popularly shown in Mandelbrot’s book [1982] are the

1. Functional iteration in the complex domain.
2. Generation of random fractals by random processes.

Iterated Function Systems (IFS): The Language of Fractals:

Fractals are nothing but self-similar object; in other words, fractals are reduced size copies of the whole. Any fractal has some infinitely repeating pattern. When creating such a fractal, you would suspect that the easiest way is to repeat a certain series of steps which create that pattern. Instead of the word "repeat" we use a mathematical synonym "iterate" and the process is called *iteration.* An IFS is another way of generating fractals. It is based on taking a point or a figure and substituting it with several other identical ones. IFS represent an extremely versatile method for conveniently generating a wide variety of useful fractal structures (Peitgen H 1992). These

iterated function systems are based on the application of a series of affine transformations, w, defined by (Barnsley 1993) .

$$w\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix} \tag{21}$$

Or, equivalently, by

$$\text{w (x, y) = (ax + by + e, cx + dy + f)} \tag{22}$$

Where a, b, c, d, e and f are real numbers. Hence the affine transformation, w, is represented by six parameters.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ and } \begin{pmatrix} e \\ f \end{pmatrix}$$

The Parameters *a, b,* c, and *d* control rotation and scaling, while parameters *e* and *f* control linear translation. Now suppose we consider $w_1, w_2, ..., w_N$ as a set of affine linear transformations, and let *A* be the initial geometry, then a new geometry, is produced by applying the set of transformations to the original geometry, A **,** the collection of results from $w_1(A), w_2(A), ..., w_N(A)$ **,** can be represented by

$$w(A) = \bigcup_{n=1}^{N} w(A) \tag{23}$$

Where, *W* is known as the Hutchinson operator (Peitgen H 1992). A fractal geometry can be obtained by repeatedly applying *W* to the previous geometry. For example, if the set A_0 represents the initial geometry, then we will have

$$A_1 = \mathrm{w}\,(A_0),\ A_2 = \mathrm{w}\,(A_1),..................,\ = \mathrm{w}\,(A_K) \tag{24}$$

An iterated function system raises a sequence that converges to a final image, A_∞, in such a way that

$$w_1\,(x,y) = \left(\frac{1}{3}x + (0)y + 0, (0)x + \frac{1}{3}y + 0\right) \tag{25}$$

$$w_2(x,y) = \left(\frac{1}{6}x - \frac{1.732}{6}y + \frac{1}{3}, \frac{1.732}{6}x + \frac{1}{6}y + 0\right) \tag{26}$$

$$w_3(x,y) = \left(\frac{1}{6}x + \frac{1.732}{6}y + \frac{1}{2}, -\frac{1.732}{6}x + \frac{1}{6}y + \frac{1.732}{6}\right) \tag{27}$$

$$w_4(x,y) = \left(\frac{1}{3}x + (0)y + \frac{2}{3}, (0)x + \frac{1}{3}y + 0\right) \tag{28}$$

$$w(A) = w_1(A) \cup w_2(A) \cup w_3(A) \cup w_4(A) \tag{29}$$

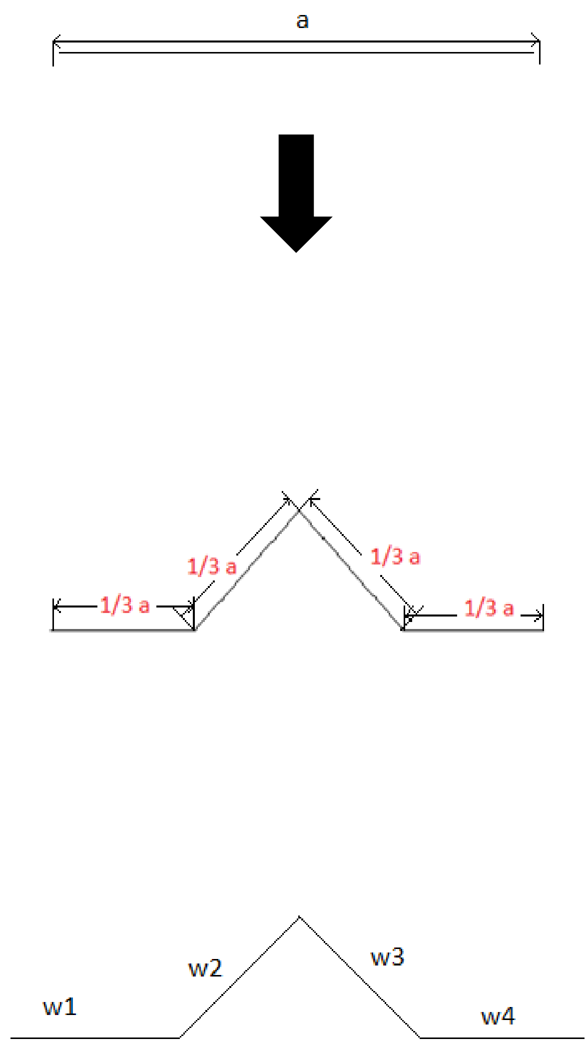


Figure 11: The standard Koch curve as an iterated function system

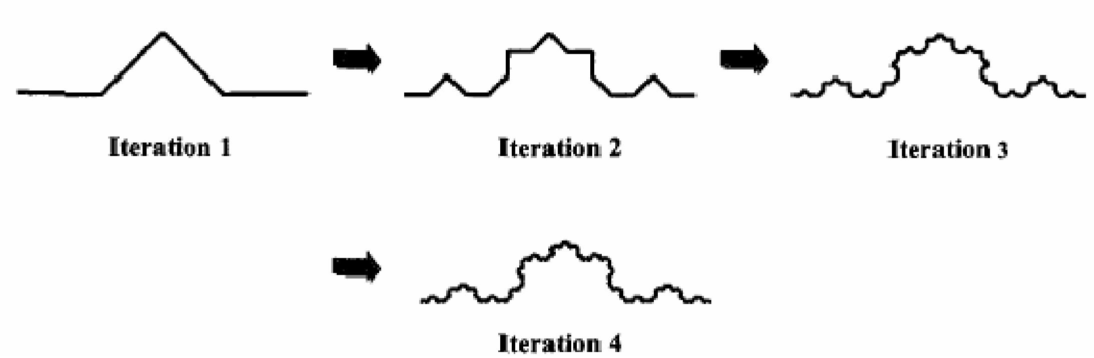


Figure 12: The first four stages in the construction of the standard Koch curve via an iterated function system (IFS) approach

The transformation is applied to each iteration to achieve higher levels of fractalization. $w(A_1\infty) = A_\infty$

This image is called the attractor of the iterated function system, and represents a "fixed point" of w. Figure 12 illustrates the iterated function system procedure for generating the well-known Koch fractal curve. In this case, the initial set, *A,* is the line interval of unit length. Four affine linear transformations are then applied to *A*. Next, the results of these four linear transformations are combined together to form the first iteration of the Koch curve, denoted by A_1 . The second iteration of the Koch curve A_2 , may then be obtained by applying the same four affine transformations to A_1 . Higher-order version of the Koch curve is generated by simply repeating the iterative process until the desired resolution is achieved. The first four iterations of the Koch curve are shown in Figure 12. We note that these curves would converge to the actual Koch fractal, represented by A_∞ , as the number of iterations approaches infinity (Ganguly). Iterated function systems have proven to be a very powerful design tool for fractal antenna engineers. This is primarily because they provide a general framework for the description, classification,

and manipulation of fractals (Barnsley 1993). In order to further illustrate this important point, the iterated function system code fix such diverse objects as a Sierpinski gasket and a fractal tree have been provided in Figure 13 and Figure 14 respectively (Peitgen H 1992).

Table 1: Iterated Function System code for a Sierpinski Gasket

a	b	c	d	e	f
0.195	-0.488	0.344	0.443	0.4431	0.2452
0.462	0.414	-0.252	0.361	0.2511	0.5692
-0.0058	-0.07	0.453	-0.111	0.5976	0.0969
-0.035	0.07	-0.469	-0.022	0.4884	0.5069
-0.637	0.0	0.0	0.501	0.8562	0.2513

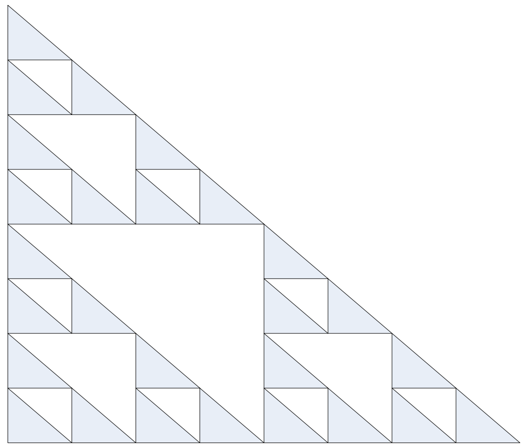


Figure 13: Sierpinski gasket **(Peitgen H 1992).**

Table 2: Iterated Function System code for a fractal tree

a	b	c	d	e	f
0.5	0.000	0.000	0.5	0.000	0.000
0.5	0.000	0.000	0.5	0.5	0.000
0.5	0.000	0.000	0.5	0.000	0.5

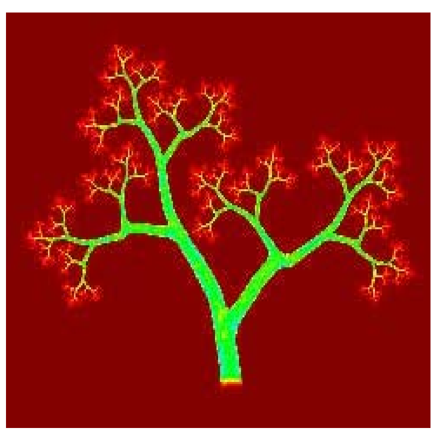


Figure 14: Fractal tree **(Peitgen H 1992).**

Fractals - A space filling geometry:

Euclidean geometries are limited to points, lines, sheets & volumes, Fractal include geometries that fall in between these distinctions. Therefore, a fractal can be line that approaches a sheet. In the case of fractal antennas, space filling properties lead to curve that are electrically very long (Jampala), but fit into a compact physical space. For the Koch curve with each iteration the length of Koch curve increases by 1/3 of the original length.

Length of Koch curve after the n^{th} iteration.

$$l_n = l_o \left(\frac{4}{3} \right)^n \tag{30}$$

Where l_n and l_o are the length after nth iteration and original length (without any iteration) respectively.

For the Sierpinski triangle, with each iteration the area of the holes and circumference of the solid pieces changes. If the area of original triangle is 1, then the first iteration removes $1/4^{th}$ of the area, second iteration removes a further $3/16^{th}$ and third iteration $9/64^{th}$.

Then total area removed after the Nth iteration.

$$A_N = 1/3 \sum_{i=1}^{N} \left(\frac{3}{4} \right)^i \tag{31}$$

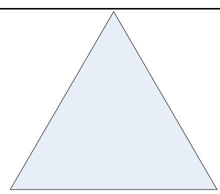
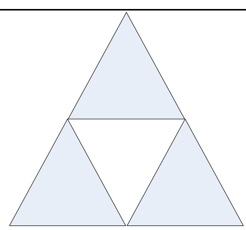
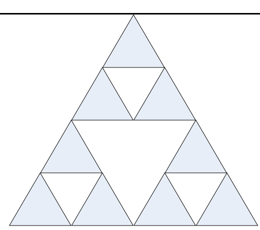
$$A_\infty = 1 \tag{32}$$

If circumference of original triangle is 1, then after first iteration the circumference increases by $\frac{1}{2}$. After second iteration it increases by $\frac{3}{4}$, after N^{th} iteration.

$$C_N = 1 + \frac{1}{3}\sum_{i=1}^{N}\left(\frac{3}{2}\right)^i \tag{33}$$

$$C_\infty = \infty \tag{34}$$

This means that the gasket has no area but the boundary is of infinite length. Figure 15 shows how with each iteration the area of holes and circumference varies.

Figure	Area	Perimeter
	$A_0 = \frac{\sqrt{3}}{4}$	$P_0 = 3$
	$A_1 = \left(\frac{3}{4}\right)A_0$	$P_1 = 3 + 3\left(\frac{1}{2}\right)$ $= 3 + \frac{3}{2}$
	$A_2 = \left(\frac{3}{4}\right)^2 A_0$	$P_2 = 3 + \frac{3}{2} + 3 * 3 * \left(\frac{1}{4}\right)$

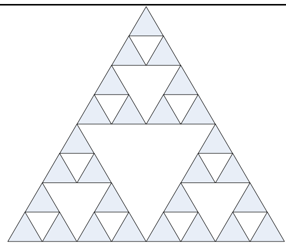
		$= 3+\frac{3}{2}+\frac{9}{4}$
	$A_3 = \left(\frac{3}{4}\right)^3 A_0$	$P_3 = 3+\frac{3}{2}+\frac{9}{4}+9*3*\frac{1}{8}$ $= 3+\frac{3}{2}+\frac{9}{4}+\frac{27}{8}$
Stage *n*	$A_n = \left(\frac{3}{4}\right)^3 A_0$	$P_n = 3+\frac{3}{2}+.....+\left(\frac{3}{2}\right)^n$
Sierpinski triangle	0	Infinity (Geometric series with r > 1)

Figure 15: Different iteration of the Sierpinski gasket and variations in areas and circumference

(KALRA)

Fractals in nature and application:

Fractals are not just complex shapes and pretty pictures generated by computers. However, any object that appears random and irregular can potentially be fractal. Fractals are the unique, irregular patterns left behind by the unpredictable movements of the chaotic world at work. In theory, one can argue that everything existent on this world is a fractal (Garg 1990), including.

- The leaves in trees,
- The veins in a hand,
- Water swirling and twisting out of a tap.

Fractals also have applications in science.

Astronomy:

Fractals may revolutionize the way that the universe is seen. Cosmologists usually assume that matter is spread uniformly across space. But observation shows that this is not true. Astronomers agree with that assumption on "small" scales, but most think that the universe is smooth at very large scales. However, a dissident group of scientist’s claims that the structure of the universe is fractal at all scales.

Nature:

Take a tree, for example. Pick a particular branch and study it closely. Choose a bundle of leaves on that branch. All three of the objects described - the tree, the branch, and the leaves - are identical. To many, the word chaos suggests randomness, unpredictability and sometimes even messiness. Weather is a favorite example for many people. Forecasts are never totally accurate, and long-term forecasts, even for one week, can be totally wrong. This is due to minor disturbances in airflow, solar heating, etc. Each disturbance may be minor, but the change it creates will increase geometrically with time. Soon, the weather will be far different than what was expected. With fractal geometry we can visually model much of what we witness in nature, the most recognized being coastlines and mountains.

Fractals previously studied, with dimensionality (more complete list than illustration above)

2.2.1, Shoreline length (Fractals and Chaos, Chapter 3, p. 43)

Although fractals are mainly discussed in mathematical forums, they exist in all parts of nature. For example Mandelbrot (Mandelbrot 1982) have discussed the basics of fractal theory, as

applied to the characteristics of a coastline. The length of a coastline depends on the size of the measuring yardstick. As the yardstick which we use to measure every turn and detail decreases in length, the coastline perimeter increases exponentially. If we view the coastline more closely, we find that within the coastline there lie miniature bays and peninsulas. As we examine the coastline on a rescaled map, we discover that each of the bays and peninsulas contain sub-bays and sub-peninsulas. There is a self-similar trait observed as we look at the coastline at various resolutions. In fact, because of the large number of irregularities, the physical length of a coastline is almost infinite.

