

2012-01-01

Mathematics and Science Integrated Instruction: with the Emphasis on the Types of Mathematical Knowledge

Barbie A. Avila

University of Texas at El Paso, email4barbie@yahoo.com

Follow this and additional works at: https://digitalcommons.utep.edu/open_etd



Part of the [Science and Mathematics Education Commons](#), and the [Teacher Education and Professional Development Commons](#)

Recommended Citation

Avila, Barbie A., "Mathematics and Science Integrated Instruction: with the Emphasis on the Types of Mathematical Knowledge" (2012). *Open Access Theses & Dissertations*. 2037.
https://digitalcommons.utep.edu/open_etd/2037

MATHEMATICS AND SCIENCE INTEGRATED INSTRUCTION:
WITH THE EMPHASIS ON THE TYPES OF MATHEMATICAL KNOWLEDGE

BARBIE AVILA

Department of Teacher Education

APPROVED:

Olga Kosheleva, Ph.D., Chair

Laura Serpa, Ph.D.

Brian Giza, Ph.D.

Benjamin C. Flores, Ph.D.
Interim Dean of the Graduate School

Copyright ©

by

Barbie Avila

2012

Dedication

The work contained herein is dedicated to all the educators of the world who work tirelessly to impact their students; and ultimately the future. Thank you...

MATHEMATICS AND SCIENCE INTEGRATED INSTRUCTION:
WITH THE EMPHASIS ON THE TYPES OF MATHEMATICAL KNOWLEDGE

by

BARBIE AVILA, B.A.

THESIS

Presented to the Faculty of the Graduate School of
The University of Texas at El Paso
in Partial Fulfillment
of the Requirements
for the Degree of
MASTER OF ARTS

Department of Teacher Education
THE UNIVERSITY OF TEXAS AT EL PASO

May 2012

Acknowledgements

I would like to acknowledge my husband for his endearing support through thick and thin; writer's block and all! Without your constant pampering and loving care, I would have taken [more] years to finish. I love you and am so proud of all of your own accomplishments. This joy is shared with you!

Mom and Dad, thank you for teaching me the value of perseverance and fortitude. You both know how easily distracted I can get! But, really, was there ever a doubt I would finish? Thank you also for fostering my love of learning early in my life.

Thank you also Dr. Kosheleva and Dr. Serpa. Reciprocally, you were my ying and yang. I needed the balance and advice you both offered. Your passion for education is inspiring. Thank you for your belief in me and my ideas.

Thank you, Dr. Giza for serving on my thesis committee; which you unknowingly became a part of when you participated in my Kaleidoscope presentation! Thank you for your continued curiosity.

Abstract

Internationally, American students continue to lag behind other countries in mathematics and science academic achievement. This is especially alarming due to the multi-faceted approaches taken in these areas by researchers and politicians, as well as the requisite schooling required by the American education system. The idea of integrating both content areas in order to enhance and deepen student understanding of each subject is a useful tactic in order to achieve the goal of improved mathematics and science academic achievement by American students. The emphasis should be on *what* the students will be *doing* to learn these content areas, so that various cognitive demands are used and developed. A science context allows for the investigation of various mathematical tasks and provides for the manipulation of the different knowledge types necessary to accomplish those tasks. By empowering teachers to think critically and evaluate integrated mathematics and science lessons, based on *what* students will be *doing*, teachers can begin to feel self-efficacious and confident in selecting and creating lessons that are most beneficial for students.

Table of Contents

Acknowledgements.....	v
Abstract.....	vi
Table of Contents.....	vii
List of Tables.....	ix
List of Figures.....	x
Chapter 1: Introduction.....	1
1.1 Rationale.....	3
1.2 National Education Goals.....	7
1.3 How Students Learn Mathematics.....	9
Chapter 2: Literature Review.....	17
2.1 Historical Perspective of Integration in Education.....	17
2.2 Challenges of Integration in Education.....	18
2.3 Mathematics and Science Integration.....	21
2.4 Previous Frameworks.....	22
Chapter 3: Methodology.....	26
3.1 Proposed Framework.....	26
3.2 Procedural Knowledge.....	32
3.3 Conceptual Knowledge.....	34
3.4 Generalized Knowledge.....	36
3.5 Research Question.....	39
Chapter 4: Lesson Plans Analysis.....	40
4.1 SABE MAS Middle School Lesson Analysis Overviews and Rubrics.....	42
4.2 SABE MAS High School Lesson Analysis Overviews and Rubrics.....	65

4.3	Published Lesson Overviews and Rubrics.....	89
Chapter 5: Discussion.....		100
5.1	Discussion of Findings.....	100
5.2	Implications for Future Research.....	105
References.....		106
Appendix 1.....		111
1.1	SABE MAS Middle School Lessons.....	111
Appendix 2.....		136
2.1	SABE MAS High School Lessons.....	136
Vita.....		156

List of Tables

Table 1.1:	Average Scores of 15-Year-Old Students on the 2006 PISA.....	6
Table 1.2	The National Research Council’s Five Identified Strands of Mathematical Proficiency.....	12
Table 1.3	Designation of Knowledge Types Rubric.....	30
Table 1.4	Tchoshanov’s Teacher Content Knowledge Cognitive Types’ Descriptors.....	38
Table 1.5	Middle School Lesson Analysis Overview.....	101
Table 1.6	High School Lesson Analysis Overview.....	102
Table 1.7	Published Lesson Analysis Overview.....	103

List of Figures

Figure 1.1	Concept Map of the TQ Grant.....	3
Figure 1.2	Mathematics/Science Continuum.....	22
Figure 1.3	Continuum of Integration of Mathematics and Science Concepts/Activities.....	23
Figure 1.4	Hierarchy of Integration Approaches.....	24
Figure 1.5	Proposed Framework of Mathematical Knowledge Types in a Science Context..	27
Figure 1.6	Percentages of Each Knowledge Type in Middle School Lessons.....	101
Figure 1.7	Percentages of Each Knowledge Type in High School Lessons.....	102
Figure 1.8	Percentages of Each Knowledge Type in Published Lessons.....	103
Figure 1.9	Percentages of Each Knowledge Type in All Lessons Analyzed.....	104

Chapter 1: Introduction

Educational reform, especially in mathematics and science, has been a constant struggle hidden behind catchy titles and reform rhetoric. The time for change is long overdue. America's children deserve a quality, innovative education that allows them to engage as active learners, preparing themselves to become successful problem solvers and critical thinkers. This can be accomplished by the sustainment of high caliber educators who are prepared and competent to educate today's students in creative and motivating ways. However, without proper preparation and training in both content knowledge and pedagogy, teacher education programs will only propagate an already alarming problem. American students have trailed behind, and continue to trail, many other countries in math and science as seen on the most recent international scores (Baldi, et al., 2007). Consequently, this enables an apathetic, low sense of teacher self-efficacy, resulting in low student performance, substantiating that the time for reform is now. Therefore, quality education and training for both pre-service and in-service teachers is the key to improved student performance, especially for mathematics and science achievement.

As an attempt to reform mathematics and science instruction in the El Paso (Texas) area, seven professors from the University of Texas at El Paso collaborated on two grants, SABE MAS Learning Academy ("KNOW MORE" in Spanish) and the Teacher Quality (TQ) Program: Connecting Math and Science Across Grade Levels. Both grants consisted of in-service elementary self-contained teachers that teach both mathematics and science; middle school mathematics and science content teachers; and high school mathematics and science content teachers.

SABE MAS attempted to foster a "community of learners to collaborate on cross-disciplinary lesson plans supported by technology, aligned to TEKS, and informed by the

teachers' needs.” (SABE MAS website, 2010). The professors’ goals were to:

- Increase mathematics and science teachers’ content and deep conceptual understanding of basic STEM topics and their connectivity in science, technology, engineering, and mathematics through experimentation, lesson plan development, and exemplary pedagogy.
- Increase the number of in-service math and science teachers in the districts with Master of Arts in Teaching (MAT) degrees and Master Teacher Certifications.
- Increase the number of pre-service math and science teachers prepared to enter the workforce.

(SABE MAS website, 2009)

Figure 1.1 shows a Concept Map of the TQ Grant. Once participants enrolled in the grant, (TQ participants also participate in SABE MAS), they were assessed using the Mathematics Teaching Efficacy Belief Instrument (MTEBI) and the Science Teaching Efficacy Belief Instrument (STEBI) self-efficacy surveys. Researchers were curious to see if self-efficacy played a factor in participant learning, and if so, to what degree? After reviewing initial pre- and post-tests, researchers decided to focus on the more relevant issue of the type of cognitive knowledge necessary to perform the mathematics concepts in participant lessons. It was believed that if participants learn how to evaluate lessons for maximum student learning, their self-efficacy would also improve. Teacher participants were also given “people” support, in the form of knowledgeable experts in the fields of mathematics and science, as well as material support, such as research-based books, flip cameras and iPads, to help with integrating mathematics and science.

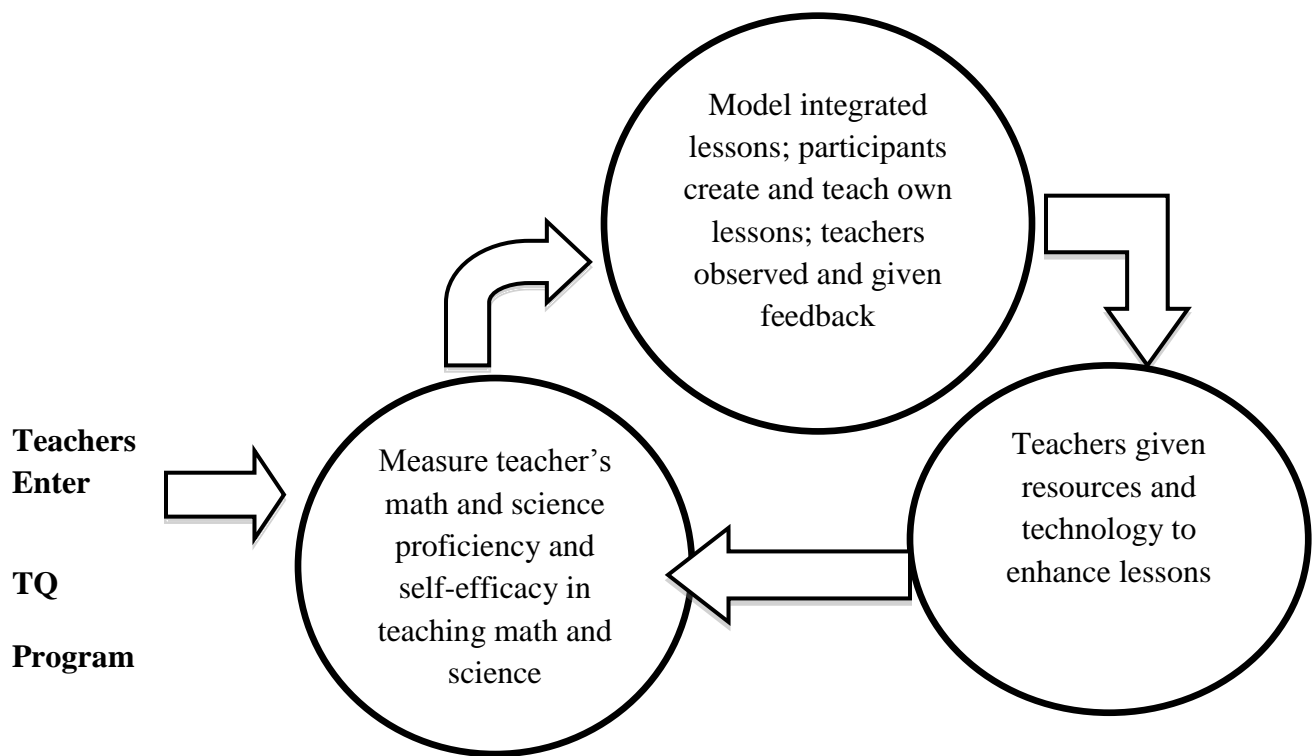


Figure 1.1 Concept Map of the TQ Grant

1.1 Rationale

Although many American schools may struggle for funding, technology, and materials, they are still among the most fortunate in the world when compared to other developing nations. According to recent research, however, American children remain stagnant in their academic understanding and development in the fields of mathematics and science (Gonzales, et al., 2009). The world has quickly evolved into a technologically revolutionary one, where technology, television, computers, and cellular phones instantly gratify students. America's students' education should reflect the same level of imaginative and enterprising methods of constructing their own understanding through engaging experiences. Thus, today's students need experiences that allow them to acquire mathematical and scientific understanding by doing and exploring mathematics and science.

Aristotle stated, "All who have meditated on the art of governing mankind have been

convinced that the fate of empires depends on the education of youth" (as cited in Halligan, 2007), which is an incredibly poignant statement. Aristotle knew that an educated society is a productive society and, similarly, an educated teacher produces an educated student. With this in mind, reform measures should naturally include an inquiry into university teacher preparation programs, to evaluate their effectiveness in producing competent, qualified, inspired, and confident teachers. Research into math and science teacher education programs has highlighted the importance of both content knowledge and pedagogy. Many students are ill-prepared to take introductory college courses in mathematics and science, therefore requiring them to take remedial mathematics and science courses to help bridge the gap (Attewell, Lavin, Domina, & Levey, 2006).

Individuals teach based on their own experiences and understandings, constructing a method for teaching it. If this method is never challenged, the individual holds on to these concepts and perceptions until they are challenged by another way of thinking (Smith III, 1996). A typical American math and/or science class will most likely consist of a teacher models, student copies method of remembering procedures and rules, completely devoid of any concrete experience or understanding of the problem; the deeper understanding of why. Teachers continue to teach the way they were taught, where some students will be fortunate enough to understand the mathematics and science concepts, while others will fail to see math and science in a way that is conducive to their learning. These students will, consequently, continue to fall further and further behind in these content areas.

Several countries are rapidly raising student achievement in math and science while American students show only minimal improvement. The United States lagged behind

Singapore, Taiwan, China, Korea, and the Netherlands in both math and science achievement (Gonzales, et al., 2009). President Obama (2010) stated, “Our competitors understand that the nation that out-educates us today will out-compete us tomorrow”. These countries are focusing on educational development and are producing educated, productive citizens as a result. The largest employer in the fields of math and science professions is the United States government (Branscomb & Johnson, 1992).

The *Programme for International Student Assessment* (PISA) is a study conducted by the *Organisation for Economic Co-operation and Development* (OECD). The OECD is an international organization whose purpose is to promote the economic and social well-being for all individuals worldwide. The PISA is given to 15-year-old students, in what most countries consider compulsory education, in order to measure their preparedness in entering the working society. PISA is unique in that it measures students’ ability to apply knowledge and skills in reading, mathematics and science and not just assess for mastery of school curricula. PISA also asks students about their motivations, beliefs and learning styles (PISA,)

As adapted from *Highlights from PISA 2006: Performance of U.S. 15-Year-Old Students in Science and Mathematics Literacy in an International Context* (Baldi, et al., 2007), Table 1.1 clearly highlights, the United States of America must re-evaluate the current mathematics and science programs, standards, and practices. If America continues on the path it has been on, the rest of the world will continue to outperform American children. Since mathematics and science rely so heavily on each other, integration of instruction in both of these content areas seems like one natural solution to the lackluster academic performance of American students.

Table 1.1 Average Scores of 15-Year-Old Students on the 2006 PISA

Science literacy scale	Math literacy scale
<i>Finland 563</i>	<i>Finland 548</i>
<i>Canada 534</i>	<i>Korea, Republic of 547</i>
<i>Japan 531</i>	<i>Netherlands 531</i>
<i>New Zealand 530</i>	<i>Switzerland 530</i>
<i>Australia 527</i>	<i>Canada 527</i>
<i>Netherlands 525</i>	<i>Japan 523</i>
<i>Korea, Republic of 522</i>	<i>New Zealand 522</i>
<i>Germany 516</i>	<i>Belgium 520</i>
<i>United Kingdom 515</i>	<i>Australia 520</i>
<i>Czech Republic 513</i>	<i>Denmark 513</i>
<i>Switzerland 512</i>	<i>Czech Republic 510</i>
<i>Austria 511</i>	<i>Iceland 506</i>
<i>Belgium 510</i>	<i>Austria 505</i>
<i>Ireland 508</i>	<i>Germany 504</i>
<i>Hungary 504</i>	<i>Sweden 502</i>
<i>Sweden 503</i>	<i>Ireland 501</i>
<i>Poland 498</i>	<i>France 496</i>
<i>Denmark 496</i>	<i>United Kingdom 495</i>
<i>France 495</i>	<i>Poland 495</i>
<i>Iceland 491</i>	<i>Slovak Republic 492</i>
United States 489	<i>Hungary 491</i>
<i>Slovak Republic 488</i>	<i>Luxembourg 490</i>
<i>Spain 488</i>	<i>Norway 490</i>
<i>Norway 487</i>	<i>Spain 480</i>
<i>Luxembourg 486</i>	United States 474
<i>Italy 475</i>	<i>Portugal 466</i>
<i>Portugal 474</i>	<i>Italy 462</i>
<i>Greece 473</i>	<i>Greece 459</i>
<i>Turkey 424</i>	<i>Turkey 424</i>
<i>Mexico 410</i>	<i>Mexico 406</i>

Italics highlight those countries scoring (statistically significant) higher than the United States.

1.2 National Education Goals

The United States Department of Education National Goals, established in 1991, expected the country to be "...first in the world in math and science achievement by the year 2000" (Branscomb & Johnson, 1992, p.95). Over a decade has passed since the proclamation of the goal, and the United States is not even in the top five on various international assessments (Baldi, et al., 2007; Brown & Brown, 2007). Within the past 20 years, Presidents have created or adapted a national educational reform shortly after taking office. In the early 1990s, Clinton's first legislative act was an attempt at educational reform entitled, *Goals 2000: Educate America Act*. In January 2001, the *No Child Left Behind Act* (NCLB) was President G.W. Bush's attempt at reform (States' Impact on Federal Education Policy Project, 2010). In essence, these are just a couple of examples of presidential attempts at education reform that have emerged within the past decade to try and achieve those national goals.

Similarly, now in 2010, President Obama's attempt at education reform is coming into focus. Obama has made spending on education a critical component of the American Recovery and Reinvestment Act to help stimulate both our economy and children's education. On March 15, 2010, President Obama presented a plan for an updated Elementary and Secondary Education Act that will revamp President Bush's *No Child Left Behind Act* (Office of the Press Secretary, 2010). Thus, the goal of improving student success rates on mathematics and science assessments remains elusive.

As President Barak Obama has taken office, he too has his own visions of reform measures necessary for improved student performance. In 2009, Obama announced the Race to the Top campaign to improve American education, setting aside an unprecedented \$4.35 billion.

President Obama claimed in his Race to the Top Fact Sheet:

America will not succeed in the 21st century unless we do a far better job of educating our sons and daughters... And the race starts today. I am issuing a challenge to our nation's governors and school boards, principals and teachers, businesses and non-profits, parents and students: if you set and enforce rigorous and challenging standards and assessments; if you put outstanding teachers at the front of the classroom; if you turn around failing schools – your state can win a Race to the Top grant that will not only help students outcompete workers around the world, but let them fulfill their God-given potential (Obama, 2009).

Another more recent reform attempt is President Obama's Educate to Innovate, which is directed at improving students' performance in the Science, Technology, Engineering, and Mathematics (STEM) content areas. This latest endeavor involves universities and various organizations working together to help promote quality STEM education (Obama, 2010).

President Obama stated in his March 13, 2010, Weekly Address:

The quality of math and science teachers is the most important single factor influencing whether students will succeed or fail in science, technology, engineering, and math...Passionate educators with issue expertise can make all the difference, enabling hands-on learning that truly engages students—including girls and underrepresented minorities—and preparing them to tackle the grand challenges of the 21st century such as increasing energy independence, improving people's health, protecting the environment, and strengthening national security (Obama, 2010, p.1).

Science for All Americans (Rutherford & Ahlgren, 1990) established the set of recommendations to ensure the improvement in scientific literacy. The document states, “It is the union of science, mathematics, and technology that forms the scientific endeavor and that makes it so successful.” The document stresses the focus on the interconnected nature of science and mathematics. The document also discusses the nature of the connections between science and mathematics. It says, “Science provides mathematics with interesting problems to investigate, and mathematics provides science with powerful tools to use in analyzing data.” When discussing mathematical inquiry, the document specifies the following three phases: “(1) representing some aspects of things abstractly, (2) manipulating the abstractions by rules of logic to find new relationships between them, and (3) seeing whether the new relationships say something useful about the original things” (Rutherford & Ahlgren, 1990). In an attempt to improve one content area, can result in the improvement of both content areas.

1.3 How Students Learn Mathematics

The National Research Council, whose members come from the councils of the National Academy of Sciences, the National Academy of Engineering, the Institute of Medicine, and the Institute of Medicine collaborated on a project to improve the understanding of how people learn. This undertaking produced *How People Learn* (National Research Council, 2000). This work spurred the creation of the book *How Students Learn: History, Mathematics, and Science in the Classroom*. The guiding principles from this research attempts to explain the critical attributes of a high caliber, cognitively demanding mathematics program, where learners are engaged with real-world situations and act as reflective thinkers, adjusting and reevaluating their initial preconceptions (National Research Council, 2000).

The three guiding principles stated in *How Students Learn: History, Mathematics, and Science in the Classroom* (2000) are as follows:

1. Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information, or they may learn them for purposes of a test but revert to their preconceptions outside the classroom.
2. To develop competence in an area of inquiry, students must (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that facilitate retrieval and application.
3. A “metacognitive” approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them.

This work states that the current mathematics instruction in the United States fails to connect, build and refine the mathematical understandings and intuitions students hold (Principle 1), in favor of instruction that is grounded in rules and procedures, disconnecting students from making meaning of the mathematics found in everyday situations. Instead of having students build their procedural proficiency in the skills necessary to conceptually understand the various mathematical concepts (Principle 2), these same skills and procedures become the lesson focus. It is for this reason of removing context and meaning from procedural knowledge that students neglect to use reflective strategies (Principle 3). Students find no need to evaluate the reasonableness of their procedure, since it is seen as only a series of rules or steps to follow, completely devoid of meaning and purpose (National Research Council, 2000).

The National Research Council (2000) established a series of strands that were, when taken as a whole, represent the strands necessary for mathematical proficiency. Table 1.2 illustrates these strands with a brief description of each. These strands are directly related to the National Research Council's Principles of *How People Learn* (National Research Council, 2000). Principle 2 states that procedural (factual) knowledge must be taught within a conceptual framework, which is the goal for the strands conceptual understanding, procedural fluency, and strategic competence. Principle 3 highlights the importance of metacognition as do the strands adaptive reasoning and productive disposition. Principle 1 serves as the starting point; where teachers build on the preconceptions and prior knowledge that students come to school clinging on to, in order to try and make sense of the world around them. People create their own strategies for responding to the world around them. These informal problem-solving strategies establish a foundation for building more abstract mathematical ideas and connections. Teachers must act on this knowledge to explicitly make the connection between the informal and formal, because many students do not see these connections.

Table 1.2 The National Research Council's Five Identified Strands of Mathematical Proficiency

Strand	Description of Strand
Conceptual understanding	comprehension of mathematical concepts, operations, and relations
Procedural fluency	skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
Strategic competence	ability to formulate, represent, and solve mathematical problems
Adaptive reasoning	capacity for logical thought, reflection, explanation, and justification
Productive disposition	habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

The National Research Council gives an excellent example of how procedural knowledge can serve as a less efficient method of answering the question:

What, approximately, is the sum of $\frac{8}{9}$ plus $\frac{12}{13}$?

Many students, and even some adults, were taught to first find the least common denominator (in this case it is not that easy) then add the fractions together. This method of solution is based on procedural knowledge. No real concept of the size of each fraction or what each fractional part represents is necessary. One only has to try and memorize a series of steps or procedures, then apply each step the best one can.

A conceptual approach to solving this problem would involve looking at each fraction as being “almost 1”, so the approximate sum is “almost 2”. It is a much more efficient method,

especially since an estimate is an appropriate response. If an exact answer is required, then procedural knowledge would be the knowledge type to call upon. If students are never given the opportunity to make the connections for using procedural knowledge as a tool in solving a conceptually understood situation, then students will struggle in becoming more efficient mathematical thinkers (National Research Council, 2000). This example shows the need for more efficient and proficient maneuvering between different types of mathematical knowledge.

