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# Towards Better Ways to Compute the Overall Grade for a Class

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# Towards Better Ways to Compute the Overall Grade for a Class

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**Abstract** Traditional way to compute the overall grade for the class is to use the weighted sum of the grades for all the assignments and exams, including the final exam. In terms of encouraging students to study hard throughout the semester, this grading scheme is better than an alternative scheme, in which all that matters is the grade on the final exam: in contrast to this alternative scheme, in the weighted-sum approach, students are penalized if they did not do well in the beginning of the semester. In practice, however, instructors sometimes deviate from the weighted-sum scheme: indeed, if the weighted sum is below the passing threshold, but a student shows good knowledge on the comprehensive final exam, it makes no sense to fail the student and make him/her waste time re-learning what this student already learned. So, in this case, instructors usually raise the weighted sum grade to the passing level and pass the student. This sounds reasonable, but this idea has a limitation similar to the limitation of the alternative scheme: namely, it does not encourage those students who were initially low-performing to do as well as possible on the final exam. Indeed, within this idea, a small increase in the student's grade on the final exam will not change the overall grade for the class. In this paper, we provide a natural idea of how we can overcome this limitation.

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## 1 How the overall grade is computed: the usual description

**How the overall grade is computed now.** Usually in the US, an overall grade is computed as a weighted linear combination of grades for all the class assignments, tests, quizzes, a comprehensive final exam, etc.

The number of points one can get for each assignment is limited. As a result, the overall grade is limited by some number $M$ – usually, 100 – corresponding to the situation when the students get a perfect grade for each assignment.

There is usually a passing threshold $P$, so that:

- if the student's overall grade $G$ for the class is $P$ or larger, the student passes the class;
- however, if the student's overall grade $G$ for the class is smaller than $P$, the student fails the class; in this case, the student usually has a chance to re-take this class next semester.

Usually, $P = 70$, which means that C is the passing grade; however, for some classes, the passing grade is $P = 60$ (equivalent to D).

**Alternative grading systems and their limitations.** In Russia (where two of us are from) and in many other countries, the overall grade for the class is just the grade $E$ for the final exam. The student still needs to maintain some minimum number of points on all the previous assignments, since without this minimum he/she will not be allowed to take the final exam. However, even if the student did poorly on all the previous assignment, when this student's grade on the final exam is perfect, this perfect grade will be the overall student's grade for this class.

The main limitation of this scheme is that, while ideally, a students should study hard the whole semester, in this scheme, many students study not so hard during the semester and then cram the material in the last few weeks – not the best arrangement, often leading to imperfect knowledge. In contrast, the US scheme penalizes students who do not study hard in the beginning, and thus, motivates them to study hard during the whole semester.

**What we do in this paper.** In this paper, we show that the actual grading by US instructors is somewhat different from the usually assumed weighted-sum scheme. Specifically, in Section 2, we explicitly describe how the actual grading is done. In Section 3, we show that, in some cases, this actual approach faces limitations similar to the limitations of the alternative grading scheme. Finally, in Section 4, we explain a natural way to modify the actual grading scheme that will allow us to overcome these limitations.

*Comment.* The results of this paper were first announced in [1].

## 2 How the overall grade is actually computed

**The problem with the usual weighted-sum approach.** The above weighted-combination scheme is what most instructors place in their syllabi and claim to follow. However, in reality, what they do is more complicated, and it is more complicated for the following very simple reason. If the final exam results show that the student has mastered the material but the weighted average leads to failing, do we really want to fail this student? If this student has mastered the material already, what is the purpose of forcing him/her to retake this class?

**So how is the overall grade actually computed?** Yes, we want to penalize the student for not working hard in the beginning of the semester, but we do not want the student to fail. As a result, if the grade on the final exam is larger than or equal to the passing grade, then, even if the weighted sum leads to failing, we still pass the student – i.e., in effect, increase the student's overall grade for this class to $P$.

**Let us describe the actual computation in precise terms.** Let us describe this scheme in precise terms. The weighted-sum overall grade $G$ is a weighted combination of all the student's grades for this class, including this student's grade $E$ on the final exam: $G = G_- + w_E \cdot E$, where $G_-$ denotes a weighted combination of grades of all previous assignments, and $w_E$ is the weight of the final exam. In these terms, the actual overall grade $A$ has the following form:

- if $G < P$ and $E \geq P$, then $A = P$;
- in all other cases, $A = G$.

