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Why Five Stages of Solar Activity, Why Five Stages of Grief, Why Seven Plus Minus Two: A General Geometric Explanation

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Abstract A recent paper showed that the solar activity cycle has five clear stages, and that taking theses stages into account helps to make accurate predictions of future solar activity. Similar 5-stage models have been effective in many other application area, e.g., in psychology, where a 5-stage model provides an effective description of grief. In this paper, we provide a general geometric explanations of why 5-stage models are often effective. This result also explains other empirical facts, e.g., the seven plus minus two law in psychology and the fact that only five space-time dimensions have found direct physical meaning.

1 Introduction

Empirical fact. A recent study of solar activity [6] has shown that the solar activity cycle can be divided into five clearly different stages, and that explicitly taking these stages into account leads to a much more effective technique for predicting future solar activity. Of course, we can further subdivide each of these five stages into sub-stages, but the gist of dynamics is already well-captured by this 5-stage description. Taking into account that five stages naturally appear in many other dynamical situations – e.g., in the well-known five-stages-of-grief model [5] – a natural conclusion is that there may be a general explanation of why five-stage models are effective.

What we do in this paper. In this paper, we provide a general geometric explanation for the effectiveness of 5-stage dynamical models.

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2 Our Explanation

What does the 5-stage description mean. On each stage, a different type of activity is prevalent. So, at each stage, a different quantity characterizes the system's behavior.

As an example, let is consider the dynamics of a simple pendulum. In its stationary state, the pendulum is positioned at its lowest point, and its velocity is 0. When the pendulum starts moving, each cycle of its motion consists of two stages following one another:

- One of them is the stage where the pendulum is close to its lowest point. On this stage, the height of the pendulum's position is close to the stationary value. What is drastically different from the stationary state is the speed.
- Another is the stage where the pendulum is close to one of its highest points. On this stage, the velocity is close to the stationary one. What is drastically different from the stationary state is the height of the pendulum's position.

So, each of the two stages is indeed characterized by its own characteristics quantity:

- velocity at the first stage, and
- height at the second stage.

When we have s stages, this means that, to provide a reasonable description of this system's dynamics, we need to trace to trace the values of s different quantities, each of which corresponds to one of these stages. In other words, to get a reasonable description of the system's dynamics, we can view this system as a system in an s-dimensional space.

In particular, the fact that the system exhibits 5 stages means that this system's dynamics can be reasonably well described by modeling this system's behavior in 5-dimensional space.

How many stages? We can use spaces of different dimension and we get different approximate descriptions of the given system. Which of these descriptions is the most informative, the most adequate?

To fully describe a complex system, we need to know the values of a large number of variables. Thus, a complex system can be in a large number of different states. When we describe the complex system as a system in an s-dimensional space, for some small s, we thus limit the number of possible states, and therefore, make the description approximate.

The fewer states we use, the cruder the approximation. Let us illustrate this natural idea on a simple example of approximating real numbers from the interval $[0, 1]$ by a finite set of points.

- If we only allow one approximating point, then the most accurate description we can achieve is if we select the point 0.5. In this case, the worst-case approximation error is attained when we are trying to approximate the borderline values 0 and 1 – this approximation error is equal to

$$|0 - 0.5| = |1 - 0.5| = 0.5.$$

- If we allow 2 points, then we can use points 0.25 and 0.75, in which case the worst-case approximation error is twice smaller – it is equal to 0.25.
- In general, if we allow n points, then we should select points

$$\frac{1}{2n}, \ \frac{3}{2n}, \ \frac{5}{2n}, \ \ldots, \ \frac{2n-1}{2n}.$$

In this case, the worst-case approximation error is equal to

$$\frac{1}{2n}.$$

The more points we use to approximate, the more accurate our approximation.

In our case, we approximate original states by states in an s-dimensional space. In physics, the values of all the quantities are bounded. For example, in general:

- velocity is limited by the speed of light,
- distance is limited by the size of the Universe, etc.

For practical applications, there are even stricter bounds.

