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How Viscosity of An Asphalt Binder Depends on Temperature: Theoretical Explanation of an Empirical Dependence

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Abstract Pavement must be adequate for all the temperatures, ranging from the winter cold to the summer heat. In particular, this means that for all possible temperatures, the viscosity of the asphalt binder must stay within the desired bounds. To predict how the designed pavement will behave under different temperatures, it is desirable to have a general idea of how viscosity changes with temperature. Pavement engineers have come up with an empirical approximate formula describing this change. However, since this formula is purely empirical, with no theoretical justification, practitioners are often somewhat reluctant to depend on this formula. In this paper, we provide a theoretical explanation for this empirical formula – namely, we should that this formula can be naturally derived from natural invariance requirements.

1 Formulation of the Problem

What is asphalt binder: a brief reminder. In most pavements, the top – asphalt – layer is formed by mixing the asphalt binder (which is usually the residue from petroleum refining) with rocks of appropriate sizes.

It is important to maintain the right viscosity of the asphalt binder. In general, if a stress is applied to a material, it first deforms and then, if the stress increases

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further, it cracks. Some materials are brittle, a reasonably small stress caused then to crack, other materials are more flexible, they undergo a reasonable large amount of deformation before they start cracking.

Materials like asphalt binders are, from the physical viewpoint, highly viscous liquids (such materials are also called *semi-solids*). Their mechanical properties are different from the properties of usual liquids like water because, in comparison to water, the asphalt binders have a very high viscosity. The asphalt binder's reaction to stress is determined by this viscosity, and it affects the quality of the resulting pavement:

- If the viscosity is too low, the pavement is deformed by traffic and this deformation will make the pavement non-smooth and thus, hinder the traffic.
- If the viscosity is too high, i.e., if the pavement is too stiff, then the stress caused by the traffic will cause cracks. As more traffic passes through the pavement segment, the cracks will grow, and the resulting large cracks will also hinder the traffic.

It is therefore important to maintain the proper value of viscosity.

How viscosity changed with temperature. For all liquids – and asphalt binders are not an exception – viscosity η decreases with temperature T. Since a pavement has to function properly at all seasons, during the wide range of outside temperatures, it is important to take this dependence into account when designing the pavement. Pavement engineering uses the following empirical formula [1, 2] to describe this dependence:

$$\ln\left(\ln\left(\frac{\eta}{\eta_0}\right)\right) = a + b \cdot \ln(T) \quad (1)$$

for some constants η_0, a, and b, where the temperature is measured in absolute units (e.g., Kelvins).

Formulation of the problem and what we do in this paper. The formula (1) is empirical. In general, empirical formulas are not as reliable as formulas that have at least some theoretical explanation – these explanations make the use of such formulas outside their original testing example more convincing. It is therefore desirable to come up with a theoretical explanation for this formula. Such an explanation is provided in this paper.

2 Our Explanation

Invariance: main idea behind our explanation. Our explanation is based on the fact that the numerical value of a physical quantity depends on the selection of the measuring unit – and sometimes, e.g., when we measure time, also on the selection of the starting point. If we replace the original measuring unit with a new unit which is λ times smaller, then each numerical value x changes to $\lambda \cdot x$: for example, 2 kg becomes $1000 \cdot 2 = 2000$ g. Similarly, if we replace the original starting point with the new one which is x_0 units lower, then each numerical value changes to $x + x_0$.

In many situations, there is no physically preferred measuring unit, selection of a measuring unit is simply a matter of convenience. In this case, it makes sense to assume that the exact form of the dependence $y = f(x)$ between the two quantities x and y should remain the same if we change the measuring unit for x. Of course, we may also need to appropriately change the measuring unit for y: e.g., the formula $y = x^2$ relating the area y of a square region to the length x of its side does not depend on what unit we use to measure length. However, to preserve this formula, when we change the unit of length from kilometers to miles, we also need to change the unit of area from square kilometers to square miles.