## 3 Limitations of the actually used scheme

**Limitation: a general description.** Suppose that we have $G_- + w_E \cdot E < P$ and $E \geq P$, then, according to this scheme, we take $A = P$. Since $G_- + w_E \cdot E < P$, if we increase $E$ by a sufficiently small amount $\varepsilon > 0$, then for the resulting value $E' = E + \varepsilon$, we will still have $G_- + w_E \cdot E' < P$. Also, since $E' \geq E$ and $E \geq P$, we will still have $E' \geq P$. Thus, according to the above-described scheme, the resulting overall student's grade for this class will be the same.

So, the student may get the grade $E$ for the class, the same student may get a larger grade $E' > E$ – in both cases, the overall grade for the class remains the same $A = P$. Thus, the student does not have any incentive to study better: he will get the same grade $P$:

- if he/she barely knows the material by the time of final exam, and
- if he/she knows much more than the required minimum.

This lack of motivation is exactly the same limitation that explains the advantages of the US weighted-sum scheme in comparison the laternative scheme, where the grade on the final exam is all that matters.

**Limitation: an example.** Let us consider a typical situation when the final exam has weight 0.35. In this case, if the student did not submit any assignments before attending the final exam, this student will get $G_- = 0$ points for all these assignments. So, according to the weighted-sum formula, the overall student's grade for this class should be $G = G_- + 0.35 \cdot E = 0.35 \cdot E$.

So, even if the student gets the perfect grade $E = 100$ on the final exam, we will have $G = 0.35 \cdot 100 = 35 < 70$. Thus, if we literally follow the weighted-sum scheme, we should fail this student.

In practice, very few instructors would fail such a student. We would definitely penalize him/her, we will not give him/her 100 points overall for the class, but we will not fail this students. In this case, according to the above description of the actual grading, we will assign $A = 70$.

However, if another student also gets 0s on all previous assignments, but gets 80 – above the passing grade of 70 – on the final exam – we also, according to the above formulas, assign the grade $A = 70$. So, whether the student gets $E = 80$ or $E = 100$ on the final exam, this will not change the student's overall grade – so there is no incentive for the student to study hard for the final exam.

## 4 How can we overcome this limitation

**The actually used grading scheme: reminder.** To analyze how we can overcome the above limitation, let us summarize the actually used grading scheme. In this scheme, if both the overall grade $G$ and the grade $E$ on the final exam are smaller than the passing grade $P$, the student fails the class anyway.

The only case when a student passes the class is when:

- either this student's weighted-sum overall grade $G$ is larger than or equal to the passing threshold $P$
- or this student's grade on the final exam is greater than or equal to $P$.

In this case:

- if $G \geq P$, we assign the grade $A = G$ to this student for this class;
- otherwise, if $G < P$ and $E \geq P$, we assign the grade $A = P$.

These two cases can be summarized into a single formula – applicable when either $G \geq P$ or $E \geq P$:

$$A = \max(G, P). \qquad (1).$$

**Analysis of the problem.** As we have mentioned, the limitation of the actually used formula (1) is that the value (1) remains the same – equal to $P$ – when we increase $G < P$ to a larger value $G' < P$, so a student has no incentive to study better for the final exam.

The smallest possible value $G$ that a student can get when $E \geq P$ is when $G_- = 0$, then $G = w_E \cdot P$. To give the student an incentive to study better, a reasonable idea is to give, to this student, a few extra points proportional to every single point beyond that, i.e., proportional to the difference $G - w_E \cdot P$. In other words, we consider, for the cases when $G \le P$, the dependence of the type

$$A = \max(G,P) + \alpha_0 \cdot (G - w_E \cdot P), \tag{2}$$

for some small value $\alpha_0 > 0$. This way, any increase in $G$ increases the final grade – so the students get the desired incentive.

The only absolute limitation on $\alpha_0$ is that we should not exceed the maximum grade $M$. Also, this value $A$ should not be equal to $M$ – otherwise, there is no incentive for a student to get perfect grades on all the assignments and on the final exam, if this student can earn the perfect grade with $G < P$. So, in this case, we must have $A < M$ for all $G \le P$. Since we consider the case when $G \le P$, the largest possible value of $G$ is $P$, and the corresponding largest possible value of the expression (2) is $P + \alpha_0 \cdot (P - w_E \cdot P)$. Thus, the requirement that this value does not exceed $M$ means that $P + \alpha_0 \cdot (P - w_E \cdot P) < M$, i.e., that

$$\alpha_0 < \frac{M - P}{P - w_E \cdot P}. \tag{3}$$

In particular, for $M = 100$, $P = 70$, and $w_E = 0.35$, we conclude that $\alpha_0 < 30/24.5 \approx 1.2$

In the limit $G = P$, the formula (2) leads to $A = P + \alpha_0 \cdot (P - w_E \cdot P)$. For $G > P$, we cannot use the same formula (2) – otherwise for $G = M$, we will have the overall grade larger than $M$. Indeed, for $G = M$, we already have $\max(G,P) = M$, so we cannot add anything to this grade. So, for $G > P$, in the formula (2), instead of a constant $\alpha_0$, we should have an expression $\alpha(G)$ that goes from $\alpha(P) = \alpha_0$ for $G = P$ to $\alpha(M) = 0$ for $G = M$.

