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Usually, Either Left and Right Brains Are Equally Active or Only One of Them Is Active: First-Principles Explanation

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Usually, Either Left and Right Brains Are Equally Active or Only One of Them Is Active: First-Principles Explanation

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Abstract It is known that in most practical situations, either both left and right brains are equally active, or only one of them is active. A recent paper showed that this empirical phenomenon can be explained by a realistic model of the brain effectiveness. In this paper, we show that this conclusion can be made without any specific assumptions about the brain, based on first principles.

1 Formulation of the problem

Empirical phenomenon that needs explaining. The brain of many living creatures – including, of course, humans – consists of two largely independent parts. One of the names for these two parts is left and right brain.

It turns out that in most practical situations:

- either left and right brains are equally active,
- or only one of them is active.

Specifically:

- when the corresponding task is not very complex, left and right brain are equally involved, but
- when the task is sufficiently complex, only one of the two brain halves is active.

How this phenomenon is usually explained. Several qualitative explanations of the above phenomenon have been proposed. Their main idea is that:

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- on the one hand, the duplication coming from using both (reasonably independent) parts of the brain to solve a problem makes the result more reliable, but
- on the other hand, using both parts of the brain to solve a problem doubles the needed amount of energy; thus, when the task is complex and thus requires a large amount of energy, the duplication is no longer affordable.

A recent quantitative explanation. A recent paper [2] come up with a reasonable model $f(\ell, r)$ of how the brain's effectiveness depends on the activity levels ℓ and r of the left and right brains. This model includes some parameters that correspond to the complexity of the task. It turns out that in this model, for each level of complexity, the maximal effectiveness is attained:

- either when $\ell = r$
- or when one of the two activity levels is equal to 0.

In this model, in full accordance with the experimental data:

- for complex tasks, the maximal effectiveness is attained when one of the activity levels is 0,
- while for less complex tasks, the maximum effectiveness is attained when $\ell = r$.

Thus, this model full explains the empirical phenomenon.

A natural question. While the model proposed in [2] is reasonable, it provides – as any other model – only an approximate description of the real-life phenomenon. To more accurately describe the actual phenomenon, we most probably need a somewhat different model. It is therefore reasonable to ask whether the explanation from this paper can be extended to a more general class of models.

What we do in this paper. In this paper, we provide a partial answer to the above question. Namely, we show that in the natural quadratic approximation, every bounded model has the property that optimal effectiveness is attained:

- either when $\ell = r$
- or when one of the activity levels is equal to 0.

2 Definitions and the Main Result

We need bounded models. In practice, there is always a limit to our effectiveness. So, it is reasonable to only consider models $f(\ell, r)$ that are *bounded*:

Definition 1. *We say that a function $f(\ell, r)$ is* bounded *if there exists a bound B for which $f(\ell, r) \le B$ for all $\ell \ge 0$ and $r \ge 0$.*

We need realistic models.

- We know that the level of activities in each half of the brain affects the effectiveness, so we want the models that reflect this, i.e., for which $f(\ell, r)$ is not equal to the constant or to the function of only one of the two variables.

- We want to make sure that in the model, it makes sense for the brain to be active, i.e., at least some values of ℓ and r leads to better effectiveness than in situations when the brain is inactive, i.e., that we have $f(\ell, r) > 0$ for some ℓ and r.
- We also want to make sure that there is a difference between the left and the right brains, i.e., that the effectiveness is not simply the function of the total activation $\ell + r$.

Definition 2. *We say that a function $f(\ell, r)$ is* realistic *if:*

- *it is not equal to a function of one variable,*
- *there exist values $\ell > 0$ and $r \geq 0$ for which $f(\ell, r) > f(0, 0)$, and*
- *it is not equal to a function of the sum $\ell + r$.*

We will consider only symmetric models. In practice, left and right brain are largely similar. So, following [2], we will only consider models in which the dependence of effectiveness on the two activity levels is symmetric:

Definition 3. *We say that a function $f(\ell, r)$ is* symmetric *if we have $f(\ell, r) = f(r, \ell)$ for all ℓ and r.*

We will consider the simplest possible models. We do not know the exact shape of the dependence $f(\ell, r)$.

Such situations are ubiquitous in physics and in other application areas. In such situations, a usual practice is to consider a general form of the function – which, in the sufficient smooth cases, can be expanded in Taylor series – and keep only a few terms in this expansion; see, e.g., [1, 3].

The simplest case if when we only keep linear terms in this expansion, i.e., consider the dependence of the type

$$f(\ell, r) = a_0 + a_\ell \cdot \ell + a_r \cdot r \tag{1}$$

for some coefficient a_0, a_ℓ, and a_r. It turns out that in this approximation, we cannot have a function with the desired properties, even if we do not require symmetry:

Proposition 1. *No linear function is bounded and realistic.*

Comment. For reader's convenience, all the proofs are placed in the special Proofs section.

Since we cannot have linear model, the next simplest are quadratic models. A general quadratic function of two variables ℓ and r has the following form:

$$f(\ell, r) = a_0 + a_\ell \cdot \ell + a_r \cdot r + a_{\ell\ell} \cdot \ell^2 + a_{\ell r} \cdot \ell r + a_{rr} \cdot r^2 \tag{1}$$

For quadratic models, we have the following result.

Proposition 2. *For any bounded realistic symmetric quadratic function $f(\ell, r)$, its largest value is attained:*

- *either when one of the values ℓ and r is equal to 0,*

- *or when* $\ell = r$.

Discussion. Thus indeed, the above-described phenomenon follows from the first principles.

