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Why Rarity Score Is a Good Evaluation of a Non-Fungible Token

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Why Rarity Score Is a Good Evaluation of a Non-Fungible Token

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Abstract One of the new forms of investment is investing in so-called non-fungible tokens – unique software objects associated with different real-life objects like songs, painting, photos, videos, characters in computer games, etc. Since these tokens are a form of financial investment, investors would like to estimate the fair price of such tokens. For tokens corresponding to objects that have their own price – such as a song or a painting – a reasonable estimate is proportional to the price of the corresponding object. However, for tokens corresponding to computer game characters, we cannot estimate their price this way. Based on the market price of such tokens, an empirical expression – named rarity score – has been developed. This expression takes into account the rarity of different features of the corresponding character. In this paper, we provide a theoretical explanation for the use of rarity score to estimate the prices of non-fungible tokens.

1 Formulation of the Problem

What is a non-fungible token. One of the new ways to invest money is to invest in *non-fungible tokens* (NFT). For many real-life objects – e.g.:

- for a song,
- for a painting,
- for a photo,

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- for a video,
- for a character in a computer game –

a special unique software object is designed called a non-fungible token.

For each real-life object, there is at most one non-fungible token.

Owning a token does not mean owning the corresponding object, but people still buy these tokens, often for high prices: for example, if a person cannot afford to buy the actual painting, he/she can still be a proud owner of this painting's non-fungible token (which is cheaper).

Non-fungible tokens are financial instruments. Non-fungible tokens have been an investment instrument – since, like many other good investments, they, on average, increase in price. At present (2022), the overall market price of all such tokens is in billions of dollars.

How to evaluate the price of a non-fungible token? Since tokens serve as investment instruments, buyers and sellers are interested in estimating the fair price for each such token.

Such an estimate is easier for token of objects that themselves have a price – e.g., songs, paintings, etc. For such tokens, it is reasonable to assume that the price of a token is proportional to the price of the actual object.

This makes sense:

- If a multi-million dollar painting by a famous artist turned to be a later forgery, the actual price of this painting plummets and, naturally, the price of its token shall fall down too.
- On the other hand, if an obscure painting turns out to be painted by a famous artist, the price of this painting skyrockets and, correspondingly, the price of its token should drastically increase.

But how can we estimate the price of a token corresponding to something that does not have a naturally defined price – e.g., a character in a computer game?

Enter rarity score. People actually sell and buy such tokens all the time. Usually, as with all other objects that can be sold and bought, after some oscillation, the market more or less settles on some price. How can we estimate this price? Such an estimation will be of great value to those who want to buy and/or sell such tokens.

Several empirical formulas have been proposed to estimate the price of such tokens; see, e.g., [1]. The most accurate estimates are based on so-called *rarity score*. To understand this notion, it is necessary to take into account that each game character has several different features that are useful in different circumstances. For example:

- a character can fly or can jump high, which is helpful in avoiding obstacles or pursuing some other dynamic goals like running away from danger;
- a character can have X-ray vision that helps this character clearly see the situation, etc.

For each of these features, we can define the rarity score as the result of dividing the overall number of characters in the given game by the number of characters with the given feature. For example, if out of 100 game characters, 5 can fly, then the rarity score of flying is

$$\frac{100}{5} = 20.$$

The rarity score of a character is then estimated as the sum of rarity scores of all its features. For example, suppose that a character:

- can fly – the rarity score of this feature is 20.0,
- has a normal vision – as well as 79 other characters, so that the rarity score is

  $$\frac{100}{80} = 1.25,$$

  and
- has a magic wand – as well as 19 other characters, so that the rarity score of this feature is

  $$\frac{100}{20} = 5.0.$$

Then, the overall rarity score of this character is

$$20.0 + 1.25 + 5.0 = 26.25.$$

But why? Rarity score provides a reasonable estimate of the marker value of the token corresponding to the character, but why is not clear.

What we do in this paper. In this paper, we provide a possible explanation of the efficiency of rarity score estimation.

2 Our Explanation

A natural analogy. For people who do not play computer games, it may be difficult to think of computer game characters. So, to explain our reasoning, let us consider a somewhat similar situation of coins or postage stamps. We all receive letters with postage stamps, we all get coins as change and use coins to buy things (although both stamps and coins are becoming rarer and rarer events.)

Most the stamps we see on our letters are mass-produced, easily available ones – but sometimes, we see an unusual stamp. Similarly, most coins we get and use are easily available ones, but sometimes, we accidentally encounter an unusual coin – e.g., a rare so-called “zinc” US penny produced in 1943. Rare stamps and rare coins are highly valued by collectors.

What is a reasonable way to estimate the price of a rare stamp or a rare coin?