In 1956, Blooms' Taxonomy of Educational Objectives was devised in order to identify student behavior in relation to curriculum standards and assessment. Six levels of student knowledge were described ranging from lower cognitive demands to higher-order thinking. Bloom's Taxonomy has been a mainstay in education since its inception and has sparked many other variations of looking at education through such a lens (Holmes, 2012).

In order of lowest to highest cognitive demands are: knowledge, comprehension, application, analysis, synthesis and evaluation. Knowledge refers to recall of information that was previously learned. Comprehension refers to the student's ability to understand the material's meaning. Application refers to the ability to use the information learned in new and existing situations. Analysis refers to the ability to break apart information into deconstructed components in an attempt to find hidden meaning. Synthesis refers to the ability to use those deconstructed parts of information in order to construct new meaning. The highest level of Bloom's Taxonomy is evaluation, which refers to the ability to make precise judgments of value for a stated purpose (Holmes, 2012). The proposed framework builds on the work of Bloom's Taxonomy. Bloom's levels of knowledge and comprehension correlate to the mathematical knowledge of *Type 1: Knowledge of Facts and Procedures*. Bloom's levels of application and analysis correlate to the mathematical knowledge of *Type 2: Knowledge of Concepts and*

Connections. Bloom's levels of synthesis and evaluation correlate to the mathematical knowledge of *Type 3: Knowledge of Models and Generalizations*.

Looking at the National Council of Teachers of Mathematics (NCTM), the Texas Essential Knowledge and Skills (TEKS) statements, and the National Common Cores Standards established by President Obama's Secretary of Education, one can use the level of knowledge required of students to decide where the standards fall- either on the lower or higher end. This becomes a useful activity for educators to help them ensure many methods of mathematics instruction are taking place. Using the mathematics strand of algebraic thinking, a strand many people often fear, an analysis of the cognitive demands can be made by focusing on the verbs.

For example, the NCTM standards for the middle school algebra strand state that students are expected to:

represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules; *relate and compare* different forms of representation for a relationship; *identify* functions as linear or nonlinear and *contrast* their properties from tables, graphs, or equations; *develop* an initial conceptual understanding of different uses of variables; *explore* relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope; *use* symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships; *recognize* and *generate* equivalent forms for simple algebraic expressions and solve linear equations (NCTM, 1998).

Identifying the verbs according to their placement on Bloom's Taxonomy allows the teacher to decide which activities will target each specific verb and the knowledge required of students. This can be critical in scaffolding student work in order to achieve the desired cognitive demand. For the NCTM algebra strand of mathematics, the only verbs on the higher end of the taxonomy are *generalize* and *generate*. This highlights the need for a more critical look at how students are learning mathematics, so that higher-order thinking becomes a desirable and achievable goal.

Using the TEKS algebra strand of mathematics as an example, students are expected to:

use algebraic thinking to *describe* how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships (Texas Education Agency, 2006).

At the 6th grade, examples of student expectations are:

use ratios to *describe* proportional situations; *represent* ratios and percents with concrete models, fractions, and decimals; *use* ratios to *make predictions* in proportional situations; *use* tables and symbols to *represent* and *describe* proportional and other relationships; *use* tables of data to *generate* formulas (Texas Education Agency, 2006).

As was the case with the NCTM standards, there are only a few verbs that are classified as higher-order thinking. In the case of the TEKS, *generate* is the only verb on the higher end of Bloom's Taxonomy and the only expectation requiring students to use the *Type 3: Knowledge of Models and Generalizations*.

Using the Common Core State Standards for Mathematics in the same manner to evaluate the level of depth of student knowledge, one can see that standards fail to maintain a steady shift through cognitive demands. The Common Core State Standards for Mathematics in algebra are:

apply and *extend* previous understandings of arithmetic to algebraic expressions; *reason* about and *solve* one-variable equations and inequalities; *represent* and *analyze* quantitative relationships between dependent and independent variables.

Analysis of these mathematics standards exemplifies how most require only the first four levels of Bloom's Taxonomy: knowledge, comprehension, application, and analysis. If students are not required to manipulate their mathematical knowledge over many cognitive demands, then they are missing an opportunity to move beyond the analysis level and build deeper mathematical understanding.

Procedural knowledge is imperative as a means for efficient problem solving. However, if students only view mathematics as a series of procedures they may fail to ever make sense of why these procedures work and what it means to *do* [emphasis added] mathematics. Teachers must teach students to use procedural knowledge as a tool to be called upon many times throughout this journey of mathematical exploration. The solution then moves from being a "request for computation" (p. 220) to being a problem situation that allows for the exploration of various knowledge types (National Research Council, 2000).

In Chapter 2, the Literature Review discusses a brief description of previous approaches to integration of mathematics and science education. In Chapter 3, the newly proposed framework is presented and explained. Chapter 4 contains the lesson plans analysis for various sources that are analyzed from the point of view of the proposed framework. The last chapter, Chapter 5, includes a discussion of the findings and implications for future research.

Chapter 2: Literature Review

2.1 Historical Perspective of Integration in Education

The idea of integrating mathematics and science instruction is not new. As a matter of fact, it dates back into the early 1900's (Czerniak, et al., 1999). With budget cuts, pressures for improved standardized testing and societal influences among some of the many obstacles educators must face in the challenge to educate America's youth, the idea of integrating mathematics and science instruction seems like a natural "fit". With their inherent value in one another, mathematics being the language of science and science the answer to questions about the world around us, why does the research on the successful implementation of an effective integrated curriculum not yet exist?

One of the reasons given by several sources is that a clear definition of what integrated instruction means is still inconclusive. In addition to no clear definition, many resources call it various names including: *inter-disciplinary, multidisciplinary, transdisciplinary, thematic, integrated, connected, nested, sequenced, shared, webbed, threaded, immersed, networked, blended, unified coordinated, and fused* (Czerniak, et al., 1999). With so many different names, it is no wonder that educators have not yet attained successful implementation of integrated mathematics and science instruction.

In addition to multiple definitions and words used to essentially describe integration, various models of integration also exist with regard to the extent a specific content area is emphasized. Some models classify lessons according to what content area is emphasized in a particular lesson. This creates opportunities for various interpretations, creating more confusion in integration.

2.2 Challenges of Integration in Education

The idea of an integrated curriculum ideally supports the claim that all content areas be taught simultaneously, in order to show students the interrelatedness and dependence of one content area upon another. This ideal model is not without its own challenges. The focus of this study is on the integration of mathematics and science, regardless of any other explicit or implied integration from another content area. The challenges of either model of integration are congruent and therefore used in this research.

In 1994, Berlin and Hillen reported the results from a study where students participated in using the Activities Integrating Mathematics and Science (AIMS) program. Data from 2,025 students in fourth, fifth, and sixth grades, from six different states was collected (Berlin & Hillen, 1994). The following conclusions were made in the cognitive, affective, and social domains. Berlin and Hillen found the following:

Some of the cognitive outcomes identified by the teacher-researchers include: increased understanding of math and science concepts; development of process skills (e.g., observation, classification, measurement, hypothesizing); using appropriate terminology; selecting and using scientific equipment; making interdisciplinary connections; application of concepts to the real world; use of the scientific method; and the development of life skills such as communication, understanding graphs, measurement, and visual/spatial relationships. In addition to the cognitive outcomes, one affective finding of interest was that students showed "increased motivation and interest in math, science, and learning in general" (p. 287). Three specific social

outcomes of interest were also found: an increase in student interaction, improvement in classroom behavior, and the development of the ability to work in a group or alone (Berlin & Hillen, 1994).

With so many potential benefits to integrating mathematics and science instruction, why has the idea remained so elusive? There are several challenges to integration. One of the challenges is the lack of research to support integration. There fails to be a consistent depth of research in favor of integration, especially when considering that the idea of integration dates back into the 1900's (Meier, Cobbs, & Nicol, 1998). Without substantial support for integration, the other challenges of integration can make the idea seem daunting and inconsequential.

Another critical barrier to integration is the cognitive level of the lessons. If one merely integrates mathematics into a science lesson for the sole purpose of integration, then the lack of cognitively demanding tasks makes the lesson unproductive and ineffective at teaching to all knowledge types. Many of the mathematical concepts that are integrated are usually very basic, cognitively low tasks (Meier, Cobbs, & Nicol, 1998). This type of integration allows for many conceptual gaps in the students' mathematical understanding. This type of mathematics lesson would be taught superficially, with concepts being included only for the sake of integration, without consideration of the implications for teaching and learning.

Assuming cognitively-structured lessons that integrate mathematics and science, while maintaining the fidelity of the knowledge types, was attainable, this creates another challenge for integration, the knowledge of both content and instruction required by the teacher. Many teachers lack depth and understanding of mathematics beyond a procedural understanding (Tchoshanov, 2010; Meier, Cobbs, & Nicol, 1998). Many secondary teachers are content

specialists in the area they teach, with no comprehensive understanding of the other content areas. Most elementary teachers have a general basic understanding of many content areas, with no in-depth understanding of any one in particular, since they are usually required to teach many subjects (Meier, Cobbs, & Nicol, 1998). This lack of cross-curricular knowledge hinders the teaching and learning of mathematics and/or science. This challenge creates a sense of doubt in the teacher, consequently creating another challenge; teacher self-efficacy.

Teachers' own awareness of a lack of an extensive understanding of more than one content area may limit how effective the teacher perceives himself/herself to be or how effective the teacher really is. Mathematics anxiety is a common problem with preservice teachers (Meier, Cobbs, & Nicol, 1998). This anxiety coupled with the idea of teaching mathematics in a scientific context may further exacerbate the challenge for many teachers. For those teachers that have an in-depth understanding of their content area, they may not find the need for integration, feeling they can teach cognitively demanding content in isolation. Yet others may feel they are teaching integrated curricula, when realistically there is no integration taking place (Meier, Cobbs, & Nicol, 1998).

2.3 Mathematics and Science Integration

The National Council of Teachers of Mathematics (NCTM) was one of the first educational organizations to publish a set of national mathematics standards. Six assumptions were made based on what was believed to be requisite of a well-designed curriculum. In *Concept-Based Curriculum and Instruction*, by H. Lynn Erickson, she states the NCTM standards as (pp. 28-29):

- Be conceptually designed. A conceptual approach enables children to acquire clear and stable concepts by constructing meaning in the context of physical situations and allows mathematical abstractions to emerge from empirical experience. A strong conceptual framework allows skills to be acquired in ways that make sense to children...; and supports the development of problem solving.
- Actively involve children in doing mathematics. “Young children...construct, modify, and integrate ideas by interacting with the physical world, materials, and other children”.
- Emphasize the development of children’s mathematical thinking and reasoning abilities. The curriculum must...instill in students a sense of confidence in their ability to think and communicate mathematically, to solve problems, to demonstrate flexibility in working with mathematical ideas and problems, to make appropriate decisions in selecting strategies and techniques, to recognize familiar mathematical structures in unfamiliar settings, to detect patterns, and to analyze data.

Clearly, these standards highlight the usefulness of integrating mathematics and science. By conducting science investigations, students have an opportunity to experience firsthand the application of various mathematics concepts and skills.

2.4 Previous Frameworks

In 1967, a conference addressing professionals in education was held in order to promote the instructional approach of integrating mathematics and science instruction. Ideas for integration were shared as well as recommendations for future research. Participants of the 1967 Cambridge Conference described five distinct categories where mathematics and science interaction vary. The five categories were:

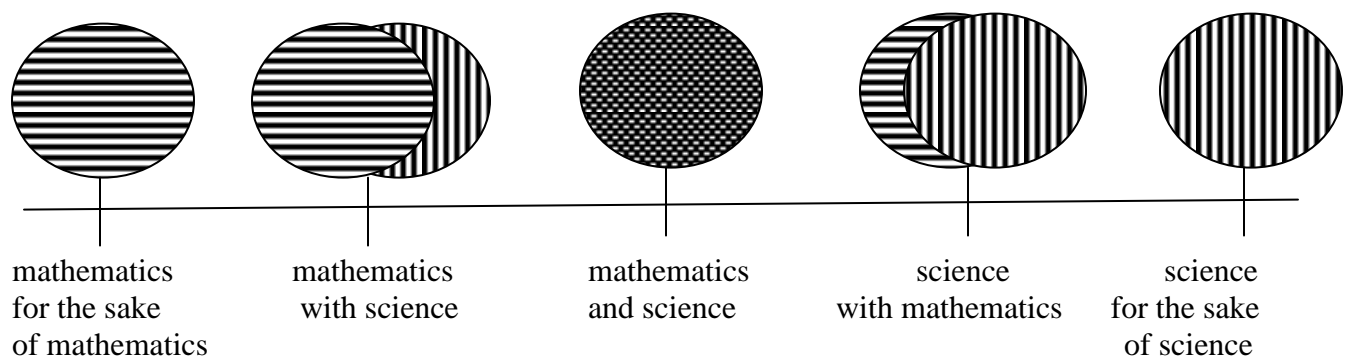


Figure 1.2 Mathematics/Science Continuum

mathematics for the sake of mathematics a mathematics course that presents mathematics as a formal system

mathematics with science a mathematics course in which science (content and/or methods) is used to establish problem context and relevance

mathematics and science mathematics (content and methods) and science (content and methods) course in which these two disciplines play synergistic roles in explaining the world

science with mathematics a science course which emphasizes mathematics (content and/or methods) as a tool for solving scientific problems

science for the sake of science a science course in which the habits and instincts of working scientists (science content and/or methods) dominate

(Education Development Center, 1969)

Lonning & DeFranco (1997) presented the following Continuum of Integration of Mathematics and Science Concepts/Activities model.

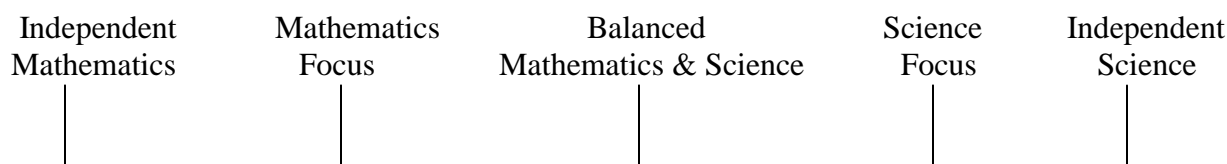


Figure 1.3 Continuum of Integration of Mathematics and Science Concepts/Activities

Independent Mathematics: This level of integration includes concepts best taught in a purely mathematical context; this includes integration within the discipline.

Mathematics Focus: This level of integration includes mathematics concepts as of primary importance. Science concepts/activities are in support of mathematics concepts.

Balanced Mathematics & Science: This level of integration includes activities that provide for integration of equally appropriate mathematics and science concepts/activities.

Science Focus: This level of integration includes science concepts as of primary importance. Mathematics concepts/activities are in support of science concepts.

Independent Science: This level of integration includes concepts best taught in a purely scientific context; this includes integration within the discipline.

Lonning & DeFranco (1997) propose that lessons fall somewhere on this continuum, based on the focus of the lesson. Lessons range from those that are entirely mathematics or science oriented, with no explicit integration of the other, to a complete balanced approach with equal emphasis on mathematics and science. The ideal situation would be a balanced program that integrates, with equal emphasis on mathematics and science, in a cognitively demanding manner.

Several theorists claim that a hierarchy can be established as lessons become more and more integrated (Erikson, 2001). This is an interesting notion. The idea that integration itself can

be classified as more or less integrated. However, is this enough to classify a lesson as high quality and cognitively demanding, or is it merely looking at how integrated lessons can be; where they fall on a hierarchy continuum? It is this question that drove the notion of looking at knowledge types in mathematics. If researchers and educators begin to look deeper into lessons and the knowledge required of students, then the possibility of a fully developed integrated approach to teaching and learning mathematics may be attainable.

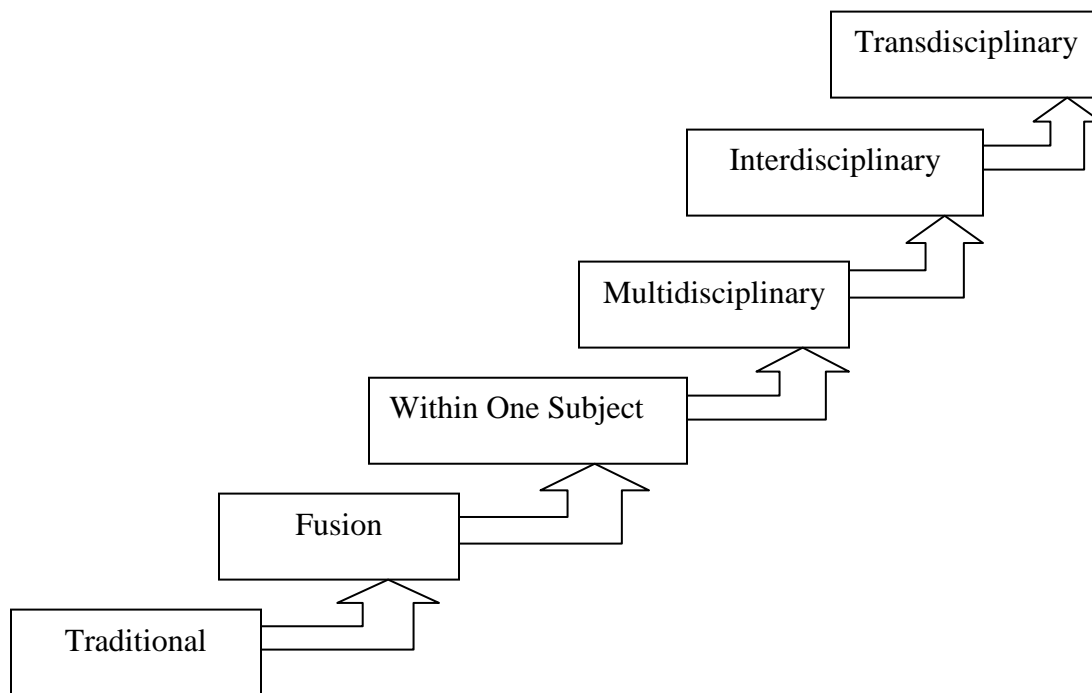


Figure 1.4 Hierarchy of Integration Approaches

In Erickson's (2001) approach to integration, a lesson becomes more integrated as it moves up the hierarchy of levels. Levels range from the traditional model of teaching subjects in isolation, to a fusion model that fuses various subjects into an existing curriculum to the most integrated model, transdisciplinary, where discipline boundaries become indistinct with the focus

on the connections within each discipline being of utmost importance (Lyublinskaya & Kerekes, 2008). Students apply integrated learning by tapping into various subjects interchangeably while solving problems in a real-world context.

Chapter 3: Methodology

3.1 Proposed Framework

As participants created lessons, another interesting question arose among the researchers, what cognitive level are the students being challenged at, and is it at the correct level of rigor? Current reform in state mandated tests requires teachers to teach a more focused and cognitively challenging curriculum that is vertically aligned, Kinder through 12th grades (Texas Education Agency, 2011). A natural conclusion was to take the opportunity to describe learning levels as a solution to the more rigorous standardized testing. New science standards were also in effect this year. Soon, the mathematics standards will be revised and more aligned to national Common Core standards (Texas Education Agency, 2011).

What cognitive level are the students being challenged at, and is it at the appropriate level of rigor so that students are challenged to think critically? This question proposes the idea that various levels of learning and thinking exist and can be qualitatively measured in order to evaluate lessons. These various levels range from basic, rote learning/thinking to a more connected way of looking at mathematics, where ideas are interrelated, to an ability to generalize and make predictions based on a profound mathematical understanding. Based partly on the work of Dr. Mourat Tchoshanov and John Van DeWalle, three distinct types of mathematical learning and thinking are identified as: *Type 1: Knowledge of Facts and Procedures*, *Type 2: Knowledge of Concepts and Connections*, and *Type 3: Knowledge of Models and Generalizations* (Tchoshanov, 2010).

Looking at the different types of knowledge required of students in various integrated lessons, educators can begin to identify lessons that allow students to flexibly use these various

knowledge types as the need arises; this is the perspective of the proposed framework. The focus is on *how* [emphasis added] the students will learn, not on the level of mathematics and science integration. This parallels the idea first developed by the 1967 Cambridge Conference participants. They recognized that mathematics and science interact in varying degrees, and this variance is how they classified instruction; by the varying degrees of mathematics and science integration. The proposed framework also recognizes the importance of teaching mathematics and science in multiple ways, however, the focus is on classifying lessons according to the facilitation of varying mathematical knowledge types.

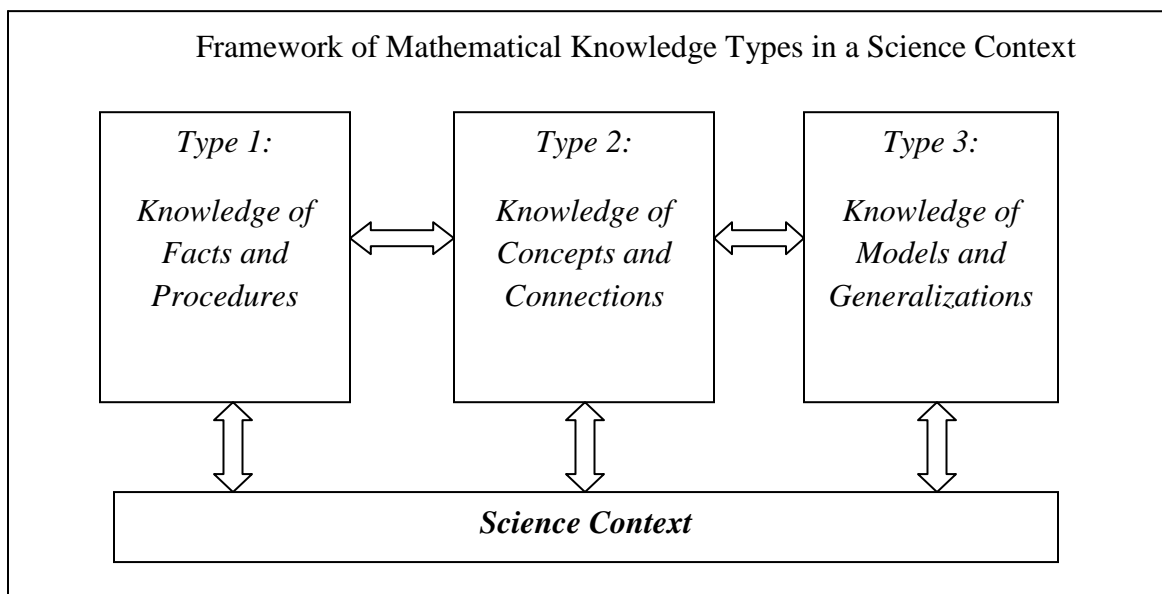


Figure 1.5 Proposed Framework of Mathematical Knowledge Types in a Science Context

The integrated Framework of Mathematical Knowledge Types in Figure 1.5 demonstrates how lessons can be created or evaluated and classified as primarily employing one type, two types or all three types of mathematical knowledge utilizing a science context. The mathematical knowledge types required varies depending on the situations and tasks students are expected to reason through. Figure 1.5 also displays how each knowledge type is important in its own right. The science context allows students to be engaged in a situation that gives them a reference

point, while allowing them to communicate and reflect on different ideas. Many of the lessons evaluated contain a distinct science context and/or investigation. According to Vygotsky (1994), communication is a cultural tool and mathematics has its own vocabulary and cultural “norms” (Steele, 2001). The *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) also identified the importance of communication in the learning and understanding of mathematics by stating “Through communication, ideas become objects of reflection, refinement, discussion, and amendment. The communication process also helps build meaning and permanence for ideas and makes them public” (p.60).

Keeping with the idea of teaching mathematics in a way that allows students the opportunity to discuss, reflect and refine their mathematical thinking and understanding, a science context provides this opportunity. Too often, lessons claim that they are integrated simply by including some very basic, procedural mathematics, such as measuring temperature. Although necessary to learn, the act of recording temperature becomes very procedural and rote if no context is given. Students are able to attach meaning to temperature, if students instead are presented with biomes around the world and asked what the difference in climate tells them; or what they can infer about these environments based on these differences in temperature; or what the central modes of tendencies represent. Students would definitely have to know how to read and write temperatures, but in the given context, students are also able to build relevance and purpose to an otherwise simple procedure, while gaining a deeper understanding of temperature ranges. A perspective is framed. Science investigations create a platform for the goals below to occur, even if the level of integration is simply the scientific method.