3 Proofs

Proof of Proposition 1.

1°. Depending on the signs of the coefficients a_ℓ and a_r, we can have the following situations:

- one of these two coefficients is positive;
- one of these coefficients is 0; or
- both coefficients are neither positive nor 0, i.e., both coefficients are negative.

Let us show that in all three cases, one of two desired conditions – that the function is bounded and realistic – is not satisfied.

2°. If at least one of the coefficients a_ℓ and a_r is positive, then the function $f(\ell, r)$ is not bounded.

3°. If at least one of these two coefficients is 0, then the effectiveness does not depend on the corresponding value, so the function is not realistic.

4°. If both coefficients are negative, then for $\ell \geq 0$ and $r \geq 0$, we cannot have $f(\ell, r) > f(0, 0)$, so the function is not realistic.

The proposition is proven.

Proof of Proposition 2. Let $f(\ell, r)$ be a bounded realistic symmetric quadratic function.

1°. Due to symmetry, we have $a_\ell = a_r$ and $a_{\ell\ell} = a_{rr}$. Thus, the expression (1) takes the following simplified form:

$$f(\ell, r) = a_0 + a_\ell \cdot (\ell + r) + a_{\ell\ell} \cdot (\ell^2 + r^2) + a_{\ell r} \cdot \ell \cdot r. \tag{2}$$

2°. Let us show that $a_{\ell\ell} \leq 0$.

Indeed, if this inequality was not true, i.e., if we had $a_{\ell\ell} > 0$, then for $r = 0$ and $\ell \to \infty$, the expression (2) would tend to infinity, thus the function $f(\ell, r)$ would not be bounded. Since we assumed that the function $f(\ell, r)$ is bounded, we have $a_{\ell\ell} \leq 0$.

3°. Let us now show, by contradiction, that we cannot have $a_{\ell\ell} = 0$, i.e., that we have $a_{\ell\ell} < 0$.

3.1°. Indeed, if $a_{\ell\ell} = 0$, the quadratic function has the form

$$f(\ell, r) = a_0 + a_\ell \cdot (\ell + r) + a_{\ell r} \cdot \ell \cdot r. \quad (3)$$

3.2°. Let us show that if $a_{\ell\ell} = 0$, then we must have $a_{\ell r} < 0$.

Indeed:

- we cannot have $a_{\ell r} = 0$, since then the function $f(\ell, r)$ will be linear, and this would contradict Proposition 1;
- we cannot have $a_{\ell r} > 0$, since then for $\ell = r \to \infty$, we would get $f(\ell, r) \to \infty$, and we assumed that the function $f(\ell, r)$ is bounded.

3.3°. Let us now show that if $a_{\ell\ell} = 0$, then we must have $a_\ell \le 0$.

Indeed, if $a_\ell > 0$, then for $\ell \to \infty$ and $r \to 0$, we will have $f(\ell, r) \to \infty$, and we assume that the function $f(\ell, r)$ is bounded.

3.4°. We have shown that if $a_{\ell\ell} = 0$, then we have $a_\ell \le 0$ and $a_{\ell r} < 0$. Thus, for $\ell \ge 0$ and $r \ge 0$, we have $f(\ell, r) \le f(0, 0)$, and we assumed that the function $f(\ell, r)$ is realistic.

4°. We are interested in values (ℓ_0, r_0) for which the function $f(\ell, r)$ attains its maximum for $\ell \ge 0$ and $r \ge 0$. In particular, this means that:

- for $r = r_0$, the function $\ell \mapsto f(\ell, r_0)$ attains its maximum for $\ell = \ell_0$, and
- for $\ell = \ell_0$, the function $r \mapsto f(\ell_0, r)$ attains its maximum for $r = r_0$.

In general, the maximum of any smooth function on an interval is attained:

- either at one of the endpoints,
- or at a point where its derivative is equal to 0.

Here, the endpoints are $\ell = 0$ and $r = 0$. Thus, if the maximum is attained at one of these endpoint, for this case, the proposition is proven.

To complete the proof, we need to consider the case when both partial derivatives – with respect to ℓ and r – are equal to 0. In this case, we have the following two equalities:

$$\frac{\partial f}{\partial \ell} = a_\ell + 2a_{\ell\ell} \cdot \ell + a_{\ell r} \cdot r = 0; \quad (4)$$

$$\frac{\partial f}{\partial r} = a_\ell + 2a_{\ell\ell} \cdot r + a_{\ell r} \cdot \ell = 0. \quad (5)$$

Subtracting (5) from (4), we get

$$2a_{\ell\ell} \cdot (\ell - r) - a_{\ell r} \cdot (\ell - r) = 0,$$

i.e.,

$$(2a_{\ell\ell} - a_{\ell r}) \cdot (\ell - r) = 0. \quad (6)$$

5°. Let us show that we cannot have $2a_{\ell\ell} - a_{\ell r} = 0$.

Indeed, in this case, we would have $a_{\ell r} = 2a_{\ell\ell}$, so the function $f(\ell, r)$ would take the form

$$f(\ell, r) = a_0 + a_\ell \cdot (\ell + r) + a_{\ell\ell} \cdot (\ell + r)^2,$$

and we assumed that $f(\ell, r)$ is not a function of the sum $\ell + r$.

Thus, we cannot have $2a_{\ell\ell} - a_{\ell r} = 0$, so $2a_{\ell\ell} - a_{\ell r} \neq 0$. Dividing both sides of the equality (6) by this non-zero number, we conclude that $\ell - r = 0$, i.e., that $\ell = r$.

The proposition is proven.

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