Invariance: precise definitions. So, in the case when we consider scaling of x and scaling of y, the corresponding *scale-scale* invariance of the relation $y = f(x)$ means that for every value $\lambda > 0$ there exists a value $\mu > 0$ – in general, depending on λ – for which $y = f(x)$ implies that $Y = f(X)$, where $X = \lambda \cdot x$ and $Y = \mu \cdot y$. Similarly:

- *shift-scale* invariance of the relation $y = f(x)$ means that for every value x_0 there exists a value $\mu > 0$ – in general, depending on x_0 – for which $y = f(x)$ implies that $Y = f(X)$, where $X = x + x_0$ and $Y = \mu \cdot y$;
- *scale-shift* invariance of the relation $y = f(x)$ means that for every value $\lambda > 0$ there exists a value y_0 – in general, depending on λ – for which $y = f(x)$ implies that $Y = f(X)$, where $X = \lambda \cdot x$ and $Y = y + y_0$; and
- *shift-shift* invariance of the relation $y = f(x)$ means that for every value x_0 there exists a value y_0 – in general, depending on x_0 – for which $y = f(x)$ implies that $Y = f(X)$, where $X = x + x_0$ and $Y = y + y_0$.

Which dependencies are invariant. For each of the above four types of invariance, we have a full description of all the functions which are correspondingly invariant (see, e.g., [3]):

- all scale-scale invariant functions have the form $f(x) = A \cdot x^a$ for some values A and a;
- all shift-scale invariant functions have the form $f(x) = A \cdot \exp(a \cdot x)$ for some values A and a;
- all scale-shift invariant functions have the form $f(x) = A \cdot \ln(x) + a$ for some values A and a; and
- all shift-shift invariant functions have the form $f(x) = A \cdot x + a$ for some values A and a.

Some dependencies are indirect. In many practical situations, while there is a dependence between quantities x and y, this dependence is not direct: the quantity x affects some other quantity z, and that quantity z affects y – and we may have an even longer chain of effects. In the case of the dependencies $x \to z \to y$, it is often reasonable to assume that both dependencies $z = g(x)$ and $y = h(z)$ are invariant. In this cases, the resulting dependence of y on x is a composition of two invariant functions $y = h(g(x))$; see [3] for examples.

What natural invariances we have in our case. In our case, both for viscosity and for temperature (when described ion absolute units), there is a definite starting point: 0 viscosity and absolute zero temperature. However, in both cases, there does not seem to be a preferred measuring unit. It is therefore reasonable to assume that for both quantities, the natural transformation is scaling – corresponding to the selection of a different measuring unit.

The observed dependence is not scale-scale invariant. Based on the natural invariances, one may expect that the dependence of the viscosity on temperature should be described by the scale-scale invariant power law $\eta = A \cdot T^a$. However, the empirical dependence (1) in different. Namely, if we apply the exponential function $\exp(z)$ to both sides of the formula (1), we get

$$\ln\left(\frac{\eta}{\eta_0}\right) = \exp(a + b \cdot \ln(T)) = \exp(a) \cdot \exp(b \cdot \ln(T)) =$$

$$\exp(a) \cdot (\exp(\ln(T))^b = A \cdot T^b, \tag{2}$$

where we denoted $A \stackrel{\text{def}}{=} \exp(a)$. By applying $\exp(z)$ to both sides of the equality (2), we conclude that

$$\frac{\eta}{\eta_0} = \exp(A \cdot T^b), \tag{3}$$

and thus, that

$$\eta = \eta_0 \cdot \exp(A \cdot T^b). \tag{4}$$

This is different from the power law.

In other words, the observed dependence of η on T cannot be explained by the direct invariance. Thus, a natural idea is to see if this empirical dependence can be explained as indirect dependence.

Indirect invariance indeed explains the empirical dependence. In line with the conclusion of the previous subsection, let us assume that η depends on some auxiliary quantity z that, in its turn, depends on T, i.e., that the dependence $\eta = f(T)$ has the form $\eta = h(g(T))$, i.e., the form $\eta = h(z)$ and $z = g(T)$.

We know that for both T and η, natural transformations are scalings. So, depending on what transformations we assume for z in both dependencies, we get the following four possible cases:

1. the dependence of z on $Y = T$ is scale-scale-invariant, and the dependence of η on z is also scale-scale-invariant;
2. the dependence of z on $Y = T$ is scale-shift-invariant, and the dependence of η on z is shift-scale-invariant;
3. the dependence of z on $Y = T$ is scale-scale-invariant, and the dependence of η on z is shift-scale-invariant;
4. the dependence of z on $Y = T$ is scale-shift-invariant, and the dependence of η on z is scale-scale-invariant.

