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How Hot Is Too Hot

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Abstract A recent study has shown that the temperature threshold – after which even young healthy individuals start feeling the effect of heat on their productivity – is $30.5° \pm 1°$. In this paper, we use decision theory ideas to provide a theoretical explanation for this empirical finding.

1 Introduction

Formulation of the problem. Humans can tolerate heat, but when the weather becomes too hot, our productivity decreases. Not only our productivity decreases: continuous exposure to high temperature stresses the organism and can lead to illness and even to death.

This is specially true for older people or for people who are not feeling well, but heat affects young healthy people as well. An important question is: what is the threshold temperature T_0 after which even young healthy people will be affected?

Comment. Of course, our perception of heat depends not only on the temperature, it also depends on humidity. Because of this, to describe human perception of heat, researchers use a characteristic known as "wet-bulb temperature": the temperature measured by a regular thermometer while the bulb is covered by a water-soaked cloth.

Traditional estimate for this threshold. Until recently, it was believed that the corresponding threshold is $T_0 = 35°$, i.e., that:

- temperatures below 35°C do not affect the productivity of young healthy individuals, while

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- higher temperatures read to a loss of productivity.

This threshold was based on the original research [10].

Recent research result. A recent study [11] used more accurate measurements of the effect of heat on humans. These more accurate measurements resulted in a conclusion that wet-bulb temperatures below 35° also affect people. Specifically, it was shown that the actual threshold above which even young healthy individuals decrease their productivity is $T_0 = 30.5^\circ \pm 1^\circ$.

Resulting problem. How can we explain this empirical threshold?

What we do in this paper. In this paper, we provide a possible explanation for this empirical finding.

2 Our Explanation

Preliminary analysis of the problem. According to the average people's opinion, the most comfortable temperature is about 24 degrees; see, e.g., [3]. This is the typical temperature of the countries and regions known as tropical paradises, where the temperature stays close to this most comfortable level all year long. Clearly, at this level, there should be no bad effect on humans.

On the other extreme, when the outside temperature reaches the average normal human body temperature of 36.6°, this would clearly make us uncomfortable, since in this case, excess hear generated by our bodies cannot dissipate in the surrounding air – as it can in lower temperatures.

So:

- the temperature of 24° is clearly below the threshold, while
- the temperature of 36.6° is clearly above the threshold.

Thus, the threshold T_0 is somewhere between these two temperatures, i.e., somewhere on the interval $[24, 36.6]$.

Let us use Laplace Indeterminacy Principle. We do not know which temperature in this interval corresponds to the desired threshold. Such situations of uncertainty are common in real life. In such situations, if we have several alternative hypotheses and we have no reason to believe that some of them are more probable than others, a natural idea is to assume that all these hypotheses are equally probable, i.e., that each of n hypotheses has the same probability of

$$\frac{1}{n}.$$

This natural idea was first formulated by Laplace, one of the pioneers of probability theory, and is thus known as the *Laplace Indeterminacy Principle*; see, e.g., [4].

In our case, possible hypotheses correspond to possible values from the interval $[\underline{T}, \overline{T}] = [24, 36.6]$. We have no reason to believe that some values from this interval are more probable than others. Thus, it is reasonable to assume that all these values are equally probable, i.e., that we have a uniform distribution on this interval.

Based on this probability distribution, what is the most reasonable estimate? We need to select a single value from this interval. Ideally, this value should be close to all the values from this interval. In practice, we only measure temperature with some accuracy ε – and we can feel temperature only up to some accuracy. This means that we cannot distinguish two temperatures – neither by measurement not by its effect on a human body – if the difference between them is smaller than ε. For example, all the values between $\underline{T}$ and $\underline{T}+\varepsilon$ are indistinguishable from each other. So, in effect, what we possibly have are the following values

$$T_1 = \underline{T}, T_2 = \underline{T}+\varepsilon, T_3 = \underline{T}+2\varepsilon, \ldots, T_k = \overline{T}.$$

We want to make sure that the selected value T_0 is close to all these values, i.e., that we have

$$T_0 \approx \underline{T}, T_0 \approx \underline{T}+\varepsilon, T_0 \approx \underline{T}+2\varepsilon, \ldots, T_0 \approx \overline{T}.$$

In other words, we want to make sure that the vector

$$(T_0, T_0, T_0, \ldots, T_0)$$

formed by the left-hand sides is close to the vector

$$\left(\underline{T}, \underline{T}+\varepsilon, \underline{T}+2\varepsilon, \ldots, \overline{T}\right)$$

formed by the right-hand sides.