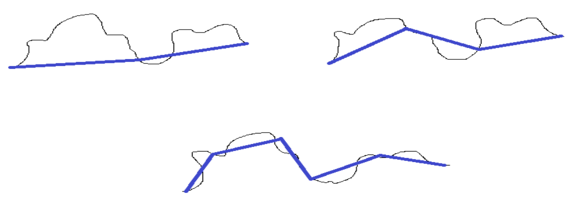


Figure 16: Different scales used to measure the shoreline length

These pictures represent an imaginary coastline of Britain. The blue lines are scales being used to measure the length of the coastline (L) with number of scales (N). These rulers are of the length (s) which can be represented as.

$$L = N\,s \tag{34}$$

When we decrease the length of s the number of times that s is used increases. These rulers shows us is that as the size of the measuring device becomes smaller the accuracy of the measurements become more and more accurate. Thus, we can make an assumption that it is quite possible to get an exact measurement of the coastline. This statement is false. As we decrease the size of the measuring device the length that we have to measure becomes greater. We can see this by zooming in on the coastline. As we get closer and closer we will notice that it looks very similar to how it looked from a greater distance away. Only now we are much closer. This observation shows the self-similarity of the coastline. Therefore as we decrease the size of the measuring device the length of the coastline will increase without limit. Thus, showing us its fractal nature. Mandelbrot showed (Mandelbrot 1967) that the difficulty can be resolved if a different type of "measure" (E) is used in place of the length, modifying Equation (34) to read.

$$E = Ns^{D} \tag{35}$$

Where,

E = Fractal extent.

D = Fractal dimension

A unique value can be found for D (the "fractal dimension") by keeping "E" constant and varying "s". Unlike the length, E has a sharply defined value for a particular stretch of coastline, once the dimension is known. The extent and dimension of an actual stretch of coastline can easily be determined by inverting Equation (35) to.

$$N = Es^{-D} \tag{36}$$

A pair of dividers is used to scale along the coastline on a map, and the number of scales (N) is counted for several different values of the scale length (s). The extent (E) and dimension (D) are then calculated using a logarithmic regression according to Equation (36). For smooth (differentiable) curves, D=1, and the "extent" which was shown in Equation. (35) is then the same as "length" from Equation. (34), which exists only in the special case. The coastlines have dimensions, mostly in the range 1.1 to 1.3; a higher value indicates a most convoluted coastline. The example of coastline shows that the coastline has a dimension greater than 1 but less than 2.

Chapter III, Fractal-COSYSMO

The pre-conditions for a fractal formulation of COSYSMO have now been set, because the investigation of fractals and COSYSMO has revealed their similar areas of application.

COSYSMO is formulated to estimate the systems engineering effort needed on complex projects. Systems engineering involves both 1) products, and 2) processes. As to products, systems engineering constructs complicated information-organizing structures that mimic the complex projects being organized. These structures, for example, tables, diagrams, matrices, architectures, and traceability trees, are structurally similar to fractal structures, because they begin in a simple fashion, as the first level, but are then the sites of aggregation at finer and finer levels of detail. As to processes, systems engineering involves all the work that systems engineers perform, in investigating the area of application, in constructing useful analogous models, and in applying the insights of the models to the real systems. In all these activities, systems engineers must repetitively visit all parts of the product design, iteratively approaching the product design at finer and finer levels of detail, until the design is sufficiently refined so as to be able to operate at the required level of sophistication. An analogy lies in bees that form a bee colony, acting both physically and culturally. The bees organize to construct a highly symmetric hive with gathered materials, and then tend the hive, as well as maintain the coherence of the bee colony throughout its life cycle. The hive is complicated, yet the task of building and maintaining it is complex.

Fractals can describe both the complicated structures of engineering projects, as well as the complex collective actions that must be orchestrated in order to organize the engineering processes of the project.

3.1, Analogies between COSYSMO Parameters and Fractals

The parameters of COSYSMO can be examined as they are grouped, and as individual parameters, in order to note their similarities to fractals.

COSYSMO SIZE PARAMETERS:

COSYSMO size parameters for a system project are either estimated numerically or counted approximately into bins -- for example, Easy, Normal, or Difficult -- and then added together into a total numerical estimate. Four size drivers, namely,

- Number of Requirements,
- Number of Interfaces,
- Number of Algorithms, and
- Number of Operational Scenarios,

"represent the functional size of a system which is believed to be a significant predictor of systems engineering effort" (Valerdi and Raj 2005).

These COSYSMO size drivers are assumed to add together. This additive formulation was decided upon the formulation of COCOMO, when the areas of application of these parameters were determined to be largely separate.

In Fractal-COSYSMO, this additive assumption is exchanged for a multiplicative assumption, because fractal-like complex systems have structures that are self-similar throughout. Further, it can be seen that in fractal structures, the basic building blocks will be the foundation for laying smaller and more numerous structures – a situation that lends itself naturally to a multiplicative formulation. The exact multiplicative formulation of Fractal-

COSYSMO Size Drivers is left open to sub-sets of application. For example, there will be a formulation of Fractal-COSYSMO tailored to city growth, and another tailored for network growth.

COSYSMO TECHNICAL COST PARAMETERS:

Cost parameters in COSYSMO are effort multipliers. “The rating of effort multiplier parameters for a completed project requires an assessment from the total project perspective” (Valerdi, Rieff et al. 2007). COSYSMO technical cost parameters include the following:

- Requirements Understanding,
- Architectural Understanding,
- Level of Service Requirements,
- Migration Complexity,
- Technology Risk,
- Documentation,
- # and Diversity of Installations/Platforms,
- # of Recursive Levels in the Design,

In Fractal-COSYSMO, Technical Cost Parameters account for the iterative re-visiting of technical aspects of the project, in order to understand and engineer those technical aspects. For example, a bee hive needs to be understood (at least by engineers!) in its many physical aspects, including the hive cells, the recursive levels in the design, the architecture of the hive, and the level of service required. Additionally, a hive design must be migrated for application in different environments, as well as installed in various locations. Also, a hive’s technical aspects must be documented.

COSYSMO TEAM COST PARAMETERS:

COSYSMO human cost parameters include:

Stakeholder Team Cohesion (Personalities and personal values)

Multisite Coordination (distance communications)

Heterogeneity (domains, cultures)

Personnel/Team Capability

Process Capability

Personnel Experience/Continuity

Tool Support

In Fractal-COSYSMO, Team Cost Parameters are practically the same as in COSYSMO, as the human teaming aspects of engineering projects are similar across engineering projects on fractal-like structures and non-fractal like structures.

Table (3) below summarizes the analogies between COSYSMO Parameters, Fractal Systems and parameters, and Fractal Mathematics.

COSYSMO Parameter	**Why Fractal-like?**	**Fractals & Methods**
COSYSMO SIZE PARAMETERS		
Number of Requirements	The number of requirements is uncertain, because requirement development can always continue at larger and	Box-Counting Method

	smaller scales.	
Number of Interfaces	Smaller objects are considered, the number of interfaces grows exponentially.	The tree-like pattern formed by DLA and DBM processes have been accepted commonly and also it has been said that many physical systems are close to this kind of form . Diffusion Limited Aggregation (DLA), Box-Counting Method
Number of Algorithms	Algorithms can always be formulated in a growing variety of ways.	Box-Counting Method
Number of Operational Scenarios	The operational environment provides an infinite number of scenarios or interactions.	Shorelines: Scaling
COSYSMO TECHNICAL COST PARAMETERS		
Requirements Understanding	Learning requires repetitively searching through entire set at smaller scales.	Any fractals
Architectural Understanding	Learning requires repetitively	Any fractals

	searching through entire set.	
Level of Service Requirements	Building in Reliability, Maintainability, etc. requires re-engineering at many scales.	Any fractals
Migration Complexity	Migration requires dis-attachment and re-attachment at many scales.	Any fractals
Technology Risk	(just a level?)	Any fractals
Documentation	Documentation requires iterative learning, describing and re-writing at many scales.	Any fractals
# and Diversity of Installations/Platforms	Fitting a product to many environmental requires fractal examination of each environment.	Any fractals
# of Recursive Levels in the Design	Design has to be engineered at various scales.	Any
	Task Completion Times	
COSYSMO TEAM COST PARAMETERS		
Stakeholder Team Cohesion	Socialization requires complete set of relationships among stakeholders.	Scale-free networks which models the social networks.

Multisite Coordination	Sites have to be connected with a fully-connected network.	Scale-free networks which models the social networks.
Heterogeneity		Self-similarity
Personnel / Team Capability	humans and human groups Learning time, learning time (cohesiveness), human thoughts.	In human knowledge of network effects.
Process Capability	No process is mastered without repetitive experience at many levels.	Awareness of fractal nature of processes and ability to counteract fractalization.
Tool Support	An integrated set of tools at all levels is necessary for complete support.	Tools must be capable of interfacing with a variety of operands.

With a bridge of analogies built between general engineering endeavors in which systems engineering is employed and projects to build strongly fractal-like products, the justification has been laid for the mathematical formulation of Fractal-COSYMO.

3.2, Mathematical Formulation

The regular COSYSMO parametric equation:

$$PM_{NS} = A \cdot (\Sigma w_e s + w_n s + w_d s)^E \cdot \prod_{j=1}^{14} EM_j \qquad (37)$$

Internalizing, distributing and specializing the exponential into the Size:

$$PM_{NS} = A \cdot (\Sigma w_e s^{d_1} + w_n s^{d_2} + w_d s^{d_3}) \cdot \prod_{j=1}^{14} EM_j \tag{38}$$

With the Size parameters combined multiplicatively, instead of additively, this becomes:

$$PM_{NS} = A \cdot (w_e s^{d_1} \cdot w_n s^{d_2} \cdot w_d s^{d_3}) \cdot \prod_{j=1}^{14} EM_j \tag{39}$$

Fractal-COSYSMO parametric equation is:

$$PM_{NS} = A \cdot \prod_{i=1}^{n} w_i s_i^{d_i} \cdot \prod_{j=1}^{14} EM_j \tag{40}$$

Where n is in general much greater than 3, because of the numerous features that fractal structures have.

If no adjustable weights are needed for the estimation of the size of the structure, or how size effects technical or team costs, the formulation becomes the most characteristic of Fractal-COSYSMO:

$$PM_{NS} = B \cdot \prod_{i=1}^{n} s_i^{d_i} \cdot \prod_{j=1}^{14} EM_j \tag{41}$$

The estimation of the pure size of a fractal structure is made possible by a reduced formula which focuses only on the power law (Bunde and Havlin 1991, p. 3) parameters of a structure.

$$PM_{NS} = B \cdot \prod_{i=1}^{n} s_i^{d_i} \tag{42}$$

If the fractal dimensionalities are condensed into one fractal dimension, this becomes:

$$PM_{NS} = B \cdot S^{D} \tag{43}$$

This has become the most basic fractal dimensionality equation, with a coefficient *B*.

3.3, Sensitivity Analysis

A sensitivity analysis was conducted in order to determine the most important parameters in the various models: COSYSMO, COSYSMO with multiplication replacing addition in the size, and Fractal-COSYSMO. The results are as follows.

COSYSMO:

$$PM_{NS} = A \cdot (\Sigma w_e \Phi_e + w_n \Phi_n + w_d \Phi_d)^E \cdot \prod_{j=1}^{14} EM_j$$

PM	=	A	We	Sigma e	Wn	Sigma n	Wd	Sigma d	E	EM1	EM2	EM3	EM4
375,812		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
394,603		1.05	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
357,022		0.95	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
383,975		1	0.3465	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
367,690		1	0.3135	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
383,975		1	0.33	10500	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
367,690		1	0.33	9500	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
383,975		1	0.33	10000	0.3465	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
367,690		1	0.33	10000	0.3135	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
383,975		1	0.33	10000	0.33	10500	0.33	10000	1.3	1.1	1.2	1.3	1.4
367,690		1	0.33	10000	0.33	9500	0.33	10000	1.3	1.1	1.2	1.3	1.4
383,975		1	0.33	10000	0.33	10000	0.3465	10000	1.3	1.1	1.2	1.3	1.4
367,690		1	0.33	10000	0.33	10000	0.3135	10000	1.3	1.1	1.2	1.3	1.4
383,975		1	0.33	10000	0.33	10000	0.33	10500	1.3	1.1	1.2	1.3	1.4
367,690		1	0.33	10000	0.33	10000	0.33	9500	1.3	1.1	1.2	1.3	1.4
683,419	**Max**	1	0.33	10000	0.33	10000	0.33	10000	1.365	1.1	1.2	1.3	1.4
206,659	**Min**	1	0.33	10000	0.33	10000	0.33	10000	1.235	1.1	1.2	1.3	1.4
394,603		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.155	1.2	1.3	1.4
357,022		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.045	1.2	1.3	1.4
394,603		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.26	1.3	1.4
357,022		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.14	1.3	1.4
394,603		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.365	1.4
357,022		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.235	1.4
394,603		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.47
357,022		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.33

Result: E is the most important parameter.

An interim conclusion is that the parameter E in COSYSMO is often adjusted in order to match the calculated size of the system with the actual size of the system. Unfortunately, the use of E is problematic in the additive COSYSMO term, because E, although not determined from data, becomes the most important parameter.

COSYSMO with multiplication instead of addition in the size driver term.

$$PM_{NS} = A \cdot (w_e s^{d_1} \cdot w_n s^{d_2} \cdot w_d s^{d_3})^E \cdot \prod_{j=1}^{14} EM_j$$

PM	=	A	We	Sigma e	Wn	Sigma n	Wd	Sigma d	E	EM1	EM2	EM3	EM4
126,721,319,005,584		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
1.33057E+14		1.05	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
1.20385E+14		0.95	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
1.35019E+14		1	0.3465	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
1.18547E+14		1	0.3135	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
1.35019E+14		1	0.33	10500	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
1.18547E+14		1	0.33	9500	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
1.35019E+14		1	0.33	10000	0.3465	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
1.18547E+14		1	0.33	10000	0.3135	10000	0.33	10000	1.3	1.1	1.2	1.3	1.4
1.35019E+14		1	0.33	10000	0.33	10500	0.33	10000	1.3	1.1	1.2	1.3	1.4
1.18547E+14		1	0.33	10000	0.33	9500	0.33	10000	1.3	1.1	1.2	1.3	1.4
1.35019E+14		1	0.33	10000	0.33	10000	0.3465	10000	1.3	1.1	1.2	1.3	1.4
1.18547E+14		1	0.33	10000	0.33	10000	0.3135	10000	1.3	1.1	1.2	1.3	1.4
1.35019E+14		1	0.33	10000	0.33	10000	0.33	10500	1.3	1.1	1.2	1.3	1.4
1.18547E+14		1	0.33	10000	0.33	10000	0.33	9500	1.3	1.1	1.2	1.3	1.4
6.1512E+14	Max	1	0.33	10000	0.33	10000	0.33	10000	1.365	1.1	1.2	1.3	1.4
2.61059E+13	Min	1	0.33	10000	0.33	10000	0.33	10000	1.235	1.1	1.2	1.3	1.4
1.33057E+14		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.155	1.2	1.3	1.4
1.20385E+14		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.045	1.2	1.3	1.4
1.33057E+14		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.26	1.3	1.4
1.20385E+14		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.14	1.3	1.4
1.33057E+14		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.365	1.4
1.20385E+14		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.235	1.4
1.33057E+14		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.47
1.20385E+14		1	0.33	10000	0.33	10000	0.33	10000	1.3	1.1	1.2	1.3	1.33

Result: E is the most important parameter.

