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Complex Numbers Explain Why in Chinese Tradition, 4 Is Bad But 8 Is Good

Luc Longpre
The University of Texas at El Paso, longpre@utep.edu

Olga Kosheleva
The University of Texas at El Paso, olgak@utep.edu

Vladik Kreinovich
The University of Texas at El Paso, vladik@utep.edu

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Complex Numbers Explain Why in Chinese Tradition, 4 Is Bad But 8 Is Good

Luc Longpré, Olga Kosheleva, and Vladik Kreinovich

Abstract In the traditional Chinese culture, 4 is considered to be an unlucky number, while the number 8 is considered to be very lucky. In this paper, we show that both "badness" and "goodness" can be explained if we take into account the role of complex numbers in the analysis of general dynamical systems.

1 Formulation of the Problem

Cultural fact. In the traditional Chinese culture, 4 is considered to be an unlucky number, to the extent that in hotels, there is often no 4th floor – just like in many European and American hotels, there is no 13th floor. On the other hand, in this culture, number 8 is considered to be very lucky; see, e.g., [3].

Usual explanation. The usual explanation for the above fact in that in Chinese, the pronunciation of number 4 is similar to the word "death", while the pronunciation of number 8 is similar to the verb "to prosper".

Our idea. It is known that sometimes ancient traditions – which at first may seem completely arbitrary – later turn out to make some sense. Because of this, we decided to see is there can be a more serious explanation for the attitude towards 4 and

Luc Longpré
Department of Computer Science, University of Texas at El Paso, 500 W. University
El Paso, Texas 79968, USA, e-mail: longpre@utep.edu

Olga Kosheleva
Department of Teacher Educat, University of Texas at El Paso, 500 W. University
El Paso, Texas 79968, USA, e-mail: olgak@utep.edu

Vladik Kreinovich
Department of Computer Science, University of Texas at El Paso, 500 W. University
El Paso, Texas 79968, USA, e-mail: vladik@utep.edu

8 – and we found it. Our explanation – based on complex numbers – is described in this paper.

2 Our Explanation

Dynamical systems: brief reminder. The attitude towards numbers 4 and 8 is also applied to time periods. In view of this, let us consider systems changing with time, i.e., dynamical systems.

In general, at each moment of time, the state of a dynamical system can be described by values of several quantities $x_1,\ldots,x_n$. The values of these quantities in the next moment of time $t+\Delta t$ are determined by their current values:

$$x_i(t+\Delta t) = f_i(x_1(t),\ldots,x_n(t)). \tag{1}$$

Possibility of linearization. In many practical situations, the changes in the state are relatively small. This means that the value of each quantity x_i describing the system's state are close to the average value $\bar{x}_i$ of this quantity, so that the differences $\Delta x_i \stackrel{\text{def}}{=} x_i(t) - \bar{x}_i$ are small. In terms of these differences, the equation (1) takes the form

$$\Delta x_i(t+\Delta t) = F_i(\Delta x_1(t),\ldots,\Delta x_n(t)), \tag{2}$$

where we denoted

$$F_i(\Delta x_1,\ldots,\Delta x_n) \stackrel{\text{def}}{=} f_i(\bar{x}_1+\Delta x_1,\ldots,\bar{x}_n+\Delta x_n).$$

Since the values Δx_i are relatively small, terms which are quadratic in Δx_i (or of higher order) can be safely ignored. Thus, we can expand the right-hand side of the equation (2) in Taylor series and keep only linear terms in this expansion. This is a usual idea in physics (see, e.g., [1, 2]). In our case, it leads to the system of linear equations:

$$\Delta x_i(t+\Delta) = a_i + \sum_{j=1}^{n} c_{ij}\cdot \Delta x_j(t). \tag{3}$$

Generic solution to the resulting linear system: reminder. A general solution to a linear system (3) is well-known: in the generic case, in each solution $\Delta x_i(t+k\cdot\Delta t)$ to this system, each quantity Δx_i is a linear combination of the expressions λ_j^k, where each λ_j is an eigenvalue (in general, complex) of the matrix c_{ij}:

$$x_i(t+k\cdot\Delta t) = C_i + \sum_{j=1}^{n} C_{ij}\cdot\lambda_j^k,$$

for some coefficients C_i and C_{ij}.

