9-1-2022

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Comments:
Technical Report: UTEP-CS-22-95a
To appear in *Proceedings of the 11th IEEE International Conference on Intelligent Systems IS'22*, Warsaw, Poland, October 12-14, 2022.

Recommended Citation

Bokati, Laxman and Kreinovich, Vladik, "Why Smaller-Size Objects Affect the Flow Much More than Larger Ones: A Geometric Explanation with Applications Ranging from Volcanoes and Tornadoes to Blood, Fish, and Building Preservation" (2022). *Departmental Technical Reports (CS)*. 1752.
https://scholarworks.utep.edu/cs_techrep/1752

Why Smaller-Size Objects Affect the Flow Much More than Larger Ones: A Geometric Explanation with Applications Ranging from Volcanoes and Tornadoes to Blood, Fish, and Buildings Preservation

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Abstract—**At first glance, the larger the object, the larger should be its effect on the surroundings – in particular, the larger should be its effect on the surrounding flow. However, in many practical situations, we observe the opposite effect: smaller-size particles affect the flow much more than larger-size particles. This seemingly counterintuitive phenomena has been observed in many situations: lava flow in the volcanoes, air circulation in tornadoes, blood flow in a body, the effect of fish on water circulation in the ocean, and the effect of added particles on seeping water that damages historic buildings. In this paper, we show that all these phenomena can be explained in natural geometric terms.**

Index Terms—**invariance, volcanoes, tornado, blood flow, fish, historic buildings**

I. INTRODUCTION

Formulation of the problem. What is the effect of objects of different size on a liquid or gaseous flow?

- At first glance, the larger the object, the more this object will affect the flow.
- However, in many different situations, empirical data shows the opposite effect: smaller-size objects have a much larger effect on the flow that larger-size ones.

This phenomenon has been observed in real-life situations, ranging from:

- the flow of lava in volcanoes and
- the flow or air in a tornado

to:

- the flow of blood in a body,
- the fish-affected flow of water in the ocean, and
- the destructive flow of seeped water in the old buildings.

The very fact that this phenomenon occurs in different physical situations seems to indicate that there should be a general explanation for all such effects, an explanation that does not depend on the specific physics. How can we find such an explanation?

What we do in this paper. In this paper, we provide such a general explanation. Our explanation is based only on simple geometry and thus, does not depend on the underlying physics.

Comment. The general explanation provided in this paper expands a geometric explanation provided in [13] for a specific case of tornadoes.

Structure of the paper. In Section 2, we describe examples of such a phenomenon. In Section 3, we describe our geometric explanation of this phenomenon.

II. EXAMPLES WHEN SMALLER-SIZE OBJECTS HAVE LARGER EFFECT THAN LARGER-SIZE ONES

Volcanoes. In volcanoes, hot lava – mixed with gases and rocks of different sizes – comes to the surface.

- In some volcanoes, lava come out as a continuous stream.
- In other volcanoes, lava accumulates and then gets released in a violent eruption.

Violent eruptions often have catastrophic consequences. It is therefore important to be able to predict whether a volcano will erupt or whether it will release its lava in a more "peaceful" way – as a continuous flow. Based on the general laws of hydrodynamics, researchers are able, in most cases, to predict whether an abrupt eruption is possible (see, e.g., [8]) –

This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and HRD-1834620 and HRD-2034030 (CAHSI Includes), and by the AT&T Fellowship in Information Technology. It was also supported by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478, and by a grant from the Hungarian National Research, Development and Innovation Office (NRDI).

978-1-6654-5656-2/22/$31.00 ©2022 IEEE

although in cases when eruption is possible, the current models can only predict the very fact of the future eruption, not its exact time.

These models assume, for simplicity, that the lava flow only contain the liquid lava and hot gases, they ignore the presence of rocks. In many cases, this simplification is well justified: irrespective of how many rocks the actual lava flows contains, these models usually provide good predictions. However, there have been quite a few puzzling cases, when:

- the model predicted a smooth continuous lava flow, but
- the volcano actually violently erupted;

see, e.g., [2], [3], [9], [15], [23].

A partial explanation of this puzzle came from a recent research [4] that showed that in all the cases of unexpected explosions, lava had a significant portion of nano-size particles, while in the cases of a smooth continuous lava flow, the overall amount of nano-size particles was much smaller. But this explanation leads to another puzzle: why do smaller-size particles affect the properties of the flow while larger-size rocks do not have such an effect?

Tornadoes. Tornadoes – rotating fast air flows – cause a lot of destruction. It is difficult to predict them, so maybe we can stop them – or at least slow them down and thus, make them less destructive – when they have already formed?

