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Why Gliding Symmetry Used to Be Prevalent in Biology But Practically Disappeared

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Why Gliding Symmetry Used to Be Prevalent in Biology But Practically Disappeared

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Abstract At present, many living creatures have symmetries; in particular, the left-right symmetry is ubiquitous. Interestingly, 600 million years ago, very fee living creatures had the left-right symmetry: most of them had a gliding symmetry, symmetry with respect to shift along a line followed by reflection in this line. This symmetry is really seen in living creatures today. In this paper, we provide a physical-based geometric explanation for this symmetry change: we explain both why gliding symmetry was ubiquitous, and why at present, it is rarely observed, while the left-right symmetry is prevalent.

1 Formulation of the Problem

Empirical fact. Many fossil creatures have unusual shapes and unusual symmetries. For example, during the Ediacaran period (635 to 541 million years ago), most creatures had the so-called *gliding symmetry* (see, e.g., [2, 4]), symmetry with respect to shift along a line followed by reflection in this line, a symmetry that is very rarely seen in biological creatures today. A simplified example of a glide-symmetric shape is given below:

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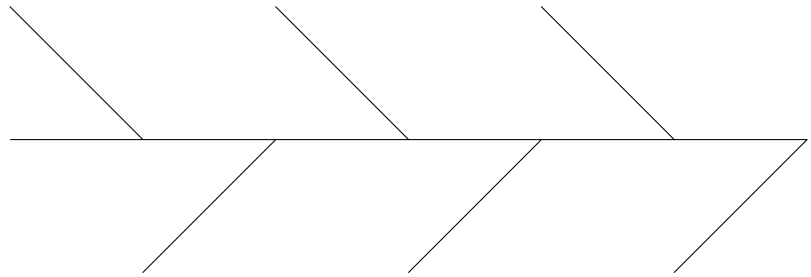


Natural question. Why did this symmetry first become prevalent and then practically disappear?

What we do in this paper. In this paper, we provide a physics-motivated geometric explanation for this phenomenon.

2 Our Explanation

Biological creatures as symmetry breaking. In general, the properties of a planetary environment – in particular, of the Earth environment – do not change if we shift horizontally, to a different location, and/or if we rotate in the horizontal plane. Without life, both ocean and atmosphere are, in general, similarly symmetric: the properties of air or water such as density do not change much if we shift horizontally to a new location or if we perform a rotation in the horizontal plane.

Biological creatures appear in some part of the ocean and not in others and thus, violate this original symmetry. In other words, the appearance of biological creatures means symmetry breaking.

Some symmetries are preserved. According to statistical physics (see, e.g., [1, 3]), when the original state is symmetric with respect to some transformations, it is more probable that the next state still has some of these symmetries– and the more symmetries are preserved, the more probable is the corresponding transition. In line with this argument, most biological creatures have some symmetries. For example, the geometric shape of most current biological creatures – including humans – is invariant with respect to reflection against a central plane: in other words, our left and right sides are very similar.

Evolution of a biological creatures sometimes simply preserves the original symmetry, and sometimes leads to new creatures with fewer symmetries. Again, the most probable transition is to new shapes that retain some of the original symmetries.

Gliding symmetry naturally appears: general idea. Let us first explain that the geometric analysis of the evolution of the biological forms naturally leads to the ubiquity of gliding symmetry.

In the beginning, we have a 6-dimensional symmetry group: the group formed by all 3-D shifts and all 3-D rotations. In addition, we have discrete symmetries: reflections against a point, reflections against a plane, etc.

As we have mentioned, some of the resulting symmetries are broken, and the resulting configuration has fewer symmetries. In the first transition, the most probable – and thus, the most frequent – creatures are the ones with the largest set of remaining symmetries. The resulting shape of a biological creature should be invariant with respect to all these remaining symmetries. Let us show, stage by stage, how the corresponding evolution will work, from the most possible shapes appearing on the first stage to the next most probable shapes appearing on the next stages.