For each term $T \stackrel{\text{def}}{=} C_{ij} \cdot \lambda_j^k$, its value at the next moment of time $t+(k+1)\cdot \Delta t$ can be obtained from its value at the previous moment of time $t+k\cdot \Delta t$ by multiplying by λ_j:

$$T(t+(k+1)\cdot \Delta t) = \lambda_j \cdot T(t+k\cdot \Delta t).$$

In general, we have

$$T(t+k\cdot \Delta t) = \lambda_j^k \cdot T(t).$$

What is good and what is not good: an example. An example when it is clear what is good and what is not good is economics:

- if it grows, it is usually good (although if it grows too fast, it may be dangerous in the long run), while
- if it shrinks, this is usually not good.

In terms of eigenvalues:

- grows corresponds to $\lambda_j > 1$, while
- a crisis corresponds to $0 < \lambda_j < 1$.

How complex numbers affect the result. Suppose that we start with an ideal good case, when λ_j is a positive real number larger than 1: $\lambda_j = r > 1$. In this case, the corresponding term in the description of economy grows:

$$T(t+k\cdot \Delta t) = r^k \cdot T(t).$$

What happens when we take into account that eigenvalues are, in general, complex numbers? This means that, in general, the eigenvalues have the form

$$\lambda_j = r + m \cdot \mathrm{i},$$

where m is a real number, and $\mathrm{i} \stackrel{\text{def}}{=} \sqrt{-1}$.

When the imaginary part m is small, i.e., when its absolute value much smaller than the real part r, the effect of this part is also small, so the situation continues to be good. We therefore need to start being careful when the imaginary part becomes comparable with the real one. To analyze this case, let us consider the situation when the imaginary part if of exactly the same size as the real one, i.e., when $|m| = r$ and thus, $m = \pm r$. In this case,

$$\lambda_j = r \pm r \cdot \mathrm{i} = r \cdot (1 \pm \mathrm{i}).$$

We start at some moment t_0 (e.g., in Year 0) with some value $T(t_0) = T_0 > 0$. Then, at the next moment of time $t_0 + \Delta t$, we get

$$T(t_0 + \Delta t) = r \cdot (1+\mathrm{i}) \cdot T_0.$$

Since this value contains both real and imaginary parts, it is not clear how it will affect the expected results, whether it will be good or bad. At Moment 2, i.e., when

$t = t_0 + 2\cot\Delta$, we similarly get

$$T(t_0 + 2\cdot\Delta t) = (r\cdot(1\pm i))^2\cdot T_0 = \pm 2\cdot \mathrm{i}\cdot r^2\cdot T_0.$$

Again, it is not clear where it is good or bad. At the third moment of time $t = t_0 + 3\Delta$, we get

$$T(t_0 + 3\cdot\Delta t) = (r\cdot(1\pm i))^3\cdot T_0 = (-2\pm 2\cdot\mathrm{i})\cdot r^3\cdot T_0,$$

still unclear. However, at the 4-th moment of time, we get

$$T(t_0 + 4\cdot\Delta t) = (r\cdot(1\pm i))^4\cdot T_0 = -4\cdot r^4\cdot T_0.$$

This time, there is no confusing imaginary part, the value is purely real – and negative. What does that mean? We started with a positive value $T_0 >$ indicating good.

- If the eigenvalue did not gave any imaginary part, at the 4-th moment of time, we would have a positive value $r^4\cdot T_0$.
- However, because the eigenvalue has an imaginary part, the value $T(t_0 + 4\Delta t)$ is now negative – i.e., bad!

This may explain why 4 is considered a bad number.

What happens after that? At moments 5, 6, and 7 we will have values

$$T(t_0 + k\cdot\Delta t) = r^k\cdot(1+\mathrm{i})^k\cdot T_0$$

with non-zero imaginary parts, so it is not clear whether it will be good or bad. The first moment when the value $T(t_0 + k\cdot\Delta t)$ becomes real again is the moment $k = 8$. At this moment, we have

$$T(t_0 + 8\cdot\Delta t) = 16\cdot r^8\cdot T_0.$$

This value is not only positive (which is good), but it is even much larger (16 times larger) than the value $r^8\cdot T_0$ that we would have gotten if the eigenvalue did not have the imaginary part. No wonder number 8 is considered to be good!

So, indeed, both “badness” of 4 and “goodness” of 8 are indeed explained by complex numbers!

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References

1. R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Addison Wesley, Boston, Massachusetts, 2005.
2. K. S. Thorne and R. D. Blandford, *Modern Classical Physics: Optics, Fluids, Plasmas, Elasticity, Relativity, and Statistical Physics*, Princeton University Press, Princeton, New Jersey, 2021.
3. Wikipedia, Chinese Numerology, https://en.wikipedia.org/wiki/Chinese_numerology, downloaded on April 5, 2023