At first glance, a natural idea is to inject dust into a tornado: this way, we increase the rotating mass, and thus, since the angular momentum remains constant, slow down the rotation.

Surprisingly, in most cases, the corresponding experiments did not work: the tornado did not slow down; see, e.g., [11], [16], [17], [22]. The only cases when the tornado did slow down somewhat was when the injected dust was fine-grained, consisting of micro-size particles; see, e.g., [18]–[20].

This result is puzzling: why do smaller-size particles affect the flow while larger-size particles do not have the same effect?

Blood. Our bodies depend on blood, blood circulates oxygen and nutrients to different parts of the body. Sometimes, blood becomes too viscous to circulate properly, and this causes serious problems. This happens, e.g., during many cases of so-called "long Covid", complications after a Covid infection.

Interestingly, in such cases, what helps is *thrombocytapheresis* – partial removal of micro-size cells (called platelets) from the patients's blood; see, e.g., [1], [6]. Here, we face the same puzzling question: why do smaller-size particles significantly affect the blood flow while the effect of larger-size particles is much smaller?

Fish. Most biological creatures depend on oxygen. Thus, for deep-water creatures to survive, it is important to have a circulation of water between different depths, so that oxygen consumed by these creatures will be replaced by oxygen coming from the surface.

Similarly, the main source of energy is – eventually – the Sun. The sunlight practically does not penetrate deep water, so it is important that circulation between different depths bring nutrients to the deeper layers.

In the ocean, there is a natural mixing of waters, but this mixing is too slow to explain the richness of deep-water life. In addition to a natural mixing, there is also mixing caused by sea creatures many of which move in all directions (including up and down), and thus, help the water circulate. At first glance, it seems like the larger the creature, the larger effect it has on water circulation. So, to check on this effect, researchers tried to measure circulation caused by large animals like whales. Surprisingly, they found out that the resulting mixing effect is too small to explain how oxygen and nutrients get into the depths.

Because of the negative results of these large-creature measurements, researchers naturally assumed that the mixing effect caused by smaller creatures will be even smaller – even if we take into consideration that fish may congregate in big shoals, so the researchers did not even bother to try to measure this effect. When they finally measured it, they got the second surprise:

- that the mixing caused by small-size sea creatures is much larger than the mixing caused by larger creatures, and
- that this small-size-creature-induced mixing can actually explain the current levels of oxygen and nutrient in the deep sea;

see, e.g., [7].

Thus, here is another case of the same puzzling question: why do smaller-size sea creatures have a large effect on water circulation that larger-size ones?

Old buildings. With time, old buildings eventually deteriorate. One of the reasons for this is that water seeps into the walls, into the foundation, and causes damage. To preserve old buildings, to slow down their deterioration, it is important to prevent water from flowing.

Many techniques have been invented for this purpose. One of the main problems with these techniques is that for historic buildings – buildings that we want to disrupt as little as possible – the existing techniques are too invasive.

Interestingly, a recent research shows that inserting nano-particles not only slows down the deterioration – it works even better than previous proposed techniques which were based on inserting larger-size water barriers [5].

Again, the puzzling question is: why smaller-size particles have a larger effect on water flow than larger-size ones?

These are the questions that we explain. In this paper, as promised, we provide a general geometric explanation for all these puzzling phenomena.

III. OUR EXPLANATION

The ideal state of a flow: a geometric description. According to statistical physics, all the processes, if undisturbed, eventually reach their most stable state – the state characterized by the maximal entropy; see, e.g., [10], [12].

Equations describing liquids and gases are invariant with respect to:

- rotations,
- shifts, and

- scalings – i.e., dilations $\vec{x} \mapsto \lambda \cdot \vec{x}$.

Thus, the most stable state of liquids and gases should also be invariant with respect to all these transformations.

Invariance with respect to shifts means that the liquid or gas should be in the homogeneous state: all the characteristics of the liquid should be the same (at least locally) in all neighboring points. Invariance with respect to rotations imply that in the stable state, liquid or gas should be stationary: otherwise, if they move, we get a preferred direction – the direction in which the liquid or gas moves – and thus, this state is not invariant with respect to rotations.

How perturbations change the state. While eventually, the object will reach its final state, this process can take a long time. For example, the Universe itself is already billions of years in the making, and it has not yet (luckily for us) reached its final stable state. According to statistical physics, this change can be sped up by appropriate perturbations – just like a small particle helps the overheated liquid to start boiling.

If the perturbation has some invariances (= symmetries), its effect on the surrounding liquid also has the same invariances and thus, brings the surrounding flow closer to the state with these invariances. From this viewpoint, the more of the desired symmetries the perturbation has, the closer it brings the surrounding flow to the desired fully invariant state. From this angle, let us compare the effect of smaller-size and larger-size particles.