How Students Learn states, “While there is surely no single best instructional approach, it is possible to identify certain features of instruction that support the above goals:

- Allowing students to use their own informal problem-solving strategies, at least initially, and then guiding their mathematical thinking toward more effective strategies and advanced understandings.
- Encouraging math talk so that students can clarify their strategies to themselves and others, and compare the benefits and limitations of alternate approaches.
- Designing instructional activities that can effectively bridge commonly held conceptions and targeted mathematical understandings”.

(National Research Council, 2000)

In order to help teachers quickly and effectively evaluate lessons for mathematics rigor, a rubric was created based on questions that describe attributes of each one of Tchoshanov’s knowledge types. It was created with the intention of being universally teacher-friendly and applicable at any grade level. Teachers must simply answer a series of yes or no questions that are based on what their students would be engaged in or asked to produce as a product of learning. This allows teachers to think about the activities students will be participating in and determine what portion of the lesson students will be working at the various knowledge types. The goal is to develop teacher proficiency in thinking critically about integrated mathematics and science lessons, so that quality lessons, where students maneuver between and within each knowledge type, are selected. This method of evaluation is focused on student outcomes, performances, and ability to think flexibly. This supports the idea of how students learn. Table 1.3 is an example of the rubric used for evaluating each lesson.

Table 1.3 Designation of Knowledge Types Rubric

Designation of Knowledge Types Rubric:

Classification: Type ____

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1:</p>	... memorize/recall facts, definitions, formulas, properties, and rules?				
	... perform procedures and computations and solve routine problems?				
	... make observations?				
	... conduct measurements, collect data and represent data in table or graph?				
<p><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2:</p>	...select and use appropriate representation; translate between multiple representations?				
	...connect two or more concepts?				
	...explain and justify solutions to problems?				
	...solve non-routine problem				
<p><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3:</p>	...generate mathematical statements and test conjectures?				
	...design mathematical models?				
	...extrapolate findings from data analysis?				
	...prove statements and theorems?				

How to use this rubric

When deciding if an integrated mathematics and science lesson is worth the investment of time, resources and student learning, evaluate the lesson on the amount of time the lesson spends on each knowledge type. A lesson that optimizes all knowledge types is to be considered an effective tool at helping students facilitate the various modes of thinking, and thus a more effective tool for student learning. When deciding if a lesson is more heavily rooted in one knowledge type, answer the different question stems under each knowledge type that begins, “Do the students...”

- A lesson is considered *Type 1 Knowledge of Facts and Procedures* if Subtotal 1 is greater or equal to 6 (50% of total available points) **AND** Subtotal 2 is less than 6 (50% of total available points) and Subtotal 3 is less than 6 (50% of total available points).
- A lesson is considered *Type 2 Knowledge of Concepts and Connections* if Subtotal 2 is greater or equal to 6 (50% of total available points) **AND** Subtotal 3 is less than 6 (50% of total available points).
- A lesson is considered *Type 3 Knowledge of Models and Generalizations* if Subtotal 3 is greater or equal to 6 (50% of total available points).

It is assumed that there could not be a lesson with only *Type 2* or *Type 3* elements, since these types would require some or most of the elements found in *Type 1*.

The **HIGHEST** knowledge type that has 50% or more of subtotal present determines the designation of a lesson.

3.2 Procedural Knowledge

A student memorizing facts and rules on how to solve an algorithm without any understanding of why it is being used or what it represents requires a basic, disconnected mathematical understanding or knowledge. This type of mathematical knowledge is classified as *Type 1: Knowledge of Facts and Procedures*. This type of knowledge may sound undesirable, however, it serves another purpose. Students must learn to use procedural knowledge to help them solve problems more efficiently as well as be able to communicate their thinking. Van DeWalle (2010) classified this type of procedural knowledge as: "...knowledge of the rules and procedures used in carrying out routine mathematical tasks and also the symbolism used to represent mathematics. It is generally accepted that procedural rules should never be taught in the absence of concepts. These procedures without a conceptual base are the rules without reasons that lead to errors and a dislike of mathematics. All mathematics procedures can and should be [emphasis added] connected to the conceptual ideas that explain why they work. Procedures for doing mathematics, symbolism, and definitions are always preceded by a strong conceptual development. Conceptually developed procedures are often indistinguishable as being conceptual or procedural knowledge. It is this complete connection and integration of concepts and procedures that should be a primary goal."

An example of a *Type 1: Knowledge of Facts and Procedures* mathematics task is:

What is the rule for fraction division?

A. $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ B. $\frac{a}{b} \div \frac{c}{d} = \frac{ab}{cd}$ C. $\frac{a}{b} \div \frac{c}{d} = \frac{cd}{ab}$ D. $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$

(Tchoshanov, 2010)

This can be classified as a *Type 1: Knowledge of Facts and Procedures* mathematics task, because only a memorization of the rule is necessary for solving. A student may answer this question correctly, with an unassuming teacher not realizing the student may not really understand what division of fractions really means or how it is used in the real world. Procedural knowledge is a means to an end, a tool for solving a problem more efficiently. (National Research Council, 2000).

3.3 Conceptual Knowledge

Building an understanding of connected mathematical ideas and relational concepts is *Type 2: Knowledge of Concepts and Connections*. When students learn conceptually they are able to make connections within the mathematics; the ideas, rules, facts, and procedures all make sense and are grounded in an understanding of when and how to implement them (Tchoshanov, 2010 & Van DeWalle, 2010).

It is the job of the teacher to ensure conceptual contexts are structured in a way, as to incorporate procedural knowledge, exemplifying the usefulness of its inherent qualities. Unlike learning a language, which is centered in various contexts to make learning comprehensible, mathematics does not explicitly make such contexts readily available; therefore, the teacher must structure lessons around the opportunity to learn mathematics in a rich context (National Research Council, 2000). A context that allows the student to grapple with new ideas and situations, identifying the source of confusion, so questions can be asked or strategies can be implemented. Teachers can then build on this knowledge to introduce vocabulary and linking relevant informal strategies with formal conventions.

An example of a *Type 2: Knowledge of Concepts and Connections* mathematics task is:

Which problem represents the operation $1\frac{1}{2} \div \frac{1}{3}$?

Correct response: Maria has $1\frac{1}{2}$ liter of juice. How many $\frac{1}{3}$ liter containers can she fill?

(Tchoshanov, 2010)

In the above example, a mere memorization of the rule for dividing fractions is insufficient for solving this type of mathematical task. Students must have some degree of connection to what division of fractions means to how it is used in a given context. If students fail to have a conceptual understanding of what is learned procedurally, then if the situation

changes, the student may fail to adapt and the procedural knowledge becomes useless (National Research Council, 2000).

3.4 Generalized Knowledge

When a student has an understanding and knowledge base of the interconnectedness of mathematics and can use this information, transferring it to new and varied situations, a student is said to have generalized knowledge. This *Type 3: Knowledge of Models and Generalizations* in conjunction with *Type 1: Knowledge of Facts and Procedures* and *Type 2: Knowledge of Concepts and Connections* is the goal of education. Students who can think in generalizations can take their mathematics learning to a new level. Does generalized knowledge imply that the thinking is higher or more valuable? No, “Cognitive types of content knowledge are not hierarchical” (Tchoshanov, 2010). All knowledge types are necessary for developing well-rounded mathematical thinkers. When one thinks of the various knowledge types as tools for becoming proficient mathematical thinkers, it becomes easier to see that they are not hierarchical.

An example of a *Type 3: Knowledge of Models and Generalizations* mathematics task is:

Some students mistakenly divide two fractions in the following way (Tchoshanov, 2010):

If a , b , c , and d are positive integers, which of the following holds true:

- A. This equation is always true.
- B. This equation is true when $c=d$.
- C. This equation is never true.
- D. This equation is true when $ad=bc$.

In the above example, the student has a deeper understanding of “why” the algorithm works, and is able to understand mathematical statements in general terms. If a student knew the procedural method for dividing fractions or made a connection between the procedure and rule, does not guarantee that the above example could be answered correctly. Being able to think

reflectively, evaluating one's own thinking and revising it as more information is gained, in order to apply the knowledge in new situations is indicative of this type of knowledge.

The second principle found in *How People Learn* highlights the significance of both conceptual knowledge and procedural knowledge. The weaknesses in the level of conceptual knowledge of American students has attributed to low performance of mathematical achievement, while also implying that the system of teaching fundamentally procedurally is inefficient at producing mathematical thinkers. However, students with very little procedural knowledge do not become efficient problem solvers. This principle of learning reinforces the idea that all types of mathematical knowledge are critical (National Research Council, 2000).

Table 1.4 below describes each of Tchoshanov's mathematical knowledge types. Tchoshanov devised these classifications based on his research of the knowledge required of teachers in becoming successful mathematical leaders in their classrooms (Tchoshanov, 2010). The proposed framework also builds on these three knowledge types as a means for identifying high-rigor mathematical lessons. If lessons incorporate all three knowledge types in a manner that allows students to think adaptively, calling on whichever knowledge type is required for various situations, then the child is developing mathematical proficiency.

Table 1.4 Tchoshanov's Teacher Content Knowledge Cognitive Types' Descriptors

Cognitive Types of Teacher Content Knowledge	Descriptors
<p>Type 1. Knowledge of Facts and Procedures</p>	<ul style="list-style-type: none"> • Recognize basic terminology and notation • Recall facts • State definitions • Name properties and rules • Do computations • Make observations • Conduct measurements • Simplify and evaluate numerical expressions • Solve routine problems
<p>Type 2. Knowledge of Concepts and Connections</p>	<ul style="list-style-type: none"> • Select and use appropriate representation • Translate between multiple representations • Transform within the same representation • Transfer knowledge to a new situation • Connect two or more concepts • Explain and justify solutions to problems • Communicate big mathematical idea • Explain findings and results from analysis of data • Solve non-routine problems
<p>Type 3. Knowledge of Models and Generalization</p>	<ul style="list-style-type: none"> • Generalize patterns • Formulate mathematical problems • Generate mathematical statements • Derive mathematical formulas • Make predictions and hypothesize • Design mathematical models • Extrapolate findings from data analysis • Test conjectures • Prove statements and theorems

(Tchoshanov, 2010)

3.5 Research Question

To what extent do the published and participant lessons, that claim to integrate mathematics and science, use the different knowledge types? It is our hypothesis that most (approximately 90%) of the integrated mathematics and science lessons created by participants, as well as published curriculum, only teach mathematics content at the very basic, disconnected level of *Type 1: Knowledge of Facts and Procedures* with some lessons reaching the *Type 2: Knowledge of Concepts and Connections*. It is important to note that one level of knowledge is not seen as “better” than another, since they all serve a very useful and important purpose; rather, the goal is to successfully transition within these levels as the need arises (National Research Council, 2000 & Tchoshanov, 2010). The metacognitive awareness to decide which mathematical knowledge or tool is required in solving any given situation is the idyllic student learning. The objective of high caliber, integrated mathematics and science lessons should strive to ensure all types of knowledge are being challenged and developed. It is hypothesized that this approach for creating and evaluating lessons will enhance students’ learning, by a positive correlation in higher achievement on state, national, and international assessments. The student would ideally shift from being a student that can or cannot “do math” to a student who is mathematically proficient.

Chapter 4: Lesson Plans Analysis

In this chapter, lessons created by SABE MAS/TQ participants are evaluated according to the proposed framework. Published lessons are also evaluated in order to attempt to identify a trend in integrated mathematics and science lessons, where the focus is on the level of mathematics knowledge required. Middle school and high school lessons from participants are evaluated; while elementary through high school lessons from published sources are evaluated. Suggested grade level(s) and/or courses were included in the analysis. If objectives were stated in the lesson, they were also included in the analysis. If participants used the 5E Model for delivering instruction, this was identified in the analysis. A brief overview of each lesson is given, with a designation of a specific mathematical knowledge type assigned to it based on the level of knowledge necessary to complete the assignment. MS.1 refers to the first middle school lesson. HS.1 refers to the first high school lesson and PM/HS refers to a published middle/high school lesson.

The designation was judged holistically with regard to integration of lessons, as shown by the objectives (where included). It became evident that many participants struggled to effectively integrate mathematics and science at all or in doing so, created watered down mathematics applications. Participants either included math and science objectives, most doing so with great disparity, i.e., several of one type of objective with a few from the other-no balance; or participants neglected to include objectives at all. There was a similarity in the published lessons. Many published lessons included math and/or science objectives, with one content area usually dominating. Many of the published lessons that were evaluated came from the Activities Integrating Mathematics and Science (AIMS) Education Foundation; except where noted.

AIMS began as project funded by the National Science Foundation in 1981. The lessons adhere to recommendations in the national education reform documents: Project 2061 Benchmarks, National Research Council Standards, and National Council of Teachers of Mathematics (NCTM) Standards 2000 (AIMS Education Foundation, 2007). AIMS has a large collection of materials ranging from Kindergarten through High School (9th grade), is easily attainable for teachers, is known nationwide, and has been around since 1981, which is why lessons from this published source were selected.

All of the AIMS lessons are from the AIMS book that participants were given at their AIMS training in the Teacher Quality (TQ) grant. The AIMS book used was, *Earth Book: Hydrosphere, Geosphere, Atmosphere, and Their Interactions* (2007). The focus of Earth science was selected due in part to state and local data trends, as well as teacher experience. Participants of the TQ grant claimed that this was the branch of science in which students had the most difficulty. Participants used the resource as needed, with no real reporting back of their effectiveness. Therefore, a random selection of lessons is used to exemplify published integrated mathematics and science lessons from an organization claiming to integrate the two disciplines.

4.1 SABE MAS Middle School Lessons

The following lessons were created by the TQ and SABE MAS participants that teach at the high school level. All participants independently decided to use the 5E format in their lesson planning. All lessons have some form of activity to engage students' interest in the content, and then an exploration and explanation are followed by an evaluation and elaboration activity. Some lessons that are included may have little to no mathematical knowledge type present. These lessons are included in order to highlight teachers' misconceptions of integrated mathematics and science lessons, since these teachers were specifically asked to design integrated lessons.

MS.1: Kinetic vs. Potential Energy, presented by SABEMAS participants

Suggested Grade Level and Course(s): 6th Grade Mathematics and Science

OBJECTIVES: Objectives not identified as mathematics or science

- Use statistical & graphical models to represent real-world data
- Communicate through graphical models
- Use logical reasoning to form conclusions
- Acknowledge relationships between force and motion
- Identify changes in position, direction, of motion and speed when acted upon by a force and measure and graphically represent it
- Sketch circle graphs by organizing, interpreting, & displaying data
- Communicate mathematic ideas using graphical models
- Make conjectures from patterns or sets of examples & non-examples
- Convert between fractions, decimals, & percents
- Accurately read information from given tables
- Section pie graph using calculator & protractor

In this lesson students define kinetic and potential energy through exploration.

Students use a triple-beam balance to weigh various balls and then drop the balls from three specific heights for three trials each. Students measure the bounce of each ball in each trial. Students then find the gravitational potential energy and the kinetic energy. Once students have their data, they transfer their data to a line graph. Upon completion, the teacher leads a class discussion based on the acquired knowledge and its practical use in science. All groups present their findings.

In this lesson students go through the scientific process coming up with a hypothesis to answer the questions on different forms of energy and if the mass has an effect on the results. In this integrated approach students are able to understand the usefulness of mathematics in interpreting scientific explanations. Students are able to gain a conceptual understanding of the science involved, kinetic versus potential energy; however, the lesson fails to deliver the same type of knowledge in mathematics. The students merely use procedural knowledge to calculate the data. If students were asked to make generalizations or conjectures on larger or smaller massed balls, then the *Type 3: Knowledge of Models and Generalizations* designation would possibly be warranted. Students collect and manipulate various data, but the lesson does not make use of all the mathematical knowledge types. The data collected could have been used in more ways than simply as a tool for interpreting the science. The level of integration is commendable, but the lack of cognitive rigor in mathematics justifies this lesson receiving a designation of a *Type 1: Knowledge of Facts and Procedures*.

Designation of Knowledge Types Rubric: *Kinetic vs. Potential Energy*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p style="text-align: center;"><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 6</p>	... memorize/recall facts, definitions, formulas, properties, and rules?		1		Formulas for kinetic and potential energies
	... perform procedures and computations and solve routine problems??		1		Use triple-beam, solve for kinetic and potential energies
	... make observations?		1		Observe ball bounces
	... conduct measurements, collect data and represent data in table or graph?			3	Find mass of balls, data represented in both tables and graphs
<p style="text-align: center;"><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 1</p>	...select and use appropriate representation; translate between multiple representations?	0.5			Data in tables and graphs
	...connect two or more concepts?	0			
	...explain and justify solutions to problems?	0.5			Kinetic energy results vs potential energy results
	...solve non-routine problem	0			
<p style="text-align: center;"><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

MS.2: Kites, presented by SABEMAS participants

Suggested Grade Level and Course(s): Sixth Grade Math and Science

OBJECTIVES:

Math:

- Use addition and subtraction to solve problems involving fractions and decimals;
- Use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates; and
- Estimate and round to approximate reasonable results and to solve problems where exact answers are not required.
- Use ratios to describe proportional situations;
- Use ratios to make predictions in proportional situations.
- Estimate measurements and evaluate reasonableness of results;
- Select and use appropriate units, tools, or formulas to measure and to solve problems involving length (including perimeter and circumference), area, time, temperature, capacity, and weight;
- Draw and compare different graphical representations of the same data;
- Use median, mode, and range to describe data;
- Solve problems by collecting, organizing, displaying, and interpreting data.

Science:

- Demonstrate safe practices during field and laboratory investigations; and
- Make wise choices in the use and conservation of resources and the disposal or recycling of materials.

- Plan and implement investigative procedures including asking questions, formulating testable hypotheses, and selecting and using equipment and technology;
- Collect data by observing and measuring;
- Analyze and interpret information to construct reasonable explanations from direct and indirect evidence;
- Communicate valid conclusions; and
- Construct graphs, tables, maps, and charts using tools including computers to organize, examine, and evaluate data.
- Analyze, review, and critique scientific explanations, including hypotheses and theories, as to their strengths and weaknesses using scientific evidence and information;
- Draw inferences based on data related to promotional materials for products and services;
- Represent the natural world using models and identify their limitations;
- Evaluate the impact of research on scientific thought, society, and the environment; and
- Connect Grade 6 science concepts with the history of science and contributions of scientists.
- Collect, analyze, and record information using tools including beakers, petri dishes, meter sticks, graduated cylinders, weather instruments, timing devices, hot plates, test tubes, safety goggles, spring scales, magnets, balances, microscopes, telescopes, thermometers, calculators, field equipment, compasses, computers, and computer probes; and
- Identify patterns in collected information using percent, average, range, and frequency.

This lesson has a suggested timeframe of 15 days. Students research and decide on a kite pattern. As a class, students predict which pattern will fly the best based on criteria that they

come up with as a class. Students then write a one-page research paper on kites, including a hypothesis on which design will fly best and why they chose that hypothesis. Students review equivalent ratios, using a scale ratio of 1:3. Students design their kites using teacher-selected materials. As a class, students generate a table for collecting data based on which method is the best (histogram? bar graph? etc...). The kites are flown outside and data is gathered. Students solve for the central modes of tendencies and explain results in a conclusion.

MS.2 is designated as a *Type 1: Knowledge of Facts and Procedures* math task. Students are engaged in mathematical tasks involving connections between scales and ratios, yet are not asked to generalize. The level of integration is commendable and allows for various aspects of each content area; however, the focus is on the mathematics involved and a conceptual understanding is necessary, using procedural knowledge simultaneously. This lesson can very well be a *Type 3: Knowledge of Models and Generalizations* math task if the criterion the class decides to base the “best kite” design on is explored. Conjectures for the “best kite” designs can be made and defended mathematically, then tested to determine the reasonableness of the stated conjecture. This transference of knowledge to a new situation, based on new dimensions, or to real world designs can then be seen as generalized mathematical knowledge.

Designation of Knowledge Types Rubric: *Kites*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p style="text-align: center;"><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 6</p>	... memorize/recall facts, definitions, formulas, properties, and rules?		1		Scaling,
	... perform procedures and computations and solve routine problems??			2	Calculate area and perimeter
	... make observations?		1		Observe different shapes flying
	... conduct measurements, collect data and represent data in table or graph?			2	Measure, convert, represent results graphically
<p style="text-align: center;"><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 4</p>	...select and use appropriate representation; translate between multiple representations?		1		Decide on which graph would be best to represent data
	...connect two or more concepts?			3	Concepts of scale and ratio.
	...explain and justify solutions to problems?	0			
	...solve non-routine problem	0			
<p style="text-align: center;"><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

MS.3: Probability, presented by SABEMAS participants

Suggested Grade Level and Course(s):

OBJECTIVES: Only math objectives were included in this lesson.

- To construct sample spaces for simple or composite experiments
- To find the probability of independent events

This lesson has a suggested timeframe of about 3-4 days. On day 1, students are given a pretest consisting of probability and sample spaces. Students then play a game involving numbered cubes and a set of rules. Students are to strategize how they can increase their chances of winning. Students play the game, collecting data as the game progresses. Students are then given new numbered cubes changing the outcomes. The teacher leads a discussion based on guiding questions. On day 2, the class reviews the prior day's activity and creates definitions for probability, sample space and outcomes. Students again engage in probability experiments, collecting data, and responding to guiding questions. On day 3, students respond to guiding questions in their interactive notebooks. Students take a post-test and reply to a teacher-created survey. The purpose of the survey is for student feedback that the teacher will use.

This lesson is designated as a *Type 2: Knowledge of Concepts and Connections* math task. The students are able to build conceptual understanding of what probability, outcomes, and sample space are given the many ways of using probability, numbered cubes, spinners, and pennies.

Designation of Knowledge Types Rubric: *Probability*
Classification: Type 2

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 7.5</p>	... memorize/recall facts, definitions, formulas, properties, and rules?			2	Rules for setting up probability, recall definitions
	... perform procedures and computations and solve routine problems?		1.5		Procedure for solving probability, routine application of probability
	... make observations?			2	Observe results of experiments
	... conduct measurements, collect data and represent data in table or graph?			2	Data collected in frequency chart, table, tree diagram
<p><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 6</p>	...select and use appropriate representation; translate between multiple representations?			3	Students will write 3 probability problems using the experimental probability from the table
	...connect two or more concepts?			2	Different types of probability and sample spaces
	...explain and justify solutions to problems?		1		Justify best way to win, etc... for various probability experiments
	...solve non-routine problem	0			
<p><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 0.5</p>	...generate mathematical statements and test conjectures?	0.5			Students compare the two probabilities in a journal entry. At this point students must be able to see that the more experimental probability events occurs the closer it gets to the number on the theoretical probability.
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

MS.4: I Walk the Graph, presented by SABEMAS participants

Suggested Grade Level and Course(s): 8-9 Math or Integrated Physics/Chemistry

OBJECTIVES:

Math:

- The student makes connections among various representations of a numerical relationship. The student is expected to generate a different representation of data given another representation of data (such as a table, graph, equation, or verbal description).
- The student uses graphs, tables, and algebraic representations to make predictions and solve problems.
- Predict, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations.

Science:

- Plan and implement experimental procedures including asking questions, formulating testable hypotheses, and selecting equipment and technology;
- Make quantitative observations and measurements with precision;
- Organize, analyze, evaluate, make inferences, and predict trends from data;
- Graph data to observe and identify relationships between variables; and
- Read the scale on scientific instruments with precision.
- Generate and interpret graphs describing motion including the use of real-time technology;
- Analyze examples of uniform and accelerated motion including linear, projectile, and circular.

Students work in groups to decide on a graph to use. One of three roles is assigned to each student: “walk the graph”, “call out the coordinates”, or “call out the time”. The rest of the class attempts to draw the graph. Students then solve for their chosen graph’s slope, writing a story to go along with the graph’s slope making sure to include mathematical and scientific vocabulary.

The mathematical knowledge is procedural in that students only need to be able to have a procedural understanding of how to make a graph. Even though slope is connected to word problems, students are not being required to “transfer knowledge to a new situation or connect two or more concepts”, which are some key attributes of knowledge *Type 2: Knowledge of Concepts and Connections* (Tchoshanov, 2010). Nor are they being asked to “generalize patterns, formulate mathematical problems, or generate mathematical statements” as are the attributes of knowledge *Type 3: Knowledge of Models and Generalizations* (Tchoshanov, 2010). Therefore, this lesson is designated as a *Type 1: Knowledge of Facts and Procedures* math task.