An interim conclusion is that the parameter E in COSYSMO is often adjusted in order to match the calculated size of the system with the actual size of the system. Unfortunately, the use of E is problematic in the multiplicative COSYSMO term, because E, although not determined from data, becomes the most important parameter. Here we get the bigger values of person months than in the previous case. Hence, we can easily find out the maximum and minimum values.

Fractal-COSYSMO:

$$PM_{NS} = A \cdot \prod_{i=1}^{n} w_i s_i^{d_i} \cdot \prod_{j=1}^{14} EM_j$$

PM	=	A	We	s1	d1	Wn	s2	d2	Wd	s3	d3	EM1	EM2	EM3	EM4
7.25472E+25		1	0.33	10000	0.8	0.33	10000	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.4
7.61745E+25		1.05	0.33	10000	0.8	0.33	10000	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.4
6.89198E+25		0.95	0.33	10000	0.8	0.33	10000	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.4
7.92066E+25		1	0.3465	10000	0.8	0.33	10000	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.4
6.6149E+25		1	0.3135	10000	0.8	0.33	10000	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.4
7.78275E+25		1	0.33	10500	0.8	0.33	10000	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.4
6.73818E+25		1	0.33	9500	0.8	0.33	10000	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.4
1.40806E+26		1	0.33	10000	0.84	0.33	10000	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.4
3.73784E+25		1	0.33	10000	0.76	0.33	10000	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.4
7.92066E+25		1	0.33	10000	0.8	0.3465	10000	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.4
6.6149E+25		1	0.33	10000	0.8	0.3135	10000	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.4
8.13212E+25		1	0.33	10000	0.8	0.33	10500	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.4
6.43419E+25		1	0.33	10000	0.8	0.33	9500	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.4
2.13118E+26		1	0.33	10000	0.8	0.33	10000	1.365	0.33	10000	1.8	1.1	1.2	1.3	1.4
2.46957E+25		1	0.33	10000	0.8	0.33	10000	1.235	0.33	10000	1.8	1.1	1.2	1.3	1.4
7.92066E+25		1	0.33	10000	0.8	0.33	10000	1.3	0.3465	10000	1.8	1.1	1.2	1.3	1.4
6.6149E+25		1	0.33	10000	0.8	0.33	10000	1.3	0.3135	10000	1.8	1.1	1.2	1.3	1.4
8.49716E+25		1	0.33	10000	0.8	0.33	10000	1.3	0.33	10500	1.8	1.1	1.2	1.3	1.4
6.14391E+25		1	0.33	10000	0.8	0.33	10000	1.3	0.33	9500	1.8	1.1	1.2	1.3	1.4
6.5316E+27	Max	1	0.33	10000	0.8	0.33	10000	1.3	0.33	10000	1.89	1.1	1.2	1.3	1.4
9.35453E+23	Min	1	0.33	10000	0.8	0.33	10000	1.3	0.33	10000	1.71	1.1	1.2	1.3	1.4
7.61745E+25		1	0.33	10000	0.8	0.33	10000	1.3	0.33	10000	1.8	1.155	1.2	1.3	1.4
6.89198E+25		1	0.33	10000	0.8	0.33	10000	1.3	0.33	10000	1.8	1.045	1.2	1.3	1.4
7.61745E+25		1	0.33	10000	0.8	0.33	10000	1.3	0.33	10000	1.8	1.1	1.26	1.3	1.4
6.89198E+25		1	0.33	10000	0.8	0.33	10000	1.3	0.33	10000	1.8	1.1	1.14	1.3	1.4
7.61745E+25		1	0.33	10000	0.8	0.33	10000	1.3	0.33	10000	1.8	1.1	1.2	1.365	1.4
6.89198E+25		1	0.33	10000	0.8	0.33	10000	1.3	0.33	10000	1.8	1.1	1.2	1.235	1.4
7.61745E+25		1	0.33	10000	0.8	0.33	10000	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.47
6.89198E+25		1	0.33	10000	0.8	0.33	10000	1.3	0.33	10000	1.8	1.1	1.2	1.3	1.33

Result: Largest *d* fractal dimension is the most important parameter.

In this case d is our exponential term which is often adjusted in order to match the calculated size of the system. While, considering the Fractal theory we should know that fractal dimension d is the most important factor because it shows the important form of the fractal structure and also it helps in quantifying the degree of irregularity. Fractal dimension d is a measure of extent to which the fractal cluster fills the space available. Here we can see that the highest value of d gives the maximum value of person month, whereas, the lower value of d gives the minimum value of person month.

Conclusion:

Any exponential or fractal dimensional parameter will be the most important parameter in the examined cost estimation formulas. Fractal dimensionality is the principle driver of cost in fractal systems.

CHAPTER IV: Applications of Fractal-COSYSMO

The purely physical cost of these systems could be estimated by an aggregation of power law expressions. However, estimation of the cost of the real system will involve factoring in COSYSMO Cost Parameters, which will be different for each system type, because these factors account for the inherent cost differences between system types. For example, although a communication network and a city may have common power laws that account for the number of components, the effort involved in constructing each unit in the two different systems will be significantly different.

4.1. Network Construction

A, Cellular networks, newly installed

- Computer networks
- Fiber optic and cable networks

B. Electric Vehicle Charging Networks

Networks can have a great variety of general topologies, including that of a bus, ring, star, tree, mesh, array, or torus, among others. Star networks joined together form tree, mesh and array network. Cui, Kumara and Albert (2010) list 10 engineered network types, including the internet, supply chains, service networks, energy and powers distribution networks, transportation networks, tourism networks, communication/mobile networks, crowd sourcing, malware heredity and provenance detection, and product networks.

A network node can be characterized by its *degree*, or the number of connections, *k*, to other nodes. A network is characterized by its *degree distribution*, that is, a function giving the probability or fraction of network nodes according to degree.

Many real networks, such as social networks and the internet, citation networks, biological networks, and airline networks are *scale-free* because their degree distribution follows a power law with the form:

$$f(x) = wx^k \tag{44}$$

Power laws are scale free because multiplying the argument by a constant preserves the shape of the function.

$$f(cx) = w(cx)^k = (wc)x^k = bx^k \tag{45}$$

Power laws follow the same functional form as fractals, with scale invariance being the mathematical expression of the self-similarity of fractals at all scales. Power laws conformance is a characteristic of complexity in the natural world. Power laws have a ‘fat tail’ that provides that phenomena of any significance have a non-vanishing probability. For example, arbitrarily large stock market movements will occur, given sufficient time.

Real phenomena that follow power laws include city size, popularity, net worth, stock market movements, natural disasters, earthquake magnitudes, power outages, the internet, service networks, and energy and power networks, to name a few.

The expected form of a fractal complexity within fractal complexity is therefore:

$$f(x_1, x_2, \ldots x_n) = w_1 x_1^{k_1} \cdot w_2 x_2^{k_2} \cdot \ldots \cdot w_n x_n^{k_n} \tag{46}$$

For example, this equation can characterize: (1) the total number of rooms, in the buildings, in the cities of a country, (2) the total number of component requirements within the sub-system requirements of a system’s requirements, or (3) the total number of assembly procedures within a sub-assembly, given a nesting of assemblies. In other words, the equation is well suited to characterize the total number of units within larger units, with the dimensionality of the units

changing as the scale changes and transition points are crossed. Thus, a multi-tiered, multi-medium, nested structure can be mathematically described.

Fractal-COSYSMO, because of its formulation as multiplication of power laws, can be employed to estimate the costs of a complex structure, including both the physical construction costs, and the engineering costs. For example, Fractal-COSYSMO can be used to estimate the costs of a cellular telephone network that is newly installed, including backbones, trucks, feeders and towers. The costs considered could include the cost for repeater stations and transmission towers, along with the cost to install, adjust and tune such equipment. Of course, rarely are cellular telephone networks installed all at once, and for this reason costs are not usually estimated on a whole-network basis, although there are many example of fiber optic or cable networks installed all at once.

A pure fractal formulation of cost estimation will ideally be applied to a fractal like system that is being newly built. That is, any system that is built incrementally over a long period of time, may be funded one piece at a time, negating a total cost estimation according to Fractal-COSYSMO. Nevertheless, there are some systems that can benefit directly from Fractal-COSYSMO cost estimation. The cost of limited networks, such a computer Local Area Network (LAN), could be expected to be needed more often. Electric Vehicle (EV) charging networks can be expected to be installed in their entirety in the near future. The sized of installed network parts are probably modeled with a power law.

4.2. Fractal Cities:

The Fractal Conceptual Foundation

Taking the whole fractal summary into account, the more interesting and challenging part that came ahead was, how this idea of self-similarity can be applied to the structure of cities? Coastlines and man-made boundaries, of course, show self-similarity; but the distribution of cities and their arrangement as central places, in terms of both size and spacing, illustrate clear and unique ordering.

Many researchers applied their theories, as well as many analogies between city and physical systems prompted by Stewart and Warntz (Stewart 1958) amongst others as 'social physics', have dominated the development of urban theory and regional science since the 1950s. Although this concern has not emphasized the morphology of city systems, and there has been a lot of research concerned with spatial form which is consistent with social physics on one hand and the economic operations of cities on the other.

The concern for forms is best seen in many analogies in geography between physical and human systems- for Example Rivers and central places, as well as in various attempts to research the growth of cities using allometry which means the measurement or growth of city. Since, the popularization of fractals during the last decade, a number of discussions in urban geography and regional science noted above have been picked up and developed with renewed vigor. The use of computer graphics in simulating the irregular form of cities in terms of land use development (Batty 1986), work on map generalization and cartography using fractal ideas, research into how central places can be generated through their hierarchy using deterministic fractals (Arlinghaus 1985) the measurement of urban boundaries and edges (Batty 1988), these are few of the applications of fractal geometry to city shape, order and form. Classification of cities according to their various shapes has always been an important influence on the way cities are planned and

conserved. Cities are frequently conceived in terms of simple Euclidean geometry, for example linear and concentric forms of ideal city proposed by Le Corbusier and other such architectural practitioners. The planning system as it grows is based upon guiding and manipulating physical change in terms of spatial, hence geometric policy instruments ranging from new towns to the containment of urban growth. This shows that fractal geometry have something important to contribute here.

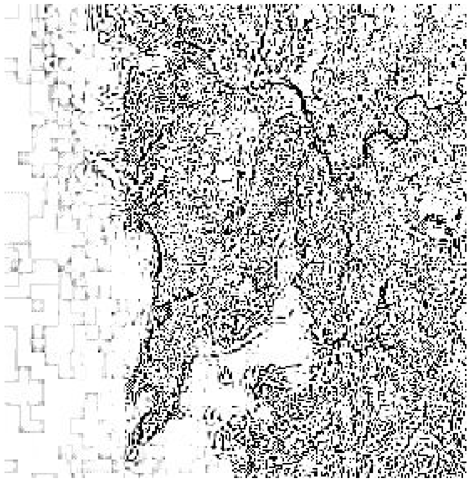
There is another reason why fractal geometry must be of concern to urban researchers and city planners. A stock of sophisticated techniques for analyzing and predicting urban structure has been assembled over last decades, but their relation that this stock or arsenal to the urban form is uncertain. There are some economic and demographic activities carried out in the city and the models for this economic and demographic activity are largely based on representing spatial form at a level of abstraction from which the city geometry cannot be easily defined. Models are built on a data at the level of the census tract, while in other cases none of the attempt is made to represent the spatial dimension whatsoever or define it using simplistic and often inappropriate conceptions of Euclidean geometry. Thus the type of fractal geometry which we used here lead to models of urban structure which are in no way alternatives to contemporary particles, but which are in every sense complementary to existing approaches.

The major approach here will be to frame how fractal processes can generate highly ordered clusters of particles in two-dimensional space, and we suggest that the methods and models of these processes recently developed for physical systems might constitute useful analogies for city growth. The processes here generate clusters which are far-from-equilibrium,

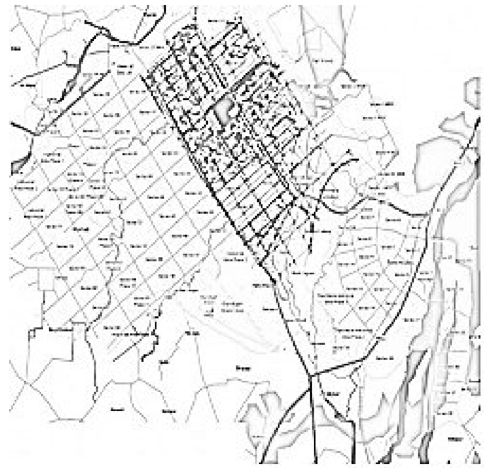
in that their growth processes are irreversible. The clusters produced here are tree-like structures-- dendrites--which shows self-similarity in their branching, and which apply to a range of physical systems all driven in some way by diffusion of particles from some source. For example physical growth such as electro-deposition, crystallization, and dielectric breakdown all have been recently simulated using fractal growth processes. A recent review is provided by Vicsek, but there has been a tremendous application since the early 1980s, Witten and Sander (Witten 1981) provided an original suggestion of a simple model of a diffusing particle whose behavior could be simulated as a random walk on a two-dimensional lattice. This model is based on the aggregation of particles, one at a time, whose diffusion is limited or constrained by a fixed field of influence around the growing cluster, and by the fact that once a particle reaches the cluster it sticks permanently. This model is called the 'Diffusion Limited Aggregation' or the DLA model.

4.2.1 Urban Form, Growth and Pattern (Scale, Size and Morphology)

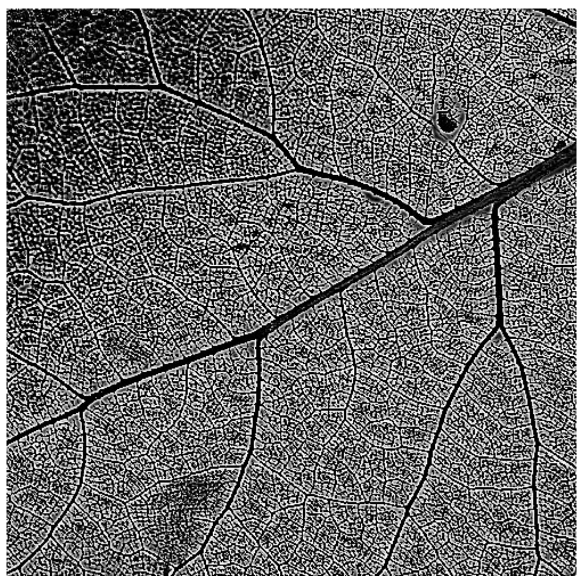
The structure or form of cities can be visualized in many different ways and at many levels of absorption. It is therefore necessary to be completely clear as to the type of form in question. In Figure 17 we have showed the urban development and the urban field of three large cities (Also can be called as western cities)-Mumbai (Including New Mumbai), and Chandigarh



(a)



(b)



(c)

Figure 17: The Morphology of larger cities in India compared to the morphology of leaf.