Comparing symmetries of smaller-size and larger-size particles leads to the desired explanation. By definition, a small-size particle is a particle whose size can be safely ignored, i.e., in effect, what physicists call a point-wise particle. A point is invariant:

- with respect to rotations around it, and
- with respect to all scalings relative to this point.

On the other hand, a larger-size body cannot be scale-invariant. Indeed, invariance with respect to a transformation means that:

- if we have a point inside the body and apply the transformation to this point,
- then we also get a point from this same body.

However, by re-scaling points in a small vicinity of the body's central point, we can get all points in the 3D space – thus, including points which are so far away from the central point that they are no longer part of the body. At best, a body can be rotation-invariant: if its shape is close to a sphere.

So, in all cases, a smaller-size particle has more desired symmetries that a larger-size one. Thus, smaller-size particles bring the state of the surrounding flow closer to the stable state (i.e., homogeneous and stationary state) than larger-size ones.

Why this is not a paradox. This larger effect of smaller-size particles would be a paradox if we were taking about injecting energy into the flow. Of course, a smaller-size particle has much fewer energy than a larger-size one, so it cannot inject more energy into the flow than the larger-size particle.

However, in our case, we are not talking about injecting energy. In all the above examples, the flow already has energy, and in examples like volcanoes or tornadoes, a huge amount of energy. What we are talking about is *not* processes that require injection of energy. Vice versa, we talk about processes like slowing down the flow, i.e., processes that decrease the flow's energy. For this purpose, we do not need to inject energy, we just need to trigger – and thus speed up – the process that, according to the Second Law of Thermodynamics, will eventually happen anyway.

Similarly, to make an overheated liquid start boiling, we do not need to inject any energy into it: all we need is a triggering particle, and this particle can be as small as possible.

How this explains all the above-described empirically observed effects. The general idea is that adding smaller-size particles slows down the flow and makes it more homogeneous.

- For tornadoes, this means that the destructive airflow slows down, and thus, becomes less destructive.
- For volcanoes, it means that nano-particles prevent the lava from flowing. Thus, instead of flowing continuously, lava stays under the volcano. As more and more lava flows from lower levels into the same space, the pressure increases and eventually, an explosion happens.

Interestingly, in these two cases, the same physical effects leads, from the human viewpoint, to opposite consequences:

- the presence of smaller-size particles make tornadoes less dangerous, but
- a similar presence of smaller-size particles makes volcanoes much more dangerous.

Similarly, we can explain all other phenomena described in the previous section:

- For blood flows, the presence of smaller-size particles slows down the blood flow – so we need to remove some of these particles to restore the healthy blood flow speed.
- For fish in the ocean, smaller-size sea creatures bring the state of the sea closure to uniform, thus, to the state in which the amounts of oxygen and nutrients are (approximately) the same at all depth – which is exactly what mixing is about.
- Finally, for historic buildings, smaller-size particles prevent water from seeping – which is exactly what is needed to slow down the building's deterioration.

ACKNOWLEDGMENTS

The authors are greatly thankful to Janusz Kacprzyk for his encouragement and to the anonymous referees for valuable suggestions.

REFERENCES

[1] P. Boddu, L. Falchi, C. Hosing, K. Newberry, P. Bose, and S. Verstovsek, "The role of thrombocytapheresis in the tontemporary management of hyperthrombocytosis in myeloproliferative neoplasms: A case-based review", *Leukemia Research*, 2017, Vol. 58, pp. 14–22.

[2] S. Campagnola, A. Vona, C. Romano, and G. Giordano, "Crystallization kinetics and rheology of leucite-bearing tephriphonolite magmas from the Colli Albani volcano (Italy)", *Chemical Geology*, 2016, Vol. 424, pp. 12–29.

[3] M. Coltelli, P. Del Carlo, and L. Vezzoli, "Discovery of a Plinian basaltic eruption of Roman age at Etna volcano", *Geology*, 1998, Vol. 26, No. 12, pp. 1095–1098.

[4] D. Di Genova, R. A. Brooker, H. M. Mader, J. W. Drewitt, A. Longo, J. Deubener, D. R. Neuville, S. Fanara, O. Shebanova, S. Anzellini, and F. Arzilli, "In situ observation of nanolite growth in volcanic melt: A driving force for explosive eruptions", *Science Advances*, 2020, Vol. 6,No. 39.