Designation of Knowledge Types Rubric: *I Walk the Graph*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p style="text-align: center;"><i>Type 1.</i> <i>Knowledge of</i> <i>Facts and</i> <i>Procedures</i></p> <p style="text-align: center;">Subtotal 1: 6</p>	... memorize/recall facts, definitions, formulas, properties, and rules?		1		Rule for finding slope
	... perform procedures and computations and solve routine problems?			2	Calculate slope
	... make observations?			2	Students are given graphs, and then “recreate” them by walking.
	... conduct measurements, collect data and represent data in table or graph?		1		Represent their graph and slope
<p style="text-align: center;"><i>Type 2.</i> <i>Knowledge of</i> <i>Concepts and</i> <i>Connections</i></p> <p style="text-align: center;">Subtotal 2: 1.5</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?		1		Connect slope to word problems.
	...explain and justify solutions to problems?	0.5			Word problem used to explain given slope
	...solve non-routine problem	0			
<p style="text-align: center;"><i>Type 3.</i> <i>Knowledge of</i> <i>Models and</i> <i>Generalizations</i></p> <p style="text-align: center;">Subtotal 3: 0.5</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0.5			determine slope
	...prove statements and theorems?	0			

MS.5: Plotting Points, presented by SABEMAS participants

Suggested Grade Level and Course(s): 7th Grade Mathematics class

OBJECTIVES: None given

Students will plot points on a coordinate plane and translate the point horizontally or vertically in a positive or negative direction. Students will then use the Mpact program to plot and translate given points.

This lesson requires cognitively low expectations. Students only need a procedural understanding of the directionality of each axis to plot the points and the steps in translating the points. Therefore, this lesson is designated as a *Type 1: Knowledge of Facts and Procedures* mathematical task.

Designation of Knowledge Types Rubric: *Plotting Points*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p style="text-align: center;"><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p style="text-align: center;">Subtotal 1: 6</p>	... memorize/recall facts, definitions, formulas, properties, and rules?		1		Must know coordinates and axes
	... perform procedures and computations and solve routine problems?			2	Translate coordinates according to directions
	... make observations?		1		Observe new points
	... conduct measurements, collect data and represent data in table or graph?			2	Translate on coordinate grid
<p style="text-align: center;"><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p style="text-align: center;">Subtotal 2: 1</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?		1		Coordinates and translations
	...explain and justify solutions to problems?	0			
	...solve non-routine problem	0			
<p style="text-align: center;"><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p style="text-align: center;">Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

MS.6: Proportions, Predictions, and your Body, presented by SABEMAS participants

Suggested Grade Level and Course(s):

OBJECTIVES:

Math:

- Students will use ratios to describe proportions.
- Students will learn how to use tables and symbols to represent and describe proportional and other relationships involving conversions and their position in the sequence.

Science:

- Students will learn the anatomical names of a few body structures.

Students work in pairs to measure each other's assigned body part by its anatomical name. Students are to first make predictions on the measure of each body part and then work with their partner to find the actual measurement. Once this data is input into a table, students convert using the Metric System. Students are also asked to write the ratio representing the various measurements.

This lesson is cognitively low in terms of using the various types of knowledge. Students measure, then convert based on a sheet that is given to them. For a seventh grade classroom, the rigor is very low, as is the level of integration. Therefore, this lesson is designated as a *Type 1: Knowledge of Facts and Procedures* mathematics task lesson. Similar lessons can be found in the high school section, as a comparison for how it can be enhanced.

Designation of Knowledge Types Rubric: *Proportions, Predictions, and your Body*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
Type 1. Knowledge of Facts and Procedures Subtotal 1: 4	... memorize/recall facts, definitions, formulas, properties, and rules?	0			
	... perform procedures and computations and solve routine problems?			2	Conversions
	... make observations?		1		Observe and record measurements
	... conduct measurements, collect data and represent data in table or graph?		1		Measure different body parts and convert
Type 2. Knowledge of Concepts and Connections Subtotal 2: 2	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?			2	Measurement conversions, ratio of body part & circumference
	...explain and justify solutions to problems?	0			
	...solve non-routine problem	0			
Type 3. Knowledge of Models and Generalizations Subtotal 3: 0	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

MS.7: Probability Spinner, presented by SABEMAS participants

Suggested Grade Level and Course(s): 8th Grade Mathematics class

OBJECTIVES: Only math objectives were included in this lesson.

- Apply concepts of theoretical and experimental probability to make predictions.
- Use theoretical probabilities and experimental results to make predictions and decisions.

Students create spinners with five different sections based on the central angle. For example, $45^\circ \div 360^\circ = 0.125 = 12.5\% = 1/8$. Students then make a table to record and describe the theoretical probability of each section occurring. After that, students write three probability problems using the words likely, unlikely, equally likely and the theoretical probability information from their table. Students use their spinners to test for experimental probability, recording their results in a new table. Students create three probability problems based on their experimental probability data. Using their two data tables, students write in their journals comparing and contrasting the two types of probability. Students are to make the conjecture that the more an experimental probability event occurs, the closer it gets to the number on the theoretical probability table.

This lesson, although not integrated, does incorporate some aspect of all the types of knowledge.

Students use a procedural understanding of how to record the data; students must also have a conceptual understanding of how to create five different sections based on the central angle and use this data to write three problems; finally students must extrapolate findings from data analysis in order to generalize the results of an experimental probability in relation to the theoretical probability. Although an aspect from each knowledge type is present, the aspects are insufficient to classify this lesson beyond a *Type 2: Knowledge of Concepts and Connections*.

Designation of Knowledge Types Rubric: *Probability Spinner*
Classification: Type 2

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p style="text-align: center;"><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p style="text-align: center;">Subtotal 1: 7.5</p>	... memorize/recall facts, definitions, formulas, properties, and rules?			2	Recall how to set-up probability
	... perform procedures and computations and solve routine problems?		1.5		Compute for results of probability experiment
	... make observations?			2	Observe results of experiment
	... conduct measurements, collect data and represent data in table or graph?			2	Data collected in frequency chart, table, tree diagram
<p style="text-align: center;"><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p style="text-align: center;">Subtotal 2: 6.5</p>	...select and use appropriate representation; translate between multiple representations?			3	Students will write 3 probability problems using the experimental probability from the table
	...connect two or more concepts?			2.5	Experimental vs. theoretical probability
	...explain and justify solutions to problems?		1		Explain their results
	...solve non-routine problem	0			
<p style="text-align: center;"><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p style="text-align: center;">Subtotal 3: 1</p>	...generate mathematical statements and test conjectures?		1		Finally students compare the two probabilities in a journal entry. At this point students must be able to see that the more experimental probability events occurs the closer it gets to the number on the theoretical probability.
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

MS.8: Scales and Pythagorean Triangles, presented by SABEMAS participants

Suggested Grade Level and Course(s): 7th Grade Mathematics class

OBJECTIVES:

- Construct a mock scale using the listed materials to find the Pythagorean Triangles from the model. Convert measurements from inches to centimeters.
- Understand how the balance on the scale works.

Students construct a mock scale according to the written instructions on the handout.

After completing the scale, students then answer questions on a worksheet about conversions and Pythagorean Triangles.

This lesson is very basic and lacks all of the various mathematical knowledge types. This lesson is designated as a *Type 1: Knowledge of Facts and Procedures* even with the fact that students explore the Pythagorean Theorem by building different triangles (designing mathematical models) and are required to extrapolate findings on various triangles. Once again the opportunity to build on acquired knowledge, so that students can transfer this knowledge to new situations, is lost. The lesson uses procedural knowledge when students solve for conversions and answer questions regarding the Pythagorean Theorem on a worksheet.

Designation of Knowledge Types Rubric: *Scales and Pythagorean Triangles*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p style="text-align: center;"><i>Type 1.</i> <i>Knowledge of</i> <i>Facts and</i> <i>Procedures</i></p> <p style="text-align: center;">Subtotal 1: 10</p>	... memorize/recall facts, definitions, formulas, properties, and rules?			2	Pythagorean Theorem
	... perform procedures and computations and solve routine problems?			2	Convert and solve for Pythagorean Theorem
	... make observations?			3	Observe Pythagorean Theorem
	... conduct measurements, collect data and represent data in table or graph?			3	Measure triangles
<p style="text-align: center;"><i>Type 2.</i> <i>Knowledge of</i> <i>Concepts and</i> <i>Connections</i></p> <p style="text-align: center;">Subtotal 2: 1</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?	0			
	...explain and justify solutions to problems?		1		Explain how Pythagorean Theorem works
	...solve non-routine problem	0			
<p style="text-align: center;"><i>Type 3.</i> <i>Knowledge of</i> <i>Models and</i> <i>Generalizations</i></p> <p style="text-align: center;">Subtotal 3: 2</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?		1		Different triangles
	...extrapolate findings from data analysis?		1		Discover Pythagorean Theorem
	...prove statements and theorems?	0			

MS.9: Solving Proportions, presented by SABEMAS participants

Suggested Grade Level and Course(s): 6th Grade Mathematics class

OBJECTIVES:

- Use ratios to describe proportional situations
- Use ratios to make predictions in proportional situations

Students are solving for unit rates and proportions using cross products. The lesson is from an Internet website and incorporates the use of an interactive whiteboard. There is not much information given in this lesson.

This lesson is designated as a *Type 1: Knowledge of Facts and Procedures*. The only knowledge required is a basic, procedural understanding of how to follow the steps in solving proportions. No explicit connection is made, nor are generalizations asked of the students.

Designation of Knowledge Types Rubric: *Solving Proportions*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p style="text-align: center;"><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 4</p>	... memorize/recall facts, definitions, formulas, properties, and rules?	0			
	... perform procedures and computations and solve routine problems?			2	Solve proportions
	... make observations?	0			
	... conduct measurements, collect data and represent data in table or graph?			2	Draw house to scale
<p style="text-align: center;"><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 0</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?	0			
	...explain and justify solutions to problems?	0			
	...solve non-routine problem	0			
<p style="text-align: center;"><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

4.2 SABE MAS High School Lessons

The following lessons were created by the TQ and SABE MAS participants that teach at the high school level. All participants independently decided to use the 5E format in their lesson planning. All lessons have some form of activity to engage students' interest in the content, and then an exploration and explanation are followed by an evaluation and elaboration activity. Some lessons that are included may have little to no mathematical knowledge type present. These lessons are included in order to highlight teachers' misconceptions of integrated mathematics and science lessons, since these teachers were specifically asked to design integrated lessons.

HS.1: Logarithmic Scales in Nature, presented by SABEMAS participants

Suggested Grade Level and/or Course(s): 9th-10th

OBJECTIVES: No Objectives given. The lesson is in the 5E Model

Engage: Students are introduced to dimensional analysis by computing a large value quantity and converting it into a logarithm.

Explore: Students use geological time to help students make sense of standard notation and logarithms by exploring the geologic timescale on a linear scale and then on a logarithmic scale.

Explain: Students do a Web Quest on logarithms and logarithmic scales.

Extend: Students conduct logarithm research and presentation

Evaluate: Students will take an assessment

HS.1 is designated as a *Type 2: Knowledge of Concepts and Connections* math task, due to the high scores earned in domain type of the rubric. The context of geological time provides a relevant context and allows students to solve non-routine problems.

Designation of Knowledge Types Rubric: *Logarithmic Scales in Nature*
Classification: Type 2

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 5.5</p>	... memorize/recall facts, definitions, formulas, properties, and rules?			2	Recall definition of logarithm, definition of standard notation.
	... perform procedures and computations and solve routine problems??			3	Perform conversions express quantities in standard notation (the student computes a large value quantity and converts it into a logarithm).
	.. make observations?	0.5			Observe two computer-based components
	... conduct measurements, collect data and represent data in table or graph?	0			
<p><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 6</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?			3	Concept of logarithms is connected to Geologic timescale model via a linear scale and logarithmic scale.
	...explain and justify solutions to problems?	0			
	...solve non-routine problem			3	Non-routine use of logarithms and logarithmic scales
<p><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?				

HS.2: An Introduction to Wave Motion, presented by SABE MAS participants

Suggested Grade Level and/or Course(s): Grades 9-10: Integrated Physics & Chemistry,
Physics, Geology

OBJECTIVES: The lesson is in the 5E Model

- Students will have a better understanding of basic wave principles, wave characteristics, and wave calculations.

Engage: Students tie a fork to one end of a pencil. Students then gently place the eraser end of the pencil against their outer ear. Another student strikes the fork with a spoon. The students are then asked to describe the sound produced in terms of pitch and volume.

Explore/Explain: Students use a Slinky toy to make waves. The students record the waves produced by moving the Slinky back-and forth, then upload the video to the Logger Pro program. Once uploaded, students set parameters to measure the wavelength and then calculate frequency. The formula for wave speed and how to calculate wave speed is then discovered as students investigate further into waves, creating and testing their own hypothesis based on what students learn about how a wave behaves. The teacher goes over how to calculate the wave frequency and speed.

Elaborate: After having gained experience with waves, wave motion, wavelengths, and frequency, students create a learning journal where they write about what they learned. Students then work in pairs to create a poster explaining what they liked best or learned from the explore activity.

Evaluate: Students will be given a homework assignment to practice the wave speed formula explored in class. The following day, homework will be checked for correct calculations,

and a short assessment will be given, testing knowledge of wave parts and how to correctly use the wave speed formula.

HS.2 designated as a *Type 1: Knowledge of Facts and Procedures* math task. Students are only expected to perform routine tasks and make simple measurements. The participants are conscious to use a science context; however, this is an example of teachers' assuming basic computation is enough to suffice for a quality integrated lesson. Students are not challenged to think mathematically and engage in rote activities.

Designation of Knowledge Types Rubric: *An Introduction to Wave Motion*
 Classification: *Type 1*

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 4</p>	... memorize/recall facts, definitions, formulas, properties, and rules?	0			
	... perform procedures and computations and solve routine problems?			2	Calculate frequencies, wave speed
	... make observations?		1		Observe wave lengths
	... conduct measurements, collect data and represent data in table or graph?		1		Measure wavelengths
<p><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 0</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?	0			
	...explain and justify solutions to problems?	0			
	...solve non-routine problem	0			
<p><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

HS.3: Bones, Bones, Bones, presented by SABE MAS participants

Suggested Grade Level and Course: High School Biology

OBJECTIVES: The lesson is in the 5E Model. (Only science objectives were included in this lesson).

- The student will collect data and make measurements with precision;
- The student will organize, analyze, evaluate, make inferences, and predict trends from data;
- The student will communicate valid conclusions;
- The student will describe the connection between biology and future careers;
- The student sequences the levels of organization in multi-cellular organisms to relate the parts to each other and to the whole; and
- The student will interpret the functions of systems in organisms including circulatory, digestive, nervous, endocrine, reproductive, integumentary, skeletal, respiratory, muscular, excretory, and immune.

Engage: Students are given index cards with the human skeleton on it. A particular bone will be highlighted on each index card. Students will use the index cards to place each highlighted bone in descending order.

Explore: After having some experience with human bone size comparisons, students measure a piece of construction paper the length of their femurs. Students then stand against a wall (without shoes) to measure their height. Students use the piece of construction paper, which represents their femurs, to determine how many pieces equal their height.

Explain: At this point, the teacher provides the femur-height relationship formula. Students collect data on a class graph.

Elaborate: Students investigate and discuss the similarities in limbs of different animals and compare them to humans. In order to further teach the concept of body proportions, students measure their arm span and compare it to their height. Measurements are compiled on a class bar graph with three columns: square, tall rectangle, and far-reaching rectangle. Based on their height and arm span, students mark the column that best matches their height-arm span relationship.

This lesson is designated as a *Type 1: Knowledge of Facts and Procedures* math task. This lesson includes very procedural mathematics, although the lesson allows for a more conceptualized understanding of what proportions are, the teachers failed to explicitly include this connection in the lesson. Teachers could have also capitalized on the opportunity to make generalizations of human bone sizes to that of other animals, by creating and testing possible algorithms for describing these relationships. This is an example of science teachers collaborating on a science lesson, excluding the great opportunities to teach some higher levels of mathematical knowledge in an integrated lesson.

Designation of Knowledge Types Rubric: *Bones, Bones, Bones*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p style="text-align: center;"><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p style="text-align: center;">Subtotal 1: 6</p>	... memorize/recall facts, definitions, formulas, properties, and rules?	0			
	... perform procedures and computations and solve routine problems??			2	Computation of femur-height & arm span-height relationships
	... make observations?			2	Observe relationships
	... conduct measurements, collect data and represent data in table or graph?			2	Measure body parts, represent data in different graphs
<p style="text-align: center;"><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p style="text-align: center;">Subtotal 2: 4</p>	...select and use appropriate representation; translate between multiple representations?			3	Use of scaling, display data
	...connect two or more concepts?		1		Connect to percentages
	...explain and justify solutions to problems?	0			
	...solve non-routine problem	0			
<p style="text-align: center;"><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p style="text-align: center;">Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

HS.4: The Vitruvian Man, presented by SABE MAS participants

Suggested Grade Level and Course: High School Biology

OBJECTIVES: The lesson is in the 5E Model.

Science:

- Plan and implement investigative procedures including asking questions, formulating testable hypotheses, and selecting equipment and technology;
- Collect data and make measurements with precision;
- Organize, analyze, evaluate, make inferences, and predict trends from data; and communicate valid conclusions.

Math:

- Geometric thinking and spatial reasoning.
- Geometric figures and their properties.
- The relationship between geometry, other mathematics, and other disciplines.
- Tools for geometric thinking.

Engage: Students make predictions on human body proportions and a collaborative discussion ensues.

Explore: Much like the previous lesson, HS.4, students work in pairs to measure each other's height and arm span. Students calculate the difference between arm span and height (arm span-height) and waist size and elbow to wrist length (waist size-wrist to elbow length).

Explain: The data is added to a class graph. Students create an Excel spreadsheet based on the class graph and use it to determine if a correlation exists between arm span and height and waist size and elbow to wrist length, as Leonardo Da Vinci had stated. Students also convert between meters to centimeters in the lesson.

Elaboration: Students will analyze the graphs, and answer high rigor questions. Also add in extensions on their own allowing them ownership.

Evaluation: Students will be given TAKS problems from Science TEKS 2, 10A and Math TEKS 2, 3, 4, and 5.

This lesson is designated as a *Type 1: Knowledge of Facts and Procedures* math task. A noticeable difference is that students measure and collect data to determine proportionality; they are not given an algorithm to use. Students use a type of knowledge and thinking for generalization of mathematical statements. Although they must test for proportionality, they are not making many connections to other areas or using multiple representations, as with a conceptual type of knowledge. It is important to reiterate that the goal of the lessons is to move within the different knowledge types to foster a well-rounded mathematical thinker.

Designation of Knowledge Types Rubric: *The Vitruvian Man*
 Classification: *Type 1*

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p style="text-align: center;"><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 7</p>	... memorize/recall facts, definitions, formulas, properties, and rules?	0			
	... perform procedures and computations and solve routine problems??			3	Arm span height relationship, other bodily relationships
	... make observations?		1		Observe ratio relationship
	... conduct measurements, collect data and represent data in table or graph?			3	Graphs are made on paper and using Excel
<p style="text-align: center;"><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 0</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?	0			
	...explain and justify solutions to problems?	0			
	...solve non-routine problem	0			
<p style="text-align: center;"><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

HS.5: Density, presented by SABE MAS participants

Suggested Grade Level and Course: Integrated Physics and Chemistry

OBJECTIVES: The lesson is in the 5E Model (Only science objectives were included in this lesson).

- The student is expected to investigate and identify properties of fluids including density, viscosity, and buoyancy.

Engage: Students compare the density of two glasses of water, where one glass had ten tablespoons of salt added to it and the other contains only water. An egg is placed into each glass and students observe how the egg floats in the salt water and sinks in the water.

Explain: Students discuss why the egg floated in one glass, but sank in the other.

Extend: The teacher, with examples of everyday substances and their densities, discusses a brief lesson on density. The teacher explains the terms density, buoyancy, and viscosity.

Explore: Students create scales (based on ones created in the SABE MAS program). Using their scales, the students would have to find the density of “mystery” liquids in opaque bottles to determine what liquid is inside.

Evaluate: Students turn in a paper with the bottle number, the density they calculated and their guess for what liquid is inside.

This lesson is designated as a *Type 1: Knowledge of Facts and Procedures* math task. The mathematics used in this lesson is procedural and does not require cognitively demanding skills. As the teacher planned this “integrated” lesson, no mathematics objectives are given, leading one to believe that by students performing some calculations the lesson is integrated.

Designation of Knowledge Types Rubric: *Density*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p style="text-align: center;"><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p style="text-align: center;">Subtotal 1: 6</p>	... memorize/recall facts, definitions, formulas, properties, and rules?			2	Formula for density
	... perform procedures and computations and solve routine problems?			2	Division, solve for density of different liquids
	... make observations?			2	Use scales to find density of mystery liquids
	... conduct measurements, collect data and represent data in table or graph?	0			
<p style="text-align: center;"><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p style="text-align: center;">Subtotal 2: 0</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?	0			
	...explain and justify solutions to problems?	0			
	...solve non-routine problem	0			
<p style="text-align: center;"><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p style="text-align: center;">Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

HS.6: Good Vibrations, presented by SABE MAS participants

Suggested Grade Level and Course: 9-12, Physical Science

OBJECTIVES: The lesson is in the 5E Model (Only science objectives were included in this lesson).

- Understand the difference in wavelength size in the electromagnetic spectrum;
- Understand that visible light makes up a very small portion of the electromagnetic spectrum.
- Understand that light travels in a straight line and refracts when it passes from one substance to another, which is the principle behind why a prism separates white light into individual colors.
- Understand the colors seen by the eye are a result of light being reflected, not absorbed.

Engage: Using a solo cup with a cut balloon stretched over it, a mirror glued to the middle of it, and a radio and laser pointers to demonstrate sound waves.

Explain: (adapted from Discoverschool.com) In this activity, students will create a model of the infrared, visible, and ultraviolet portions of the electromagnetic spectrum. The model they create will be made to scale based on wavelength. Students will convert measurements, using the scale 1 nanometer equals 1 millimeter.

Explore: Students look into the “Shoebox of Fun”; a shoebox with a square hole cut into the lid that is covered with red cellophane. Three gumballs colored red, white and green are glued down to the inside. Students describe what they see. The teacher then places primary colored transparencies on an overhead and asks students why the combined colors make black.

Elaborate: The teacher asks the students, “Why does the color change when chemicals are added to a fire, for example, Epsom salt makes a fire extremely orange?” and then performs a demonstration of this for the students.

Evaluate: A data sheet is given to students where various wavelengths are given in meters and students must convert them into millimeters and nanometers.

This lesson is designated as a *Type 1: Knowledge of Facts and Procedures* math task. The mathematics used in this lesson is very procedural and does not require cognitively demanding skills. Once again, students are expected to merely make procedural conversions, using low-level mathematics.

Designation of Knowledge Types Rubric: *Good Vibrations*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p style="text-align: center;"><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p style="text-align: center;">Subtotal 1: 8</p>	... memorize/recall facts, definitions, formulas, properties, and rules?			2	How to convert, scale
	... perform procedures and computations and solve routine problems?			2	Conversions and scale factor for model in Explanation
	... make observations?			2	Observe various wavelengths and what it implies
	... conduct measurements, collect data and represent data in table or graph?			2	Measure wavelengths, working with strips of colored paper to represent the visible spectrum.
<p style="text-align: center;"><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p style="text-align: center;">Subtotal 2: 5.5</p>	...select and use appropriate representation; translate between multiple representations?			3	Translate between various wavelengths, which have different scales
	...connect two or more concepts?		2		Science concept of light energy and math conversions and measuring
	...explain and justify solutions to problems?	0			
	...solve non-routine problem	0.5			Science context is non-routine, but the mathematics is routine
<p style="text-align: center;"><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p style="text-align: center;">Subtotal 3: 4</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?			2.5	Students design a model for wavelength using scale factor
	...extrapolate findings from data analysis?		1.5		Light spectrum data used to discover various wavelengths; more scientific than mathematical
	...prove statements and theorems?	0			

HS. 7: Logarithmic and Exponential Applications, presented by SABE MAS participants

Suggested Grade Level and Course: 11th and 12th grade, Algebra II or Pre-Calculus and/or

10th and 11th grade – I.P.C., Chemistry

OBJECTIVES: The lesson is in the 5E Model.

