11-1-2022

# Resource Allocation for Multi-Tasking Optimization: Explanation of an Empirical Formula

Alan Gamez
*The University of Texas at El Paso*, agamez8@miners.utep.edu

Antonio Aguirre
*The University of Texas at El Paso*, aaaguirre10@miners.utep.edu

Christian Cordova
*The University of Texas at El Paso*, cjcordova@miners.utep.edu

Alberto Miranda
*The University of Texas at El Paso*, aamiranda4@miners.utep.edu

Vladik Kreinovich
*The University of Texas at El Paso*, vladik@utep.edu



Comments:

Technical Report: UTEP-CS-22-113

# Resource Allocation for Multi-Tasking Optimization: Explanation of an Empirical Formula

Alan Gamez, Antonio Aguirre, Christian Cordova, Alberto Miranda, and Vladik Kreinovich

**Abstract** For multi-tasking optimization problems, it has been empirically shown that the most effective resource allocation is attained when we assume that the gain of each task logarithmically depends on the computation time allocated to this task. In this paper, we provide a theoretical explanation for this empirical fact.

## 1 Formulation of the Problem

**Formulation of the practical problem.** In many practical situations, we have several possible objective functions. For example, when we design a plant:

- we can look for the cheapest design,
- we can look for the most durable design,
- we can look for the most environmentally friendly design, etc.

To make a decision, it is desirable to find designs which are optimal with respect to each pf these criteria.

- This way, if we find, e.g., that the most environmentally friendly design is close to the cheapest one, we can add a little more money and make the design environmentally friendly.
- On the other hand, if these two designs are too far away, we can apply for an environment-related grant to make the design environment friendly.

Optimizing different functions on the same domain involves several common domain-specific computational modules. Thus, it makes sense to perform all these optimization tasks on the same computer. In this case, the important question is

---

Alan Gamez, Antonio Aguirre, Christian Cordova, Alberto Miranda, and Vladik Kreinovich
Department of Computer Science, University of Texas at El Paso, 500 W. University
El Paso, Texas 79968, USA, e-mail: agamez8@miners.utep.edu, aaaguirre10@miners.utep.edu, cjcordova@miners.utep.edu, aamiranda4@miners.utep.edu, vladik@utep.edu

how to distribute resources between different tasks: e.g., when one task is close to completion:

- this task does not require many resources,
- while other tasks may require a lot of resources.

**Let us formulate this problem in precise terms.** At each time period, we need to distribute, e.g., the available computation time $\Delta T$ between tasks, i.e., find value $\Delta t_k$ for which

$$\Delta t_1 + \Delta t_2 + \ldots = \Delta T.$$

We should select the values $\Delta t_k$ for which the overall gain is the largest

$$\sum_k g_k(t_k + \Delta t_k) \to \max,$$

where:

- $t_k$ is the time already spent on task $t$, and
- $g_k(t)$ indicates how much the $k$-th task will gain if it is allowed computation time $t$.

**Related challenge and how to overcome it.** We do not know exactly how each gain $g_k$ will change with time. So, a natural idea is:

- to select a family of functions $g(t)$ depending on a few parameters;
- for each task, to find parameters that lead to the best fit;
- then use these parameters to predict the value $g_k(t_k + \Delta t_k)$.

**Empirical fact.** An empirical fact is that among 2-parametric families, the best results are achieved for $g(t) = a \cdot \ln(t) + A$; see, e.g., [2].

**Remaining problem.** How can we explain this empirical fact?

**What we do in this paper.** In this paper, we provide a theoretical explanation for this empirical fact.

## 2 Our Explanation

**Invariance: the main idea behind our explanation.** The numerical value of each physical quantity depends:

- on the selection of the measuring unit, and
- on the selection of the starting point.

Specifically:

- If we replace the original measuring unit with the one which is $\lambda$ times smaller, all numerical values multiply by $\lambda$: $x \mapsto \lambda \cdot x$.

- If we select a new starting point which is $x_0$ units smaller, then we get $x \mapsto x + x_0$.

In many cases, there is no preferable measuring unit. In this case, it makes sense to assume that the formulas should remain the same if we change the measuring unit.

**Let us apply this idea to our case.** In our case:

- there is a clear starting point for time: the moment when computations started;
- however, there is no preferred measuring unit.

Thus, it is reasonable to require that if we change $t$ to $\lambda \cdot t$, we will get the same resource allocation.

**Clarification and the resulting requirement.** Invariance does not necessarily mean that $g(\lambda \cdot t) = g(t)$, since functions $g_k(t)$ and $g_k(t) + \text{const}$ lead to the same resource allocation. Thus, we require that for every $\lambda > 0$, we have $g(\lambda \cdot t) = g(t) + c$ for some constant $c$ depending on $\lambda$.

**This requirement explains the above empirical fact.** It is known (see, e.g., [1]) that every measurable solution to this functional equation has the form $g(t) = a \cdot \ln(t) + A$. This explains the empirical fact: dependencies $g(t) = a \cdot \ln(t) + A$ are the only ones that do not depend on the choice of a measuring unit for time.

## 3 Acknowledgments

This work was supported in part by the National Science Foundation grants:

- 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and
- HRD-1834620 and HRD-2034030 (CAHSI Includes).

It was also supported by the AT&T Fellowship in Information Technology, and by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478.

The authors are thankful to the participants of the 2022 UTEP/NMSU Workshop on Mathematics, Computer Science, and Computational Science (El Paso, Texas, November 5, 2022) for valuable discussions.

## References

1. J. Aczél and J. Dhombres, *Functional Equations in Several Variables*, Cambridge University Press, 2008.
2. T. Wei and J> Zhong, "Towards generalized resource allocation on evolutionary multitasking for multi-objective optimization", *IEEE Computational Intelligence Magazine*, November 2021, pp. 20–36.

## 4 How to Prove the Result about the Functional Equation

This result is easy to prove when the function $f(X)$ is differentiable. Indeed, Suppose that

$$g(\lambda \cdot X) = g(X) + c(\lambda).$$

If we differentiate both sides with respect to $\lambda$, we get

$$X \cdot g'(\lambda \cdot X) = c'(\lambda).$$

In particular, for $\lambda = 1$, we get

$$X \cdot g'(X) = a, \text{ where } a \stackrel{\text{def}}{=} c'(1).$$

So:

$$X \cdot \frac{dg}{dX} = a.$$

We can separate the variables if we multiply both sides by $dX$ and divide both sides by $X$, then we get

$$dg = a \cdot \frac{dX}{X}.$$

Integrating both sides of this equality, we get the desired formula

$$g(X) = a \cdot \ln(X) + C.$$