Let us analyze these cases one by one.

1. In the first case, we have $z = A_1 \cdot T^{a_1}$ for some A_1 and a_1, and $\eta = A_2 \cdot z^{a_2}$. Substituting the expression for z into the formula describing the dependence of η on z, we conclude that
$$\eta = A_2 \cdot (A_1 \cdot T^{a_1})^{a_2} = (A_2 \cdot A_1^{a_2}) \cdot T^{a_1 \cdot a_2}, \tag{5}$$
i.e., $\eta = A \cdot T^a$, where we denoted $A \stackrel{\text{def}}{=} A_2 \cdot A_1^{a_2}$ and $a \stackrel{\text{def}}{=} a_1 \cdot a_2$. Thus, we get a power law – and we have already mentioned that the actual dependence is different from the power law.
2. In the second case, we have $z = A_1 \cdot \ln(T) + a_1$ for some A_1 and a_1, and $\eta = A_2 \cdot \exp(a_2 \cdot z)$. Substituting the expression for z into the formula describing the dependence of η on z, we conclude that
$$\eta = A_2 \cdot \exp(a_2 \cdot (A_1 \cdot \ln(T) + a_1)) = A_2 \cdot \exp(a_2 \cdot A_1 \cdot \ln(T)) \cdot \exp(a_2 \cdot a_1) =$$
$$(A_2 \cdot \exp(a_2 \cdot a_1)) \cdot (\exp(\ln(T))^{a_2 \cdot A_1} = (A_2 \cdot \exp(a_2 \cdot a_1)) \cdot T^{a_2 \cdot A_1}, \tag{6}$$
i.e., $\eta = A \cdot T^a$, where we denoted $A \stackrel{\text{def}}{=} A_2 \cdot \exp(a_2 \cdot a_1)$ and $a \stackrel{\text{def}}{=} a_2 \cdot A_1$. Thus, we also get a power law.
3. In the third case, we have $z = A_1 \cdot T^{a_1}$ for some A_1 and a_1, and $\eta = A_2 \cdot \exp(a_2 \cdot z)$. Substituting the expression for z into the formula describing the dependence of η on z, we conclude that
$$\eta = A_2 \cdot \exp(a_2 \cdot A_1 \cdot T^{a_1}) = A_2 \cdot \exp((a_2 \cdot A_1) \cdot T^{a_1}), \tag{7}$$
i.e., we get the formula (4) for $\eta_0 = A_2$, $A = a_2 \cdot A_1$, and $b = a_1$.
4. In the fourth case, we have $z = A_1 \cdot \ln(T) + a_1$ for some A_1 and a_1, and $\eta = A_2 \cdot z^{a_2}$. Substituting the expression for z into the formula describing the dependence of η on z, we conclude that
$$\eta = A_2 \cdot (A_1 \cdot \ln(T) + a_1)^{a_2}. \tag{8}$$
At first glance, this expression does not look like the desired formula (4), but it actually describes, under this formula, the dependence of T on η. Indeed, in this case, taking logarithm of both sides of the formula (3), we get
$$\ln(\eta) - \ln(\eta_0) = A \cdot T^a,$$
hence
$$T^a = A^{-1} \cdot \ln(\eta) - A^{-1} \cdot \ln(\eta_0),$$
and
$$T = (A^{-1} \cdot \ln(\eta) - A^{-1} \cdot \ln(\eta_0))^{1/a}.$$
This formula can be described as
$$T = A_2 \cdot (A_1 \cdot \ln(\eta) + a_1)^{a_2}, \tag{8a}$$

if we take $A_2 = 1$, $A_1 = A^{-1}$, $a_1 = -A^{-1} \cdot \ln(\eta_0)$, and $a_2 = 1/a$.

So, indeed, indirect invariance explains the desired formula – to be more precise, it explains either the dependence of η on T or the dependence of T on η. Which of the two formulas (7) and (8) should be applied to the dependence of η on T should be determined experimentally, just like the numerical values of all the corresponding parameters.

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