Mathematics:

- Formulate equations and inequalities based on exponential and logarithmic functions;
- Use a variety of methods to solve them;
- Analyze the solutions in terms of the situation.

Science:

- Know the properties and behavior of acids and bases.

Engage: The teacher leads a discussion on how exponential formulas and logarithms apply in both science and math.

Explore: Students work on a handout involving both subject areas.

Explain: Student groups compare answers and help each other as preparation for the quiz.

Elaborate: Teacher compares all the problems, bridging gaps of logarithmic concepts for students.

Evaluate: A quiz with a variety of questions from both science and math.

This lesson is designated as a *Type 2: Knowledge of Concepts and Connections* math task. Students are shown how logarithms and exponential functions are used to solve problem-based science situations: Richter scale readings; sound's relative intensity; and the pH levels of various common items. Students learn the process for solving logarithms and exponential functions in various real-world contexts developing mathematic relevance.

Designation of Knowledge Types Rubric: *Logarithmic and Exponential Applications*
Classification: Type 2

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p style="text-align: center;"><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 6</p>	... memorize/recall facts, definitions, formulas, properties, and rules?			2	Students need to recall many formulas (see handout)
	... perform procedures and computations and solve routine problems??			3	Computing functions
	... make observations?	0			
	... conduct measurements, collect data and represent data in table or graph?	0			
<p style="text-align: center;"><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 6</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?			3	Connect concepts of “compound interest”, “population growth”.
	...explain and justify solutions to problems?		1		Explaining to one another in teams
	...solve non-routine problem			3	Understanding “logarithm model”, non-routine applications of this model (earthquakes, Richter scale, sound, chemistry examples
<p style="text-align: center;"><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 2.5</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?			2.5	Exponential, logarithmic growth model.
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

HS.8: pH Titration and Light Absorbance presented by SABE MAS participants

Suggested Grade Level and Course: 9th - 10th Grade, Chemistry

OBJECTIVES: The lesson is in the 5E Model and only science objectives included.

- Use UV-visible spectroscopy or other form of spectroscopy to monitor the properties of anthocyanins in fruit juices;
- To use a pH meter to monitor changes in pH in a juice sample;
- To understand the relationship between pH and absorbance in fruit juices.

Engage: Demonstration of Red Tide USB Spectrometer System

Explore: Red Cabbage pH Indicator Activity

Explain: Anthocyanins in Fruit Juices Lab

Elaborate: Inquiry Activity

Evaluate: Group Presentations

This lesson is designated as a *Type 1: Knowledge of Facts and Procedures* math task.

Students go through a series of activities that will help the students construct an understanding of pH and titration as they apply to the various sciences. Starting with a demonstration of new technology, the students then explore some of the oldest technologies relating to pH measurement, through a more quantitative approach to pH measurement and make the connection between pH and absorbance in the spectrophotometer. The cognitively demanding mathematics is lacking. This lesson was more focused on the interdisciplinary aspects of science without regard to the mathematics.

Designation of Knowledge Types Rubric: *pH Titration and Light Absorbance*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p style="text-align: center;"><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 5.25</p>	... memorize/recall facts, definitions, formulas, properties, and rules?		1.25		How to read a pH indicator, least to greatest, etc...
	... perform procedures and computations and solve routine problems?	0			
	... make observations?			2	Observe the pH change of different liquids and wavelength times
	... conduct measurements and represent data in table or graph?			2	Measure various liquids
<p style="text-align: center;"><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 0</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?	0			
	...explain and justify solutions to problems?	0			
	...solve non-routine problem	0			
<p style="text-align: center;"><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

HS.9: Fruit Fly Population Study, presented by SABE MAS participants

Suggested Grade Level and Course: Middle School (grades 6-8) and high school (9-12).

Environmental Science and Biology Courses

OBJECTIVES: None stated. The lesson is in the 5E Model.

Engage: Students discuss whether a population can grow faster than the available food. Student pairs share with their partner switching roles speaking, listening and writing to create a discussion web on the focus question of population growth versus available food. A discussion web with the focus question

Explore: Students set up their own experiment with their own selection of jars, fruit, ecosystems, cover and flies.

Explain: Students compose sentence strips with definitions and pictures to define terms necessary for the lesson. Students then engage in a cooperative strategy with classroom made cards that they share with one another to review material.

Elaborate: Students respond to the following questions: “What would happen to the graph if additional food is introduced? What would happen to the graph if food was removed? What if a predator introduced into the ecosystem? How would the introduction of a spider affect the population curve?”

Evaluate: Students read an article with visuals showing the carrying capacity within a population. Cards are distributed to each student for discussion and explanation while performing the “mix and match”, where students stand in line and exchange information on their assigned cards every 2 minutes. Students are assessed formally using

standardized test questions at the end of the lesson. Students create an ecosystem of jars, fruit, a cover and flies, with the selection of their own variables; students monitor the daily population growth; collect data; draw the graph and make valid conclusions. Students are asked to generalize in the elaborate portion based on predictions using data, making the mathematics and science cognitively more demanding. Students also use fruit fly ecosystem as a non-routine method to explain exponential function, logarithmic function. These characteristics of the lessons make a compelling argument for a higher classification, however the amount of time spent on each task is not enough to designate the lesson as a *Type 3: Knowledge of Models and Generalizations*. Therefore, this lesson is designated as a *Type 2: Knowledge of Concepts and Connections* math task. It can be a slippery slope to look at lessons superficially and assume that a lesson uses more than one knowledge type throughout the lesson, which is why a scale including numbers between whole numbers is used.

Designation of Knowledge Types Rubric: *Fruit Fly Population Study*
Classification: Type 2

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 8.5</p>	... memorize/recall facts, definitions, formulas, properties, and rules?			2.5	Explain consists entirely of vocabulary recall
	... perform procedures and computations and solve routine problems??	0			
	... make observations?			3	Monitor daily growth
	... conduct measurements, collect data and represent data in table or graph?			3	Daily collection and display of data in graphs
<p><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 6</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?		2		Population growth to available resources; science connection
	...explain and justify solutions to problems?			2	Students use graphs to explain what would happen if different variables arise
	...solve non-routine problem		2		Use of fruit fly ecosystem non-routine method to explain exponential and logarithmic function
<p><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 2</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?			2	In Elaborate, students asked "what if..." statements based on data
	...prove statements and theorems?	0			

4.3 Published Lesson Analysis Overviews and Rubrics

PL.1: Science and Mathematics in Balance, by Peggy A. House

Overview: The lesson involves learning about the center of gravity using a children's toy known as *Blockhead!* (Pressman Toy Corp. 1992) and some important geometric principles.

Designation of Knowledge Types Rubric: *Science and Mathematics in Balance*
Classification: Type 3

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<i>Type 1. Knowledge of Facts and Procedures</i>	... memorize/recall facts, definitions, formulas, properties, and rules?		1		Medians and area size recall
	... perform procedures and computations and solve routine problems?			2	Calculate scales
	... make observations?			2	Observe where plumb line falls to identify centroids
	Subtotal 1: 7			2	Measure centroids
<i>Type 2. Knowledge of Concepts and Connections</i>	...select and use appropriate representation; translate between multiple representations?			2	Switching between 2D and 3D representations.
	...connect two or more concepts?			3	Dilations, ratios
	...explain and justify solutions to problems?			3	Justify how they found centroids
	Subtotal 2: 11			3	Applying to non-routine situations.
<i>Type 3. Knowledge of Models and Generalizations</i>	...generate mathematical statements and test conjectures?			3	Different ways of finding Centroid of the shape. Using unusual, sophisticated shapes.
	...design mathematical models?			3	Drawing models on paper.
	...extrapolate findings from data analysis?			3	Making general conclusion.
	Subtotal 3: 9	0			

PL.2: A Science Application of Area and Ratio Concepts, by Virginia M. Horak

Overview: Students use footprints, hoofprints and paw prints in different surfaces to draw conclusions about the animals' size and weight.

Designation of Knowledge Types Rubric: *A Science Application of Area and Ratio Concepts*
Classification: Type 2

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 6</p>	... memorize/recall facts, definitions, formulas, properties, and rules?		1		Formulas for area, proportions
	... perform procedures and computations and solve routine problems??		2		Solve for area and proportions
	... make observations?		1		Observe different animal prints in relation to surface area
	... conduct measurements, collect data and represent data in table or graph?			2	Collect data on different animal prints
<p><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 10</p>	...select and use appropriate representation; translate between multiple representations?			2	Ratios, "sinking values"
	...connect two or more concepts?			3	Estimation, proportional reasoning, area
	...explain and justify solutions to problems?			2	Justify "sinking values" of weight vs. area of imprint
	...solve non-routine problem			3	"Sinking value" of different imprints
<p><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

PL.3: A Worthwhile Task to Teach Slope, by Lauren L. Wagener

Overview: By introducing students to biomedical engineering, students are given the opportunity to conduct a biomechanical test and use math and science to analyze data and answer the question, "Does knotting a suture strengthen or weaken it?"

Designation of Knowledge Types Rubric: *A Worthwhile Task to Teach Slope*
Classification: Type 2

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 4</p>	... memorize/recall facts, definitions, formulas, properties, and rules?		1		Slope "rise over run"
	... perform procedures and computations and solve routine problems?		1		Solve for slope
	... make observations?		1		Observe Hooke's law in action
	... conduct measurements, collect data and represent data in table or graph?		1		Represent data
<p><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 9</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?			3	Slope, Hooke's law, discrete and continuous patterns of change
	...explain and justify solutions to problems?			3	Explain discrete and continuous patterns of change
	...solve non-routine problem			3	Context non-routine for Hooke's law
<p><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

PL.4: See the Math behind the Medicine, by Marnie M. Saunders

Overview: By introducing students to biomedical engineering, they are engaged in a hands-on biomedical test. Students use math and science to analyze data and answer the question, "Does knotting a suture strengthen or weaken it?"

Designation of Knowledge Types Rubric: *See the Math behind the Medicine*
 Classification: Type 2

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
Type 1. <i>Knowledge of Facts and Procedures</i> Subtotal 1: 4	... memorize/recall facts, definitions, formulas, properties, and rules?	0			
	... perform procedures and computations and solve routine problems?		1		Computation to compare Load and "breaking point"
	... make observations?		1		Observe the "breaking point" of various sutures
	... conduct measurements, collect data and represent data in table or graph?			2	Collect data on various suture ruptures
Type 2. <i>Knowledge of Concepts and Connections</i> Subtotal 2: 6	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?	0			
	...explain and justify solutions to problems?			3	Use data to explain breaking points
	...solve non-routine problem			3	Context and use of biomedical devices non-routine
Type 3. <i>Knowledge of Models and Generalizations</i> Subtotal 3: 0	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

PL.5: An Investigation of Solar Moon by, H. Bruce Stewart, Maria E. Reininger, Walter A. Smudzinski

Overview: Students use principles of geometry, measurement and calculations to interpret the sun's apparent movement across the sky.

Designation of Knowledge Types Rubric: Solar Moon
Classification: Type 2

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1. Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 8</p>	... memorize/recall facts, definitions, formulas, properties, and rules?		1		Rules for proportions
	... perform procedures and computations and solve routine problems?			2	Computations and proportions
	... make observations?			2	Observe shadows in relation to angles
	... conduct measurements, collect data and represent data in table or graph?			3	Collect shadow lines and measure angles, represent data
<p><i>Type 2. Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 9</p>	...select and use appropriate representation; translate between multiple representations?			3	Geometric representations, setting proportions.
	...connect two or more concepts?			3	Geometric constructions are connected to concept of proportion.
	...explain and justify solutions to problems?	0			
	...solve non-routine problem			3	"Solar noon" not typical and is compared to "clock noon"
<p><i>Type 3. Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

PL.6: Where is the Moon Tonight? by Susann M. Mathews, Kevin F. Cornell, Beth A. Basista

Overview: After studying the Earth's rotation, students studied the Moon and its patterns and relationships between Earth, Moon, and Sun using 2D and 3D representations; developed units to measure 3D angles, and worked with data analysis and multiple representations.

Designation of Knowledge Types Rubric: *Where is the Moon Tonight?*

Classification: Type 2

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 4</p>	... memorize/recall facts, definitions, formulas, properties, and rules?		1		How to measure angles
	... perform procedures and computations and solve routine problems?		1		Shade proper amount of "moon"
	... make observations?		1		Observe moon phases to correlate with angles
	... conduct measurements, collect data and represent data in table or graph?		1		Measure angles, altitude
<p><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 8</p>	...select and use appropriate representation; translate between multiple representations?			3	Translating between 3-D and 2-D
	...connect two or more concepts?			2	2D angles to 3D angles
	...explain and justify solutions to problems?			2	Justify angles in relation to moon position
	...solve non-routine problem		1		Non-routine use of angles
<p><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

PL.7: Predator/Prey Simulation by Irina Lyublinskaya, J. Kerekes

Overview: This is an outdoor activity, students simulate the predator/ prey roles of bobcats and rabbits. Students observe changes to population as resources increase and decrease. Students represent the data graphically and make predictions based n this graph.

Designation of Knowledge Types Rubric: *Predator/Prey Simulation*
Classification: Type 3

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 8</p>	... memorize/recall facts, definitions, formulas, properties, and rules?	0			
	... perform procedures and computations and solve routine problems?		2		Compute number of predators and prey
	... make observations?			3	Students observe data trends
	... conduct measurements, collect data and represent data in table or graph?			3	Represent data in tables and graphs
<p><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 9</p>	...select and use appropriate representation; translate between multiple representations?			2	Translate between different representations
	...connect two or more concepts?			2.5	Connect computation, graphing, reasoning, predictions
	...explain and justify solutions to problems?			2	Justify why data represents activity
	...solve non-routine problem			2.5	Context non-routine
<p><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 6</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?			3	Model of competition for food
	...extrapolate findings from data analysis?			3	Data used to predict for future seasons
	...prove statements and theorems?	0			

PL.8: Vertical Ocean Currents from AIMS Earth Book: Hydrosphere, Geosphere, Atmosphere, and Their Interactions

Overview: Students observe what happens when cold water meets hot water in a container filled with room temperature water.

Designation of Knowledge Types Rubric: *Vertical Ocean Currents*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 2</p>	... memorize/recall facts, definitions, formulas, properties, and rules?	0			
	... perform procedures and computations and solve routine problems?	0			
	... make observations?			2	Students observe "currents"
	... conduct measurements, collect data and represent data in table or graph?	0			
<p><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 0</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?	0			
	...explain and justify solutions to problems?	0			
	...solve non-routine problem	0			
<p><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 0</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?	0			
	...prove statements and theorems?	0			

PL.9: Layers of the Earth from AIMS Earth Book: Hydrosphere, Geosphere, Atmosphere, and Their Interactions

Overview: Students make “core samples” out of centicubes, then make a paper slice of the Earth’s layers and a paper clip compass to answer questions regarding the layers.

Designation of Knowledge Types Rubric: *Layers of the Earth*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 7</p>	... memorize/recall facts, definitions, formulas, properties, and rules?	0			
	... perform procedures and computations and solve routine problems?		1		Create a paper clip compass
	... make observations?			3	Students make many observations to construct layers of Earth
	... conduct measurements, collect data and represent data in table or graph?			3	Use paper clip compass to collect data on different layers of Earth
<p><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 4</p>	...select and use appropriate representation; translate between multiple representations?		1.5		3D Earth to 2D Earth
	...connect two or more concepts?	0			
	...explain and justify solutions to problems?	0			
	...solve non-routine problem			2.5	Creating layers based on angles and scale
<p><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 4</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?			3	Model of Earth’s layers used to identify arcs, angles, etc...
	...extrapolate findings from data analysis?		1		Data used to explain composition of layers
	...prove statements and theorems?	0			

PL.10: Sensational Changes from AIMS Earth Book: Hydrosphere, Geosphere, Atmosphere, and Their Interactions

Overview: Students use different Earth materials to test which one will absorb the most heat and which one will cool the fastest. Students make predictions and compare to actual results.

Designation of Knowledge Types Rubric: *Sensational Changes*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 7</p>	... memorize/recall facts, definitions, formulas, properties, and rules?	0			
	... perform procedures and computations and solve routine problems?		1		Basic procedures to collect temperature
	... make observations?			3	Activity mostly consists of observations
	... conduct measurements, collect data and represent data in table or graph?			3	Collect temperatures and display
<p><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 1</p>	...select and use appropriate representation; translate between multiple representations?	0			
	...connect two or more concepts?		1		What effects rate of temperature change
	...explain and justify solutions to problems?	0			
	...solve non-routine problem	0			
<p><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 2</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?			2	Students use data to answer questions
	...prove statements and theorems?	0			

PL.11: Geofossils from AIMS Earth Book: Hydrosphere, Geosphere, Atmosphere, and Their Interactions

Overview: Geometric shapes are used to model fossils of foraminifera in order to identify their habitats and how long ago they lived. The “geofossils” are classified by angles and a census for each genus and species is completed.

Designation of Knowledge Types Rubric: *Geofossils*
Classification: Type 1

	Do the students...	Very Little Present $0 \leq n < 1$	Some Present $1 \leq n < 2$	Mostly Present $2 \leq n \leq 3$	Comments
<p><i>Type 1.</i> <i>Knowledge of Facts and Procedures</i></p> <p>Subtotal 1: 10</p>	... memorize/recall facts, definitions, formulas, properties, and rules?			2	Students must recall what they know about angle measurements
	... perform procedures and computations and solve routine problems?			3	Geometric shapes used in specific ways
	... make observations?			2	Observe the shapes for clues to past
	... conduct measurements, collect data and represent data in table or graph?			3	Data collected and displayed in different ways
<p><i>Type 2.</i> <i>Knowledge of Concepts and Connections</i></p> <p>Subtotal 2: 4</p>	...select and use appropriate representation; translate between multiple representations?		1		Translate between different representations
	...connect two or more concepts?		1		Concept of angles and how they are measured
	...explain and justify solutions to problems?	0			
	...solve non-routine problem			2	Non-routine use of “geofossils”
<p><i>Type 3.</i> <i>Knowledge of Models and Generalizations</i></p> <p>Subtotal 3: 1.5</p>	...generate mathematical statements and test conjectures?	0			
	...design mathematical models?	0			
	...extrapolate findings from data analysis?		1.5		Data used to try to classify habitats
	...prove statements and theorems?	0			

Chapter 5: Discussion

5.1 Discussion of Findings

Results from the data analysis indicate a profound need for improved lessons that aim to integrate mathematics in science, especially when considering the mathematical knowledge required. Mathematics is not used in isolation in the real-world. Students must be given the opportunity to engage in relevant mathematics that requires careful analysis of the students' own thinking. Opportunities that allow for the fine-tuning of procedural knowledge; the application and construction of conceptual knowledge; and the ability to use generalized knowledge in order to make conjectures based on their understandings of the mathematics, while being aware that they may need to make revisions to their current thinking, is what will ultimately improve this nation's students' academic achievement.

Analysis of the following pie charts shows that approximately half of the lessons that were analyzed, use mostly the lowest type of mathematical knowledge. The good news is that around 50% of all the lessons were found to be Knowledge Types 2 & 3. However, Type 3 comprises only 7%, which is insufficient to fulfill recommendations from science education documents. As Science for All Americans states, "Mathematical inquiry goes hand in hand with scientific investigation and requires thinking abstractly, manipulating the abstract by rules of logic to find new relationships between them." This is my understanding of what is meant by Mathematical Knowledge Type 3.

A limitation of the study is in the access to the lessons that integrate math and science. The author collected most of the math and science integrated lessons that were available to the graduate student through the university.

Table 1.5 Middle School Lesson Analysis Overview

Middle School Lesson Number	Classification	Type 1 Subtotal (Maximum 12)	Type 2 Subtotal (Maximum 12)	Type 3 Subtotal (Maximum 12)
MS.1	1	6	1	0
MS.2	1	6	4	0
MS.3	2	7.5	6.5	0.5
MS.4	1	6	1.5	0.5
MS.5	1	6	1	0
MS.6	1	4	2	0
MS.7	2	7.5	6.5	1
MS.8	1	10	1	2
MS.9	1	4	0	0

Percentages of Each Knowledge Type in Middle School Lessons

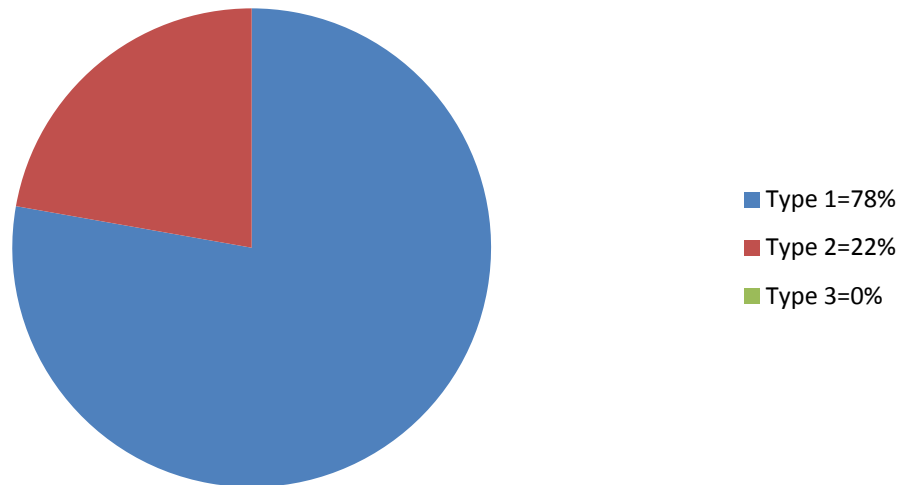


Figure 1.6 Percentages of Each Knowledge Type in Middle School Lessons

Table 1.6 High School Lesson Analysis Overview

High School Lesson Number	Classification	Type 1 Subtotal (Maximum 12)	Type 2 Subtotal (Maximum 12)	Type 3 Subtotal (Maximum 12)
HS.1	2	5.5	6	0
HS.2	1	4	0	0
HS.3	1	6	4	0
HS.4	1	7	0	0
HS.5	1	6	0	0
HS.6	1	8	5.5	4
HS.7	2	6	6	2.5
HS.8	1	5.25	0	0
HS.9	2	8.5	6	2

Percentages of Each Knowledge Type in High School Lessons

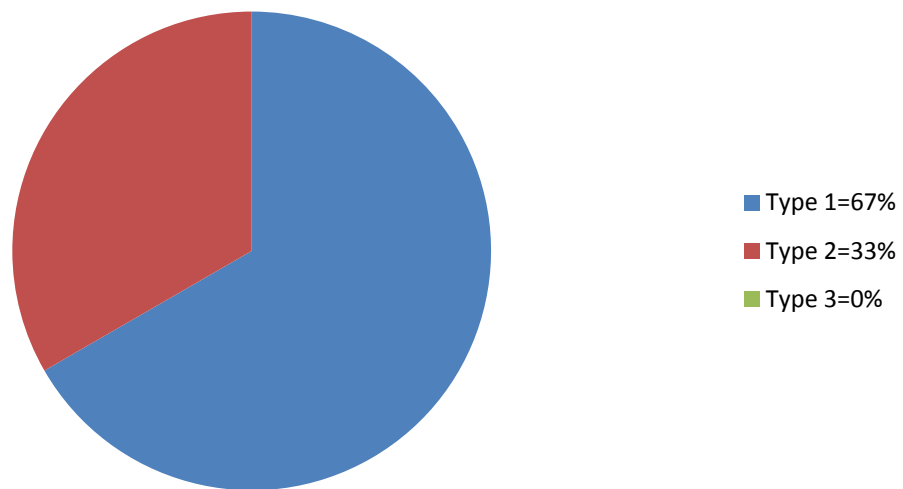


Figure 1.7 Percentages of Each Knowledge Type in High School Lessons

Table 1.7 Published Lesson Analysis Overview

Published Lesson Number	Classification	Type 1 Subtotal (Maximum 12)	Type 2 Subtotal (Maximum 12)	Type 3 Subtotal (Maximum 12)
PS.1	3	7	11	9
PS. 2	2	6	10	0
PS.3	2	4	9	0
PS.4	2	4	6	0
PS.5	2	8	9	0
PS.6	2	4	8	0
PS.7	3	8	9	6
PS.8	1	2	0	0
PS.9	1	7	4	4
PS.10	1	7	1	2
PS.11	1	10	4	1.5

Percentages of Each Knowledge Type in Published Lessons

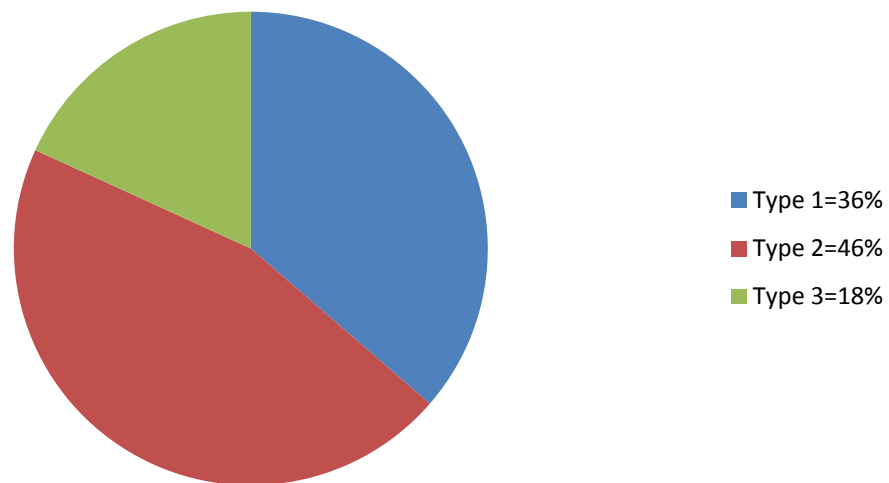


Figure 1.8 Percentages of Each Knowledge Type in Published Lessons

Percentages of Each Knowledge Type for All Lessons Analyzed

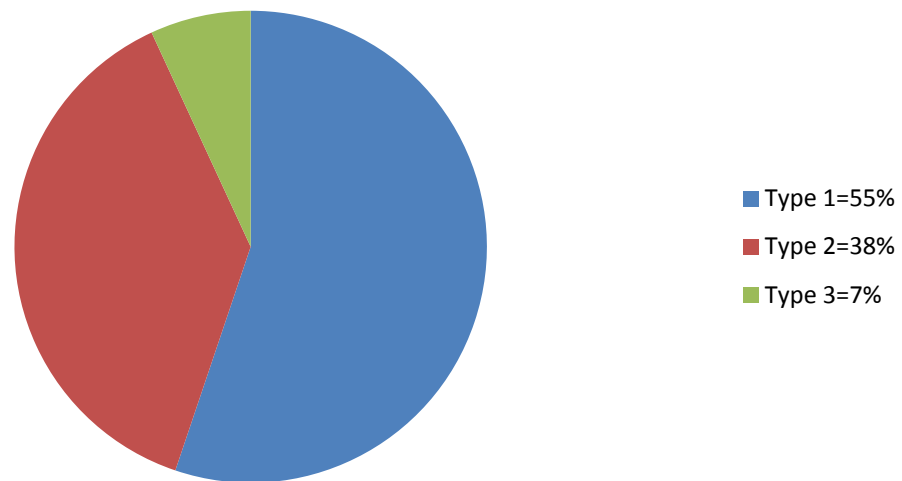


Figure 1.9 Percentages of Each Knowledge Type in All Lessons Analyzed

5.2 Implications for Future Research

Future research should involve the exploration of shifting the lens from focusing on the mathematics knowledge types to those knowledge types necessary for science. This would enhance the idea of integrated lessons that focus on conceptually rich and rigorous situations that students can investigate. Eventually a holistic approach to evaluating integrated lessons, where all content areas are integrated at cognitively high knowledge types is the goal for future research. It is believed that once this manner of evaluating lessons for depth and complexity is implemented, teachers can become empowered to select the most time-efficient lessons that incorporate more than one content area; for this is how students engage with these concepts in the real-world.