First stage in the evolution of shapes. As we have mentioned, in the beginning, we have a 6-dimensional symmetry group. The only subset of the space that is invariant with respect to all these symmetries is the space itself. So, any biological creature that occupies only a part of the space must have only some of these symmetries – i.e., some original symmetries must be broken. In line with the above general idea, the most probable – and thus, the most frequent – shape corresponds to the case when the largest number of symmetries are preserved. Let us therefore discuss what are possible shapes, and which one is the most symmetric – and thus, more frequent – shape.

The original symmetry group contained a 3-D subgroup of all possible shifts. The list of remaining symmetries cannot contain shifts in all 3 direction: in this case, if we have a perturbation at one spatial point, then, since all other points can be obtained from this one by an appropriate 3-D shift, we should have a perturbation in all the 3-D points – and we are analyzing the case when the biological creature occupies only some space. So, the resulting configuration must be invariant with respect to either 2-D or 1-D subgroup of the group of all shifts – or with respect to no shifts at all.

If the configuration is invariant with respect to a 2-D group of shifts, then with each point, it also contains the whole plane. So, this shape is either a plane or a set consisting of several parallel planes. For this configuration, in addition to shifts, we have other symmetries: rotation in the plane and reflections. So, overall, we have a 3-dimensional group of symmetries, in which in two of the dimensions – that correspond to shifts – the set of corresponding parameters is the whole real line, while in the third dimension – corresponding to rotations – the set of parameters is limited to a circle, i.e., in effect, to an interval on the real line.

If the configuration is invariant with respect to a 1-D group of shifts, then with each point, it contains the whole 1-D line. For a line, the only remaining symmetries are shifts along this line and rotations around this line – i.e., we have a 2-D group of symmetries. In this case, there are fewer symmetries than in the case of 2-D group of shifts, so this case is not the most probable one.

If the configuration is not symmetric with respect to any shifts, then the most symmetric case is when it is symmetric with respect to all remaining symmetries – i.e., symmetric with respect to all rotations. Thus, with each point, the configuration should contain all the points obtained from this one by rotations – i.e., it must contain the whole sphere. The sphere has a 3-D group of symmetries – all rotations. In this group, each of the corresponding parameters is limited to an interval. In this sense, this group is "smaller" that the group corresponding to the plane.

Thus, in the first stage in the evolution of shapes, the most symmetric – and thus, the most probable – shape is the shape of a plane.

Second stage in the evolution of shapes. The plane – the most probable shape on the first stage – has a 3-D group of symmetries: shifts in both planar directions and rotations (plus some discrete symmetries). We are now looking for shapes which are invariant with respect to some subgroup of this group of symmetries.

The original symmetry group contained a 2-D subgroup of all possible shifts in the plane. The list of remaining symmetries cannot contain shifts in both direction: in this case, if we have a perturbation at one spatial point, then, since all other points on a plane can be obtained from this one by an appropriate 2-D shift, we should have a perturbation in all the points on this plane – this is the shape that we already analyzed. So, the resulting configuration must be invariant with respect to either 1-D subgroup of the group of all shifts – or with respect to no shifts at all.

If the configuration is invariant with respect to a 1-D group of shifts, then with each point, it contains the whole 1-D line. For a line, the only remaining symmetries are shifts along this line – i.e., we have a 1-D group of symmetries. In this group, the set of the corresponding parameters is the whole real line.

If the configuration is not symmetric with respect to any shifts, then the most symmetric case is when it is symmetric with respect to all remaining symmetries – i.e., symmetric with respect to all planar rotations. Thus, with each point, the configuration should contain all the points obtained from this one by these rotations – i.e., it must contain the whole circle. The circle has a 1-D group of symmetries – all planar rotations. In this group, each of the corresponding parameters is limited to an interval. In this sense, this group is "smaller" that the group corresponding to the 1-D line.

Thus, in the second stage in the evolution of shapes, the most symmetric – and thus, the most probable – shape is the shape of a straight line. For this shape, in addition to shifts, we also have discrete symmetries: reflections in the line, and reflections in a point on this line.

