11-1-2022

Dielectric Barrier Discharge (DBD) Thrusters -- Aerospace Engines of the Future: Invariance-Based Analysis

Alexis Lupo
The University of Texas at El Paso, alupo@miners.utep.edu

Vladik Kreinovich
The University of Texas at El Paso, vladik@utep.edu

Comments:

Technical Report: UTEP-CS-22-114

Dielectric Barrier Discharge (DBD) Thrusters – Aerospace Engines of the Future: Invariance-Based Analysis

Alexis Lupo and Vladik Kreinovich

Abstract One of the most prospective aerospace engines is a Dielectric Barrier Discharge (DBD) thruster – an effective electric engine without moving parts. The efficiency of this engine depends on the proper selection of the corresponding electric field. To make this selection, we need to know, in particular, how its thrust depends on the atmospheric pressure. At present, for this dependence, we only know an approximate semi-empirical formula. In this paper, we use natural invariance requirements to come up with a theoretical explanation for this empirical dependence, and to propose a more general family of models that can lead to more accurate description of the DBD thruster's behavior.

1 Formulation of the Poblem

1.1 How to Fly on Mars: Dielectric Barrier Discharge (DBD) Thrusters

Need to fly on other planets. A large amount of information about Earth comes from air-based observations. It is therefore desirable to have similar studies of planets with atmosphere.

Related challenge. One of the problems is that on Earth, flying devices use fuel-based engines. On other planets, we do not have ready sources of fuel, and bringing fuel from Earth is too expensive.

Alexis Lupo
Department of Physics, University of Texas at El Paso, 500 W. University
El Paso, Texas 79968, USA, e-mail: alupo@miners.utep.edu

Vladik Kreinovich
Department of Computer Science, University of Texas at El Paso, 500 W. University
El Paso, Texas 79968, USA, e-mail: vladik@utep.edu

The main source of energy for planetary missions is electricity. Electricity can be generated by solar batteries and/or by radioactive energy sources. It is therefore desirable to use electricity to power the flying devices.

How to design electricity-powered flying machines. A natural idea is to use electrostatic forces between two electrodes.

When the voltage is high, an electric arc appears – as in a lightning. The arc means that atmospheric atoms are ionized, into negatively charges electrons and positively changed ions;

- ions move towards the negative electrode, while
- electrons move towards the positive electrode.

Since the ions move towards the negative electrode, the density near the negative electrode becomes smaller than nearby. Thus, the atmospheric gases are sucked into this area. These gases also become ionized, so they also move towards the negative electrode.

The mass of ions is much larger than the mass of electrons, so the ion flow produces momentum and thus, thrust. This is the main idea behind what is called *Dielectric Barrier Discharge (DBD)* Thrusters; see, e.g., [1, 2, 5].

1.2 DBD Thrusters Are Useful on Earth Too

The DBD thrusters are useful on Earth too.

- They do not have moving parts, so they are durable and reliable.
- They do not burn fuel, so they do not pollute the environment.
- They have a higher efficiency.

Let us explain why they have higher efficiency. In fuel-using flying devices, energy is wasted on two stages:

- when fuel is burning – a large part of energy goes into useless heat, and
- when turbines are used – a part of energy is spent on friction.

In contrast, in an electric device, there are only one stage, so fewer energy is wasted.

1.3 What Electric Field E Should We Select

For a given design the given value E of the electric field, the efficiency of a thruster changes with atmospheric pressure p:

- When the pressure is too low, we do not have enough ions to generate thrust.
- On the other hand, when the pressure is too high, the air resistance becomes too strong: the atmosphere is very dense, moving through it becomes practically impossible.

For each value E of the electric field, there is an optimal pressure at which the thruster is the most efficient. So, for each value of the atmospheric pressure p, we should select this optimal E. In particular, since the atmospheric pressure decreases with height, so we should thus have E changing with height.

To find the optimal value E of the electric field, we need to know how the thrust F depends on the pressure p.

1.4 Remaining problems

As we have mentioned, to find the optimal value E of the electric field, we need to know how the thrust F depends on the pressure p. At present, we only have an approximate semi-empirical formula $F(p) = c \cdot p \cdot \exp(a \cdot p)$ based on a simplified model; see, e.g., [6]. It is therefore desirable to provide a theoretical explanation for this formula.

Another issue is that this formula provides a rather crude approximation to the data. It is desirable to come up with more accurate formulas.

These are the two problem with which we deal in this paper.

2 Analysis of the Problem

We should select a family of functions $F(p)$. For different designs, for different values of the electric field E, we have, in general, different dependencies $F(p)$. So, we cannot have a *single* function $F(p)$, we should select a *family* of functions.

A natural way to represent a family of functions. A natural way to describe a family of functions is:

- to select "basic" functions $e_1(p), \ldots, e_n(p)$, and
- to consider all possible linear combinations of these basic functions, i.e., functions of the type
$$C_1 \cdot e_1(p) + \ldots + C_n \cdot e_n(p).$$

Example. For example, when $e_1(p) = 1$, $e_2(p) = p$, $e_3(p) = p^2$, we get a family of quadratic polynomials.

Resulting question. Which family should we select? In other words, which basic functions should we select?

Shift-invariance. An important feature of pressure is that its effects, in some sense, do not depend on the selection of the starting point. For example, when there are no cars on the road:

- we say that the pressure on the pavement is 0,
- while in reality, there is always a strong atmospheric pressure.