During my personal observations as a Mathematics and Science Curriculum Coordinator, I realized that there is a dire need for teachers to have a tool that is easy to implement and use to evaluate lesson quality. This framework came about as a way to improve the teaching of mathematics and science using any existing curriculum. It is a change that can realistically occur in schools with ease of teacher use via grants and teacher professional development. Fortunately, Dr. Olga Kosheleva and Dr. Laura Serpa received grant funding to begin a new cohort of Teacher Quality participants. Participants will learn how to integrate mathematics and science, specifically geosciences, into cognitively demanding lessons. As a part of this research, grant participants will learn how to use this framework to create and teach integrated mathematics and science lessons.

References

- AIMS Education Foundation. (2007). *Earth book: hydrosphere, geosphere, atmosphere, and their interactions*. Fresno, CA: AIMS Education Foundation.
- Attewell, P., Lavin, D., Domina, T., & Levey, T. (2006). New evidence on college remediation. *The Journal of Higher Education*, 77(5) 886-924.
- Baldi, S., Jin, Y., Skemer, M., Green, P.J., and Herget, D. (2007). *Highlights from PISA 2006: Performance of U.S. 15-year-old students in science and mathematics literacy in an international context* (NCES 2008-016). National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education. Washington, DC.
- Basista, B., & Mathews, S. (2002). Integrated science and mathematics professional development programs. *School Science and Mathematics*, 102(7), 359-370.
- Beane, J.A. (1995). Curriculum integration and the disciplines of knowledge. *The Phi Delta Kappan*, 76(8), 616-622.
- Berlin, D. R., & Hillen, J. A. (1994). Making connections in math and science: Identifying student outcomes. *School Science and Mathematics*, 94(6), 283-290.
- Berlin, D.F., & Hyonyong, L. (2005). Integrating science and mathematics education: Historical analysis. *School Science and Mathematics*, 105(1), 15-24.
- Berlin, D.F., & White, A.L. (1995). Using technology in assessing integrated science and mathematics learning. *Journal of Science Education and Technology*, 4(1), 47-56.

- Bosse, M.J., Lee, T.D., Swinson, M., & Faulconer, J. (2010). The NCTM process standards and the five Es of science: Connecting math and science. *School Science and Mathematics*, 110(5), 262-276.
- Branscomb, L.M., & Johnson, R.B. (1992). A government prepared: Organizing for math and science education reform. *Journal of Science Education and Technology*. 1(2), 95-104.
- Brown, A. S., & Brown, L. L.V. (2007). What are science & math test scores really telling U.S.? *THE BENT of Tau Beta Pi*, Winter 2007, 13-17.
- Czerniak, C.M., Weber, W.B.Jr., Sandmann, A., & Ahern, J. (1999). A literature review of science and mathematics integration. *School Science and Mathematics*, 99(8), 421-430.
- Davison, D.M., Miller, K.W., & Metheny, D.L. (1995). What does integration of science and mathematics really mean? *School Science and Mathematics*, 95(5), 226-230.
- Erickson, H.L. (2001). *Stirring the head, heart, and soul: Redefining curriculum and instruction*. (2nd ed.) Thousands Oaks, CA: Corwin Press.
- Fact Sheet: Race to the Top. Retrieved July 25, 2011. <http://www.whitehouse.gov/the-press-office/fact-sheet-race-top>
- Frykholm, J., & Glasson, G. (2005). Connecting science and mathematics instruction: Pedagogical context knowledge for teachers. *School Science and Mathematics*, 105(3), 127-141.
- Gonzales, P., Williams, T., Jocelyn, L., Roey, S., Kastberg, D., & Brenwald, S. (2009). *Highlights from TIMSS 2007: Mathematics and science achievement of U.S. fourth and eighth-grade students in an international context*.

- Horak, V. M. (2006). A science application of area and ratio concepts. *Mathematics Teaching in the Middle School*, 11(8), 360-366.
- Holmes, V.L. (2012). Depth of teachers' knowledge: Frameworks for teachers' knowledge of mathematics. *Journal of STEM Education*, 13(1), 55-70.
- House, P. A. (2006). Science and mathematics in balance. *Mathematics Teaching in the Middle School*, 11(9), 453-459.
- Huntley, M.A. (1998). Design and implementation of a framework for defining integrated mathematics and science education. *School Science and Mathematics*, 98(6), 320-327.
- Hurley, M.M. (2001). Reviewing integrated science and mathematics: The search for evidence and definitions from new perspectives. *Springer Science + Business Media*, 101(5), 259-268.
- Koirala, H.P., & Bowman, J.K. (2003). Preparing middle level preservice teachers to integrate mathematics and science: problems and possibilities. *School Science and Mathematics*, 103(3), 145-154.
- Lonning, R.A., & DeFranco, T.C. (1997). Integration of science and mathematics: A theoretical model. *School Science and Mathematics*, 97(4), 212-215.
- Lonning, R.A., DeFranco, T.C., & Weinland, T.P. (1998). Development of theme-based, interdisciplinary, integrated curriculum: a theoretical model. *School Science and Mathematics*, 98(6), 312-318.
- Lyublinskaya, I., & Kerekes, J. (2008). *Integrating mathematics, science, and technology in the elementary classroom*. Oceanside, NY: Whittier Publications, Inc.

- Mathews, S. M., Cornell, K. F., & Basista, B. A. (2006). Where is the moon tonight?
Mathematics Teaching in the Middle School, 11(9), 467-475.
- Meier, S.L., Cobbs, G., & Nicol, M. (1998). Potential benefits and barriers to integration. *School Science and Mathematics*, 98(8), 438-447.
- National Research Council. (2005). *How Students Learn: Mathematics in the Classroom*.
Committee on *How People Learn*, A Targeted Report for Teachers, M.S. Donovan and
J.D. Bransford, Editors. Division of Behavioral Social Sciences and Education.
Washington, DC: The National Academies Press.
- Obama, B. Presidential Office, Office of the Press Secretary. (2010). *Weekly address: President Obama to send updated elementary and secondary education act*. Washington, DC:
Government Printing Office.
- Pang, J.S., & Good, R. (2000). A review of the integration of science and mathematics:
Implications for further research. *School Science and Mathematics*, 100(2), 73-82.
- Rutherford, F. J., & Ahlgren, A. (1990). *Science for all Americans*. New York: Oxford
University Press, USA. Retrieved from
<http://www.project2061.org/publications/sfaa/default.htm>
- Sakshaug, L.E., & Wohlhuter, K.A. (2010). Journey toward teaching mathematics through
problem solving. *School Science and Mathematics*, 110(8), 397-409.
- Saunders, M. M. (2010). See the math behind the medicine. *Mathematics Teaching in the Middle School*, 16(4), 240-246.
- Smith III, J.P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform.
Journal for Research in Mathematics Education, 27(4), 387-402.

- Steele, D.F. (2001). Using sociocultural theory to teach mathematics: A Vygotskian perspective. *School Science and Mathematics*, 101(8), 404-416.
- Stewart, H. B., Reininger, M. E., & Smudzinski, W. A. (2008). An investigation of solar noon. *Mathematics Teaching in the Middle School*, 14(3), 140-144.
- Stinson, K., Harkness, S.S., Meyer, H., & Stallworth, J. (2009). Mathematics and science integration: Models and characterizations. *School Science and Mathematics*, 109(3), 153-161.
- Tchoshanov, M.A. (2010). Quantitative study on teacher quality: Case of middle grades mathematics. *International Journal for Studies in Mathematics Education*, 3, 44-73.
- Tchoshanov, M.A. (2010). Relationship between teacher knowledge of concepts and connections, teaching practice, and student achievement in middle grades mathematics. *Springer Science + Business Media*, 76. doi: 10.1007/s10649-010-9269-y
- Texas Education Agency (TEA). (2011, October 21). STAAR resources. Retrieved from <http://www.tea.state.tx.us/>
- Van de Walle, J.A. (2007). *Elementary and middle school mathematics: Teaching developmentally*. (7th ed.) Boston, MA: Pearson Education, Inc.
- Wagener, L. L. (2009). A worthwhile task to teach slope. *Mathematics Teaching in the Middle School*, 15(3), 168-174.

Appendix 1

1.1 SABE MAS Middle School Lessons

MS.1

Lesson Plan: Kinetic vs. Potential Energy

TEKS:

The Student will:

- 6.10 Use statistical & graphical models to represent real-world data
- 6.12 Communicate through graphical models
- 6.13 Use logical reasoning to form conclusions
- 6.6 Acknowledges relationships between force and motion
- 6.6A,B Identifies changes in position, direction, of motion and speed when acted upon by a force and measure and graphically represent it
- 6.10C,D Sketches circle graphs by organizing, in-terpreting, & displaying data
- 6.12A Communicates mathematic ideas using graphical models
- 6.13A Makes conjectures from patterns or sets of examples & nonexamples

TEKS Student Expectations written in “student friendly” language to be written on board:

- *Convert between fractions, decimals, & percents
- *Accurately read information from given tables
- *Section pie graph using calculator & protractor

Vocabulary: sketch, model, percent, protractor, display, table, calculate, justify, circle graph, ratio, convert

Guiding Questions:

1. What is the difference between kinetic and potential energy?
2. Will you experience between the acceleration of motion during or after dropping each ball?
3. What is the relationship between both energies and mass?

- *Higher order thinking Q&A
- What could you expect...?
- What would you conclude?
- *PowerPoint presentations
- *Hands-on projects
- *Computer Lab/Websites
- *Interactive Notebooks
- *Word Wall – Vocabulary
- *Promethean Interactive White Board

Assignment/Activity/Lab:

- * Define kinetic and potential energy
- * Use triple beam scale to weigh specific size balls to be considered in the experiment (record in centimeters)
- * Drop balls from 3 specific heights for three trials each (record in centimeters)
- * Measure the bounce in each trial for each ball (record in centimeters)
- * Find the GPE by mass x height
- * Find KE by $\frac{1}{2}MV$
- *Transfer all data to line graph

Interventions:

- *Documenting homework on both agendas/Interact NB
- *Extra time on assignments after school
- *Parent notification
- *Small group Math /Science Lab
- *Disaggregation of data to help identify areas of needs and students that struggle

Enrichment:

- *Higher order thinking Q&A
- What could you expect...?
- What would you conclude?
- *PowerPoint presentations
- *Hands-on projects
- *Problem of the Day
- *Computations
- *Science Investigations

Closure: Students will...

Have a ten minute discussion based on acquired knowledge and its practical use in science.

Presentation: Students will...

Present in groups their findings to the class on a different date

Kinetic vs. Potential Energy

Ball Type	Ball Mass (172 g)				
Softball	Gravitational Potential Energy (GPE)	Bounce Height			
Drop Height (cm)	GPE = mass x height	Drop 1	Drop 2	Drop 3	Average Bounce Height
40	$172 \times 29 = 4988$	28 cm	31 cm	29 cm	29 cm
50					
60	$172 \times 42 = 7224$	41 cm	39cm	45 cm	42 cm
70					
80					
90	$172 \times 51 = 8772$	53 cm	49 cm	51 cm	51 cm
100					

Ball Type	Ball Mass (g) = 538.6 grams				
Basketball	Gravitational Potential Energy (GPE)	Bounce Height			
Drop Height	GPE = mass x height	Drop 1		Drop 2	Drop 3
40					
50	$538.6 \times 37 = 19,943$	36 cm	35 cm	39 cm	37 cm
60					
70	$538.6 \times 55 = 29,645$	57 cm	53 cm	55 cm	55 cm
80					
90					
100	$538.6 \times 73 = 39,347$	70 cm	75 cm	73 cm	73 cm

KINETIC VS POTENTIAL

HYPOTHESIS: THE SIZE AND MASS OF THE DIFFERENT BALLS WILL IMPACT THE KINETIC ENERGY

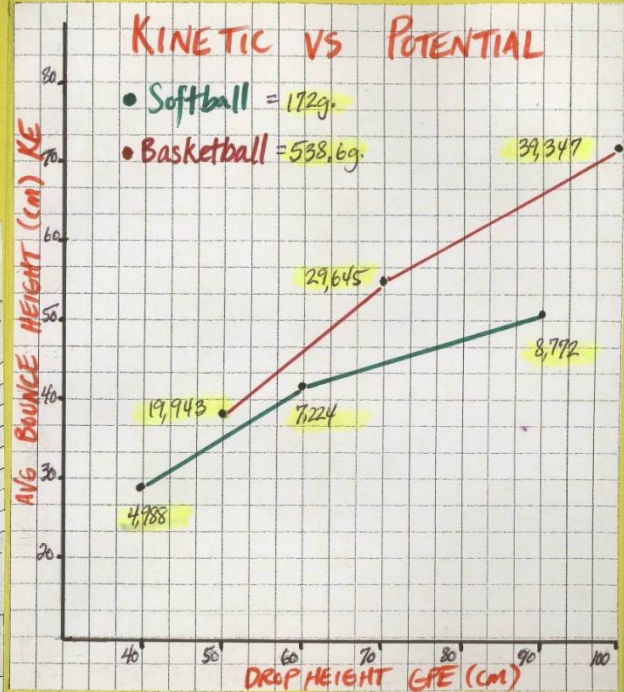
TABLE:

Ball Type:	Ball Mass (g) = 172g	Bounce Height (cm)				
Softball	Gravitational Potential Energy (GPE)					
	GPE = mass x height	Drop 1	Drop 2	Drop 3	Average Bounce Height	
	40	172 x 29 = 4,988	28cm	31cm	29cm	29cm
	50					
	60	172 x 42 = 7,224	41cm	39cm	45cm	42cm
	70					
	80					
	90	172 x 51 = 8,772	53cm	49cm	51cm	51cm
	100					

Ball Type:	Ball Mass (g) = 538.6g	Bounce Height (cm)				
Basketball	Gravitational Potential Energy (GPE)					
	GPE = mass x height	Drop 1	Drop 2	Drop 3	Average Bounce Height	
	40					
	50	538.6 x 37 = 19,943	36cm	35cm	39cm	37cm
	60					
	70	538.6 x 55 = 29,645	57cm	53cm	55cm	55cm
	80					
	90					
	100	538.6 x 73 = 39,347	70cm	75cm	73cm	73cm

NAME:

Graph:



CONCLUSION: THE GRAPH DEMONSTRATES THAT THE HIGHER THE DROP HEIGHT OF GRAVITATIONAL POTENTIAL ENERGY, THE HIGHER THE AVG. BOUNCE HEIGHT OF KINETIC ENERGY

Lesson Plan: Kite Lesson Plan

Objective: The student will create a portfolio that includes research, hypothesis, scale drawings, results, conclusions, and graphs about the kite they design and build.

Math TEKS:

The student is expected to:

- (A) use addition and subtraction to solve problems involving fractions and decimals;
- (B) use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates; and
- (C) estimate and round to approximate reasonable results and to solve problems where exact answers are not required.
- (D) use ratios to describe proportional situations;
- (E) use ratios to make predictions in proportional situations.
- (F) estimate measurements and evaluate reasonableness of results;
- (G) select and use appropriate units, tools, or formulas to measure and to solve problems involving length (including perimeter and circumference), area, time, temperature, capacity, and weight;
- (H) draw and compare different graphical representations of the same data;
- (I) use median, mode, and range to describe data;
- (J) solve problems by collecting, organizing, displaying, and interpreting data.

Science TEKS:

The student is expected to:

- (A) demonstrate safe practices during field and laboratory investigations; and
- (B) make wise choices in the use and conservation of resources and the disposal or recycling of materials.
- (C) plan and implement investigative procedures including asking questions, formulating testable hypotheses, and selecting and using equipment and technology;
- (D) collect data by observing and measuring;
- (E) analyze and interpret information to construct reasonable explanations from direct and indirect evidence;
- (F) communicate valid conclusions; and
- (E) construct graphs, tables, maps, and charts using tools including computers to organize, examine, and evaluate data.
- (F) analyze, review, and critique scientific explanations, including hypotheses and theories, as to their strengths and weaknesses using scientific evidence and information;
- (G) draw inferences based on data related to promotional materials for products and services;
- (H) represent the natural world using models and identify their limitations;
- (I) evaluate the impact of research on scientific thought, society, and the environment; and

- (J) connect Grade 6 science concepts with the history of science and contributions of scientists.
- (K) collect, analyze, and record information using tools; and
- (L) identify patterns in collected information using percent, average, range, and frequency.

Vocabulary:

Scale, ratios, prediction, hypothesis, portfolio, results, mean, median, mode, perimeter, area, measurement, conclusions, bar graph, histogram, scatterplot, line graph

Materials:

Blue prints, graph paper, rulers, history of kites, colored pencils, permanent markers, string, strong plastic, file folders, wood glue, dowels, kite handles (paper towel rolls), masking tape, large chart paper

Activities:

Day 1:

1. Explain the project and how it is going to be graded. (Suggest Rubrics)
2. Show video on kites and the history of them. Talk about importance of kites. Have students take notes.
3. Have students look through designs and pick out pattern they want to make. The website for the blue prints are <http://www.my-best-kite.com/index.html>
4. As a class make predictions about which kite would fly better, what criteria we are going to judge it by & complete the tables.
5. Have students write 1 page research paper on kites. Have students write 1 page on hypothesis and why they have that hypothesis.
6. If time review equivalent ratios.

Day 2:

1. Review equivalent ratios.
2. Talk about scale as a class. What does scale mean? Where do we see it?
3. Review scale. Have students do review worksheet.
4. Have students create designs on graph paper
5. Have students scale up design using 1:3 ratio
6. Close class by discussing how this relates to making our kites.

Day 3:

1. Review scale
2. Have students design their kites' rough sketches. No gang or drug references aloud.
3. As a class scale down the measurement given for the three kite designs so they will fit on an 8 x 11 grid paper. The students must draw an outline of their kite precisely on the grid paper. They must then put their designs on the grid paper along with the colors they want to incorporate in their kite.

Day 4:

1. Have students recreate their scaled down version on large grid paper at a 1:4 ratio. (preferably everything is in centimeter). The students must recreate their pictures to scale.

Day 5:

1. Finish sketching and designing.

Day 6:

1. Discuss safety issues. Remind them of safe conduct.
2. Have students cut dowels to measurement for scaled up kite.
3. Have students wrap & glue sticks together (for delta and diamond). They need to sit for 2 days.
4. Have students use scaled up drawing as a guide for cutting out and measuring plastic for kites.
5. Have students cover edges with light weight tape.

Day 7:

1. Students should transfer design and color with permanent markers or paint. Remember to remind students that weight plays an issue with how our kite will fly.

Day 8:

1. Students must measure out string. Measure out at least 100 yards. Have students mark every three feet with a marker.
2. Start assembling kites.

Days 9 & 10:

1. Finish Kites

Day 11:

1. Create Class table to collect data.
2. As class what are we going to measure? What is going to affect how far a kite flies? Have students calculate area and perimeter for kites to add to data table.
3. Finish anything needed for flying.
4. Go over rules for flying the kite. What behaviors are expected, what behaviors will not be tolerated.
5. Watch a video on how to fly kites.

Day 12:

1. Fly Kites. Take down data

Day 13:

1. Post Data in large tables for students to see.
2. Students must create a table or graph that displays the information correctly. Discuss with students what would be appropriate and why. (bar graphs, histogram, circle graph, line graph, etc.)

3. Have students find mean, median and mode for the data. Then have students write 1 page on the results.
4. Have students write 1 page conclusion.

Days 14 & 15:

1. Finish Portfolios.

Portfolios should include:

1 page research on kites

1 page hypothesis and explanations

Worksheet on equivalent ratios

Worksheet on scale

Original sketch

8 X 11 graph paper design

Scaled up design

Data Table & Graph

1 page on results

1 page on conclusions

Lesson Plan: Probability

TEKS:

7.10a To construct sample spaces for simple or composite experiments

7.10b To find the probability of independent events.

Vocabulary:

Outcomes possible independent events sample space

Materials:

12 sets of “yellow” dice (marked with 1,1,1,1,2,2 on each dice)

12 sets of “blue” dice (marked with 2,2,2,2,4,4 on one dice and 3,3,3,3,5,5 on the other)

6-8 game boards marked with 1-12 on them

Red markers

Blue markers

Overhead transparency for game rules

Worksheet 1

Spinners, Regular dice, Pennies

Video on sample space

Probability problem worksheet

Construction Paper

Glue

Surveys

Pre Assessment

Post Assessment

Activities:

Day 1

1. Hand out pre-test. Give students 15 minutes to take and then pick it up. (15 min)
2. Put students in pairs. Put up the rules of the game on the overhead and review with students. (10 min)
3. Hand out materials for game giving students “yellow” set of dice. Give students 5 minutes to strategize with their partner.
4. Have students play 1 round. When they are finished, give them worksheet 1. After they have finished filling out the worksheet, have students play one more round with the “yellow” dice. At the end of the game pick-up the worksheet. (20 min)
5. Have students play again with the “blue” dice. Repeat steps 3 & 4. (10 min)
6. Guiding Questions (10 min)
 - What kind of strategies did you come up with to try to win the game?
 - What ideas did you get from the frequency table?
 - How did the two sets of dice differ from each other?
 - Did that affect your strategy?
 - What did you need to know to win the game? Is there a way that we can make sure to get all of the possible outcomes?

