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Towards a Psychologically Natural Relation Between Colors and Fuzzy Degrees

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Abstract A natural way to speed up computations – in particular, computations that involve processing fuzzy data – is to use the fastest possible communication medium: light. Light consists of components of different color. So, if we use optical color computations to process fuzzy data, we need to associate fuzzy degrees with colors. One of the main features – and of the main advantages – of fuzzy technique is that the corresponding data has intuitive natural meaning: this data comes from words from natural language. It is desirable to preserve this naturalness as much as possible. In particular, it is desirable to come up with a natural relation between colors and fuzzy degrees. In this paper, we show that there is exactly one such natural relation, and we describe this relation.

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1 Formulation of the Problem

Need for processing fuzzy information. In many real-life situations – be it driving a car or treating a patient – humans provide a good quality control, often control which is of better quality than what current automatic controllers can provide. It is therefore desirable to incorporate the knowledge and experience of these human controllers into the automatic systems, to make these systems more effective.

Such incorporation is not easy, and one of the main reasons why it is not easy is that human experts often describe their control strategies not in precise easy-to-program mathematical form, but by using imprecise ("fuzzy") words from natural language such as "small". To transform this knowledge into precise computer-understandable terms, Lotfi Zadeh invented a special technique that he called *fuzzy*; see, e.g., [2, 6, 7, 9, 10, 23].

In this technique, to describe an imprecise property like "small", we ask the expert to assign, to each possible value x of the corresponding quantity, a degree – from the interval $[0,1]$ – describing to what extend this value has the given property (e.g., is small).

Need for discrete fuzzy degrees. From the purely mathematical viewpoint, there are infinitely many degrees in the interval $[0,1]$, so nothing should prevent experts to assign degrees like 0.2134 or 0.6778. However, in practice, people cannot meaningfully distinguish between many degrees. According to psychology, they can distinguish between 7 plus minus 2 different degrees; see, e.g., [8, 12]. For most people, this number is 7, so it makes sense to use seven different degrees.

So, in addition to absolute "yes", absolute "no", and a neutral situation when we have exactly as many arguments for "yes" as for "no", we should four degrees corresponding to some degree of confidence in a statement or it is negation. Thus, we arrive at the following natural list of degrees:

no	very probably no	probably no	neutral	probably yes	very probably yes	yes

Need for fast computations. In many practical problems, especially in problems related to real-time control, it is desirable to perform computations as fast as possible.

Need for optical computing. One of the factors that limits the computation speed is the speed with which signals pass from one part of the processing unit to another. It is therefore desirable to make sure that this signal transfer occurs at the highest possible speed – and according to physics, the fastest possible speed is the speed of light; see, e.g., [5, 14]. Thus, to speed up computations, it is desirable to use, in computations, electromagnetic waves – in particular, visible light. Such computations are known as *optical computing*.

Need for color optical computing. It is well known that every optical signal can be represented as a combination of several signals of pure color. This fact that discovered by Newton, who showed that each light can be decomposed into pure colors

– and then these pure colors can be assembled back, into the original signal. Thus, a natural idea is to use signals of different color. Such computations are known as *color optical computing*.

In particular, it is desirable to use color optical computing to process fuzzy data; see, e.g., [15, 16, 17, 18, 19, 20, 21, 22].

Historical comment. From the mathematical and signal processing viewpoints, Newton's discovery about colors was the first empirical proof of what is now known as Fourier techniques (see, e.g., [11, 13]) – whose main idea is that each signal can be presented as a combination of sinusoids with different frequency (and pure colors are exactly signals of fixed frequency).

Problem: it is desirable to have a natural relation between colors and fuzzy degrees. To use color optical computing for processing fuzzy data, it is desirable to assign different colors to different fuzzy degrees. Fuzzy techniques process intuitively clear notions like "small". It would be advantageous to extend this naturalness to data processing as well, i.e., to make the relation between colors and fuzzy degrees natural.

What we do in this paper. In this paper, we show how to come up with such a natural relation between colors and fuzzy degrees.

2 Towards the Desired Natural Relation

What colors should we consider? It is known that in our color perception, we have three basic colors – red (R), blue (B), and green (G). From our viewpoint, every other color can be represented as a combination of these three basic colors:

- a combination of red and green (R + G) leads to yellow (Y),
- a combination of red and blue (R + B) leads to magenta (M),
- a combination of green and blue (G + B) leads to cyan (C), and
- a combination of all three colors (R + G + B) leads to white (W).