By appropriately re-scaling each quantity, we can make sure that each bound is 1. After this re-scaling, instead of considering the whole space, we only need to consider a unit ball in this space.

The number of possible states is proportional to the volume of this unit ball – to be more precise, it is equal to the ratio between the volume of the unit ball and the volume of a small cell in which states are indistinguishable.

Thus, to get the most accurate description of the original system, we need to select the dimension s for which the unit ball has the largest possible volume. What is this dimension?

At first glance, it may look like the larger the dimension, the larger the volume. Indeed:

- The 1-dimensional volume (i.e., length) of the 1-dimensional unit ball – i.e., of the interval $[-1, 1]$ – is 2.
- The 2-dimensional volume (i.e., area) of the 2-dimensional unit ball – i.e., of the unit disk – is $\pi > 1$.
- The 3-dimensional volume of the 3-dimensional unit ball is

$$\frac{4}{3} \cdot \pi > \pi,$$

etc.

However, this impression is false. It turns out that the increase continues only up to the $s = 5$ dimensions, after which the volume starts decreasing – and tends to 0 as the dimension increases; see, e.g., [11]. So, the largest volume of the unit ball is attained when the dimension is equal to 5.

In view of the above, this means that 5-dimensional approximations – corresponding to 5-stage descriptions – indeed provide the most adequate first-approximation description of a dynamical system. This explains why 5-stage descriptions are effective in area ranging from solar activity to grief.

3 Other Applications of This Conclusion

Five-dimensional and higher-dimensional models of space-time. Originally, physicists believed assumed that there is a 3-dimensional space and there is 1-dimensional time. Special relativity showed that it is convenient to combine them into a single 4-dimensional space-time. This way, for example, it became clear that electric and magnetic fields are actually one single electromagnetic field; see, e.g., [2, 13].

A natural idea is to try to see if adding additional dimensions will enable us to combine other fields as well. This idea immediately lead to a success – it turns out (see, e.g., [3, 4, 7, 12]) that if we formally write down the equation of General Relativity Theory in a 5-dimensional space, then we automatically get both the usual equations of gravitational field *and* Maxwell's equations of electrodynamics.

Since then, many other attempts have been made to add even more dimensions – but so far, these attempts, while mathematically and physically interesting, did not lead to a natural integration of any additional fields; see, e.g., [13].

From this viewpoint, dimension 5 seems to be a natural way to describe physical fields – in perfect accordance with the above result.

Seven plus minus two law. It is known that when we classify objects, we divide them into five to nine categories; see, e.g., [9, 10]. In particular, when we divide a process into stage, we divide it into five to nine stages:

- some people tend of divide everything into fix stages;
- some people tend to divide everything into six stages;
- . . . , and
- some people tend to divide everything into nine stages.

The need to have at least five stages – and not four – again can be explained by the fact that, as we have shown, 5-stage representations provide the most adequate description of complex systems.

But why not ten? The above result explains why people use models that have at least 5 stages, i.e., it explains the lower bound 5 on the number of stages. But how can be explain the upper bound? Why at most 9 stages? Why cannot we divide the process into 10 or more stages?

For this question, we only have partial answers which are not as clear as the above explanation of why 5 stages. Actually, we have the following three answers explaining that starting with dimension 10 (corresponding to 10 stages) situation becomes completely different.

- The paper [1] shows that while we can feasibly analyze dynamical systems in dimensions up to 9, analysis of 10-dimensional dynamical systems is, in general, not feasible.
- Another specific feature of dimension 10 is that this is the smallest dimension in which we can have a consistent quantum field theory – which is not possible in all dimensions up to 9 [13]. This shows that 10-dimensional space are drastically different from lower-dimensional ones.
- Finally, when we consider cooperative situations with 10 agents – i.e., when the space of possible actions of all the players it 10-dimensional – there are situations in which no stable solution (also known as von Neumann-Morgenstern solution) is possible [8], while no such situations were found for smaller dimensions (i.e., for smaller number of agents).

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