Day 2

1. Review activity from the day before. (5min)
2. As a class define probability, sample space and outcomes. (10 min)
3. Put students in pairs. Give each pair a spinner, die and a coin. Have students spin the spinner, flip the coin and roll the die 20 times. Have them create a table that records each experiment. (15 min)
4. Introduce the counting principle with the students. ($4 \times 6 \times 2$) Then have the students count their outcomes. (5min)
5. Show short video on sample space. (10 min)
6. Use overhead transparency and as a class make a tree diagram showing all of the outcomes. (10 min)
7. Guiding Questions (10 min)
 - Did we find all of the outcomes? How do we know whether we have found them all? --
 - What outcomes did you miss?
 - Why do you think some outcomes showed up more often than others?
 - Where in the real world do we see probability?
 - How can we use our tree diagram to write out all of the possible outcomes?
 - Can we decide the probability of A, an odd number and heads from our tree diagram?

Day 3

Review activities from previous 2 days.(5 min)

1. Hand students square pieces of construction paper and 1 of the problems from the probability problems page.
2. Instruct students to fold in each corner and label each part according to overhead transparency. They will glue this in their interactive notebooks. (5min)
3. The activity will take place as a simultaneous round robin activity. Each student will work on a table, a tree diagram, the counting principle and justify their answer to the problem. The students should end up working on 4 unique problems. (20 min)
4. Guiding Questions (10 min)
 - When would using a tree diagram be the most efficient use of your time? A table? The counting principle?
 - How about the probability of an outcome based on certain criteria?
 - Can you see the connections between the tree diagram, the table and all possible outcomes?
 - What is probability?
 - Give Post test. (20 min)
 - Give survey. (15 min)

Pre Assessment:

1. Mrs. Boudreaux made lunch for her family. She made chicken noodle soup and vegetable soup. She made peanut butter sandwiches and grilled cheese sandwiches. Which lists shows all possible outcomes if a person picked one soup at random and one sandwich at random?

A. (chicken noodle, peanut butter), (vegetable, peanut butter) (chicken noodle, grilled cheese), (vegetable, grilled cheese)

B. (chicken noodle, peanut butter), (chicken noodle, grilled cheese) (chicken noodle, vegetable), (vegetable, peanut butter) (vegetable, grilled cheese), (vegetable, chicken noodle)

C. (chicken noodle, peanut butter), (vegetable, grilled cheese)

D. (chicken noodle, peanut butter), (vegetable, peanut butter) (peanut butter, grilled cheese), (chicken noodle, vegetable)

2. You toss two coins. What are all the possible outcomes that could occur?

Post Assessment:

1. Bonita made pepperoni pizza and cheese pizza for a party. She also made sugar cookies, chocolate cookies, and coconut cookies. Which list shows all possible outcomes if a person picked 1 pizza at random and 1 cookie at random?

A. (pepperoni, sugar), (cheese, chocolate) (pepperoni, chocolate), (cheese, coconut)

B. (pepperoni, sugar), (cheese, sugar) (pepperoni, chocolate), (cheese, coconut)

C. (pepperoni, sugar), (cheese, sugar) (pepperoni, chocolate), (cheese, chocolate) (pepperoni, coconut), (cheese, chocolate)

D. (pepperoni, sugar), (cheese, sugar) (pepperoni, chocolate), (cheese, chocolate) (pepperoni, cheese), (chocolate, coconut)

2. Andy played a game with a fair number cube and a fair coin. The fair number cube is labeled 1 through 6. List all the possible unique outcomes of rolling an even number and tossing a head or a tail.

3. Brandon has 2 dimes, 2 nickels, and 2 pennies in his pocket. List all the possible unique outcomes if Brandon chooses 3 coins at a time from his pocket.

Rules for Dice Game

To win the game: Remove all of your markers before the opponent team.

Materials: 2 dice, 12 red markers, 12 blue markers, a game board numbered 1-12

Rules:

1. Place your markers on whichever number(s) you choose. One player is red, the other blue.
2. Roll two dice and find the sum. Record the sum in a table.
3. If the sum is equal to a number that has a marker on it, you may remove one marker.
4. Continue until all markers have been removed, recording after each roll.

Worksheet 1

1. What strategies did your team use to place the markers on the board?
2. Which strategies worked? Why or why not?
3. Are you going to change your strategies for the second round? How is it going to change?

Probability Problem:

1. If a person rolls two dice, what is the probability of getting a five as the sum?

Survey:

Answer the following with complete sentences.

- 1) How much knowledge on probability did you have before the lesson?
- 2) Will you use visual representations to solve a probability problem?
- 3) Which representation do you find the most useful to solve probability problems?
- 4) Did these activities change any knowledge you had before the lessons began?
- 5) What did you believe was true and how did that understanding change?

Rate the following question from 1-5. 1 being the lowest and 5 being the highest.

Dice Game

- 1) How much did the activity interest you? 1 2 3 4 5
- 2) Did this activity help you understand sample space? 1 2 3 4 5
- 3) Did this activity help you understand finding all the possible outcomes? 1 2 3 4 5

Spinner, Dice, and Coin Activity

- 1) How much did the activity interest you? 1 2 3 4 5
- 2) Did this activity help you understand sample space? 1 2 3 4 5
- 3) Did this activity help you understand finding all the possible outcomes? 1 2 3 4 5

Round Robin Activity

- 1) How much did the activity interest you? 1 2 3 4 5
- 2) Did this activity help you understand sample space? 1 2 3 4 5
- 3) Did this activity help you understand finding all the possible outcomes? 1 2 3 4 5

Lesson Plan: I Walk the Graph

TEKS:

Physics:

The student is expected to:

- (A) plan and implement experimental procedures including asking questions, formulating testable hypotheses, and selecting equipment and technology;
- (B) make quantitative observations and measurements with precision;
- (C) organize, analyze, evaluate, make inferences, and predict trends from data;
- (D) communicate valid conclusions;
- (E) graph data to observe and identify relationships between variables; and
- (F) read the scale on scientific instruments with precision.
- (G) generate and interpret graphs describing motion including the use of real-time technology;
- (H) analyze examples of uniform and accelerated motion including linear, projectile, and circular;

Math:

The student is expected to:

- (A) generate a different representation of data given another representation of data (such as a table, graph, equation, or verbal description).
- (B) predict, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations;

Frameworks:

- Students need to understand the basic concept of speed and the graphs that accompany them. They should also understand point coordinates, calculate slope and have the ability to incorporate the proper use of math vocabulary into word problems.

Data:

- Students typically have problems distinguishing the differences between speed graphs, slope, point coordinates and the literary terms used in graphing.

Compare It:

- This lesson is able to fit the three most basic learning types of students. It benefits auditory learner because they are able to hear the coordinates being called out, visual learners are able to see the graph being walked, and kinesthetic learners are able to draw the graphs on their own.

Materials:

- stopwatches
- master number lines
- 3-4 small graphs per group
- calculator

Best Practice:

- Group work is better suited for this activity because students can participate and cooperate on learning the concepts behind the physics and math involved with graphing.

Technology:

- Photo gates and the use of computers to analyze the graphs can be incorporated if the technology is available.

Design It:

- The teacher will introduce the concept of coordinates, slope, x-axis, y-axis and graphs prior to the activity.
- 3 Roles will be implemented in this activity.
 - One student will walk the graph
 - One student will call the coordinates.
 - One student will call the time.
- Groups will pick a graph and present it to the class and the class will attempt to draw the graph based on the “instructions” the presenting group makes.
- Students will now calculate the slope on their selected graph and write a story based on the graph using mathematical and scientific vocabulary.

Assessment:

- Students will peer assess their classmates graphs on a scale of 1-5 (1-worst and 5-best) based on the accuracy of their drawing according to the instructions given.
- The teacher will assess the students’ graphs based on their calculations of slope and how well they implemented the vocabulary into their story based on given rubric the students receive beforehand.

Plan it:

- 10-15 minutes with lecture covering concepts of graphs, speed, distance, time, slope, and vocabulary
- 5 minute to assemble groups and distribute roles
- 15 minutes of exploring graphical methods
- 20 minutes for presentation of the graphs
- 5 minutes for peer assessment

Extension:

- Students can be introduced to velocity and acceleration graphs and solve motion problems.

Getting Familiar with the Coordinate Plane... Plotting Points

1. Students will review plotting points in all four quadrants.
2. Understand the placement on the horizontal axis, x-axis, is the first number in a coordinate pair.
3. Understand that the placement on the vertical axis, y-axis, is the second number in a coordinate pair.
4. Understand that together the x and y coordinate pair gives a location on the coordinate plane, represented by a point.
5. Students will use the mpact program to plot points on the coordinate plane.

Translating Points on the Coordinate Plane

1. Student will plot points on the coordinate plane.
2. Students will translate a point in a horizontal direction in a positive or negative direction.
3. Students will translate a point in a horizontal direction in a positive or negative direction.
4. Students will translate a point in both a horizontal and vertical positive or negative direction.
5. Students will use the mpact program to plot and translate points on the coordinate plane.

MS.6

Lesson Plan: Proportions, Predictions, and your Body

Teaching Goals:

The students will learn the anatomical names of a few body structures, and use conversions of measurements in Metric System.

TEKS:

(7.3) (A) Students will use ratios to describe proportions.

(7.4) (C) Students will learn how to use tables and symbols to represent and describe proportional and other relationships involving conversions and their position in the sequence.

Materials: Conversion charts, Worksheets, Yarn 2.5 yards long, Ruler/ yard stick, Pencil

Procedure:

1. Lecture on conversions with demonstration of converting inches into meters, centimeters into meters, and vice versa.
2. After lecture, break students into couple (same gender preferably).
3. Begin to identify the different body parts that will be measured.
4. Have students measure the identified parts with yarn.
5. Wrap yarn around thumb twice and the measure with ruler.
6. When measurement is obtained, record on prediction chart and continue predicting the rest of body measurements.
7. Once chart is completed. Follow instructions and record actual measurements of body areas.
8. See if prediction is same as actual measurements.
9. Begin converting measurements.

Assessment:

50% of the grade will be based on participation and 50% and the other from the work sheet.

US WEIGHT AND MEASURES

12 INCHES (IN)	1 FOOT (FT)
3 FEET (FT)	1 YARD(YR)
5,280 FEET (FT)	1 MILE (MI)
4 GILLS (GI)	1 PINT (PT)
2 PINTS (PT)	1 QUART (QT)
4 QUARTS (GT)	1 GALLON (GAL)
27 11/32 GRAINS	1 DRAM (DR)
16 DRAMS (DR)	1 OUNCE (OZ)
16 OUNCES (OZ)	1 POUND(LB)
100 POUNDS (LBS)	1 HUNDREDWEIGHT (CWT)
20 HUNDRED (2000 LBS)	1 TON (T)

METRIC WEIGHT AND MEASURES

1 CENTIMETER(CM)	10 MILLIMETERS (MM)
1 DECIMETER (DM)	10 CENTIMETERS (CM)
1 METER (M)	10 DECIMETERS (DM)
1 DEKAMETER(DAM)	10 METERS (M)
1 HECTOMETER(HM)	10 DEKAMETERS (DAM)
1 KILOMETER (KM)	10 HECTOMETERS (HM)
1 CENTILITER(cL)	10 MILILITERS (ML)
1 DECILITER(dL)	10 CENTILITERS (CL)
1 LITER(L)	10 DECILITERS (DL)
1 DEKALITER(DAL)	10 LITERS (L)
1 HECTOLITER(HL)	10 DEKALITERS (DAL)
1 KILOLETER(KL)	10 HECTOLITERS (HL)
1 CENTIGRAM(CG)	10 MILIGRAMS (MG)
1 DECIGRAM(DG)	10 CENTIGRAMS (CG)
1 GRAM (G)	10 DECIGRAMS (DG)
1 DEKAGRAM(DAG)	10 GRAMS (G)
1 HECTOGRAM(HG)	10 DEKAGRAMS (DAG)
1 KILOGRAM (KG)	10 HECTOGRAMS (HG)
1 TON (T)	1,000 KILOGRAMS (KG)

NAMES: _____ DATE ____/____/____

BODY CONVERSIONS WORKSHEET

INSTRUCTIONS: Follow the pattern and sequence. If you need help ask your partner, if he/she does not know how to do it, ask your teacher.

Example:

If the circumference of the head is 3.5 times the size of the wrist; Then how much does my head measure if my wrist measures 8 cm?

$3.5 * \text{Wrist} = \text{Head}$

$3.5 * 8 \text{ cm} = 28 \text{ cm}$

The ratio of the wrist to my head is

8 cm to 28 cm or 8 : 28 or 8/28

RATIOS YOU NEED TO KNOW

	WRIST	HEAD	ARM SPAN	HEIGHT	FACE		
Comparison							
Thumb	2 TIMES AROUND THUMB=WRIST	3.5 TIMES AROUND WRIST= HEAD	3.5 TIMES AROUND HEAD= ARMSPAN	1 ARMSPAN = HEIGHT	1 HEIGHT = ARMSPAN	1 PALM = FOREHEAD	2.5 TIMES FOREHEAD = FACE

NAMES: _____ DATE ____/____/____

Now measure the actual size of these body parts.

PART	MEASUREMENT
THUMB	
WRIST	
HEAD	
ARMSPAN	
HEIGHT	
PALM	
FOREHEAD	
FACE	

Use the two charts above to answer the following questions. (Write them in written form, and colon form and fraction form.)

Were your predictions really close to the actual measurements of your body parts?

What is the ratio of your thumb to your wrist?

What is the ration of your wrist to your arm span?

What is the ratio of your thumb to your head?

What is the ratio of your arm span to your height?

MS.7

Lesson Plan: Probability Spinner

TEKS: 8.11 (B) Probability and statistics

The student applies concepts of theoretical and experimental probability to make predictions. Use theoretical probabilities and experimental results to make predictions and decisions.

This TEKS will help students to:

Use the results of an experiment to determine the experimental probability (EP) of an event.

$$\frac{\text{\# of times event occurs}}{\text{Total \# of trials}}$$

Students can also determine the theoretical probability (TP) of an event.

$$\frac{\text{\# of favorable outcomes}}{\text{total \# possible outcomes}}$$

Materials:

Color construction paper circles

Rule

Protractor

Pencil and paper

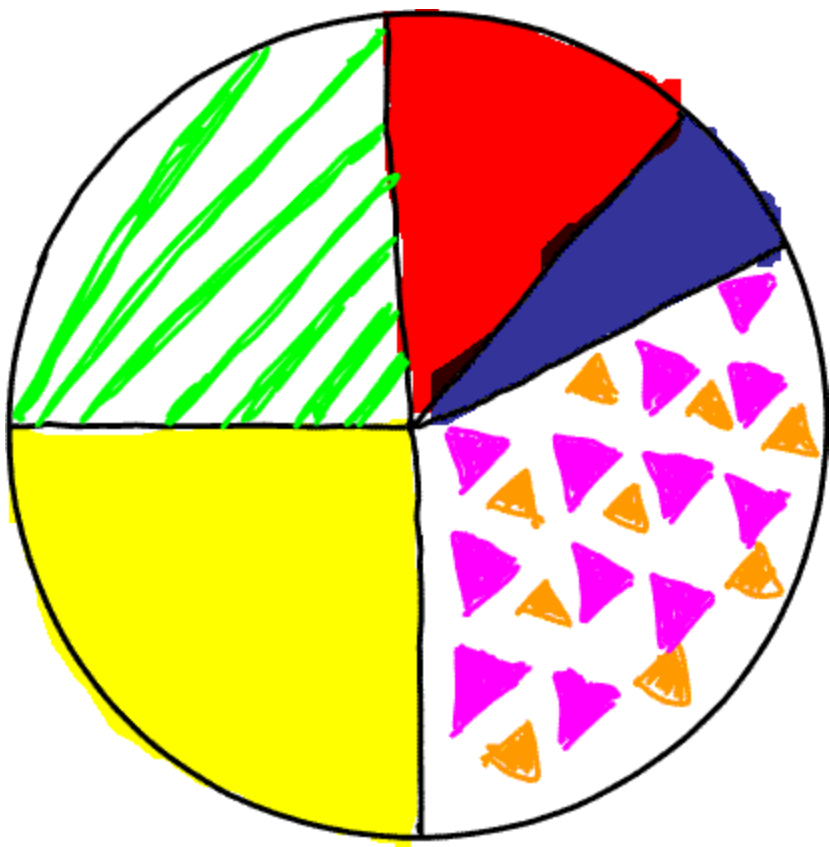
Paper clip

Color pencil

Different types of decorative items to decorate the spinner.

Step 1:

Students will create their own spinner by dividing the circle into 5 different sections using the central angle. Then they need to add 3 different colors and two different designs. Finally decorate you spinner. Please make your spinner colorful and fun to watch!!!!



Step 2:

Students will create a table describing each section on the spinner with different numerical representations, to express the theoretical probability of each section on the spinner.

Example: $45^\circ \div 360^\circ = 0.125 = 12.5\% = 1/8$

$$90^\circ \div 360^\circ = 0.25 = 25\% = 1/4$$






$$112.5^\circ \div 360^\circ = 0.3125 = 31.25\% = 5/16$$

$$22.5^\circ \div 360^\circ = .0625 = 6.25\% = 1/16$$

$$90^\circ \div 360^\circ = 0.25 = 25\% = 1/4$$

After the table is complete students need to write 3 probability problems using the theoretical probability from the table. Also they need to include one of the words likely, unlikely or equally likely for each problem.






Theoretical Probability

				
45° 0.125 12.5% $\frac{1}{8}$	90° 0.25 25% $\frac{1}{4}$			

Step 3:

Students will create another table to record will tally marks the experimental probability at 10, 20, and 50 times.

Experimental Probability

				
I	I	II	III	III
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{3}{10}$
20	20	20	20	20
50	50	50	50	50

Tallys
 10 times
 20 times
 50 times

After the table is complete students need to write 3 probability problems using the experimental probability from the table. Also they need to include one of the words likely, unlikely or equally likely for each problem.

Finally students compare the two probabilities in a journal entry. At this point students must be able to see that the more experimental probability events occurs the closet it gets to the number on the theoretical probability.

Lesson Plan: Scales and Pythagorean Triangles

Objective: Students' will be able to construct a mock scale using the listed materials then be able to find the Pythagorean Triangles from the model. Students' will also be able to convert measurements from inches to centimeters and be able to understand how the balance on the scale works.

TEKS: 7.1(A,B), 7.2(A-G), 7.3(B), 7.6(A-C), 7.8(A), 7.9(A,C), 7.10,7.13 and 7.14.

Materials: String/Yarn, Yard Stick, Small 8oz. water bottles, 16oz. water bottles, Holders for yard stick, Pencil and Paper.

Procedure:

1. Students' will break off into groups of four (4).
2. One student from each group will pick up materials and bring them back to their group.
3. Students' will construct the scale according to the written instructions on the handout.
4. After completing scale, students' will then answer questions on ditto sheet about conversions and Pythagorean Triangles.
5. If students' are finished with their work, have them turn in completed assignment. If they are not, have them turn the assignment in and keep the assignment until next class time.

Support for S.P.E.D. Student(s):

1. Written and verbal instructions.
2. Each student has a separate ditto sheet similar to the regular ones but some information is already filled in.
3. Also must turn in completed work along with log book.
4. Will work in groups with others.

Assessment: Once completed work is turned in, it is graded according to the correct amount of problems answered. There are a total of 15 questions on the regular ditto sheet for the students' to answer, 10 for the S.P.E.D. students'. The total amount of points is 90.

Scale:

- 80 – 90: A
- 70 – 79: B
- 60 – 69: C
- 50 – 59: D
- 49 or less: F

Lesson Plan: Solving Proportions

TEKS:

- To use ratios to describe proportional situations
- To use ratios to make predictions in proportional situations

1. Plan: Math Background and Lesson Resources

- You can solve a proportion in many ways. You can use unit rates to find the missing quantity. You can use number sense and write the ratio in simplest form, then cross-multiply. You can use cross products to solve a proportion.

2. Teach: Bell Ringer Practice (10 minutes)

TAKS Daily Review 51, *Use transparency. 2009 TAKS test*

3. Teach: Lesson Content (50 minutes)

Objective 1: To solve proportions using number sense and cross products

Use Examples 1, 2 and 3 and Quick Checks.

4. Teach: Differentiated Instruction

Special Needs

Ask students to draw a picture to solve the More Than One Way example, and compare the picture to the methods Jessica and Michael used. Ask them to decide which method their picture best represents. Learning Style: Visual

Below Level

As students write each cross product, have them say “top value times bottom value” and “bottom value times top value.” Also have students use a different color to write each pair of cross products. Learning Style: Verbal

Advanced Learners

Explain whether the following equation is a proportion.

English Language Learners (ELL)

Explain what a *hybrid* car is and why it uses less gas than a regular car. Learning Style: Verbal

5. Teach: Closure (10 minutes)

How does a unit rate help you solve a proportion?

How can you use cross products to solve a proportion?

6. Practice: Assignment Guide

Make house drawing to scale 1 inch = 10 feet

7. Assess: Lesson Quiz (20 minutes)

Use this lesson quiz to assess students' mastery and understanding of the math skills and concepts taught in this lesson.

Appendix 2

2.1 SABE MAS High School Lessons

HS.1

Lesson Plan: Logarithmic Scales in Nature

5-E Phase and standards (describe): This unit includes all five stages of the 5-E format.

Guiding Question(s):

Where do we find logarithmic scales in the world around us?

How do logarithms help us to understand the world we live in?

How do logarithms help us explain or describe what we observe in the world?

Expected outcomes:

At the end of this unit, students will be able to describe the use and importance of logarithmic scaling in various fields of inquiry.

They will be able to express quantities in standard notation.

They will be able to perform unit conversions.

They will be able to relate various quantitative scales to each other and to their own lives.

Materials List:

Adding machine roll paper

Colored pencils

Meter sticks

Computer Lab access

Calculators

Summary of Lesson with estimated times:

Engage (Activity 1, 30 minutes) Introduction to Dimensional Analysis

Explore (Activity 2, 50 minutes) Pages of Time

Explain (Activity 3, 90 minutes) Web Quest on Logarithms and Logarithmic Scales

Elaborate (Activity 4, 90 minutes) Logarithm Research and Presentation

Evaluate (Activity 5, Homework) Assess Logarithms

Lesson Description/Teacher Instructions:

We want to guide students through a series of activities that will help the student construct an understanding of logarithms and logarithmic scales as they apply to the science curriculum.

Starting with a dimensional analysis, the student computes a large value quantity and converts it into a logarithm.

We next explore the geologic timescale on a linear scale and then on a logarithmic scale.

Computer lab access is required for the Web Quest activity, the Research and Presentation activity and the Assess activity.

It is recommended that the Research and Presentation activity be assigned at the end of the Web Quest, so the students can do their research as homework and present to the class on the day reserved for the activity. You might want to insert an additional activity on the day after the Web Quest to allow more time for the students to research for their presentations.

Assessment plan:

As always, ongoing formative assessment should be going on throughout the unit. Each activity has proposed assessment components including grading rubrics for the Web Quest and Presentation. The unit assessment concludes with the test discussed in the Assess activity

Tell us what someone observing your class during this lesson would expect to see:

An observer to this classroom should see students engaged in cooperative learning and peer teaching. The sound level should be moderate, but silence would indicate that students are not performing as expected.

Lesson Plan: An Introduction to Wave Motion

Expected outcomes:

Students will have a better understanding of basic wave principles, wave characteristics, and wave calculations.

Guiding Questions:

1. What is a wave?
2. How do you hear?
3. How do you see?
4. Why are waves important?
5. What are the parts of a wave?
6. How can we use math to understand and learn about waves?

Lesson Description:

This lesson should be done as a follow up to a general introduction about waves. Students should be reviewed on the 2 main waves (*electromagnetic and mechanical*), types of waves (*longitudinal and transverse*), wave characteristics (*reflection, refraction, diffraction, interference*), and parts of a wave (*trough, crest, amplitude, etc...*) This lesson should reinforce their knowledge about waves and demonstrate how math can be used to calculate wave speed. The main focus is that the students learn the formula for wave speed and how to calculate it.

