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Why Would Anyone Invest in a High-Risk Low-Profit Enterprise?

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Abstract Strangely enough, investors invest in high-risk low-profit enterprises as well. At first glance, this seems to contradict common sense and financial basics. However, we show that such investments make perfect sense as long as the related risks are independent from the risks of other investments. Moreover, we show that an optimal investment portfolio should allocate some investment to this enterprise.

1 Formulation of the Problem

Puzzle. Once in a while, in our city, we encounter a struggling enterprise – e.g., a restaurant or a store – whose profit level is clearly low, and risk level is high.

Interestingly, not only this enterprise exists, but it often even manages to get companies and people investing some money in it.

Why would anyone want to invest in a high-risk low-profit enterprise? This seems to contradict all financial basics – and common sense.

What we do in this paper. In this paper, we provide an explanation for this puzzle. Specifically, we show that:

- while, of course, it does not make any sense for an investor to invest *all* his/her money into this enterprise,
- it makes perfect sense to invest *some* of the money into it – as long as its risks are independent of the risks of other enterprises.

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2 Analysis of the Problem and the Resulting Explanation

Optimal portfolio selection: a brief reminder. In general, there are different financial instruments in which we can invest, ranging:

- from low-risk low-profit financial instruments like bonds
- to high-risk high-profit instruments like risky stocks.

A smart investor distributed his/her money between different instruments, so as to maximize the expected profit under the condition that the risk remains tolerable.

Risk means that the actual profit may different from its expected value. In statistics, the difference between the actual value of a random variable and its expected value is characterized by its variance V. Thus, variance is a natural measure of investment risk.

From this viewpoint, each investor has a tolerance level V_0, so that for the selected portfolio, the variance V should not exceed this level:

$$V \le V_0. \tag{1}$$

Under this constraint (1), we need to select a portfolio that provides the largest possible expected gain.

Each financial instrument i can be characterized by its expected gain μ_i and its variance V_i. The investor distributes his/her money between these instruments, so that each instrument gets a portion $w_i \ge 0$ of the overall invested amount, with

$$\sum_{i=1}^{n} w_i = 1. \tag{2}$$

The expected gain μ of a portfolio is, therefore, the sum of gains of each investments, i.e.,

$$\mu = \sum_{i=1}^{n} w_i \cdot \mu_i. \tag{3}$$

In the simplified case when all risks are independent, the overall risk V can be obtained as

$$V = \sum_{i=1}^{n} w_i^2 \cdot V_i. \tag{4}$$

A similar — but slightly more complex – formula describes the overall variance in situations when there is correlation between different risks.

In these terms, the problem is to maximize the expected gain (3) under the condition that the variance (4) satisfies the inequality (1).

This problem was first formulated in the 1950s by Harry Markowitz, who came up with an explicit solution to this optimization problem – for which he got a Nobel Prize in Economics in 1990; see, e.g., [1, 2, 3, 4, **?**, 6].

Let us use this portfolio optimization problem to solve our puzzle. In portfolio terms, what we want to show is that in the optimal portfolio, the portion w_p allocated to the puzzling low-profit high-risk enterprise is positive.

To show this, we will show that:

- if the portfolio does not include this weird investment,
- then we can get a better portfolio by investing some money into this investment.

This would mean that a portfolio that excludes the given enterprise cannot be optimal, i.e., that the optimal portfolio *must* include the given enterprise.

Indeed, let us assume that we have an optimal portfolio that does not include the given puzzling enterprise p. Let $\mu_p > 0$ be the given enterprise's expected gain and let V_p be this enterprise's variance. An optimal portfolio usually includes an almost-sure investment a for which the gain is μ_a is relatively small – in particular, smaller than μ_p – but the risk V_a is also very small (and reasonably independent from all other risks). Let w_a be the portion of the overall investment that goes into this instrument.

Let us show that for a sufficiently small $\varepsilon > 0$, if we re-allocate a portion ε from investment a to our investment p, we will get a better portfolio – i.e.:

- the expected gain will increase while
- the variance will decrease.

Indeed, after the reallocation, in the expected gain (3), the original term $w_a \cdot \mu_a$ in the sum (3) will be replaced by the sum of the

$$(w_a - \varepsilon) \cdot \mu_a + \varepsilon \cdot \mu_p. \quad (5)$$

The difference between the new sum and the old term – and, thus, between the new and the old values of the expected gain – is equal to

$$(w_a - \varepsilon) \cdot \mu_a + \varepsilon \cdot \mu_p - w_a \cdot \mu_a = \varepsilon \cdot (\mu_p - \mu_a). \quad (6)$$

Since $\mu_p > \mu_a$, this difference is always positive. So, for all $\varepsilon > 0$, reallocating the investment indeed increases the expected gain.

Let us now analyze what happens to the portfolio's variance under such a reallocation. In this case, the original term $w_a^2 \cdot V_a$ in the sum (4) is replaced by the sum

$$(w_a - \varepsilon)^2 \cdot V_a + \varepsilon^2 \cdot V_p. \quad (7)$$

The difference between the new sum and the old term – and, thus, between the new and the old values of the variance – is equal to

$$(w_a - \varepsilon)^2 \cdot V_a + \varepsilon^2 \cdot V_p - w_a^2 \cdot V_a = -2\varepsilon \cdot V_a + \varepsilon^2 \cdot V_a + \varepsilon^2 \cdot V_p. \quad (8)$$

For small ε, the main term in this expression is the linear term. Thus, the difference is negative and reallocating the investment indeed decreases the portfolio's variance.

Conclusion. We have shown that in the optimal portfolio, some portion of the investment should be allocated to the given enterprise.

In other words, contrary to our intuition, investing some money in a low-profit high-risk enterprise makes perfect sense – it is even *required* by the desired to have an optimal portfolio.

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