We can select different starting points, e.g., we can take 0 as vacuum or 0 as atmospheric pressure.

If we replace the original starting point with the one which is p_0 units smaller, all numerical values are shifted: $p \mapsto p + p_0$. The selection of a starting point is rather arbitrary. So, it makes sense to select an approximating family that does not change under such shifts.

What we can conclude based on shift-invariance. Shift-invariance implies, in particular, that for each function $e_i(p)$, its shift $e_i(p + p_0)$ belongs to the same family. This means that for some coefficients $C_{ij}(p_0)$, we get:

$$e_i(p + p_0) = C_{i1}(p_0) \cdot e_1(p) + \ldots + C_{in}(p_0) \cdot e_n(p).$$

If we differentiate both sides with respect to p_0, we get

$$e_i'(p + p_0) = C_{i1}'(p_0) \cdot e_1(p) + \ldots + C_{in}'(p_0) \cdot e_n(p).$$

In particular, for $p_0 = 0$, we get

$$e_i'(p) = c_{i1} \cdot e_1(p) + \ldots + c_{in} \cdot e_n(p),$$

where we denoted $c_{ij} \stackrel{\text{def}}{=} C_{ij}'(0)$.

So, we get a system of linear differential equations with constant coefficients. It is known (see, e.g., [3, 4]) that all solutions to such systems are linear combinations of functions $t^m \cdot \exp(k \cdot t)$, where:

- k is an eigenvalue of the matrix c_{ij}, and
- m is a non-negative integer which is smaller than k's multiplicity.

What are the simplest cases. The simplest case if when $n = 1$. In this case, we get functions $F(p) = C_1 \cdot \exp(k \cdot p)$. However, this does not satisfy the condition $F(0) = 0$ – meaning that when there is no atmosphere, there will be no thrust.

So, to satisfy this condition, we need to take $n \geq 2$. The simplest of such cases is $n = 2$. In this case, we have three possible options:

- we may have a single eigenvalue of multiplicity 2;
- we may have two complex-valued eigenvalues; and
- we may have two different real-valued eigenvalues.

Let us consider these three options one by one.

First option explains the semi-empirical formula. if we have an eigenvalue of multiplicity 2, we get

$$F(p) = C_1 \cdot \exp(k \cdot p) + C_2 \cdot p \cdot \exp(k \cdot p).$$

The condition $F(0) = 0$ implies that $C_1 = 0$, so we have exactly the semi-empirical formula from [6].

Second option is not applicable. When eigenvalues are complex-valued $k = a \pm b \cdot \mathrm{i}$, then

$$F(p) = \exp(a \cdot p) \cdot (C_1 \cdot \cos(b \cdot p) + C_1 \cdot \sin(b \cdot p)).$$

This expression is not applicable to our case; indeed:

- this expression is negative for some p, while
- the thrust force is always non-negative.

Third option explains how to get more accurate formulas. When eigenvalues $k_1 < k_2$ are real and different, we get

$$F(p) = C_1 \cdot \exp(k_1 \cdot p) + C_2 \cdot \exp(k_2 \cdot p).$$

The condition $F(0) = 0$ implies $C_2 = -C_1$, so

$$F(p) = C_1 \cdot (\exp(k_1 \cdot p) - \exp(k_2 \cdot p)).$$

In the limit $k_2 \to k_1$, we get the expression $F(p) = c \cdot p \cdot \exp(k \cdot p)$. In general, we get a 3-parametric family that may lead to a better description of experimental data.

Comment. If this will be not accurate enough, we can use shift-invariant families with $n = 3, 4, \ldots$

3 Acknowledgments

This work was supported in part by the National Science Foundation grants:

- 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and
- HRD-1834620 and HRD-2034030 (CAHSI Includes).

It was also supported by the AT&T Fellowship in Information Technology, and by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478.

The authors are thankful to the participants of the 2022 UTEP/NMSU Workshop on Mathematics, Computer Science, and Computational Science (El Paso, Texas, November 5, 2022) for valuable discussions.

References

1. T. C. Corke, M. L. Post and D. M. Orlov, “SDBD plasma enhanced aerodynamics: concepts, optimization and applications", *Progress in Aerospace Sciences*, 2007, Vol. 43, Nos. 7–8, pp. 193–217.

2. A. D. Greig, C. H. Birzer, and M. Arjomandi, “Atmospheric plasma thruster: theory and concept", *AIAA Journal*, 2013, Vol. 51, No. 2, pp. 362–371.
3. J. C. Robinson, *An Introduction to Ordinary Differential Equations*, Cambridge University Press, Cambridge, UK, 2004.
4. M. Tenenbaum and H. Pollard, *Ordinary Differential Equations*, Dover Publ., New York, 2022.
5. F. Thomas, T. Corke, M. Iqbal, A. Kozlov, and D. Schatzman, “Optimization of dielectric barrier discharge plasma actuators for active aerodynamic flow control", *AIAA Journal*, 2009, Vol. 47, pp. 2169–2178.
6. Z. Wu, J. Xu, P. Chen., K. Xie, and N. Wang, “Maximum thrust of single dielectric barrier dishcarge thruster at low pressure”, *AIAA Journal*, 2018, Vol. 56, No. 6, pp. 2235–2241.