This shows how cities grow into each other, as well as the fact that cities of different sizes at different scales have clear self-similarity of form when all three examples in Figures (17 a) to (17 c) are compared.

It is clearly possible to detect the tentacles of development coming out from the central core of the cities in question shown as in Figure (17) above. These tentacles have a dendritic form in that they follow the main lines of transport or other forms of communication from the core, or Central Business District (CBD), to the suburbs. The cities suburbs are opened up through the provision of transport infrastructure radiating out from the core, and most western cities which rely heavily on the automobile for personal transport are characterized by such strip or ribbon development. In one sense it is the transportation system which has the fractal structure of dendrites on which the rest of the city's development is hung. There is a self-similarity in the way the dendritic are been seen.

West and his colleagues have recently shown that as cities grow in size, physical networks tend to grow more slowly than city size; that is, the physical infrastructure used to move resources around does not increase as fast as the number of such resources, whereas key economic activities such as the number of innovations as measured through financial services, patents, and scientific products increase faster than city size in terms of population. Thus, big cities appear more attractive to the most productive industries, but it is easier to move resources around in small cities. The, urban form is not merely the architectural form of the city (Lozano 1990). It is also a cultural manifestation.

Land ownership patterns, technology, transportation, communication and socioeconomic relationships influence urban patterns. Intricacies in relationships have increased the complexity of the urban form over time. The pattern of spatial distribution is recognizable in most contemporary cities (Alexander 1987) Thus urban social patterns are complex manifestations of underlying cultural values intermingled with global economic forces (McGee 1971).

Although details may not be identical, every city has certain elements. Doxiadis defines five elements in the study of human settlements. They are nature, human beings, society, buildings and infrastructure. Urban spatial patterns occur because of the repetitive spatial distribution of these elements. The patterns have similarities, which may be universal or local. "The typical sector represents the formal characteristics found throughout the area and thus acquires some universality" (Lozano 1990). Since the characteristics are universal (within the frame of study) they may be studied by a spatial representative sector. This representative sector is defined as the smallest area that exhibits the characteristics of the urban settlement. In most studies this unit is the neighborhood which displays both physical and social aspects of the whole urban development. Urban patterns represent a continuity of time and space. Time and place may provide them with different characteristics making each city unique and dynamic.

4.2.2 Factors Affecting the Urban Form:

Many factors influence the form of cities. Traditional settlements were shaped by (Lozano 1990):

- the way in which nature and man-made features satisfy needs for protection and defense
- the way in which physical and economic landscape allows for communication with other regions
- the way in which the topography of a site suggests the construction of a human settlement
- the way in which climate leads to building solutions

These factors influence the cultural and spiritual form of the cities as well. Traditional cities have used physical forms to interpret cultural and religious beliefs (Lozano 1990). For Example, a hill top site was the utilitarian response to any important building - a fort or a religious building. These features contributed to a particular urban and social pattern. The

physical form is a variable of the social and built pattern of the city. The built form is influenced by factors as (Alexander 1987):

- land ownership
- existing land use
- planning regulations
- street patterns
- economic considerations
- political and historical events

The physical expansion of the city is always bound and guided by land ownership, and natural and manmade obstacles. A city replaces existing land use. Thus, it is necessary to determine existing land use as a pre-condition to urban growth and form. The change of land use from rural to urban depends on the existing land use, and the ownership. Plots of varying sizes and shapes influence the layout of the streets and of individual buildings (Knox 1995). Planning controls influence development to a great extent. Master plans and regional plans provide long-range strategies for development. Various economic, social and political circumstances influence the social pattern (Scargill 1979). While some processes are culture-specific, others are global in scope. These factors are [(Alexander 1987), (Kosambi 1986)]:

- ethnic composition of the city
- religion
- race
- migration
- economic considerations
- political and historical events

4.3 Perfect Alternator of Urban Growth: (Box Counting Method)

From Figure (18) it is clear that cities do not grow in dense compact clusters; which means that their fractal dimension lies between 1 and 2. In Figure (18) we have compared the morphology of larger cities with the structure of leaf (Zoomed in). If we compare the structure of leaf to cities, the growth of dendrites in cities is same as that appears to be on leaf, these might also be one reason that the architecture's might have considered this thing into account and must have planned the cities. In short, the dendrites of growth which extend out into the countryside or urban field find the space between them from receiving further growth.

In this topic we need to be clear about what constitutes development; this includes all built structures which at any particular instant in time require complex demolition or institutional change for the space they occupy to be released for new growth. Thus we challenge urban development to be irregular, in that the urban fabric is likely to be slotted with holes or sites which have not been developed so far. The model which we are going to propose now meets the requirements. Following Vicsek (Vicsek 1989), case of a static structure will be examined first which has already been grown. A growing fractal's self-similarity will be examined first and then we will compute its self-similarity before going on to the details. In Figure18 we show a particular type of structure across four levels of infer and finer details or scale. We can also imagine that the instrument we have at our disposal for detecting the structure can be magnified to these finer scales and is thus able to see more details. In Figure (18A) as we go down scale we see that the pattern at each scale repeats itself and is clearly self-similar. Also, as we know that the fractals have space filling properties, it does not fill the whole of the space available. Here, we will use some mathematics. If you look at the pattern, the pattern is based on a regular

generator in which the units of space at one level k contain five units of development at the next scale down, k + 1. If we assume that the structure has a one-dimensional length scale L which is the side of the square at scale k = 0, then the unit of space at scale k, ξ_k is given as

$$\xi_k = \frac{L}{n_k} \tag{47}$$

Where,

n_k = Is the number of equal unit lengths into which L is divided.

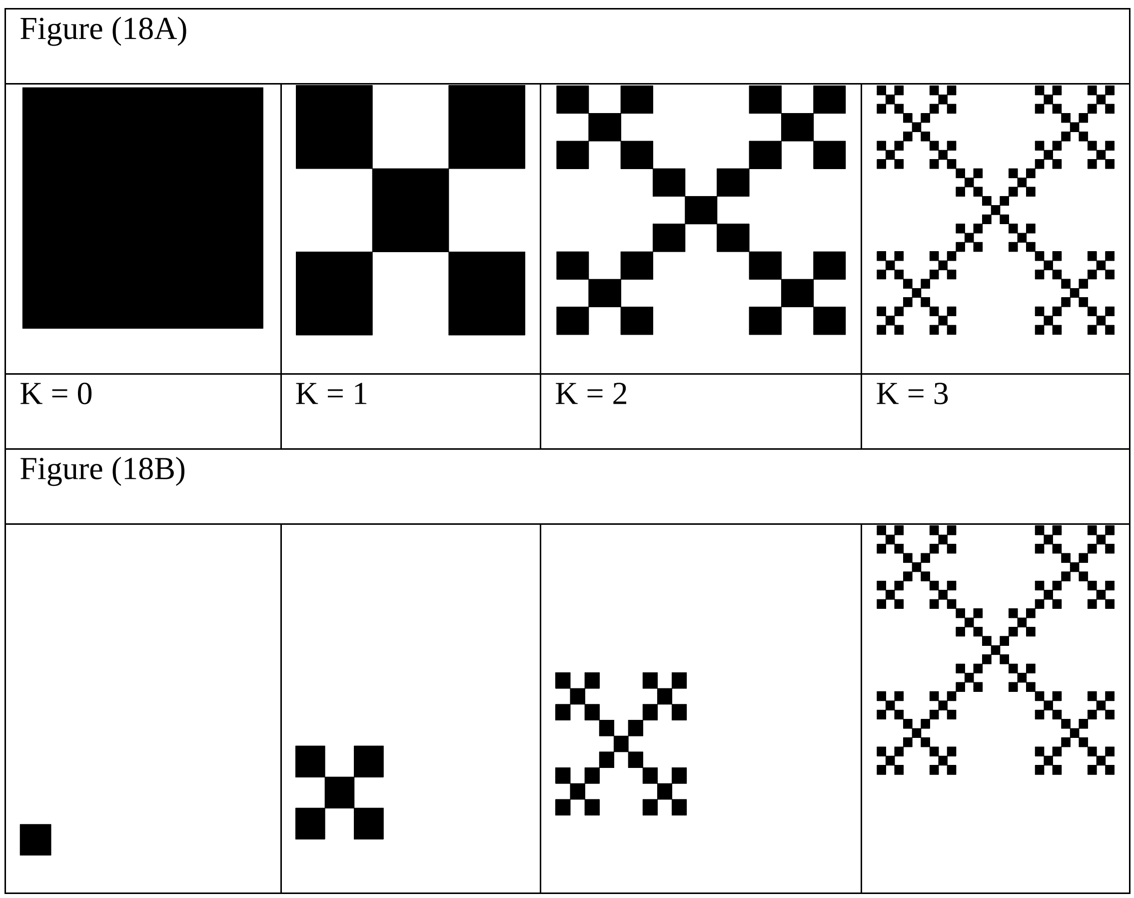


Figure 18: Scale in Static and Growing fractal

Clearly from Figure 18, where k = 0, n_k = 1; where k = 1, n_k = 3; where k = 2, n_k = 9 and so on. In short, $n_k = 3^k$, and this represents the regular pattern of subdivision. Therefore, at each level k

the total number of units into which the area L^2 is divided into $n_k^{\ 2}$. In fact, the number of squares which are occupied or developed is given as N_k which is less than $n_k^{\ 2}$. Thus the number of developed units can be written as

$$N_k = n_k^{\ D} \tag{48}$$

Where D is the fractal dimension and 1 < D < 2. In terms of scale ξ_k this relation can be written as

$$N_k = \left(\frac{L}{\xi_K}\right)^D = G\xi_k^{\ -D} \tag{49}$$

Where the constant of Proportionality $G = L^D$. If the fractal is deterministic it is regular at each scale, as the object clearly is over the range k = 0, 1, …, k, then from Equations. (48) and (49) the fractal dimension can be written directly as.

$$D = -\lim_{k\to\infty} \frac{\log N_K - \log G}{\log \xi_k} = -\lim_{k\to\infty} \frac{\log N_K}{\log \xi_k - \log L} \tag{50}$$

This reduces to

$$D = -\lim_{k\to\infty} \frac{\log N_K}{\log \xi_k} \tag{51}$$

If L, which is arbitrary, is set equal to 1, which is the usual convention.

For the regular fractal in Figure (18A), with K = 1 and $N_k = 5^k$ and $\xi_k = 3^{-k}$, substituting into Equation (47) gives $D = \dfrac{\log(5)}{\log(3)} \approx 1.465$, which is the fractal dimension, a measure of the extent to which the fractal in Figure (18A) fills its available space. What this effectively means is that although the number of units compromising the square object increases as the square of the

scale, the total number of units occupied or 'developed' increases at less than this, as the fractal dimension D, which is less than the square but greater than the unit power. This means that the density of occupation in fact decreases as the overall size of the fractal increases, this is a point we will be taking later.

The method discussed above is quiet acceptable for determining the dimension of existing cities shown in Figure (18). But it should be adopted to fit in some statistical estimation of D across a range of scales, due to the fact that the self-similarity in such fractals would vary randomly within some range. But in order to examine the dimension of a growing object, the logic or idea which was developed earlier can be stretched in a different although matching way. In Figure (18B) we show the creation of a fractal object across four scales, from a unit of development when k = 0 to a structure which is 27 times as large when k = 3. In Figure (18B), what we tried to show is, we grow the fractal from its basic seed, which now has a linear dimension ξ, to a cluster of seeds which spans the whole of the fixed space with linear dimension L, within which growth takes place (Batty 1986). This is a unique model of how a city might grow

It is clear from the figure that the structure can continue to grow, but we have terminated it at $k = 3$ so that it can be same to the static structure in Figure (18A) at this stage of its growth. It is important to note that k only represents the space scale, although in some cases it might be thought of k as representing both space and time scale. We can define the number of units containing the linear dimension of the cluster as

n_k and L_k as the linear scale of the cluster grown up to k. So,

$$n_k = \frac{L_k}{\xi} \quad \text{or} \tag{52}$$

$$L_k = n_k \xi \tag{53}$$

From Figure (18B) we can say that the total number of units N_k containing the structure increases at less than the square of its dimension L_k but greater than unity. Assuming the growth power is D then N_k is given as.

$$N_k = L_k{}^D = (n_k \xi)^D \tag{54}$$

If we assume ξ as constant and equal to 1, Equation. (54) Is then identical to Equation. (48). A simple manipulation of Equation. (50) Yields Equation. (45). Hence it is clear that the fractal Dimension D is the same as shown in Equation. (50) and Equation. (51), an assumption is made that the physical scale of N_k is different, and Figure (18) confirms this geometrically, depending on whether the analysis is for a static or growing fractal.

Finally, when we look at the density of occupation of the growing fractal, we see that the analysis is identical for the static case. Let n_k be some measure of radial distance within the cluster defined as R_k, and that the area of space within which the cluster is growing at any stage k is proportional to $R_k{}^2 = n_k{}^2$. The density of occupation for the growing fractal defined as ρ_k assuming that $\xi = 1$ is,

$$\rho_k = \frac{N_k}{R_k{}^2} = R_k{}^{D-2} \tag{55}$$

Equation (55) implies that the density decreases as the cluster grows larger. One might expect this kind of physics in a growing city.

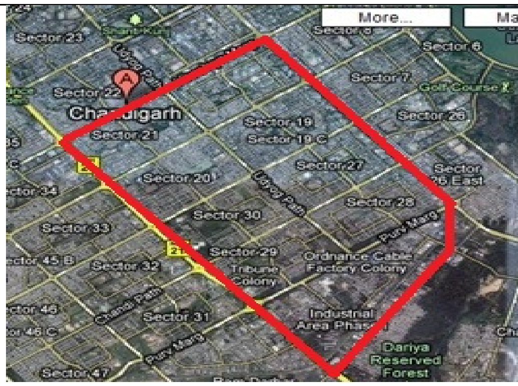
Because of the deterministic model described here one thing is clear that, the growth process is irreversible, in which once a unit of development has been located there is no process whereby this can be changed. In the cities of the West, new growth can account for as little as 20 percent of the change per annum in location patterns, although if only new physical development is

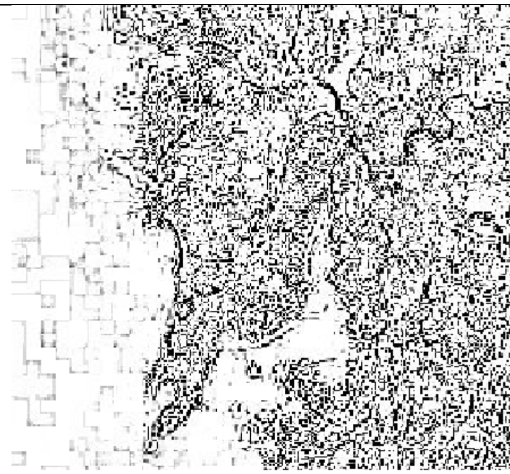
considered then this proportion can be as large as 90 percent (Batty 1986). The details shown in the urban pattern in Figure (17), because of new growth, major changes would be seen in cities over time. City generated through its social and economic progress cannot be handled by this type of theory; some different theory is used for city generated through social and economic progress. In order to understand the growth of fractals in more details a model called as ‘Diffusion Limited Aggregation’ (DLA) model was proposed by Witten and Sander which can be applied to the growth of cities. Before this model let us take a look on design methods of cities built on box-counting method.