Also, the absence of colors corresponds to black (Blc). These are the colors that we need to assign to different fuzzy degrees.

What is a natural analog of "neutral"? The most easy thing is to assign a color to the degree "neural". Indeed, in this degree, the expert has exactly as many argument for as arguments again. In other words, in such situation, the expert cannot decide, i.e., in effect, has no contribution to the decision process. It is therefore natural to associate this no-information degree with no-information color – i.e., with the color Black.

What remains to be done is to associate appropriate colors with the remaining six degrees:

no	very probably no	probably no	probably yes	very probably yes	yes
?	?	?	?	?	?

What is known. Psychologists have studied the relation between colors and decision making, and they found an association of some colors with decisions; see, e.g., [1, 3, 4]. (One has to be cautious since this research is mostly based on US customers, but still, these associations are largely universal.)

Let us sort this information in the order in which these colors appear in the spectrum:

- The color Yellow is associated with Inexpensive and Low Quality.
- The color Green is associated with Good Taste.
- The color Blue is associated with High Quality and Reliability.

Also, the color Black is associated with Expensive.

How can we translate these associations into fuzzy degrees?

- Let us start with Yellow. Low Quality is not good. However, because of Inexpensive, it is not a clear No: sometimes, we buy low quality things because we cannot afford things of better quality. We are not happy when we do it, so a natural idea is to assign, to this color, the degree “very probably no”.
- Good Taste is a positive characteristic, but it is only one – and often, not the most important characteristic. More important is how usable it is, taste is secondary. It is therefore natural to associate, to this color, the degree “probably yes”.
- Blue has only positive characteristics, both are important, so let us associate it with “yes”.
- What about Black? The fact that something is Expensive does not mean that we should buy it – only snobs buy things just because they are expensive. It also does not mean that we should *not* buy it: expensive is a price that we pay for high quality. In other words, from the viewpoint of decision making, the very fact that something is expensive does not help to decide whether we but it or not, it is neutral – which is in perfect agreement with our previous association of Black with “neutral”.

Let us summarize the above arguments by placing the corresponding colors into our table:

no	very probably no	probably no	probably yes	very probably yes	yes
?	R + G	?	G	?	B

How to extend this assignment to other colors: a natural idea. We need to extend the above psychologically natural relation between colors and fuzzy degrees to the remaining degrees. A natural idea is that the degree corresponding to the mixture of two colors should be located between the degrees corresponding to these two colors. So, we come up with the following conclusions.

According to the above idea, Y = R + G should be between R and G. In our table, the degree corresponding to G is to the right of the degree corresponding to Y. So, the degree corresponding to R should be to the left of the degree "most probably no" corresponding to Y. The only degree which is to the left of "very probably no" is "no". So, we assign, to Red, the degree "no":

no	very probably no	probably no	probably yes	very probably yes	yes
R	R + G	?	G	?	B

Similar, the degree corresponding to G + B should be in between the degrees corresponding to G and B, i.e., in between the degrees "probably yes" and "yes". There is one degree in between: "very probably yes". So, we assign this degree to the color G + B:

no	very probably no	probably no	probably yes	very probably yes	yes
R	R + G	?	G	G + B	B

The only remained un-assigned degree is "probably no", and the only remaining un-assigned color is R + B, so we naturally match them. (This matching, by the way, is in perfect accordance with our idea, since this degree is in between degrees "no" and "yes" corresponding to Red and Blue.) Thus, we arrive at the following conclusion.

Conclusion. The following is the natural association of colors and fuzzy degrees:

no	very probably no	probably no	probably yes	very probably yes	yes
R	R + G	R + B	G	G + B	B

To make this relation clearer, let us reformulate it with full names of the resulting colors:

color	structure	degree
Red (R)	R	no
Yellow (Y)	R + G	most probably no
Magenta (M)	R + B	probably no
Black		neutral
Green (G)	G	very probably yes
Cyan (C)	G + B	probably yes
Blue (B)	B	yes

3 Discussion

Can we make this natural relation less subjective? Our relation relies on the results of empirical psychological studies. A natural question is: can we make our conclusions less subjective, based only on some objective characteristics – so that we will not need to rely on these empirical results? In this section, we will try to do it.