How to estimate the price of a rare stamp or a rare coin: a natural idea. A natural way to look for a rare penny – unless we want to simply buy it – is to inspect every penny that we have, until we find the desired one. The rarer the coin, the more time we will need to spend to find it – i.e., the more work we will have to perform. It is therefore reasonable to set up a reasonable price for a rare coin by a per-hour pay, i.e., proportional to the average time that a person needs to spend to find this rare coin.

Let us transform this idea into a precise estimate.

Towards a precise estimate. Let us estimate the average time needed to find a rare coin. Let us denote the proportion of rare coins among all the coins by p. Then, if we inspect coins one by one, a rare coin can appear:

- as the first one in our search; in this case we spend 1 unit of effort – by having to look at just one coin;
- as the second one in our search; in this case we spend 2 units of effort – by having to look at two coins; etc.

Thus, the average number a of units of effort to find a rare coin can be computed as follows:

$$a = p_1 \cdot 1 + p_2 \cdot 2 + \ldots, \tag{1}$$

where p_1 denotes the probability the first coin is rare, p_2 is the probability that the first coin is not rare but the second coin is rare, etc.

In general, for every positive integer k, p_k is the probability that the first $k-1$ coins were not rare but the k-th coin is rare. The probability that a randomly selected coin is rare is equal to p, the probability that this coin is not rare is equal to $1-p$. Different coins are independent, so the probability p_k is equal to the product of $k-1$ terms equal to $1-p$ and one term equal to p:

$$p_k = (1-p)^{k-1} \cdot p. \tag{2}$$

Substituting the expression (2) into the formula (1), we conclude that

$$a = p \cdot 1 + (1-p) \cdot p \cdot 2 + (1-p)^2 \cdot p \cdot 3 + \ldots + (1-p)^{k-1} \cdot p \cdot k + \ldots \tag{3}$$

All the terms in this formula have a common factor p, so we can conclude that

$$a = p \cdot A, \tag{4}$$

where we denoted

$$A \stackrel{\text{def}}{=} 1 + (1-p) \cdot 2 + (1-p)^2 \cdot 3 + \ldots + (1-p)^{k-1} \cdot k + \ldots \tag{5}$$

To estimate the value of A, let us multiply both sides of the formula (5) by $1-p$, then we get:

$$(1-p) \cdot A = (1-p) \cdot 1 + (1-p)^2 \cdot 2 + \ldots + (1-p)^{k-1} \cdot (k-1) + \ldots \tag{6}$$

If we subtract (6) from (5), then we get 1 and also, for each k, we subtract the term $(1-p)^{k-1}\cdot(k-1)$ from the term $(1-p)^{k-1}\cdot k$, resulting in

$$(1-p)^{k-1}\cdot k-(1-p)^{k-1}\cdot(k-1)=(1-p)^{k-1}\cdot(k-(k-1))=(1-p)^{k-1}. \quad (7)$$

Thus, the difference

$$A-(1-p)\cdot A=(1-(1-p))\cdot A=p\cdot A \quad (8)$$

takes the form

$$p\cdot A=1+(1-p)+(1-p)^2+\ldots+(1-p)^{k-1}+\ldots \quad (9)$$

To compute the sum in the right-hand side of the formula (9), we can use the same trick: multiply both side of this formula by $1-p$ and subtract the resulting equality from (9). Multiplying both sides of the equality (9) by $1-p$, we conclude that

$$(1-p)\cdot p\cdot A=(1-p)+(1-p)^2+\ldots+(1-p)^{k-1}+\ldots \quad (10)$$

Subtracting (10) from (9), we conclude that

$$p\cdot A-(1-p)\cdot p\cdot A=(1-(1-p))\cdot p\cdot A=p^2\cdot A=1. \quad (11)$$

Thus,

$$A=\frac{1}{p^2}, \quad (12)$$

and so, due to formula (4),

$$a=p\cdot A=p\cdot\frac{1}{p^2}=\frac{1}{p}. \quad (13)$$

How is this estimate related to the rarity score of a feature. The probability p is equal to the ratio between the number of rare coins divided by the overall number of coins. Thus, the inverse expression (13) is the ratio of the number of all the coins to the number of rare coins – exactly what is called rarity score of a feature.

From rarity score of a feature to rarity score of the character. To explain how to go from rarity scores of different features to the rarity score of the character with these features, let us use a different analogy. Suppose that at an international airport, there are three souvenir stores:

- one sells US souvenirs and only accepts US dollars,
- the second one sells Canadian souvenirs and only accepts Canadian dollars, and
- the third one sells Mexican souvenirs and only accepts Mexican pesos.

If we have a wallet with money of all these three countries, then the overall price of our wallet is the sum of the prices of all these three components.

Similarly, since different features can be used in different situations – as different money in the above example can be used in different stores – it is reasonable to estimate the overall price of a character as the sum of prices corresponding to these different features, i.e., as the sum of the corresponding rarity scores. This is exactly how the character's rarity score is estimated.

Thus, we have indeed found an explanation for this estimate.

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