Third stage in the evolution of shapes. The straight line – the most probable shape on the second stage – has a 1-D group of symmetries: shifts along this line (plus some discrete symmetries). We are now looking for shapes which are invariant with respect to some subgroup of this group of symmetries.

The original group of symmetries was 1-dimensional, so the proper subsets of this group should be discrete. The only proper (closed) subgroup of the group of all shifts is the group of all shifts proportional to some fixed number $h > 0$. Thus, we are looking for shapes which are invariant with respect to all such shifts – plus some discrete transformations. Thus, we get shapes that are invariant with respect to all such shifts – i.e., shapes that are periodic with a period h.

Fourth stage in the evolution of shapes explains the ubiquity of gliding symmetry. Periodic shapes – the most probable shapes on the third stage – have symmetries consisting of shifts proportional to h and discrete symmetries. We are now looking for shapes which are invariant with respect to some subgroup of this group of symmetries.

Any proper subgroup of the group of all shifts proportional to h consists of all the shifts proportional to $k \cdot h$ for some integer $k > 1$. The smaller k, the more symmetries are preserved, so the most probable shape – corresponding to the most frequent shape – corresponds to the case when the value k is the smallest possible, i.e., when $k = 2$. In this case, we may have an additional symmetry – shift by h combined with the reflection in the line. The most symmetric case is when we do have this additional symmetry – and this is exactly the case of gliding symmetry that has been ubiquitous in the ancient creatures. Thus, we indeed have an explanation for the ubiquity of this shape in the ancient times.

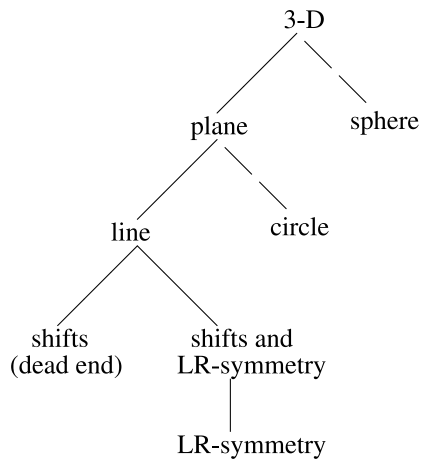
But why did this shape eventually disappear: a symmetry-based explanation. How will the creatures evolve next? Creatures appearing on the fourth stage have a shape that is invariant with respect to shifts proportional to some value $k \cdot h$. The only subgroups of this symmetry group include all shifts proportional to some multiple $k' \cdot (k \cdot h)$ of this number – i.e., in effect, to the same shapes. Thus, if this shape is not perfect, we cannot get a better-fit shape by symmetry breaking.

So what will happen? Remember that so far, on each stage, we only traced creatures corresponding to the most probable symmetry group – i.e., to the symmetry group retaining the largest number of symmetries from the shapes that are prevalent on the previous stage. On each stage, in addition to such prevailing shapes, there are also shapes which fewer symmetries – they are less probable and thus, less frequent, but they are still possible.

In particular, from the shape prevalent on the third stage – which are invariant with respect to all the shifts proportional to h and with respect to reflections in the line – we can also have shapes that are only invariant with respect to reflections – i.e., that have left-right (LR) symmetry. This are indeed the most ubiquitous shapes now.

Comment. In addition to the geometric reason, there is also a biological reason for why left-right-symmetric shapes are better fit than shapes with gliding symmetry: similarly to Adam Smith's argument in economics about the efficiency of specialization – when everyone does what he/she can do best – it is more efficient for a biological creature to have specialized organs. Invariance with respect to shifts would means that for each organ, we have a large number of similar organs in shifted positions, which would be a big waste of resources. In contrast, for left-right symmetry, we have at most two duplicate organs – which makes more sense.

Summary. The following picture summarizes our description of the evolution of shapes. In this picture, a regular line means the most probable transition; some of the less probable transitions are described by dashed lines.



Acknowledgments

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