11-1-2022

Hunting Habits of Predatory Birds: Theoretical Explanation of an Empirical Formula

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Comments:

Technical Report: UTEP-CS-22-115

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Abstract Predatory birds play an important role in an ecosystem. It is therefore important to study their hunting behavior, in particular, the distribution of their waiting time. A recent empirical study showed that the waiting time is distributed according to the power law. In this paper, we use natural invariance ideas to come up with a theoretical explanation for this empirical dependence.

1 Formulation of the Problem

It is important to study hunting habits of predatory birds. Predatory birds are an important part of an ecosystem. Like all predators, they help maintain the healthy balance in nature.

This balance is very delicate, unintended human interference can disrupt it. To avoid such disruption, it is important to study the hunting behavior of predatory birds.

A recent discovery. The hunting behavior of most predatory birds is cyclic. Most predatory birds like owls spend some time waiting for the prey, and then either attack or jump to a new location.

For the same bird, waiting time w changes randomly from one cycle to another. Researchers recently found how the probability $f(t) \stackrel{\text{def}}{=} \text{Prob}(w \geq t)$ that the waiting time is $\geq t$ depends on t:

$$f(t) \approx A \cdot t^{-a};$$

see [2].

Problem. How can we explain this empirical observation?

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What we do in this paper. In this paper, we provide a theoretical explanation for this empirical formula.

2 Our Explanation

We need a family of functions.

- Some birds from the same species tend to wait longer.
- Other birds tend to wait less.

So:

- We cannot have a *single* formula that would cover all the birds of the same species.
- We need a *family* of functions $f(t)$.

What is the simplest family. The simplest family if when:

- we fix some function $F(t)$, and
- consider all possible functions of the type $C \cdot F(t)$.

Resulting question. What family should we choose?

Invariance: idea. To find the appropriate family, let us take into account that the numerical value of waiting time depends on the selection of the measuring unit.

In precise terms: if we replace the original measuring unit with the one which is λ times smaller, then all numerical values multiply by λ: $t \mapsto \lambda \cdot t$.

It looks like there is no preferable measuring unit. So, it makes sense to assume that the family $\{C \cdot F(t)\}_C$ should remain the same if we change the measuring unit.

Invariance: precise formulation. In other words, after re-scaling, we should get the exact same family of functions, i.e., the families $\{C \cdot F(\lambda \cdot t)\}_C$ and $\{C \cdot F(t)\}_C$ should coincide, i.e., consist of exactly the same functions.

This invariance requirement leads to the desired theoretical explanation for the empirical formula. The fact that the family remains the same implies, in particular, that for every $\lambda > 0$, the function $F(\lambda \cdot t)$ should belong to the same family. Thus, for every $\lambda > 0$, there exists a constant C – depending on λ – for which $F(\lambda \cdot t) = C(\lambda) \cdot F(t)$.

It is known (see, e.g., [1], see also Appendix A) that every measurable solution to this functional equation $F(\lambda \cdot t) = C(\lambda) \cdot F(t)$ has the form $F(t) = A \cdot t^a$.

This is exactly the empirical probability distribution – it is only one which does not depend on the selection of the measuring unit for time. So, we indeed have the desired theoretical explanation.

3 Acknowledgments

This work was supported in part by the National Science Foundation grants:

- 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and
- HRD-1834620 and HRD-2034030 (CAHSI Includes).

It was also supported by the AT&T Fellowship in Information Technology, and by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478.

The authors are thankful to the participants of the 2022 UTEP/NMSU Workshop on Mathematics, Computer Science, and Computational Science (El Paso, Texas, November 5, 2022) for valuable discussions.

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4 How to Prove the Result about the Functional Equation

The above result about a functional equation is easy to prove when the function $F(t)$ is differentiable. Indeed, suppose that

$$F(\lambda \cdot t) = C(\lambda) \cdot F(t).$$

If we differentiate both sides with respect to λ, we get

$$t \cdot F'(\lambda \cdot t) = C'(\lambda) \cdot F(t).$$

In particular, for $\lambda = 1$, we get

$$t \cdot F'(t) = a \cdot F(t),$$

where we denoted $a \stackrel{\text{def}}{=} C'(1)$. So, we get

$$t \cdot \frac{dF}{dt} = a \cdot F.$$

We can separate the variables if we multiply both sides by dt and divide both sides by t and by F; then we get:

$$\frac{dF}{F} = a \cdot \frac{dt}{t}.$$

Integrating both sides of this equality, we get

$$\ln(F) = a \cdot \ln(t) + C.$$

By applying $\exp(x)$ to both sides, we get

$$F(t) = \exp(a \cdot \ln(t) + C) = A \cdot t^a,$$

where we denoted $A \stackrel{\text{def}}{=} e^C$.