What is a natural objective characteristic of a color? As we have mentioned, a color is described by its frequency. We are talking about using color to communicate. From this viewpoint, the higher the frequency, the most information we can transfer by using signals of this color. So, the larger the frequency, the more positive our attitude to different colors. Let us show how this idea helps.

Let us start with the ordering of three basic colors. Among the three basic colors Red, Green, and Blue, Red corresponds to the smallest frequency, Green to the intermediate one, and Blue to the largest one, so we have:

$$\mathrm{R} < \mathrm{G} < \mathrm{B}.$$

Let us extend this order to combinations of basic colors. It is natural to assume – as we did before – that the quality of a combination of two colors should be in between the corresponding colors, i.e., that

$$\mathrm{R} < \mathrm{R}+\mathrm{G} < \mathrm{G} < \mathrm{G}+\mathrm{B} < \mathrm{B}$$

and

$$\mathrm{R} < \mathrm{R}+\mathrm{B} < \mathrm{B}.$$

It is also reasonable to require that if we replace one color in a combination with a better color, the quality of the combination improves. As a result, we get

$$\mathrm{R}+\mathrm{G} < \mathrm{R}+\mathrm{B} < \mathrm{G}+\mathrm{B}.$$

Resulting order. By combining the above comparisons, we end up with the following order:

$$\mathrm{R} < \mathrm{R}+\mathrm{G} < \mathrm{G}, \mathrm{R}+\mathrm{B} < \mathrm{G}+\mathrm{B} < \mathrm{B}. \quad (1)$$

This is *almost* what we got by using the empirical data, but not exactly: by using the general ideas, we cannot decide which is better: Green (G) or Magenta (M = R + B).

Why cannot we get the full order this way? It is easy to see why we cannot get the full order based only on the objective characteristics. Indeed, the original situation remains the same if we reverse the order, i.e., if we:

- swap R and B, while keeping G intact, and
- replace each $<$ with $>$.

One can easily see that the resulting relations (1) do not change under this transformation. However:

- we cannot have R + B $<$ G, since this transformation will transform it into its opposite R + B $>$ G, and these two inequalities cannot be both true;
- similarly, we cannot have G $<$ R + B, since this transformation will transform it into its opposite G $>$ R + B, and these two inequalities cannot be both true.

Comment. This may be related to fact that, in contrast to Green and to many other colors, magenta (R + B) is *not* a real color – in the sense that there is no frequency corresponding to magenta. The only way to get a perception of magenta is to mix several colors. This is different from other combinations, such as Yellow:

- yes, in the Red-Green-Blue scheme, Yellow is a combination of two colors,
- but there is also a wavelength range 565–590 THz corresponding to Yellow.

Similarly, there is a wavelength range 600–620 THz corresponding to Cyan (which we represented as G + B).

But maybe we can get the full order this way? What if we take into account not only the *order* between the frequencies corresponding to three basic colors, but also the *numerical ranges* of these frequencies:

- 440–480 THz for Red, centered at 440,
- 530–600 THz for Green, centered at 565, and
- 620–670 THz for Blue, centered at 645,

we can see that the central point (565) of the Green frequency range is much further to the right than the average $(440+645)/2 = 542.5$ of the central points of the frequency ranges corresponding to Red and Blue. From this viewpoint, it is not surprising that G feels better that R + G.