Assessment Plan:

Students will be given a homework assignment so they can apply the wave speed formula to what they have observed today in class.

The following class period should involve some time at the beginning for the students to review the assignment and check calculations and units.

A short quiz should also be given to assess that the students have learned the important parts of the wave and how to use the wave speed formula.

What you would see in class:

Class today would appear very chaotic and noisy at first because of the engage activity with the forks and spoons.

Students would be very involved in their own learning and may be curious to try different ways of testing waves.

The teacher should be guiding and facilitating the student's learning experience.

Materials:**Per Student:**

Metal spoon and fork per student
String (cut into 50cm pieces)
Homework assignment

Per Group

Slinky
Digital Camera
Meter Stick
Computer
Logger Pro Software

Engage:

1. Students will begin by taking a fork and tying one end of a piece of string around it, and the other end to a pencil as shown:
2. Students will carefully place the eraser end of the pencil against their outer ear.
3. Then, using the other hand, students hold a spoon and strike the fork with their own spoon.
4. The sound from the vibrating fork should travel through the string to the ear.
Ask students to describe the sound they hear (in terms of pitch and volume - students might say the sound is "like a church bell.").

Explore/ Explain:

1. Place meter stick and slinky on table
2. Move slinky side to side and record with digital camera
3. Open Logger Pro and load video
4. Set-up scale and origin and trace the marker on slinky using "Set active point"
5. Measure wavelength and calculate frequency and wave speed

Elaborate:

1. At this point students would have completed the notes and the activity which means they should be more knowledgeable about waves, wave motion, wavelengths, frequency etc.
2. Therefore, for this part in the lesson plan we want to try and ask students to create a learning journal and elaborate on what they learned.
3. This is a chance for you to see what students caught on to, what they know much about, and what students need more work on.
4. To tie everything together students can then partner up with the person they worked with on the activity to make a poster board of what they wrote in their journal or any part of the exploration that they liked most. The poster board is a chance for students to be creative.
5. They can draw waves and explain, they can show some calculations of wave speed or frequency, or they can explain how we hear and see with pictures.
6. They can do whatever they want as long as they are creative, and they are using what they learned during the lesson. This is a chance for students to review the lesson on their own.
7. Just observe the students, guide them along, and answer questions if needed, but make sure they are coming up with their own ideas. If everything goes smoothly students should need about 15 minutes but if needed they can take this home and finish for homework.

Lesson Plan: Bones, Bones, Bones

Materials:

masking tape
construction paper
crayons
metric ruler
meter sticks
Calculators
chart-size graphing grid
Sticky notes

Guiding Questions:

- 1.) What do you think is the longest bone in your body?
- 2.) Which is easier to measure, your height or your femur?
- 3.) How accurate do you think the measurements are?
- 4.) How do your percentages compare with that of the class?
- 5.) What statements can you make by looking at the graphs?
- 6.) Are the results the same in males and in females?
- 7.) Do you think there are other relationships between other pairs of body measurements?

Engage:

Students will be given index cards with a human skeleton on it. Each card, will have a particular bone in the body that will be highlighted. Students will then place the cards in order from largest bone to smallest bone. In doing this, students will get a sense of the size of bones in the human body and their relationship to other parts of the body.

Explore:

After scale sizing of the bones, students will then begin an activity that involves them measuring the length of their femur and focus on the relationship of that length with their height. Students will work in pairs and measure the length of their femurs and cut a piece of construction paper that matches that length. They will then measure their height by standing against a wall without shoes. Students will need to mark the wall with a piece of tape to mark their height. They will then take the construction paper with their femurs and see how many equal their height.

Students will need to use the following calculations to find the femur-height relationship.

Males: $(1.9 \times \text{femur length}) + 81.3 \text{ cm}$

Females: $(1.9 \times \text{femur length}) + 72.8 \text{ cm}$

Students will need to record their results on the their activity sheet and the class graph.

Experiment:

Investigate the arm and height correlation.

Students will measure each other's height and arm span. Students will record their measurements on a class bar graph with three columns, one labeled square, one labeled tall rectangle, and the last one labeled far-reaching rectangle. Based on their height and arm span, students will mark the column that best matches their height-arm span relationship. Students can also take the information and create a pie graph.

In doing this activity, students should find a relationship between their height and arm span. Students should understand that there are relationships between different pairs of body parts. Students might extend this by measuring another class or adults to see how their class results might relate to the results of another group.

Elaborate:

Students can extend this activity by labeling a diagram of the human skeleton, measure the femur of their pet and compare to animals height or length, and comparing and contrasting the human skeleton with those of other animals.

Students can also extend the activity by beginning a discussion on homologous structures and by learning how to draw cladograms.

Students will investigate and discuss the similarities in limbs of different animals and compare them to humans. For example, birds and bats have the same bones as humans in their wings but use them for different purposes

Students will then have the opportunity to draw cladograms and see some of the evolutionary processes involved in some of our limbs and other animals limbs.

Evaluate:

Check students' ratios of femur vs. height and wingspan vs. height!!!

<http://www.pbs.org/saf/1203/segments/1203-3.htm>

Lesson Plan: The Vitruvian Man

The learner will:

- gather measurements of a peer's body to construct a chart using the scientific method
- metric measurements in meter converted to cm
- compare and contrast human body parts
- correlate a manual and computer line graph
- Analyze data and critically decipher high rigor questions

Time Required:

- 1 A or B day (130 minutes)

Engagement:

Prediction/Hypothesis on human body results with a collaborative discussion.

Exploration:

Team/2 people measurement lab

Explanation:

Students will gather data, place on table, graph (manual/computer).

Elaboration:

Students will analyze the graphs, and answer high rigor questions. Also add in extensions on their own allowing them ownership.

Evaluation:

Students will be given TAKS problems from Science TEKS 2, 10A and Math TEKS 2, 3, 4, 5

Focus question:

What is the relationship between the wrist to elbow versus the waist and a person's wingspan versus their height?

Prediction:

Will the above measurements be the same, different or show a correlation?

Lab Procedure:

1. Write an hypothesis based on Vitruvius' theory relating arm span and height.
2. Working with a partner, measure your arm span by standing against a flat surface and spreading your arms out as far as possible. Have your partner measure the distance from the longest finger on hand to the tip of the longest finger on the other hand. Record the measurements in Data Table 1.
3. Repeat step two on your partner.
4. Remove your shoes and have your partner measure your height as you stand against a flat surface. Measure the distance from the top of your head to the floor. Record your measurements in data Table 1.
5. Repeat step 4 on your partner.
6. Calculate the difference between your arm span and your height (arm span-height)
7. If the difference is within +/-2 cm -- we will say that it is close enough.
8. Go back and make a conclusion on your original hypothesis.

Making a Graph Manually:

1. Give each student a sheet of graphing paper.
 2. Identify what type of graph students may use (Line, Bar)
 3. Use your data from to make graph of their findings (Height/Arm length and waist/2 times the elbow length.)
- Also...Teacher can make a table for all students and plot on computer

Making a Graph on Excel:

1. Start up Microsoft Excel.
2. Label cell B1 as "height," and cell C1 as "arm span," and enter students' names beginning in cell A3 and going down.
3. For each student, record their height and arm span in centimeters.
4. Highlight the numbers in the height and arm span columns.
5. Click on "insert" and then select the line graph.

High Rigor Questions:

1. Look at your graph, analyze & conclude results
2. Are any of these patterns age dependent, environment dependent, or genetic dependent.

Assessment:

TAKS Questions

Lesson Plan: Density

Objectives:

The student knows relationships exist between properties of matter and its components.
The student is expected to: (A) investigate and identify properties of fluids including density, viscosity, and buoyancy

Materials:

1. two clear glasses
2. water
3. two eggs
4. table salt
5. measuring spoon

Engagement:

1. Fill first glass halfway with water and add one egg.
Note: the egg should rest at the bottom of glass.
2. Fill the second glass halfway with water and add 10 tablespoons of salt and then place egg.
Note: the egg should float

Explain:

Have the students talk about what is happening in the engagement activity.
Have them form their own hypothesis on why the egg floated in cup two but not in cup one.
Ask them what would happen if they were to add more salt in the second cup.
Would the egg still float?

Extend:

Here I would give a short lesson on Density and give examples of everyday substances and their densities.
Explain terms like density, buoyancy, and viscosity.

Explore:

Here I would have the students create their own scales, like we did in SABEMAS.
I would also have solid color bottles (so the students don't know what is inside) with different liquids. For example alcohols, milk, water from tap, water and salt.
Using their scales the students would have to find the density of the liquids in the bottles and determine what liquid is inside. I would provide them with a list of liquids and their densities in order to help them identify the liquids in the bottles.

Evaluate:

The students would have to turn in a paper with the bottle number, the density they calculated and their guess for what liquid is inside. I would evaluate as far as how close they are to the actual liquid, if they have the correct units, and if they have the correct significant figures.

Lesson Plan: Good Vibrations

Objectives:

Students will understand:

1. the difference in wavelength size in the electromagnetic spectrum.
2. that visible light makes up a very small portion of the electromagnetic spectrum.
3. that light travels in a straight line and refracts when it passes from one substance to another, which is the principle behind why a prism separates white light into individual colors.
4. the colors seen by the eye are a result of light being reflected, not absorbed.

Engage:

Using a solo cup with a cut balloon stretched over it, and a mirror glued in the middle, in combination with a radio and laser pointer to demonstrate sound waves. Review from prior teaching, that sound is made by vibrations and makes waves with different wavelengths. Explain that light energy from the sun travels in wave form, the distance between two successive wave crests being called the wavelength of the energy. Solar energy consists of a wide array of energies of differing wavelengths.

(Introduce light spectrum) Place clear cup filled with water on the overhead and turn off the lights. View light spectrum visible on the roof.

Explanation:

adapted from Discoverschool.com

1. In this activity, students will create a model of the infrared, visible, and ultraviolet portions of the electromagnetic spectrum. The model they create will be made to scale based on wavelength. In order to complete this lab, students should understand the metric system and be able to convert between different metric units. They should also understand the concepts of wavelength and frequency.
2. Hand out data sheets to each student and divide the class into small groups of two or four. Make sure each group has the materials necessary for the activity.
3. Explain to students that the wavelengths for the visible, infrared, and ultraviolet portions of the spectrum are represented in meters on their data sheets. Students will need to complete a metric conversion calculation to find the length of the waves in nanometers. Explain to students that one nanometer is 10^{-9} of a meter. To put this length in perspective, tell them that the diameter of a penny is 19 billion nanometers. The scale that will be used to build their model of the spectrum is 1 nanometer equals 1 millimeter. So if a wavelength is X-nanometers, the model for that wavelength should measure X-millimeters. Students will need to show the work they've done on their calculations in the space provided on the data sheet.
4. Work together as a class on the metric conversion calculation for red light. It is good to begin with red light rather than infrared, which is listed first on the data sheet, because the length of the scale model for infrared light is significantly longer than the scale models of any of the visible light colors. It is nice to let students discover this for themselves.

5. Have students fill in the scale length in the millimeters column on their data sheet for red light. Remind students that this column should always be the same as the final answer for wavelength in nanometers.
6. Now explain to students that the colored strips of paper will be used to represent the different colors in the visible spectrum. Red paper will be used for the wavelength of red light, orange paper for orange light, and so on. White paper will represent infrared, and black paper will represent ultraviolet.
7. Have each group cut a strip of red paper that is the same length as the number they have written in the column for scale length in millimeters. (If standard 8.5×11 -inch paper was used to make the strips, one strip by itself will not be long enough to make the model. Point out to the groups that they may need to tape more than one strip together to get a long enough length of paper.)
8. Once groups have a piece of red paper that is 750 millimeters (75 centimeters) long, have them mark the *actual* wavelength of red light, 7.5×10^{-7} meters, on the strip. At this point, walk around the room and check on each group's progress.
9. Each group should now complete a metric conversion calculation and cut strips for each of the electromagnetic waves represented on the data sheet. When the groups have finished, they should have eight strips of paper of different lengths and colors in their model.
10. Have groups align their strips horizontally, directly underneath each other, with the longest strip (which should be infrared) on top and the shortest strip (which should be ultraviolet) on the bottom. Tape all of the strips together to make one large sheet. Hang all groups' models of the spectrum around the classroom.
11. When students have finished, collect the data sheets from each group. Data sheets will serve as part of the evaluation for this activity.

Exploration

adapted from:

<http://www.eduref.org/cgi-bin/printlessons.cgi/Virtual/Lessons/Science/Physics/PHS0210.html>

Have students look into the "shoebox of fun." "Shoebox of fun" -A shoebox with a eye-hole cut out of one end and a square cut out on the lid. Cover the hole on the lid with red cellophane. Inside the box glue down three gumballs -- red, green, and white. " What do they see? Students write down their answers on a piece of paper and set aside.

Review: (5 min.)

Go over the previous class lesson (wavelengths). Write the spectrum on the board in order of decreasing wavelength (radio, infrared, visible light, ultraviolet, x- ray, Gamma rays)."All matter has color. Do you think light has anything to do with color?" On the board, break down the light section into colors. "As we just observed, light is made up of all the colors in the rainbow (ROY G BIV). There are three primary colors of light (make up all the other colors): red, green, and blue. Red + green + blue = white light. When we see a color, that means the object repels that color and absorbs all the other colors of the spectrum. So if something is white, what colors does it absorb? If we see light blue, what colors are absorbed? What about yellow and pink?"

Discussion: (5 min.)

Refer back to the box and have students hypothesize what is going on in the box, using the knowledge they now have. Refresh students' memories about what they saw in the box. Reveal

what is inside the box. Have students hypothesize why the balls are not the colors that they saw with the lid on. "What kind of light was coming in the box with the lid on? With the lid off?" The lid acted as a filter. A filter is a transparent material that transmits one or more colors of light and absorbs all the other colors of the spectrum. "Why do you think the green balls looked black?"

Individual Exercise--Mixing of Colors: (at least 5 min.)

Place the primary color transparencies on the overhead. Have students take out a piece of paper. Ask, "Why does the mixing of the three colors give you black? What is happening to white light?" Students write their ideas on a piece of paper.

Discussion: (5 min.)

When everyone is finished, ask for a volunteer to answer the questions. Get several students' feedback. Define the term **pigment**: colored material that absorbs some colors and reflects others. Mixing of colors is used in our everyday life (Example: Sunday comics).

Elaboration:

Ask the students "Why does the color change when chemicals are added to a fire—for example—Epsom salt which makes a fire extremely orange?" (perform demo of this for the students)

- With this question a teacher can review the atom, its parts, what happens when electrons become excited and so on.
- This is also good for reviewing properties of photons and also wavelengths... it shows how certain elements have certain properties that emit certain wavelengths
- Stoichiometry can also be introduced or reviewed

Apart from incombustible elements, the color of a hydrocarbon flame is primarily dependent on the richness of the flame – that is, on how much oxygen there is to combust the fuel. In practice*, when the mixture is slightly lean (has more oxygen than required for complete combustion), the color of the combustion zone is generally blue-violet due to large amounts of high-energy radical carbon and hydrogen compounds. When the mixture is slightly rich (slightly too much fuel and not enough oxygen), the color is sometimes green due to C₂ molecules breaking free, and the high-temperature products can glow red from the CO₂ and H₂O produced during combustion. When the mixture is very fuel rich (a poor flame, with not enough oxygen to burn properly), carbon particles form and an intense yellow radiation results from their being heated in the flame. In very rich flames – often you see this in candles – soot particles may impart a black color to the outer edge of the yellow flame.

Flame color comes from the energy released by the electrons of the atoms of burning gas as they are raised to higher energy states during combustion, then fall back to lower energy states. Some of this energy is released in the form of visible light. The color corresponds to frequency, which is a function of the amount of energy released. (Work with me on this.) Low energy, low-frequency light is red; medium-frequency, medium-energy light is orange, yellow, or green; and high-energy, high-frequency light is blue or violet. If the energy levels are spread over a wide range of the visible spectrum, the light will appear as white. Many thermodynamics and chemistry texts state that adiabatic flame temperature is highest when the flame is at perfect

stoichiometry (exactly enough air to burn the fuel). Since mixing and other practical effects require extra air to ensure combustion, the hottest flames in practice tend to be slightly lean (slightly more oxygen than needed).

Chemical Flame Colors

If you want to purposely color your fire then dissolve one of these chemicals in water, soak a log or pinecones in it, and let it dry.

Orange —————Calcium Chloride (a bleaching powder)

Yellow —————Sodium Chloride (table salt)

Yellow-Green—Borax (a laundry powder)

Green—————Copper Sulfate (for swimming pools)

Blue—————Copper Chloride

Purple————Potassium Chloride (fertilizer)

White—————Magnesium Sulfate (Epsom salts)

Use only one chemical per log/set of pinecones (mixing will not produce “multiple colors”) however, you may put more than one type of “colorized log” in the fire at once.

CAUTION!! Some of these chemicals are not very pleasant to get all over yourself, your clothes, the living room, or the dog. Use proper precautions (goggles etc.)

DATA SHEET

In this activity we will be building a model of the infrared through ultraviolet portions of the visible spectrum. Below you will find values for the wavelengths that will be included in the model. The wavelengths are given in meters. The scale you will use for your model is 1 nanometer equals 1 millimeter. You are to calculate the values for the waves in nanometers and then in your model measure one millimeter for every nanometer. You are to show all of your calculations in the space provided below.

Wave	Actual Wavelength in Meters (m)	Calculation	Actual Wavelength in Nanometers (nm)	Scale Wavelength in millimeters (mm)
Infrared	1×10^{-6}			
Red	7.5×10^{-7}			
Orange	6.25×10^{-7}			
Yellow	5.75×10^{-7}			
Green	5.25×10^{-7}			
Blue	4.5×10^{-7}			
Violet	4.0×10^{-7}			
Ultraviolet	3×10^{-8}			

Colors observed from the prism (listed in order from top to bottom):

1. If white light is shined on a red shirt, what colors of light are reflected and what colors are absorbed?
2. Explain why on a comic strip you see the color green but there is not green ink on the paper.

HS.7

Lesson Plan: Logarithmic and Exponential Applications

Mathematics TEKS for Algebra II:

(11) Exponential and logarithmic functions. The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

Science TEKS for Chemistry:

(14) Science Concepts. The student knows the properties and behavior of acids and bases.

Materials:

Graphing Calculators

Graph paper

Guiding Questions:

How can we apply logarithmic and exponential equations to everyday calculations?

How do these graphs behave differently than linear or other functions?

Engagement:

Discussion on how exponential formulas & logarithms apply in both science and math

Exploration:

A handout involving both subject areas will be used.

Explanation:

Teacher will group the students with into teams to compare answers. They will teach their classmates how to complete the problems as preparation/review for the quiz. (Teacher will guide each group of course)

Elaboration:

Teacher will then compare all the problems and bridge gaps of logarithmic concepts for students.

Evaluation:

Quiz with sample problems of both math and science nature, with a variety of questions from both subjects.

Lesson Plan: pH Titration and Light Absorbance

Guiding Question(s):

When do is pH a critical quantity?

How can we measure pH?

How can we use light absorbance technology to measure pH?

What do systems of measuring pH have in common and what is different among them?

Expected outcomes:

At the end of this unit, students will be able to describe the use and importance of pH various fields of inquiry. They will be able to express pH quantitatively and accurately describe the differences between different quantities. They will be able to perform pH titrations. They will be able to relate various pH values to substances and systems in their own lives.

Engage (Activity 1)

Demonstration of Red Tide USB Spectrometer System

Explore (Activity 2)

Red Cabbage pH Indicator Activity

Explain (Activity 3)

Anthocyanins in Fruit Juices Lab

Elaborate (Activity 4)

Inquiry Activity

Evaluate (Activity 5)

Group Presentations

Materials List:

Red Tide USB Spectrophotometers and related equipment

Red Cabbage Activity materials (see attached)

Anthocyanin in Fruit Juices Lab materials (see attached)

Computer Lab access

Additional materials as requested by students

Lesson Description/Teacher Instructions:

We want to guide students through a series of activities that will help the student construct an understanding of pH and titration as they apply to the various sciences. Starting with a demonstration of new technology, the students then explore some of the oldest technologies relating to pH measurement in the red cabbage lab. The fruit juices lab drives a more quantitative approach to pH measurement, and to make the connection between pH and

absorbance in the spectrophotometer. It is important for the teacher doing this activity for the first time to actually perform the activity first to ensure they are using the optimal wavelength settings for the hardware available. At this point, the students should be directed to a group inquiry activity. The aim of this activity is to have the students identify a solution system they are interested in to apply these techniques to. Their purpose should be to explore and meaningfully describe such a system through the use of these tools. Teachers should maintain close communication with all student groups, especially during the first part of this activity, to guide students down productive pathways and avoid blind alleys and dead ends. The unit evaluation then will be based on a group presentation of the group's inquiry activity.

Assessment plan:

As always, ongoing formative assessment should be going on throughout the unit. Each activity needs an assessment component such as a grading rubric for the Anthocyanin Lab Report and Presentation. Students should answer included questions in the Red Cabbage Activity.

Tell us what someone observing your class during this lesson would expect to see:

An observer to this classroom should see students engaged in cooperative learning and peer teaching. The sound level should be moderate, but silence would indicate that students are not performing as expected.

Lesson Plan: Fruit Fly Population Study

5-E Phase and standards (describe):

Engage:

Discuss whether a population can grow faster than the available food? A discussion web will follow with the focus question where two students pair share with their partner switching roles with speaking, listening and writing.

Explore:

Allow students to set up their own experiment with their own selection of jars, fruit, ecosystems, cover and flies.

Explain:

Students will compose sentence strips with definitions and pictures to define the following terms: independent variable, dependent variable, carrying capacity, limiting factors, exponential function, logarithmic function, inflection, predation, predator, prey, scientific method, analyze, conclusion, data, results, ecology, and extinct. Students will conduct as a class an Inside-Outside SIOP strategy with classroom made cards and share with one another to review material.

Elaborate:

What would happen to the graph if additional food was introduced? What would happen to the graph if food was removed? What if a predator introduced into the ecosystem? How would the introduction of a spider affect the population curve?

Evaluate:

SPED/SIOP Modifications: Students will read an article with visuals showing carrying capacity within a population. Cards will be distributed to each student for discussion and explanation while doing the doing mix and match (students stand in a line and exchange information on their assigned cards every 2 minutes). Students will be assessed formally via TAKS questions at the end of the lesson.

Guiding Question(s):

What would happen to the graph if additional food was introduced? What would happen to the graph if food was removed? What if a predator introduced into the ecosystem? How would the introduction of a spider affect the population curve?

Expected outcomes:

The student will exponential growth of a population and how limiting factors define the carrying capacity.

Materials List:

Jars, cheese cloth, fruit, twigs, leaves, grass, flies, graph paper, 3 x 5 index cards, glue, sentence strips

Lesson Description: Teacher Instructions

The student will create an ecosystem with the selection of their own variables, monitor the daily population growth, collect data, draw the graph and conclude.

Guiding questions:

1. What would happen to the graph if additional food was introduced?
Answers will vary
2. What would happen to the graph if food was removed?
Answers will vary
3. What if a predator introduced into the ecosystem?
Answers will vary
4. How would the introduction of a spider affect the population curve?
Answers will vary

Assessment plan:

Informal SIOP on students completing a Jeopardy TAKS tests questions
Inside-Outside Circle strategy with assessment cards

Tell us what someone observing your class during this lesson would expect to see:

Students will learn the scientific method hands-on inquiry of collecting data, analyzing, making graphical representations, making predictions, using inference and observation skills in a real life model.

Vita

Barbie Avila was born in William Beaumont Army Medical Center in El Paso, Texas; the eldest of three children. She graduated from Burges High School in El Paso, Texas in the spring of 1994 and entered The University of Texas at El Paso in the fall, playing for the university's soccer team her freshman year. Upon graduating with a Bachelor of Arts in Psychology, she worked as a play therapist for the Family Place Shelter for abused women and children. The work was rewarding, yet very stressful. She decided to pursue work as a school counselor which drew her into the education profession. She taught at an inner city elementary school in Houston, Texas, which is where she fell in love with the career of educating children. She has been teaching for 11 years and is now working as a Mathematics and Science Elementary Curriculum Coordinator.

Permanent Address:

12197 Saint Lucia

El Paso, Texas 79936

Or

Email4barbie@yahoo.com