The distance between the two vectors $a = (a_1, \ldots, a_k)$ and $b = (b_1, \ldots, b_k)$ is naturally represented by the Euclidean formula

$$d(a,b) = \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2 + \ldots + (a_k-b_k)^2}.$$

In particular, in our case, the desired distance d between the two vector takes the form

$$d = \sqrt{D},$$

where we denoted

$$D \stackrel{\text{def}}{=} \left(T_0 - \underline{T}\right)^2 + \left(T_0 - (\underline{T}+\varepsilon)\right)^2 + \left(T_0 - (\underline{T}+2\varepsilon)\right)^2 + \ldots + \left(T_0 - \overline{T}\right)^2.$$

One can see that this expression D is related to the general expression for the integral sum

$$\int_a^b f(x)\,dx \approx f(x_1)\cdot\Delta x + f(x_2)\cdot\Delta x + f(x_3)\cdot\Delta x + \ldots + f(x_k)\cdot\Delta x,$$

where

$$x_1 = a, x_2 = a+\Delta x, x_3 = a+2\Delta x, \ldots, x_k = b.$$

This expression is very accurate for small Δx.

Specifically, the expression D is very similar to the expression for the integral sum for the integral

$$\int_{\underline{T}}^{\overline{T}} (T_0 - T)^2 \, dT$$

corresponding to the values

$$T_1 = \underline{T}, T_2 = \underline{T} + \varepsilon, T_3 = \underline{T} + 2\varepsilon, \ldots, T_k = \overline{T}$$

that has the following form:

$$\int_{\underline{T}}^{\overline{T}} (T_0 - T)^2 \, dT \approx$$

$$\left(T_0 - \underline{T}\right)^2 \cdot \varepsilon + \left(T_0 - (\underline{T} + \varepsilon)\right)^2 \cdot \varepsilon + \left(T_0 - (\underline{T} + 2\varepsilon)\right)^2 \cdot \varepsilon + \ldots + \left(T_0 - \overline{T}\right)^2 \cdot \varepsilon.$$

All the terms in the right-hand side have a common factor ε. By separating this common factor, we get

$$\int_{\underline{T}}^{\overline{T}} (T_0 - T)^2 \, dT \approx$$

$$\varepsilon \cdot \left(\left(T_0 - \underline{T}\right)^2 + \left(T_0 - (\underline{T} + \varepsilon)\right)^2 + \left(T_0 - (\underline{T} + 2\varepsilon)\right)^2 + \ldots + \left(T_0 - \overline{T}\right)^2\right).$$

The sum in the right-hand side of this formula is exactly our expression D, so

$$\int_{\underline{T}}^{\overline{T}} (T_0 - T)^2 \, dT \approx \varepsilon \cdot D,$$

and thus

$$D \approx \frac{1}{\varepsilon} \cdot \int_{\underline{T}}^{\overline{T}} (T_0 - T)^2 \, dT.$$

So, minimizing the distance $d = \sqrt{D}$ means minimizing the expression

$$\sqrt{D} \approx \sqrt{\frac{1}{\varepsilon} \cdot \int_{\underline{T}}^{\overline{T}} (T_0 - T)^2 \, dT}.$$

One can check that this expression attains its smallest value if and only if the integral

$$\int_{\underline{T}}^{\overline{T}} (T_0 - T)^2 \, dT$$

attains its smallest value.

Differentiating this integral with respect to the unknown T_0 and equating the derivative to 0, we conclude that

$$\int_{\underline{T}}^{\overline{T}} 2 \cdot (T_0 - T)\, dT = 0.$$

Dividing both sides by 2 and taking into account that the integral of the difference is equal to the difference of integrals, we get

$$\int_{\underline{T}}^{\overline{T}} T_0\, dT - \int_{\underline{T}}^{\overline{T}} T\, dT = 0.$$

The first integral in this expression is the integral of a constant, so it is equal to $T_0 \cdot (\overline{T} - \underline{T})$. The second integral is

$$\int T\, dT = \frac{1}{2} \cdot T^2,$$

so the second integral is equal to

$$\int_{\underline{T}}^{\overline{T}} T\, dT = \frac{1}{2} \cdot T^2 \Big|_{\underline{T}}^{\overline{T}} = \frac{1}{2} \cdot \left(\left(\overline{T}\right)^2 - \left(\underline{T}\right)^2 \right).$$

Thus, the resulting equation takes the form

$$(\overline{T} - \underline{T}) \cdot T_0 - \frac{1}{2} \cdot \left(\left(\overline{T}\right)^2 - \left(\underline{T}\right)^2 \right) = 0,$$

hence

$$T_0 = \frac{\frac{1}{2} \cdot \left(\left(\overline{T}\right)^2 - \left(\underline{T}\right)^2 \right)}{\overline{T} - \underline{T}} = \frac{\underline{T} + \overline{T}}{2}.$$

This conclusion is in perfect accordance with the recommendations of the general decision theory (see, e.g., [1, 2, 5, 6, 7, 8, 9]) according to which a rational decision maker should gauge the quality of each alternative by the mean value of the corresponding utility. In our case, we have a uniform distribution on the interval $[\underline{T}, \overline{T}]$, and it is known that the mean value of the corresponding random variable is equal to the midpoint

$$\frac{\underline{T} + \overline{T}}{2}$$

of this interval.

In our case, $\underline{T} = 24$ and $\overline{T} = 36.6$, so we get

$$T_0 = \frac{24 + 36.6}{2} = \frac{60.6}{2} = 30.3.$$

This number is in perfect accordance with the empirical value 30.5 ± 1. Thus, we have indeed explained this empirical value.

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