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References

1. M. M. Aslam, “Are you selling the right colour? A cross-cultural review of colour as a marketing cue”, *Journal of Marketing Communications*, 2006, Vol. 12, No. 1, pp. 15–30.
2. R. Belohlavek, J. W. Dauben, and G. J. Klir, *Fuzzy Logic and Mathematics: A Historical Perspective*, Oxford University Press, New York, 2017.
3. P. A. Bottomley and J. R. Doyle, “The interactive effects of colors and products on perceptions of brand logo appropriateness”, *Marketing Theorey*, 2006, Vol. 6, No. 1, pp. 63–83.
4. *Color Psychology*, Wikipedia, https://en.wikipedia.org/wiki/Color_Psychology, accessed on August 1, 2023
5. R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Addison Wesley, Boston, Massachusetts, 2005.
6. G. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic*, Prentice Hall, Upper Saddle River, New Jersey, 1995.
7. J. M. Mendel, *Uncertain Rule-Based Fuzzy Systems: Introduction and New Directions*, Springer, Cham, Switzerland, 2017.
8. G. A. Miller, “The magical number seven plus or minus two: some limits on our capacity for processing information”, *Psychological Review*, 1956, Vol. 63, No. 2, pp. 81–97.
9. H. T. Nguyen, C. L. Walker, and E. A. Walker, *A First Course in Fuzzy Logic*, Chapman and Hall/CRC, Boca Raton, Florida, 2019.
10. V. Novák, I. Perfilieva, and J. Močkoř, *Mathematical Principles of Fuzzy Logic*, Kluwer, Boston, Dordrecht, 1999.
11. B. G. Osgood, *Lectures on the Fourier Transform and Its Applications*, American Mathematical Society, Providence, Rhode Island, 2019.
12. S. K. Reed, *Cognition: Theories and Application*, SAGE Publications, Thousand Oaks, California, 2022.
13. E. M. Stein and R. Shakarchi, *Fourier Analysis: An Introduction*, Princeton University Press, Princeton, New Jersey, 2003.
14. K. S. Thorne and R. D. Blandford, *Modern Classical Physics: Optics, Fluids, Plasmas, Elasticity, Relativity, and Statistical Physics*, Princeton University Press, Princeton, New Jersey, 2021.
15. V. Timchenko, Y. Kondratenko, and V. Kreinovich, “Efficient optical approach to fuzzy data processing based on colors and light filter”, *International Journal of Problems of Control and Informatics*, 2022, Vol. 52, No. 4, pp. 89–105.
16. V. Timchenko, Y. Kondratenko, and V. Kreinovich, “Decision support system for the safety of ship navigation based on optical color logic gates”, *Proceedings of the IX International Conference “Information Technology and Implementation” IT&I-2022*, Kyiv, Ukraine, November 30 – December 2, 2022, CEUR Workshop Proceedings, Vol. 3347, pp. 42–52, http://ceur-ws.org/
17. V. Timchenko, Y. Kondratenko, and V. Kreinovich, “Implementation of Optical Logic Gates Based on Color Filters”, In: Z. Hu et al. (Eds.), *Proceedings of the he 6th International Conference on Computer Science, Engineering and Education Applications ICCSEEA 2023, Warsaw, Poland, March 17–19, 2023*, Lecture Notes on Data Engineering and Communications Technologies, 2023, Vol. 181, https://doi.org/10.1007/978-3-031-36118-0_12
18. V. L. Timchenko, Y. P. Kondratenko, and V. Kreinovich, “The architecture of optical logical coloroid with fuzzy computing”, *Proceedings of the 4th International Workshop on Intelligent Information Technologies & Systems of Information Security IntelITSIS’2023*, Khmelnitsky, Ukraine, March 22–24, 2023, CEUR Workshop Proceedings, 2023, Vol. 3373, pp. 638–648.
19. V. L. Timchenko, Y. P. Kondratenko, V. Kreinovich, and Olga Kosheleva, “Natural Color Interpretation of Interval-Valued Fuzzy Degrees”, *Proceedings of the 20th World Congress of the International Fuzzy Systems Association IFSA’2023*, Daegu, South Korea, August 20–24, 2023.
20. V. Timchenko, Y. Kondratenko, Oleksiy Kozlov, and V. Kreinovich, “Fuzzy Color Computing Based on Optical Logical Architecture”, *Proceedings of the International Conference on Intelligent and Fuzzy Systems INFUS’2023*, Istanbul, Turkey, August 22–24, 2023.

21. V. L. Timchenko, Y. P. Kondratenko, and V. Kreinovich, “Interval-Valued and Set-Valued Extensions of Discrete Fuzzy Logics, Belnap Logic, and Color Optical Computing”, *Proceedings of the Biennial Conference of the European Society for Fuzzy Logic and Technology EUSFLAT’2023*, Palma de Mallorca, Spain, September 4–8, 2023.
22. V. L. Timchenko, Y. P. Kondratenko, and V. Kreinovich, “Why Color Optical Computing”, In: Nguyen Hoang Phuong and V. Kreinovich (eds.), *Deep Learning and Other Soft Computing Techniques: Biomedical and Related Applications*, Studies in Computational Intelligence, Vol. 1097, Springer, Cham, Switzerland, 2023, pp. 227–233.
23. L. A. Zadeh, “Fuzzy sets”, *Information and Control*, 1965, Vol. 8, pp. 338–353.