Which function $\alpha(G)$ should we choose? The simplest functions are linear functions. A linear function is uniquely determined by its values at two points, so we have

$$\alpha(G) = \frac{M - G}{M - P} \cdot \alpha_0, \tag{4}$$

and thus,

$$A = G + \alpha_0 \cdot \frac{M - G}{M - P} \cdot (G - w_E \cdot P). \tag{5}$$

We need to make sure that the larger $G$, the larger the resulting grade $A$. The expression (5) has the form

$$A = G + \alpha_0 \cdot \frac{M \cdot G - M \cdot w_E \cdot P - G^2 + G \cdot w_E \cdot P}{M - P}. \tag{6}$$

Thus, the derivative of this expression (5) with respect to $G$ must be positive, i.e., that we should have

$$1 + \alpha_0 \cdot \frac{M - 2G + w_E \cdot P}{M - P} \ge 0 \tag{7}$$

for all $G$. The expression for this derivative (7) is decreasing with $G$. So, to guarantee that this derivative is positive for all $G$, it is sufficient to make sure that it is positive for the largest possible value $G = M$. THis leads to the following inequality

$$1+\alpha_0 \cdot \frac{-M+w_E \cdot P}{M-P} \geq 0, \tag{8}$$

i.e., equivalently, to

$$\alpha_0 \leq \frac{M-P}{M-w_E \cdot P}. \tag{9}$$

In particular, for $M = 100$, $P = 70$, and $w_E = 0.35$, this condition takes the form $\alpha_0 \leq 0.4$.

Now, we have two inequalities bounding $\alpha_0$: inequalities (3) and (9). However, since $P < M$, we have

$$\frac{M-P}{M-w_E \cdot P} < \frac{M-P}{P-w_E \cdot P}, \tag{10}$$

so if (9) is satisfied, the inequality (3) is satisfied too. Thus, it is sufficient to satisfy the inequality (9).

Let us summarize the resulting scheme.

**Resulting proposal.** Let $P$ be the passing threshold, $M$ be the maximum possible numerical grade, and $w_E$ be the weight of the final exam. To specify the proposed arrangement, we need to select a positive real number $\alpha_0$ for which

$$\alpha_0 \leq \frac{M-P}{M-w_E \cdot P}. \tag{9}$$

Suppose now that for some student, the grade for the final exam is $E$ and that the weighted-sum combination of this grade and all the grades for the previous tests and assignments is $G$. Then:

- If $G < P$ and $E < P$, the student fails the class.
- If $G \leq P$ and $E \geq P$, we assign, to this student, the grade

$$A = P + \alpha_0 \cdot (G - w_E \cdot P). \tag{11}$$

- If $G > P$, then we assign, to this student, the grade

$$A = G + \alpha_0 \cdot \frac{M-G}{M-P} \cdot (G - w_E \cdot P). \tag{5}$$

**Numerical example.** Let us assume that $M = 100$, $P = 70$, and $w_E = 0.35$. Let us take $\alpha_0 = 0.05$.

The worst-case passing student is when $G_- = 0$ and $E = P = 70$. In this case, $w_E \cdot E = 0.35 \cdot 70 = 24.5$, and the formula (11) leads to $A = 70$ – the smallest possible passing grade, exactly as planned.

Suppose now we have a solid C student, i.e., a student for whom $G = E = 70$. For this student, the formula (11) leads to

$$A = 70 + 0.05 \cdot (70 - 0.35 \cdot 70) = 70 + 0.05 \cdot (70 - 24.5) = 70 + 0.05 \cdot 45.5 =$$

$$70 + 2.275 \approx 72.$$

For a solid B student, for whom $G = E = 80$, the formula (12) leads to

$$A = 80 + 0.05 \cdot \frac{100 - 80}{100 - 70} \cdot (80 - 0.35 \cdot 70) = 80 + 0.05 \cdot \frac{2}{3} \cdot (80 - 24.5) \approx 82.$$

For a solid A student, for whom $G = E = 90$, the formula (12) leads to

$$A = 90 + 0.05 \cdot \frac{100 - 90}{100 - 70} \cdot (90 - 0.35 \cdot 70) = 80 + 0.05 \cdot \frac{1}{3} \cdot (90 - 24.5) \approx 91.$$

Finally, for a perfect student for whom $G = E = 100$, the formula (12) leads to the expected perfect value $A = 100$.

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