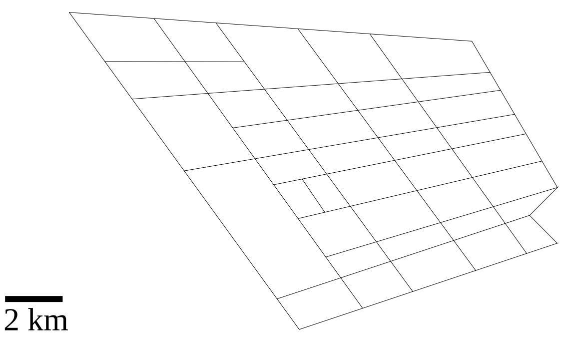
4.3.1 Example of Box-Counting Concept Applied to the Indian Cities- Helps In Knowing The Urban Form of The City:

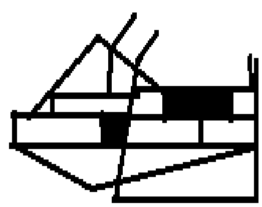
Figure (19) shows the example of the cities where the part of city under observation is shown and a box-counting method is applied**.** The theoretical design of Navi Mumbai was developed at the height of Modernism Le Corbusier had played an important role in the design of Chandigarh in Punjab in the mid-1950s (Corbusier 1961). Some of the interesting part in this design principle plan was sector planning, hierarchy of roads and important buildings of a gargantuan scale (Fry 1977). The sector was based on the Spanish *cuadra* (city blocks) of 110 to 100 meters. Each of these *cuadras* was an independent unit with primary schools, community centers and residential areas. Le Corbusier's approach was based on standardization, that is, the creation of a modular system and the use of rough concrete and glass.

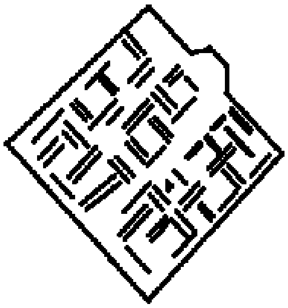
Comparison of Planned Cities in India Depending on Box-counting Method (Chandigarh and Navi Mumbai)	
Chandigarh City Foundation: 1966 Population: 900,635 Area : 114 km^2 Architect: Le Corbusier Le Corbusier's basis for the plan was 'sector'. A classified circulation pattern resulted from his theory of the 7 V's. There were 17 sectors included in the first phase of the plan each $1200m \times 800m$ in area. Ideal city	Navi Mumbai City Foundation : 1972 Population : 2,100,000 Area : 163 km^2 Architect : CIDCO This city was initially planned by CIDCO one of the biggest company, with a specific purpose, this city acted as an alternative city for great Mumbai city. The planning was to build large buildings and industrial areas which separate from each other. The planning of large buildings was to fit in such a huge population. The length of this city is almost the same as that of Mumbai.

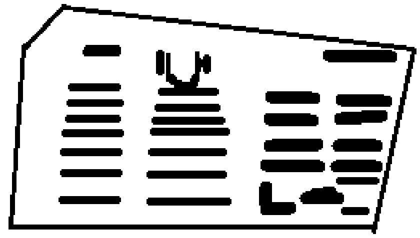

More...
Sector 23
Sector 22
Sector 21
Sector 19
Sector 19 C
Sector 7
Sector 6
Golf Course
Sector 26
Sector 27
Sector 20
Sector 26 East
Sector 28
Sector 30
Sector 33
Sector 29
Ordnance Cable Factory Colony
Tribune Colony
Sector 32
Sector 31
Industrial Area Phase I
Dariya Reserved Forest
Sector 47
Purv Marg
Udyog Path

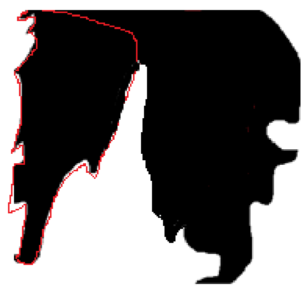


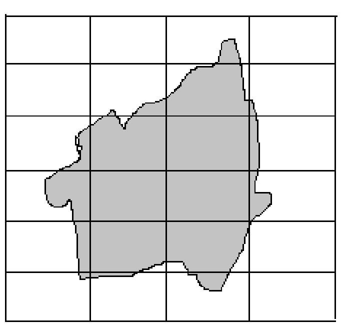

2 km











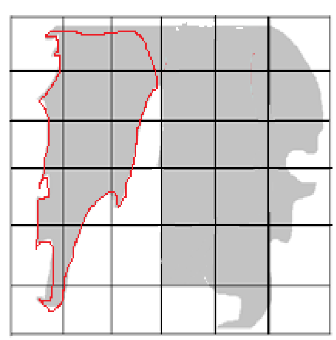


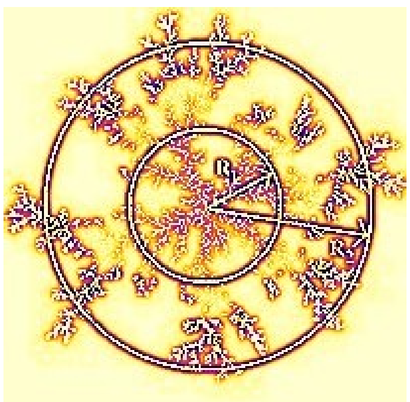
Figure 19: Box- Counting principles used to design Indian cities like Chandigarh and Navi Mumbai

4.4 Process for Growth of Fractal:

Diffusion-Limited Aggregation: The DLA model (Radial Analysis)

During 1900s, using a very simple form of diffusion and aggregation Witten and Sander [(Witten 1981), (Witten 1983)] suggested a primary hypothetical model which reproduced increasingly rich, complex but regardless ordered, dendritic clusters of particles. DLA is a process which generates a single cluster of particles over a network of sites. The network is often referred to as a lattice, which is nothing but a normal arrangement of points which fills the space. The model's name suggests a pattern whereby simple particles stick to a self-revolving structure. This sticking is determined by a set of rules. This aggregation of particles can be said as diffusive processes which are handled by specific laws that limit the development of the cluster over the lattice. Our main interest is a typical system based on a square two- dimensional lattice; we have already shown an example in Figure 18, in which a particle is placed at the center. At a larger distance from this central point a particle is launched and begins a random walk to close by lattice sites, one step at a time, entering the field of influence around the growing cluster which is from the launch point. Once the field of influence (which may or may not be synchronous with the launch site zone) is entered, the particle is either destroyed if it leaves the field or attached to the growing cluster when it reaches a lattice site at the edge of cluster (Batty 1989). In this way the cluster grows, until a certain size is reached. The result of the cluster obtained is dendritic; the dendritic cluster is obviously self-similar and thus is fractal. Now the question which arises is how this cluster is formed? When the seed is first planted each of its adjacent sides have an

opportunity of being occupied. Once this selection is made, then the side adjacent to the one which was chosen has a slightly higher probability of being chosen at the next point there is a much more probability of the other adjacent side. As this process continues further the clusters goes on increasing with theirs tips having much more probability of being chosen than those sites which are located in the opening between. Figure 21 below shows a simulated cluster using a DLA model on a 500 $\times$ 500 where something over 10,000 particles have been clustered.



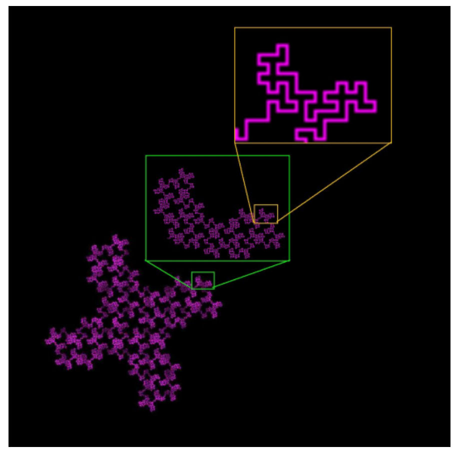


Figure 20: A simulated cluster using DLA model.

Figure (21): A simulated cluster using DLA model.

The structure in Figure (18) and Figure (20) are similar having the fractal properties. Here we can see that the density of cluster decreases as it gets larger, same as that was suggested in the case of deterministic fractal of the previous section. In comparison to Equation. (51), the number of particles at radial distance R from the seed site, given as N (R) scales as

$$\mathrm{N}\,(R) = \mathrm{G}\,R^{D} \tag{56}$$

And the density scales as

$$\rho(R) = ZR^{D-2} \tag{57}$$

G and Z are constant of proportionality, and D the fractal dimension, must be less than 2 but greater than 1. Witten and Sander's (Witten 1981) has done a lot of research into such process. One of the most important finding was that the fractal dimension D appears to be approximately 1.71 ± 0.02, this method has been used in many cases for the lattice up to about 1000×1000 in size. Latest research has shown that the fractal dimension depends upon the size of lattice and the number of particles grown in the cluster of lattice.

A lot of research has been carried out in order to extend the simulations for three or more number of dimensions. Muthukumar (Muthukumar 1983)suggested that the fractal Dimension D for the two-dimensional system gives a value of $D = 1.66\ldots$ After all, the exact value of dimension is not important, most important is how, particles compromising the clusters and their density scale behaves in a simple way with the dimension.

Another model called the Dielectric Breakdown Model (DBM) can also be used for the city growth process. The way the growth occurs in DBM is opposite to that of the DLA model, in DBM the growth takes place from the center and expands out, whereas, in DLA model the growth is formed by the movement of the particles from the boundary of the field towards the center of cluster. In these terms the DBM approach would appear to be more characteristic of city growth, although the process in both approaches is one of balancing push with pull (Batty 1989). Niemeyer, Pietronero and Wisemann (Niemeyer 1984) provided the first and most complete model in this form for the dielectric breakdown problem. Both the processes DLA and DBM produce statistical or stochastic fractals. The tree-like pattern formed by this processes have been accepted commonly and also it has been said that many physical systems are close to

this kind of form. From, the study of city form it has been clear that there are much kind of cities which are both complex and scattered than the cluster produced by fractal growth.

4.5 Evaluating the Fractal Dimension:

As per the discussion, we can say that DLA model is one of the important and useful method for the fractal growth process, also it is a simple model to be used. Now, let us talk about evaluating the fractal dimension. The dimension, D is one of the important measure of the extent to which the fractal cluster fills the space available, the dimension, D can vary from 1 to 2 and is 1.71 in the pure fractal case. The dimension is usually estimated from Equation (56) and (57). Equation (56) shows that the collective number of particles N(R) at a distance R scale at R^D. There are also other function which can be used to determine the fractal dimension, for example the branch counts based on {$dN(R)/dR$} proposed by Pietronero, Evertsz, Weismann (Pietronero 1986). The details of all the above relationship and their calculations are given in Batty, Longley and Fotheringham (Batty 1989), where it is clear that the ***fractal dimension can be easily derived from the slopes of the associated log-log regression.***

There are several problems associated with the above discussed statistics.

1. There are severe boundary effects in the relationships between population and distance.
2. At, the boundary of the growing cluster more and more sites which would develop ultimately have not yet been developed because the cluster is still growing; because of which the fractal dimension decreases.

It is therefore, necessary to eliminate the large area of this growing band from the calculations. Second, at short distance there are also some severe effects from the points around which these different relationships are formed, and this too must be eliminated.

Because of all this eliminated zones the fractal dimensionality can vary quiet essentially in the same cluster. Different values of D for the cluster shown in Figure 20 for DLA and DBM models are given in the table below.

Table 2: Showing different values for DLA and DBM methods for the cluster shown in Figure 20

DLA	DBM
• The values of D for the one-point functional relationships ranged from 1.174 to 1.686 for the entire cluster and from 1.659 to 1.739 for the cluster with short and long range edge effects • For the two-point relations, D varied from 0.161 to 1.136 for the original cluster and from 1.640 to 1.677 for the modified one	• The value of D for unmodified one-point functions varied from 0.994 tp1.317, while the modified form gave the value of D ranging from1.376 to1.737 • For the two-point function the unmodified range is much wider, from 0.005 to 1.311, while the modified range is from1.537 to1.646, where 1.646 was the best estimate with a perfect fit of $r^2 = 0.999$

Each simulation produces different and unique cluster. When the simulation is carried out it is most important to take the averages after several runs, this method should be carried out in order to obtain the best overall dimension.

Appendix:

4.6. Space, Time and Fractal Analysis of the Urbanization of Indian Mega Cities Useful For Cost Estimation of Cities. (Useful for Cost Estimation)

4.6.1 Space and Time Analysis*:*

Cities are considered to be complex systems with a distinct hierarchical order, because of the complex and emergent structure of the cities lot of question have been raised during the past two decades upon the understanding of Urban phenomena.

It is necessary to show the space and time analysis of the urbanization of Indian mega cities. The reason why I selected this analysis as a part of my thesis was its importance, which was reflected from the previous case studies, which helped in understanding the urbanization of the Indian mega cities, the space and time data proved to be helpful in understanding the city's complex urban growth and, also how the city grows forcefully on the land which is available, how the analysis of the different parameters and other remote sensing data proved to be independent and cost -effective. Finally, the research helped me to prove "Cities as fractals"

The space and time analysis of the urbanization of Indian mega cities using the time series of Land-sat data will tell us about the urban footprints and the changes in two Indian mega cities like Mumbai and Delhi, since the 1970s. Also, the spatiotemporal analysis (both in space and time) helps us analyze different parameter's like Urban growth rates, built-up densities, direction or growth, or landscape metrics like the shape index or patch density which enables us to identify the similarities and dissimilarities in urban characteristics of Indian mega cities, the main idea behind this approach was to understand the emerging growth pattern to support the planning process in future. In other words, we can also say that for the space and time series analysis of the large urban areas of the Indian mega cities all this data proved to be an ***independent*** and ***cost***

-effective. Land-sat gives us the series of earth observations than have been continuously available since 1972. Measurement of areal coverage and spatial distribution are both needed to describe the morphology of an urban area adequately (Schweitzer et al. 1998). Later on a accuracy assessment of the classification of Land-sat data was carried out for the Indian mega cities which is shown in the Table 3

Table 3: Accuracy assessment of the classification of Landsat

	Landsat MSS (Multispectral Scanner)	Landsat TM (Thematic Mapper)	Landsat ETM (Enhanced Thematic Mapper)
Mumbai	87.0%	90.4%	90.8%
Delhi	89.4%	-	91.8%

Note :

Landsat MSS (Multispectral Scanning) : The effective IFOV (Instantaneous Field of View) of the MSS detector in the cross-track direction is considered to be 68 meters which corresponds to a nominal picture element (pixel) ground area of 68 by 83 meters at the satellite nadir point. Using the effective IFOV in area calculation eliminates the overlap in area between adjacent pixels. (http://www.iwmidsp.org/dsp2/rs-gis-data/River-basins/Limpopo/01-Landsat-MSS 52x79m--basin-mosaic-streached/ReadMe/readme-first.pdf)