[5] J. Dziadkowiec, H.-W. Cheng, M. Ludwig, M. Ban, T. P. Tausendpfund, R. von Klitzing, M. Mezger, and M. Valtiner, "Cohesion gain induced by nanosilica consolidants for monumental stone restoration", *Langmuir*, 2022, Vol. 38, No. 22, Paper 6949.

[6] A. M. Elhassan, A. Alsaud, M. A. Yassin, M. Aldapt, L. Riaz, F. Ghori, A. Bin Ahmad, and M. Abdulla, "Thrombocytapheresis in patient with essential thrombocythemia: a case report", *Case Reports in Oncology*, 2020, Vol. 13, No. 2, pp. 675–679.

[7] B. Fernández Castro, M. Peña, E. Nogueira, M. Gilcoto, E. Broullón, A. Comesaña, D. Bouffard, A. C. Naveira Garabato, and B. Mouriño-Carballido, "Intense upper ocean mixing due to large aggregations of spawning fish", *Nature Geoscience*, 2022, Vol. 15, pp. 287–292.

[8] H. M. Gonnermann and M. Manga, "The fluid mechanics inside a volcano", *Annual Review of Fluid Mechanics*, 2007, Vol. 39, pp. 321–356.

[9] B. F. Houghton, C. J. N. Wilson, P. Del Carlo, M. Coltelli, J. E. Sable, and R. Carey, "The influence of conduit processes on changes in style of basaltic Plinian eruptions: Tarawera 1886 and Etna 122 BC", *Journal of Volcanology and Geothermal Research*, 2004, Vol. 137, No. 1-3 pp. 1–14.

[10] K. Huang, *Introduction to Statistical Physics*, CRC Press, Boca Raton, Florida, 2001.

[11] F. P. Kazakevich and A. M. Krapivin, "Investigations of heat transfer and aerodynamical resistance in tube assemblies when the flow of gas is dustladen", *Izvestiya Vysshikh Uchebnykh Zavedenii Energetika*, 1958, No. 1, pp. 101–107 (in Russian).

[12] L. D. Landau and E. M. Lifschitz, *Statistical Physics*, Elsevier Butterworth-Heinemann, Oxford, UK, 2005.

[13] O. Lerma, O. Kosheleva, and V. Kreinovich, "Why injecting fine dust into a tornado is more promising than injecting coarse dust: a geometric explanation", *Geombinatorics*, 2016, Vol. 25, No. 3, pp. 118–122.

[14] M. Pompilio, A. Bertagnini, P. Del Carlo, and A. Di Roberto, "Magma dynamics within a basaltic conduit revealed by textural and compositional features of erupted ash: The December 2015 Mt. Etna paroxysms", *Scientific Reports*, 2017, Vol. 7.

[15] J. E. Sable, B. F. Houghton, P. Del Carlo, and M. Coltelli, "Changing conditions of magma ascent and fragmentation during the Etna 122 BC basaltic Plinian eruption: Evidence from clast microtextures", *Journal of Volcanology and Geothermal Research*, 2006, Vol. 158, No. 3-4, pp. 333–354.

[16] P. G. Saffman, "On the stability of laminar flow of a dusty gas", *Journal of Fluid Mechanics*, 1962, Vol. 13, pp. 120–128.

[17] P. G. Saffman, *Vortex Dynamics*. Cambridge University Press, Cambridge, UK, 1995.

[18] P. A. Shmitt, *Numerical and Analytical Analyses of a Tornado Model*, Master's Thesis, Department of Mathematical Sciences, Texas Tech University, Lubbock, Texas, 1999.

[19] V. I Shubov, "Control of Vortex Flow and Tornado Models", *Abstracts of the 1999 SIAM Annual Meeting*, Atlanta, Georgia, May 12–15, 1999, Section MS65.

[20] V. I. Shubov and D. S. Gilliam, *Mathematical Analysis of Tornado Dynamics*, Technical Report, Department of Mathematical Sciences, Texas Tech University, Lubbock, Texas, 1999.

[21] S. C. Simmons, J. Schwartz, and H. P. Pham, "Therapeutic thrombocytapheresis", In: B. H. Shaz, C. D. Hillyer, and M. Reyes Gil (eds.), *Transfusion Medicine and Hemostasis*, Elsevier, Amsterdam, Netherlands, 2019, pp. 481–482.

[22] W. T. Sproull, "Viscosity of dusty gases", *Nature*, 1961, Vol. 190, pp. 976–978.

[23] G. P. L. Walker, S. Self, and L. Wilson, "Tarawera 1886, New Zealand — A basaltic plinian fissure eruption", *Journal of Volcanology and Geothermal Research*, 1984, Vol. 21, No. 1-2 pp. 61–78.