6-1-2022

Why Decreased Gaps Between Brain Cells Cause Severe Headaches: A Symmetry-Based Geometric Explanation

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Comments:
Technical Report: UTEP-CS-22-69

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Abstract When we analyze biological tissue under the microscope, cells are directly neighboring each other, with no gaps between them. However, a more detailed analysis shows that in vivo, there are small liquid-filled gaps between the cells, and these gaps are important: e.g., in abnormal situations, when the size of the gaps between brain cells decreases, this leads to severe headaches and other undesired effects. At present, there is no universally accepted explanation for this phenomenon. In this case, we show that the analysis of corresponding geometric symmetries empirical phenomenon leads to a natural explanation for this effect.

1 Decreased Gaps Between Brain Cells Cause Severe Headaches: Empirical Fact

Empirical fact. It is well known that living creatures consist of cells, cells is all you see when you look at any tissue under a microscope – and this is how scientists understood the structure of the living creatures. However, starting with the 1960s, it was determined that in living creatures, there is always a liquid-filled gap between cells – a gap that closes when the matter is no longer alive. The gaps were first

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discovered when, to preserve the structure as much as possible, researchers instantaneously froze the cells culture. Since then, new techniques have been developed to study these gaps.

The resulting studies showed that these gaps play important biological roles. For example, in abnormal situations, when the gaps between brain cells drastically decrease, a patient often experiences several headaches and other undesired effects; see [2] and references therein.

Why? As of now, there are no universally accepted explanations for this empirical fact. In this paper, we show that this empirical phenomenon naturally follows from symmetry-based geometric ideas.

2 Analysis of the Problem and the Resulting Explanation

Normal case: geometric description. Let us first consider the case when there are gaps between cells. We want to know how cells interact with each other.

All interactions are local. Thus, to analyze the interaction between the cells, we need to consider a small area on the border between two neighboring cells.

The border of each cell is usually smooth. Locally, each smooth surface can be well-approximated by its tangent plane – the smaller the area, the more accurate the approximation. Thus, with good accuracy, we can locally represent the border of each cell by a plane.

The border of the neighboring cell is also represented by a plane. Since we consider the situation when there is a gap, the borders do not intersect. When the two planes do not intersect, this means that they are parallel to each other. Thus, the normal configuration can be described by two parallel planes corresponding to two neighboring cells.

Abnormal case: geometric description. In the abnormal case, the gaps drastically decrease, to the extent these gaps become undetectable. Thus, with good accuracy, we can conclude that in this case, there is, in effect, no gaps between the two cells – and thus, that both cells can be described by a single plane, the plane that serves as a common border of the two cells.

We want to study dynamics. For a living creature, there is a usually a stable state, and then there are dynamic changes, when a change in one cell causes changes in others. To study dynamics, we therefore need to study how disturbances propagate. Since, according to physics, all interactions are local (see, e.g., [1, 3]), for a perturbation in one cell to reach another cell, this perturbation first need to reach the border of the original cell.

The simplest perturbation is when the perturbation is located at a single point on the cell's border – every other perturbation of the border can be viewed as a combination of such point-wise perturbations corresponding to all affected points. So, to study how general perturbations propagate, it is necessary to study how point-wise perturbations propagate.

Role of symmetries. Most physical processes do not change if we apply:

- shift,
- rotation, or
- scaling – i.e., replace, for some $\lambda > 0$. each point with coordinates $x = (x_1, x_2, x_3)$ with a point with coordinates $(\lambda \cdot x_1, \lambda \cdot x_2, \lambda \cdot x_3)$.

Physical processes are invariant with respect to such geometric symmetries. Thus, if we start with the initial configuration which is invariant with respect to some of these symmetries, the resulting configuration will also be invariant with respect to the same geometric transformations. Let us analyze how this idea affects the propagation of perturbations between the cells.

Normal case: what are the symmetries and what are possible dynamics. Let us first consider the normal case, when we have:

- two parallel planes and
- a point in one of these planes – the location of the original perturbation.

One can see that out of all above-listed geometric symmetries, the only symmetries that keep this configuration invariant are rotations around the fixed point in the first plane – i.e., in 3D terms, rotations around the axis that goes through this point orthogonally to the plane (and, since the planes are parallel, orthogonally to both planes).

Since the initial configuration has this symmetry, the resulting configuration – observed after some time – should also has the same symmetry. In particular, with respect to what perturbations we can have on the other plane – i.e., on the border of the neighboring cell:

- we can have a single point (on the same axis),
- or we can have a rotation-invariant planar region (e.g., a disk centered at this point) that reflects inevitable diffusion.

These cases correspond to usual information transfer between the cells.

One of the possibilities is that the resulting configuration will involve all the points of the second plane, but this is not the only configuration resulting from diffusion.

Abnormal case: what are the symmetries and what are possible dynamics. Let us now consider the abnormal no-gaps case, when we have:

- a single plane – a joint boundary between the cells, and
- a point in this plane, which is the location of the original perturbation.

In this case, in addition to rotations, the corresponding configuration has an additional symmetry – scalings around the given point. Thus, the resulting configuration should be invariant not only with respect to the rotations, but also with respect to these scalings.

Due to inevitable diffusion, we expect the plane part of the resulting configuration to include more than a single point. However, one can easily see that every two points on a plane – which are both different from the original point – can be obtained from each other by an appropriate rotation and scaling. Thus, once the resulting rotation- and scale-invariant configuration contains at least one point which different from the original point, it will automatically include all the points in the plane.

In other words, in the presence of even small diffusion, a local point-wise perturbation will lead to a perturbation of the whole boundary. This perturbation will spread to other cells – and cause a global all-cells-involving perturbation, which is exactly what corresponds to a severe headache, when many cells are affected.

Summarizing: this explains the observed phenomenon. Our analysis shows the following.

- In the normal case – when there are gaps between brain cells – while we can have global-brain effects like severe headache, this is not inevitable: we can also have a usual information transfer between cells.
- On the other hand, in the no-gaps case, effects like severe headache are inevitable.

Thus, the thinner the gaps, the closer the resulting configuration is to the no-gaps case, the more probable it is that severe headaches (and other global effects) will occur – which is exactly what is observed. So, our symmetry-based geometric analysis indeed explains the observed phenomenon.

Acknowledgments

This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and HRD-1834620 and HRD-2034030 (CAHSI Includes), and by the AT&T Fellowship in Information Technology.

It was also supported by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478, and by a grant from the Hungarian National Research, Development and Innovation Office (NRDI).

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