Landsat TM (Thematic Mapper): Landsat TM can provide users with coarse scale imagery that covers large areas at a relatively low cost. (http://www.fas.org/irp/imint/docs/remote-factpg1r.pdf)

Figure 21 shows the result of change detected in Indian mega cities, showing the urban footprints and their spatiotemporal growth since the 1970s.(Berger 2007; Taubenböck 2007; Taubenböck 2008)

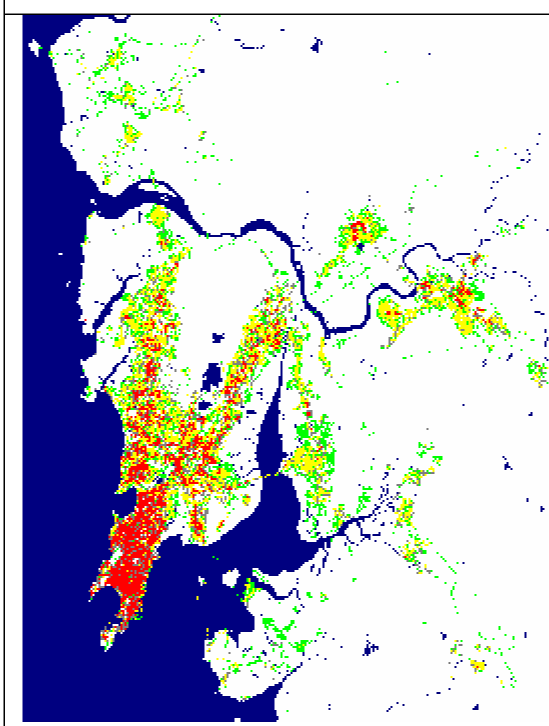
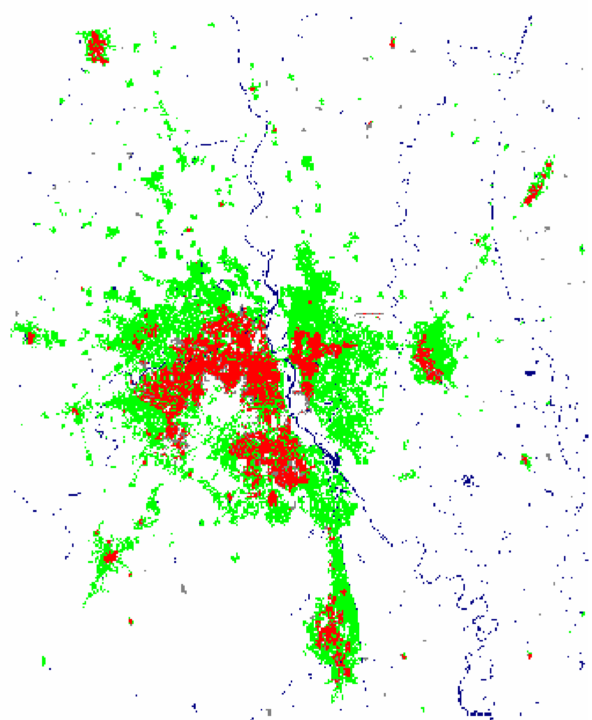
Land Cover Changes in Mumbai	Land Cover Changes in Delhi
 Mumbai 1973 1990 2001	 Delhi 1977 1999

Figure 21: Change detection of urbanized areas in Mumbai and Delhi

The result, from this change detection for the two Indian mega cities was very different. If, you look at the change detections of Mumbai you can see that the urban foot prints of Mumbai is determined by the coastal and hilly orography (Physical Geography dealing with Mountains), whereas, Delhi is not subjected to any orographic restriction.(Berger 2007; Taubenböck 2007; Taubenböck 2008)

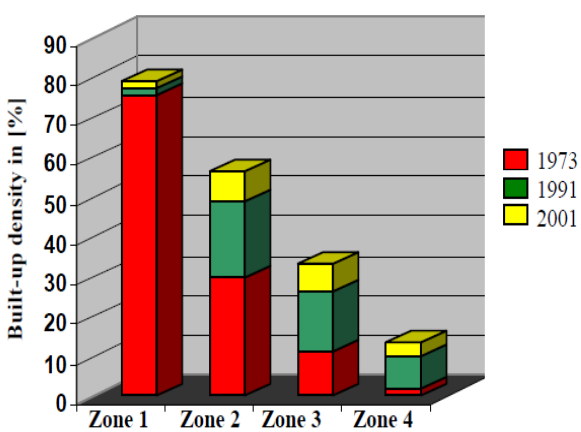
The peninsula of Mumbai forces the urbanized areas on available land, with an axial growth in the outskirts caused by transportation networks and hilly barriers. The polycentric structure and the development of the satellite cities in the 1970s steadily increased due to land shortage in the urban center and dramatic population pressure. The result which we see is a complex urban footprint, spatially polycentric with axial growth lines. The urban footprint of Delhi which shows a very little orography, results in a concentric urban ring-shaped growth with axial growth sectors caused by transportation networks.

This space and time study uses Landsat data for large-area analysis to survey urban growth and its form based on built-up and non-built-up areas. To obtain very high details of the structural analysis of the heterogeneous inner structure of urban morphology, satellite data with higher geometric resolution (f. e. Ikonos or Quickbird), are needed. We can say that urbanization may be attached with the details related to the transportation, land use, social structure and economic type, but is generally related to demography and economy in a city. In this topic, urbanization is analyzed by spatial urban form and its change over time. We have chosen the parameters like aerial growth, built-up densities for the gradient (rise) analysis. We also chose

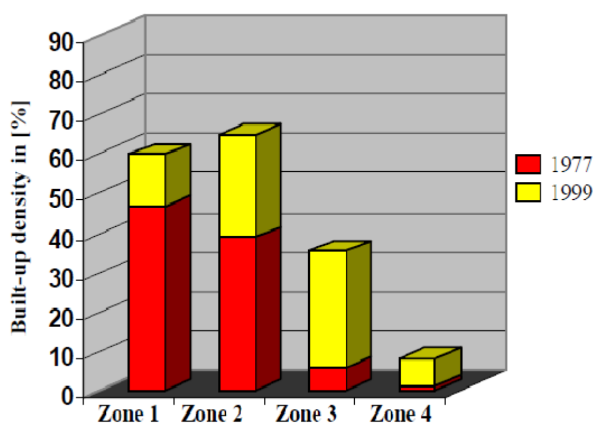
spatial metrics such as shape index, patch density and largest patch index to describe the structures and pattern of mega city landscapes. In general, spatial metrics can be defined as quantitative and aggregate measurements derived from digital analysis of thematic categorical maps showing spatial heterogeneity at a specific scale and resolution (McGarigal K. 2002; Herold 2003). The main idea behind this was to learn the system of the complex process of spatial urban growth by finding relationship and difference between cities past development.

Results of the Landsat data obtained for different parameters:

Built-up densities: Built-up density is the measure to characterize spatial urban pattern and structure. Densities vary substantially from city to city and from urban center to periphery areas. Here the built-up densities are being calculated for the zones 1-4. The Landsat data shows the built-up densities of particular zone. Figure 22 shows the built up densities for two Indian mega cities Mumbai and Delhi.



a) Mumbai



b) Delhi

Figure 22: Space and time analysis of built-up densities

From this Built-up densities graph, we can say that the two Indian mega cities clearly show a decreasing and similar built-up density gradient with distance to the main urban center, although their urban footprints are different from each other

Landscape shape index: The Landscape shape index (LSI) provides a standardized measure of the perimeter length of all patches of one land cover type (Urbanized areas in this case) in the landscape. (McGarigal K. 2002; Schneider A. 2005). If the urbanized area is composed by simple geometric rectangles, the LSI will be small, approaching 1.0. If the landscape contains dispersed patches with complex and convoluted shapes the LSI will be large. Table 4 shows the spatiotemporal results of the LSI calculation of Mumbai and Delhi. (Berger 2007; Taubenböck 2007; Taubenböck 2008)

Table 4: Spatiotemporal (both in space and time) results of the LSI

LSI	$\approx$1975	$\approx$1990	$\approx$2000
Mumbai	7,07	16,51	22,00
Delhi	7,61	-	22,62

Patch Density: The patch density (PD) which is the number of urban patches is a measure of different urban areas in the landscape and is expected to increase during periods of rapid urban nuclei development, but may decrease if urban areas expand and merge into continuous urban fabric (McGarigal K. 2002).

Table 5: Spatiotemporal (both in space and time) results of the PD

Patch density	$\approx$1975	$\approx$1990	$\approx$2000
Mumbai	9,74	19,09	15,01
Delhi	3,89	-	4,12

<u>Largest Patch Index (LPI):</u> The Largest patch index (LPI) gives the proportion of total area occupied by the largest patch (Luck et al, 2002). It is a measure that represents the separation of the urban landscape into smaller individual patches versus a dominant urban core. Table 6 shows the temporal characteristics of the LPI in Mumbai and Delhi

Table 6: Spatiotemporal (both in space and time) results of the LPI

Largest patch index	$\approx$1975	$\approx$1990	$\approx$2000
Mumbai	0,49	1,77	3,22
Delhi	1,28	-	5,95

The difference in the patch density can be seen easily from this table for the Indian mega cities. The case study has tested that urbanization and its spatiotemporal (both in space and time) form, pattern and structure can be quantified and compared across the cities using a combination of gradient analysis and spatial metrics. Landsat data with respect to the limited geometric resolution proved to be an independent data source for the analysis of fast changing and large areas of Indian mega cities. From this case study, data was generated showing that, the urban patterns are highly complex systems revealing self-similarity across scales. Hence, we can say that "Cities are fractals". As, we have proved that the cities are complex systems revealing self-

similarity at each scales, we can carry out the fractal analysis to find the fractal dimension of Mumbai city by applying different fractal geometries.

4.6.2 Fractal analysis:

The application of fractal geometry at the analysis of the urban development patterns has been widely investigated during the last two decades (Benguigui B. 2001). Fractals have been widely used in order to distinguish between different urban typologies (Frankhauser P. 1998,) to measure the degree of urban expansion and examine the way cities expand in space and time (Batty M. 1994). Although the researches on this field is new there are certain remains which still needs to be fixed, the results coming from various application all over the world confirm the fact that urban systems present a fractal structure. It therefore became necessary to describe the structural and functional properties of these systems. It is therefore necessary that the structural and functional properties of these systems should be described, and new properties which until now where considered too complex to be analyzed should be examined.

The main objective of this part in the Thesis is to examine the basic relations between fractal analysis and the urbanization at the outskirts of the city. The spatiotemporal study has found that, there has been a forceful process of growth at the edges of metropolitan areas, which reveals a high degree of complexity. Those areas can be considered as the complex, fractal urban structures constituting a promising field of inquiry for the new geometry (Lagarias A. 2005 (In Greek)). Which will help in proving the fact that the "City is a Fractal ". This, part will provide us with insights of the cities that helps in proper Urbanization of cities and giving us more better understanding of the city structure and population using space and time series data, making the complex cities more easy to understand which will further help in estimating the cost of the particular urban area under investigation using the concepts or theory of COSYSMO.

The box-counting method and the DLA (Radial analysis) method are two commonly applied models calculating the fractal dimension of an urbanized area ((Frankhauser P. 1998,),(Benguigui B. 2000,)). Sometimes the urban built-up patterns can better be described as multi-fractals (Frankhauser P. 1998,). In this case the value of D can change across space and scale and also through time (Lagarias A. 2005 (In Greek)). There are two general ways for identifying a fractal structure, first either by identifying changes in form by varying the scale and keeping the size fixed, or, second by keeping the scale fixed and varying the size (Batty 1994,). Fractals by definition are complex, hierarchically ordered structures revealing self-similarity across scales (Batty M. 1994). The method can be applied both at the study of whole urbanized surfaces and at the study of inner and outer urban boundaries. The value of the dimensions is estimated in the range $1<D<2$ (Batty M. 2005). The relation between the fractal dimension of a surface and the corresponding boundary can be understood if we consider a perfect homogeneous circle: The value of D_a (dimension of the surface) is equal to 2 while the value of D_b (dimension of the boundary) is equal to 1. The more fragmented a pattern appears, the smaller is the value of D_a and the greater of D_b. The fractal dimension therefore describes how the built up area is distributed on the surface, and how dense or fragmented is the spatial patterning of the city. The two methods do not give identical results when applied over same areas; the reason why these two methods give different results is the fact that the urban pattern displays a property of multi-fractals. Each one however, measures a different attribute of the area under investigation and therefore they should be applied in combination. In order, to prove this point we will take an example later to show the different results given by these two methods.

Note: Why Fractal Dimension D in the measurement of urban structure important?

- **Fractal Dimension D is the basic parameter of fractal showing the important form of its structure, because it quantifies the degree of irregularity or fragmentation.**
- **It also indicates the level of complexity and describes the distribution across the mass from the center [Sobreira & Filho 2002].**
- **It also describes how the built up area is distributed on the surface, and how compact and broken is the spatial patterning of the city.**

The case study presented here concentrates on the box-counting dimension of the built-up area and the corresponding boundary of Mumbai city. Table 7 and Table 8 present the basic theoretical relationships between those fractal dimensions and the urban context to which they refer.

Table 7: Change of values in space

	Central, densely built-up areas	Irregular outskirts characterized by recent sprawl (spread)	Densely built-up, homogeneous outskirts with the fill-in of existing vacant land
D_a	High values (Frankhauser P. 1998,)	Low values	High values
D_b (Inverse Values)	-	High values	Low values

Table 8: Change of values through time

	Infill of existing suburban areas- intensification	Irregular outskirts characterized by resent sprawl (spread)-leapfrog development
Change in D_a	+	-
Change in D_b	-	+

4.6.3 Application of fractal analysis at the outskirts of Mumbai city.

Mumbai city at present leads the ranking in one of the largest metropolitan area in India with a total population of 21,900,976. The form of the city has been extremely compact until approximately the end of the 19th century, while during the 20th century the city went through rapid urbanization expanding almost in every direction and forming a complex and dynamic urban structure.

The exact geographical location of Mumbai lies in the west coast of the state of Maharashtra facing the Arabian Sea. It is an interesting fact that one fourth of the city lies below the sea level. Mumbai is spread across a total area of 440 sq. kilometers. An important particularity was the creation of numerous spontaneous settlements on the eastern and western areas forming a disconnected network around the central area. The gradual incorporation of these areas in the city plan and rapid urbanization of the whole area led to a complete transformation and created a dense, unified urban area with expanding boundaries.

Figure 23: Satellite image of Mumbai city showing the area under investigation

The area chosen for analysis is Dharavi which is located at the central part of Mumbai city and connected to it are the major transportation axis of other areas of surrounding it. Figure 24 shows the satellite image of Dharavi.

Figure 24: Satellite image of biggest slum area in Mumbai (Dharavi)

Dharavi is one of the biggest slums of Mumbai city (one of the biggest in Asia). Fractal analysis to find out the dimension, was first carried out for the whole Mumbai city and then for the area under consideration. The case study tells us that the Fractal dimension of the biggest slum area Dharavi in Mumbai city (one of the biggest in Asia) can be calculated by using fractal analysis, and it is quite possible to carry out the sensitivity analysis using the Fractal-COSYSMO formulation that was generated especially from COSYSMO, which will help in giving exact cost estimation needed for the development of this slum area. Figure 25 below shows the aerial photo and the extracted built up area of Mumbai and Dharavi.

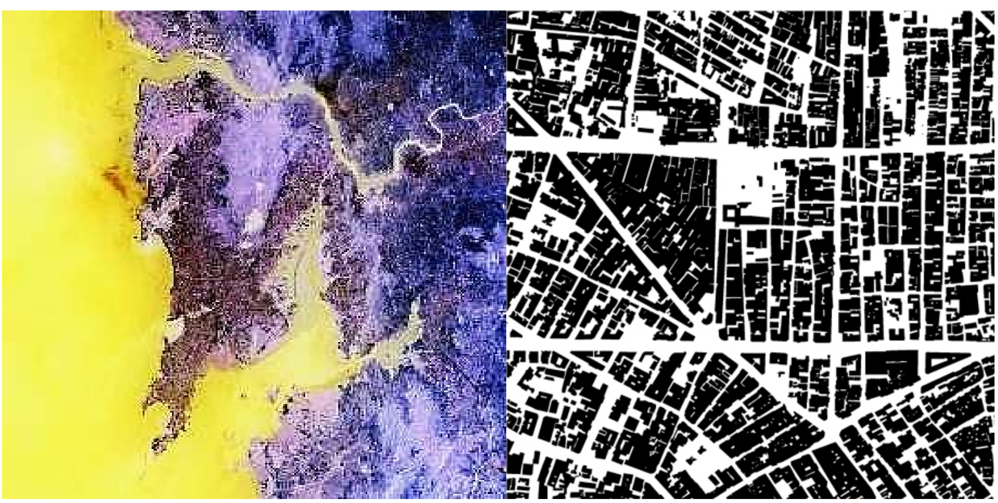


Figure 25: Aerial photo and the extracted built-up areas

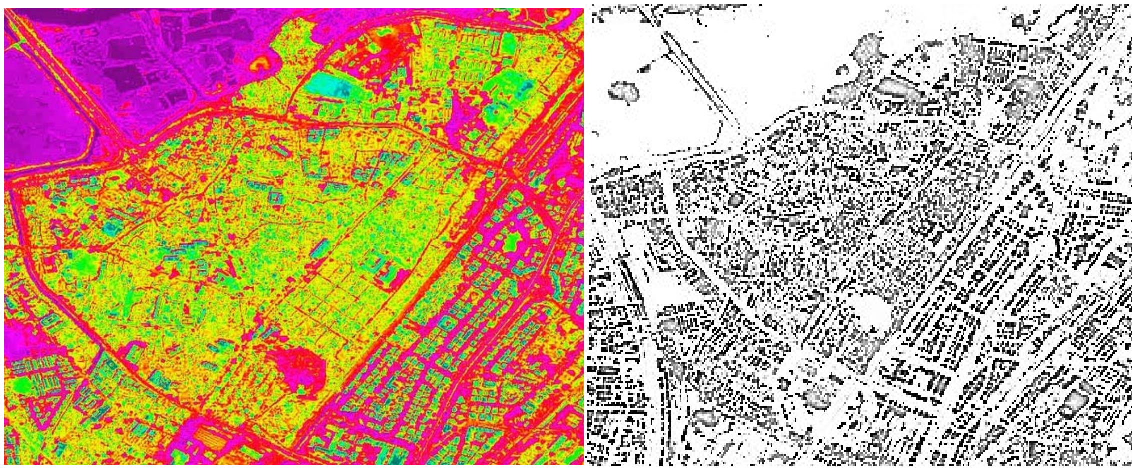


Figure 26: Heat map and black and white image of Dharavi showing the extracted built-up area

The next process will be the calculation of fractal dimension of Mumbai city and Dharavi area in Mumbai city using fractal analysis. The box-counting method was applied to the selected region using a sequence of grid sizes 1-2-4-8-16-32-64. The software ‘Fractalyse’ was used and the method ‘box-testing was chosen which consists in finding the least number of square of size ε needed to cover all black pixels. The number of iterations was fixed at 8. Figure 27 below shows the fractal dimension calculation of Mumbai city.

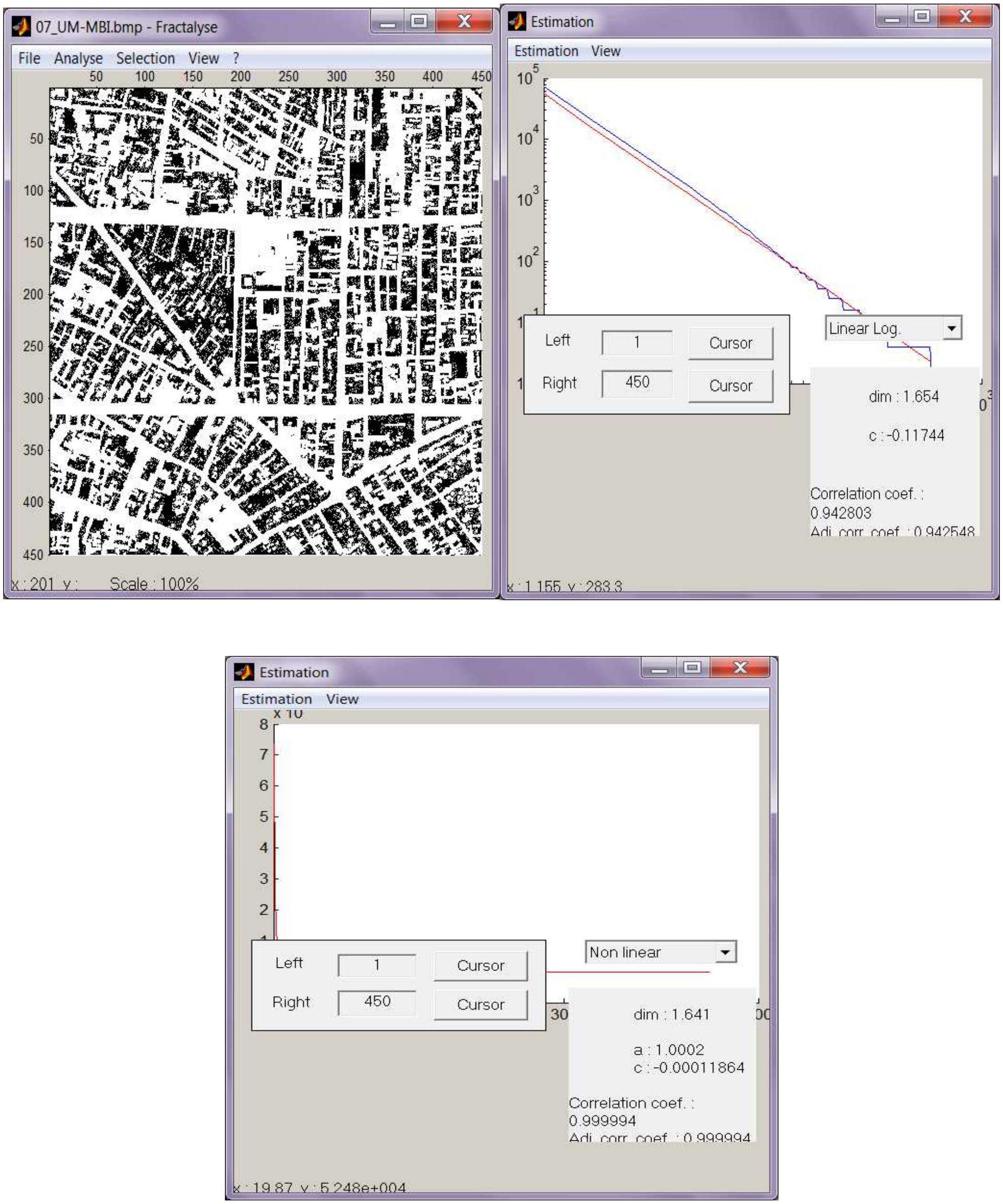
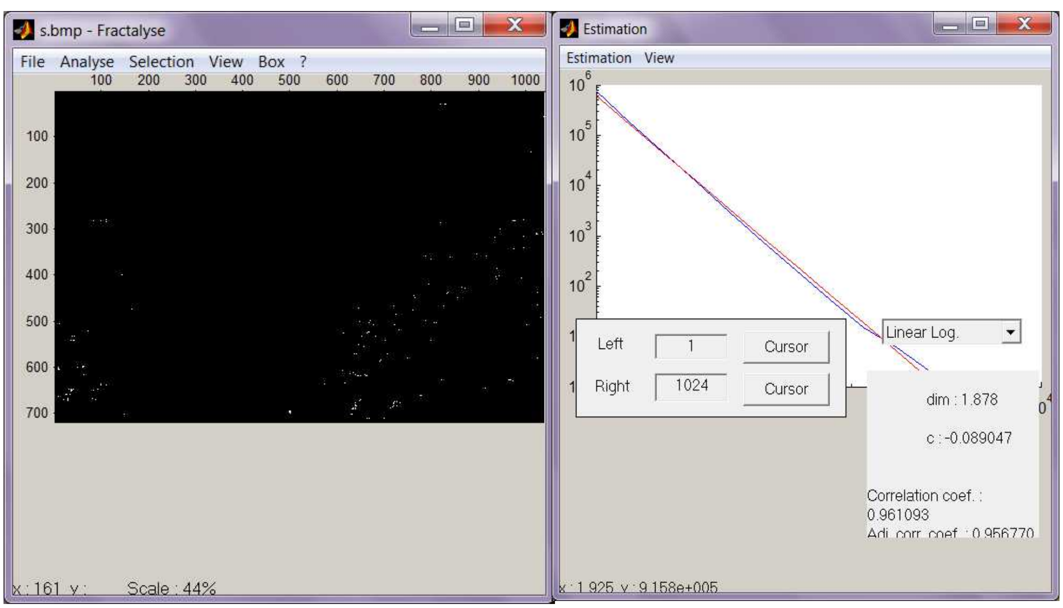


Figure 27: Linear and Non-linear log regression for Mumbai city showing its dimension and correlation co-efficient

A linear log regression was performed on the logarithmic plot of N against S (Where N is the number of grid cells in which a part of the object appears is counted and S the corresponding

size of the grid cell) and the fractal dimension was estimated equal to D=1.654, if we see Figure 27 we can see that for the non-linear regression the value of D=1.641 with a constant a= 1.0002. It must be noted that the reliability of the calculated D value is examined by the correlation coefficient r, which in present case is 0.9428 for linear regression and non-linear regression. The value of coefficient can be considered good in the calculation of fractal dimension. Hence, we can state that the urban pattern under investigation clearly displays a fractal structure. Also, we can state that any exponential or fractal dimensional parameter will be the most important parameter in the examined cost estimation formulas. Hence, Fractal dimensionality is the principle driver of cost in fractal systems.

The same method was used to find the fractal dimension for the area under investigation which is Dharavi (one of the biggest slum area in Mumbai). Figure 28 shows the linear and non-linear log regression graph of Dharavi showing its dimension and correlation coefficient.



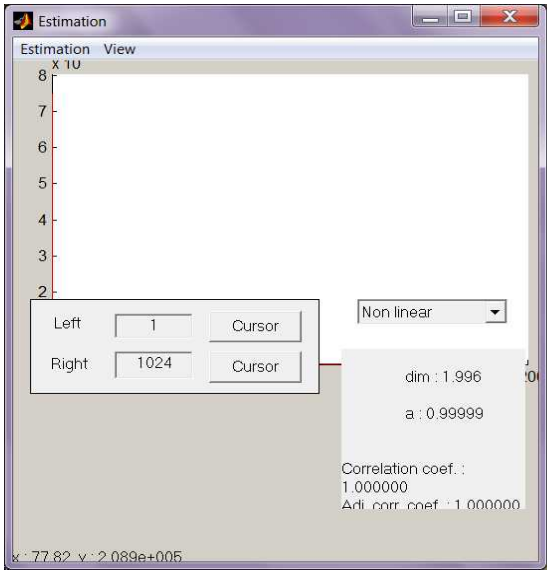


Figure 28: Linear and Non-linear log regression for Dharavi area showing its dimension and correlation co-efficient

A linear log regression was performed on the logarithmic plot of N against S (Where N is the number of grid cells in which a part of the object appears is counted and S the corresponding size of the grid cell) and the fractal dimension was estimated equal to D=1.878, if we see Figure 28 we can see that for the non-linear regression the value of D=1.996 with a constant $a \approx 1$. It must be noted that the reliability of the calculated D value is examined by the correlation coefficient r, which in present case is 0.9611 for linear regression and non-linear regression

4.7. Education: Student Person Months

Estimating Student Person Months with Fractal COSYSMO

Student time commitments as well as distractions have grown steadily over the century. While it is clear that students are interacting with a great variety of organizations and educational interests, their time commitment has not been adequately assessed and determined. Borrowing from a systems engineering parametric cost estimation model, an elaboration is made of student

time usage by counting tasks, interfaces and effort multipliers. Analogies are drawn between a student and a practicing engineer in industry, who must interact with a body of knowledge that must be assimilated and integrated, as well as an environment that expands the number of interactions with which the student must contend. The result of this work is a better characterization of the educational factors that shape a student's experience.

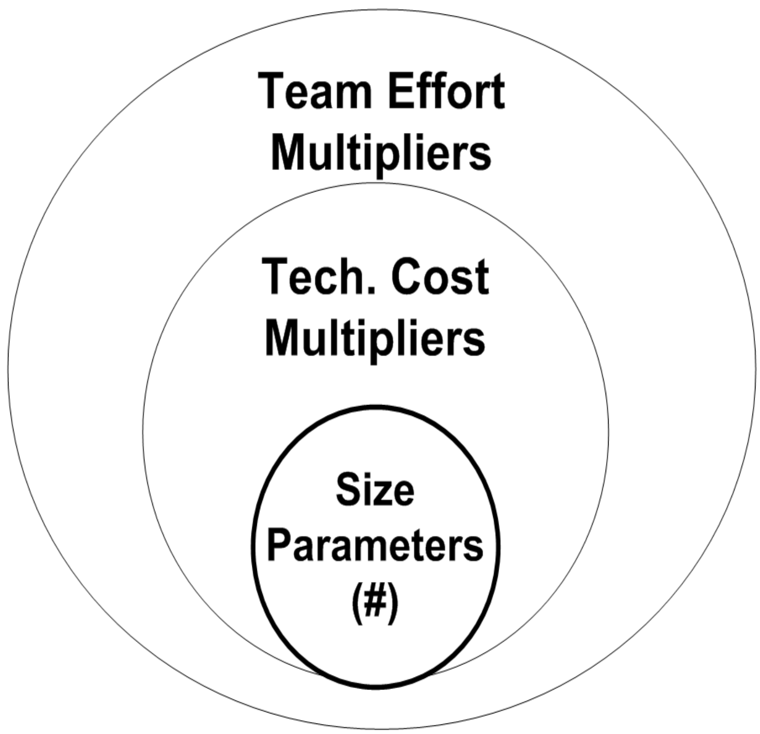


Figure 29: Estimation of Student Person Months using Fractal-COSYSMO

The regular COSYSMO parametric equation:

$$PM_{NS} = A \cdot (\Sigma w_e s + w_n s + w_d s)^E \cdot \prod_{j=1}^{14} EM_j \qquad (58)$$

Table 9: Estimating student person month's with COSYSMO

COSYSMO SIZE PARAMETERS		**LEARNING SIZE PARAMETERS**
Number of Requirements	S	Number of Facts
Number of Interfaces	S	Curriculum Interrelations
Number of Algorithms	S	Number of Mental Schemas
Number of Operational Scenarios	S	Number of Applications
COSYSMO TECHNICAL COST PARAMETERS		**LEARNING TECHNICAL COST PARAMETERS**
Requirements Understanding	**EM**	**Pre-Exposure to Academic Areas**
Architectural Understanding	**EM**	**Curriculum Understanding**
Level of Service Requirements	**EM**	**Standards for Course Passage**
Migration Complexity	**EM**	**Field of Major Change**
Technology Risk (just a level?)	**EM**	**Technological Support**

Documentation	**EM**	**Textbook Support**
# and Diversity of Installations/Platforms	**EM**	**# of Departments/ Programs**
# of Recursive Levels in the Design	**EM**	**Depth of Understanding Required**
COSYSMO TEAM COST PARAMETERS		**LEARNING TEAM COST PARAMETERS**
Stakeholder Team Cohesion	**EM**	**Faculty and Student Cohesion**
Multisite Coordination	**EM**	**Multisite Coordination (distance education)**
Heterogeneity (domains, cultures)	**EM**	**Heterogeneity (domains, cultures)**
Personnel/Team Capability	**EM**	**Personnel/Team Capability**
Process Capability	**EM**	**Learning Process Capability**
Personnel Experience/Continuity	**EM**	**Personnel Experience/Continuity**
Tool Support	**EM**	**Software Tool Support**

Table 9 above shows the comparison of COSYSMO size parameters with the Student’s learning size parameters where the COSYSMO size parameters are exactly compared to the size parameters of student’s academic life showing the application of COSYSMO. Hence, we can conclude that COSYSMO can be best applied to all such possible situations.

CHAPTER V: CONCLUSION

This thesis has filled a void in the area of the formulation of cost estimation for complex projects. While cost estimation existed for systems engineering effort, separately for engineering projects, there was previously a lack of coverage for projects on inherently complex products, which involve similarly complex matching engineering activities. Surprisingly, the task of producing Fractal-COSYSMO involved the re-examination of the mathematical assumptions of previous cost estimation models, and the relatively simple mathematical reformulation of the central CER applied. The presence of Fractal-COSYSMO should now be fruitful for the re-examination of costs and efforts in a wide variety of complex projects.

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APPENDIX 1: Fractal Antennae

What is an antenna? Stutzmann and Thiele (W.L. Stutzmann) define an antenna as transducer designed to transmit or receive the electromagnetic radio waves. In other words antennas convert electromagnetic waves into electric currents and vice versa. Many different structures can act as antennas. Generally, antennas are constructed out of conducting material of some nature and can be constructed in many shapes and sizes. The size is related to the wavelength of operation of the antenna. The structural design of an antenna operating at 10 kHz is always much larger than the structural design of antenna operating at 10 GHz, for example. Transmission lines are used to guide the power from the transmitter to the antenna and should be impedance matched to both the transmitter and the antenna.

There are many different parameters that are used to characterize antennas out of which are,

- “Gain: Which make the apparent power greater than the actual transmitted power in a given direction.
- Directivity: Directivity is defined as the ratio of the maximum radiation intensity to the average radiation intensity.

Gain and directivity have a very strong relation to each other, because the gain is equal to directivity if the efficiency of the antenna is 100 percent” (W.L. Stutzmann). Gain is a directional function; it changes with position around the antenna and is defined as

$$G(\theta,\phi)=\frac{4\pi U(\theta,\phi)}{P_{in}} \quad , \tag{59}$$

Where, $U(\theta,\phi)$ is the radiation intensity, and P_{in} is the input power to the antenna. Gain is also equal to the efficiency multiplied by the directivity (W.L. Stutzmann). Gain is usually measured in decibels with reference to another antenna, either an isotropic radiator (dBi) or to a simple dipole (dBd). An isotropic radiator is an antenna that radiates equally in all directions and is just a theoretical model (L.C. Shen). Efficiency for an antenna is defined as the ratio of the radiated power to the input power. Gain, directivity, and beam width are parameters that a radio systems engineer would use to choose an antenna for a specific job. Front-to-back isolation is another antenna parameter that is of concern to communication engineers when the antenna is to be used in a crowded frequency band. Front-to-back isolation is defined as the difference in gain from the front of the antenna and the gain from the back of the antenna. If an antenna had a forward gain of 10 dBi and a gain of 2 dBi off of the back, the front-to-back isolation for this antenna would be 8 dB. Input impedance is another important antenna parameter. The input impedance of the antenna should be matched to the impedance of the transmission line for maximum power transfer. It is also important that the input impedance of the antenna is mostly resistive, so that most of the power introduced to the antenna is radiated. Input impedance has real and complex parts and its general form is

$$Z_{in} = R_{in} + jX_{in} \tag{60}$$

The R_{in} term represents the resistance or power radiating portion of the impedance, X_{in} represents the reactive portion or power storage component of the impedance (W.L. Stutzmann). When the impedance is purely resistive, the antenna dissipates the power presented to it. Power can be dissipated from an antenna two ways. The first, and least useful, is ohmic or heating

losses from the antenna structure. Second, power that leaves the antenna as electromagnetic waves at the desired frequency is another form of dissipation. On most antennas, the ohmic losses are very small compared to the radiation losses. The first generation antennas was narrowband (small range of the order of a few percent around the designed operating frequency) and were often arrayed to increase directivity. During same period broadband antennas were also developed. The Yagi- Uda antenna remained king until after the war. Yagi antennas with added corner reflectors and/or UHF elements are commonly used for reception of television broadcasts. Amateur radio operators frequently use front- to- back isolation as a parameter when comparing Yagi-Uda antennas they are also widely used by amateur radio operators for communication on frequencies from short wave, through VHF/UHF, and into microwave bands.

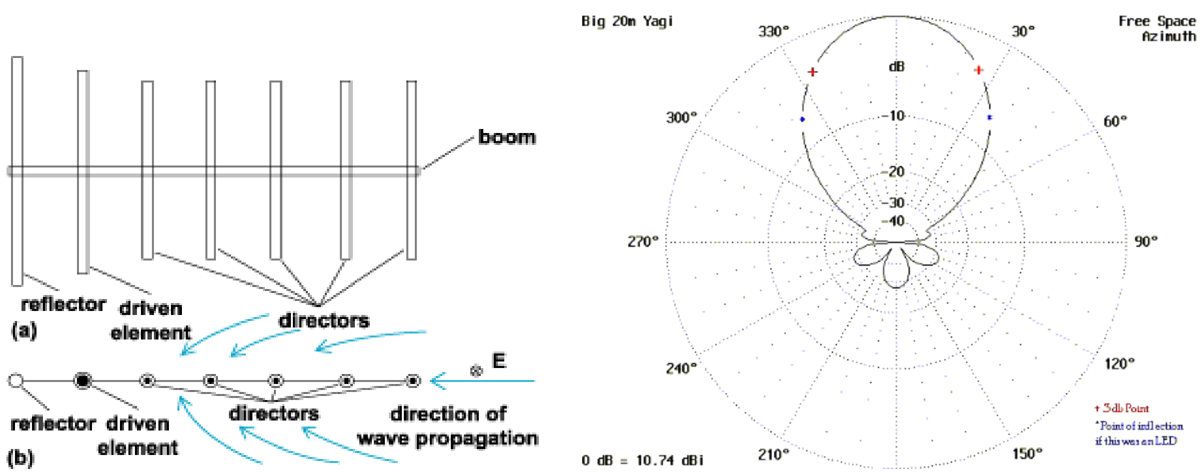


Figure 30: Yagi antenna and its plot

Fractal Antenna Technology and Engineering Research:

Fractal Antenna:

As we see fractals have been studied for about a hundred years and antennas have been in use for as long. Fractal antennas are new on the scene. Nathan Cohen, a radio astronomer at Boston University, was a fractal antenna pioneer who experimented with wire fractal antennas (von Koch curves) and fractal planar arrays (Sierpinski triangles). He built the first known fractal antenna in 1988 when he set up a ham radio station at his Boston apartment. Cohen, founder of Fractal Antenna Systems [www.fractenna.com, 2000], is now working with the Amphenol T&M Antennas, located in a Chicago suburb, to make cellular antennas for Motorola cellular phones. In Motorola's application, the fractal arrays have proven to be 25% more efficient than the conventional helical antenna (rubber-ducky) (Musser July 1999). Lot of research has been carried out on fractal antennas in different parts of the country. Today the best tools for fractal antenna analysis are *hybrid* type antenna modelers based on traditional Method of Moment (MoM) analysis (Rahmat-Samii February 1997). Developed over the last 20 years, fractal antennas have proven to be the first fundamentally important breakthrough in antenna technology in the last half century. Simply put, fractal antennas radically alter the traditional relationships between bandwidth, gain and size—permitting antennas that are more powerful, versatile and compact. In our search for the "super" wideband antenna we are driven by two desires:

(1) Make an antenna for a given frequency band as small as possible, and

(2) Make an antenna cover several (many) bands.

The fractal antenna has performance parameters that repeat periodically with an arbitrary "fineness" dependent on the iteration depth. Therefore, although the finite iteration depth fractal antenna is not frequency independent, it can cover frequency bands arbitrary close together! Also, remembering that radiation comes from accelerating charges, the typical fractal shape (with all those little bends and kinks) makes for good radiation (higher radiation resistance) because of all that acceleration going on as the charges are forced to negotiate all those sharp turns.

The space-filling and multiple scale properties of the Fractus allows it to produce maximum antenna performance with minimum antenna space.

The key benefits of fractal antenna technology are:

- Reduced antenna size
- Multi-band functionality
- Improved antenna performance

Fractal antenna technology is geometry-based, not material based. Therefore, fractal antennas are manufactured from standard materials and substrates, using standard processes. OEMs, ODMs and CEMs are able to take advantage of maximum flexibility and cost-effectiveness, from design through to final assembly, with no need to change processes or deal with special materials to produce Fractus fractal antennas.

Lot of efforts have been made by several researchers around the world to combine fractal geometry with electromagnetic theory which have lead to a plenty of new and innovative antenna designs. As we have said before in this thesis that fractals have no characteristic size, they are self-similar and generally composed of many copies of themselves at different scales. These unique properties of fractals have been exploited in order to develop a new class of antenna-

element designs that are multi-band and/or compact in size. On the other hand, fractal arrays are a subset of thinned arrays, and have been shown to possess several high desirable properties, including multi-band performance, low sidelobes level and the ability to develop rapid beam performing algorithms based on the recursive nature of fractals. There has been an ever-growing demand, in both the military as well as in commercial sectors, that possess the following highly desirable attributes:

- Compact size
- Low profile
- Conformal
- Multi-band or broadband

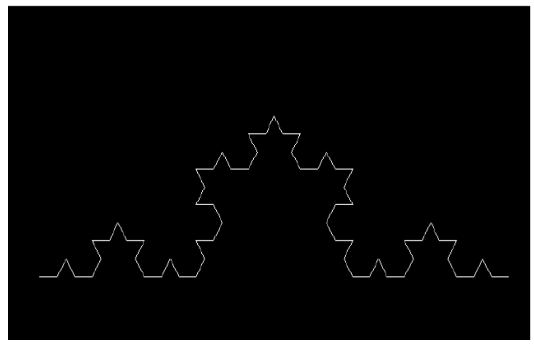
There are a variety of approaches that have been developed over, the years, which can be utilized to achieve one or more of these design objectives. “Fractal antenna engineering” is new and rapidly growing field of research. As fractal geometry is an extension of classical geometry, its previous and recent introduction provides a new knowledge to engineers, and a chance to discover a limitless number of previously unavailable configurations in the development of new and innovative antenna designs. The fractal antenna engineering primarily consist of two active areas of research which include.

1. The study of fractal-shaped antenna elements, and
2. The use of fractals in the design of antenna arrays.

In many cases, the use of fractal element antennas can simplify circuit design. Another beneficial of fractal antennas is that, fractal antennas are in form of a printed on the circuit board (PCB) (KALRA).

Some useful geometry for Fractal Antenna Engineering:

This section will tell us about a brief overview of some of the more common fractal geometries that have been found to be useful in developing new and innovative designs for antennas. There are few fractals that have been proved beneficial in antenna engineering like the Sierpinski gasket, Koch curve. The construction process of both this fractals have been explained in the previous section.



Final stage of Sierpinski gasket **Final stage of Koch snowflake**

Figure 31: Final stages of Sierpinski gasket and Koch snowflake used as fractal antennas

There is one more fractal which is very important from the fractal antenna engineering point of view called as ternary fractal tree.

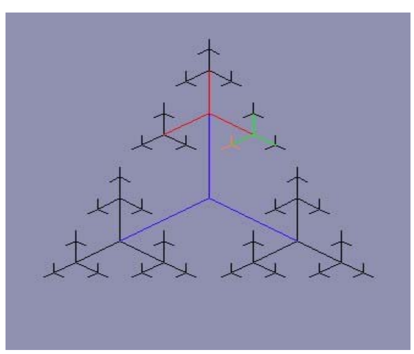


Figure 32: Ternary fractal tree used as fractal antenna

Conclusion: Fractal antennas generally falls into two categories called omnidirectional and directional. Omnidirectional antennas or only weakly directional antennas receives or radiates more or less in all directions these omnidirectional antennas are also used at lower frequencies where the directional antennas are not needed which indirectly reduces the cost. All these characteristics of the fractal antennas proves that it is cost effective if considered on a large scale

CURRICULUM VITA

Born in Mumbai, Maharashtra, India in a bright Sunday, March 14th, 1987, Manish Shivram Khadtare was the eldest son of Shivram Gopal Khadtare and Tara Shivram Khadtare. In the fall of 2008, he obtained his Bachelor of Engineering under the major of Electronics and Telecommunication Engineering in Terna Engineering College associated with Mumbai University, Mumbai, India.. In the fall of 2009, he enrolled into the pursuing the Master of Science degree in Electrical Engineering at The University of Texas at El Paso. During his master's he worked with Dr. Eric Smith in Systems Engineering area at The University of Texas at El Paso. In 2010, he presented his research as a speaker as given below.

- ***Conference Proceedings***

  Manish Khadtare and Eric D. Smith, *Fractal Dimensionality Considerations for Use with the Constructive Systems Engineering Cost Model (COSYSMO)*, Live Virtual Constructive (LVC) Conference, International Test & Evaluation Association (ITEA), El Paso, TX, 2011.

  Eric D. Smith, David Delgado and Manish Khadtare, *Latency Modeling for Testing and Evaluation*, Live Virtual Constructive (LVC) Conference, International Test & Evaluation Association (ITEA), El Paso, TX, 2010.

- ***Submitted Proceedings***:

  Manish Khadtare and Eric D. Smith, *Fractal-COSYSMO Systems Engineering Cost Estimation for Complex Projects*, Complex Adaptive Systems Conference, Chicago, IL, 2011.

- ***Abstracts***

Manish Khadtare and Eric D. Smith, *Estimating Student Person Months with Fractal COSYSMO*, Sun Conference on Teaching and Learning, El Paso, TX 2011.

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