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In the Absence of Information, the Only Reasonable Negotiation Scheme Is Offering a Certain Percentage of the Original Request: A Proof

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In the Absence of Information, the Only Reasonable Negotiation Scheme Is Offering a Certain Percentage of the Original Request: A Proof

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Abstract In the case of complete information, a reasonable solution to a negotiation process is Nash's bargaining solution, in which we maximize the product of all agents' utility gains. This is the only solution that does not depend on the order in which we list the agents, and does not change if we use a different scale for describing each agent's utility. In this paper, we apply similar invariance criteria to a situation when practically all information is absent, and all we know is the smallest and largest possible gains. We show that in this situation, the only invariant negotiation strategy is to offer, to each agent, a certain percentage of the original request – and to select the percentage for which all such reduced requests can be satisfied.

1 Formulation of the Problem

How people make decisions: a brief reminder. According to decision theory (see, e.g., [1, 2, 4, 5, 8, 9, 10]), decisions of a rational person can be described by assigning, to each alternative, a numerical value known as *utility*, so that in each situation, the alternative selected by the decision maker is the one with the largest utility value.

Utility can be described, e.g., by selecting:

- a very bad alternative A_0 – worse than any actual alternatives, and
- a very good alternative A_1 – better than any actual alternative.

To find the utility of an alternative A, we can compare it with lotteries $L(p)$ in which:

- the user gets A_1 with probability p and

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- the user gets A_0 with the remaining probability $1 - p$.

Due to our choice of A_i:

- for $p = 1$, we have $L(1) = A_1$, which is better than A (we will denote it by $L(1) > A$), and
- for $p = 0$, we have $A > L(0) = A_0$.

One can prove that there exists a threshold value $u(A)$

- for which $L(p) > A$ for $p > u(A)$ and
- for which $L(p) < A$ for $p < u(A)$.

This threshold value is known as the utility of the alternative A.

The numerical value $u(A)$ depends on our choice of the two extreme alternatives A_i. It turns out that if we select a different pairs of extreme alternatives, then the new values of the utility $u'(A)$ can be obtained from the previous values $u(A)$ by a linear transformation $u'(A) = a + b \cdot u(A)$, for some constants a and $b > 0$.

Comment. When the outcomes are purely financial, in the first approximation, we can view the money amounts as utility values. However, it is important to take into account that many negotiations involve non-financial issues as well, and that even for financial issues, utility is not always proportional to money.

Cooperative group decision making: case of full information. Cooperative decision making is when we have a status quo situation S, and several (n) agents are looking for alternatives that can make the outcomes better for all of them.

In many real-life situations, everyone known everyone's utility $u_i(A)$ for each alternative A. Since utility values are defined only modulo a linear transformation, it makes sense to consider decision strategies for which:

- the resulting solution would not change if we apply linear re-scaling $u_i(A) \to a_i + b_i \cdot u_i(A)$ to one of the utilities, and
- the resulting solution would not change if we simply rename the agents.

It turns out that the only such not-changing (= invariant) scheme is when the agents maximize the product of their utility gains, i.e., the value

$$(u_1(A) - u_1(S)) \cdot \ldots \cdot (u_n(A) - u_n(S));$$

see, e.g., [5, 6, 7]. This scheme was first proposed by the Nobelist John Nash and is thus known as *Nash's bargaining solution.*

Sometimes, we do not have the full knowledge. In some situations – e.g., when two countries have a territorial dispute – we do have full information of what each side wants. However, in other situations – e.g., in negotiations between companies – agents are reluctant to disclose their utilities: while in this case, they are collaborating, in the future, they may be competing, and any information about each other leads to a competitive advantage.

In many cases, the only two things we know for each agent i are:

- the agent's utility $u_i(S)$ corresponding to status quo – which is usually known because of the reporting requirements, and
- the agent's original offer $u_i^{(0)}$ – which usually means the best outcome that the agent can reach:

$$u_i^{(0)} = \overline{u}_i \stackrel{\text{def}}{=} \max_A u_i(A).$$

In other words, for each agent, we know the largest value $g_i^{(0)} \stackrel{\text{def}}{=} u_i^{(0)} - u_i(S)$ that this agent can gain. Based on this information, how can we make a joint decision?

General idea. In practice, it is never possible to each agent to get the largest possible gain, there is usually a need for a trade-off between agents. Since agents cannot all get their maximum gain, a natural idea is to somewhat lower their requests, from the original values $g_i^{(0)}$ to smaller values $g_i^{(1)} = f_i\left(g_i^{(0)}\right)$ for some function $f_i(g)$ for which $f_i(g) < g$. Then:

- if it is possible to satisfy all reduced requests, then this is the desired joint solution;
- if it is not possible to satisfy all reduced requests, then the request amounts should be reduced again, to values $g_i^{(2)} = f_i\left(g_i^{(1)}\right)$, etc.

The procedure should be fair, meaning that the same reducing function $f(g)$ should be applied for all the agents: $f_i(g) = f(g)$ for all i.

Question. The main question is: what reducing function $f(g)$ should we use?

2 Which Reducing Function Should We Use

Natural requirement: reminder. As we have mentioned in our description of Nash's bargaining solution, since utilities are only known modulo linear transformations, a natural requirement is that the resulting decision not change if we use re-scale an agent's utilities, from the original values $u_i(A)$ to new values $a_i + b_i \cdot u_i(A)$. Let us apply the same invariance criterion to our problem.

Which reducing functions are invariant under re-scaling? Under the linear transformation $u_i(A) \mapsto a_i + b_i \cdot u_i(A)$, we get $u_i(S) \mapsto a_i + b_i \cdot u_i(S)$ and $u_i^{(0)} \mapsto a_i + b_i \cdot u_i^{(0)}$. Thus, the difference $g_i = u_i^{(0)} - u_i(S)$ get transformed into the difference

$$g_i \mapsto \left(a_i + b_i \cdot \cdot u_i^{(0)}\right) - (a_i + b_i \cdot u_i(A)) = b_i \cdot \left(u_i^{(0)} - u_i(S)\right) = b_i \cdot g_i.$$

So, using a new scale means multiplying all gain values g_i by b_i. (Vice versa, to transform the new-scale gain value into the original scale, we need to divide this new-scale value by b_i.)

Let us see when the reducing function is invariant under such re-scaling.

- In the original utility scale, we transform the gain $g_i^{(k)}$ into the reduced gain $g_i^{(k+1)} = f\left(g_i^{(k)}\right)$. In the new scale, the reduced gain takes the form $b_i \cdot f\left(g_i^{(k)}\right)$.
- Let us see what happens if we apply the reducing function to the values described in the new scale. The original gain $g_i^{(k)}$ in the new scale has the form $b_i \cdot g_i^{(k)}$. When we apply the reducing function to this value, we get the new-scale value

$$f\left(b_i \cdot g_i^{(k)}\right).$$

Invariance means that both new-scale gains should be equal, i.e., that we should have

$$f\left(b_i \cdot g_i^{(k)}\right) = b_i \cdot f\left(b_i \cdot g_i^{(k)}\right).$$

This equality should hold for all possible values of b_i and $g_i^{(k)}$. In particular, for any number $g > 0$, for $b_i = g$ and $g_i^{(k)} = 1$, we get $f(g) = c \cdot f(g)$ for some constant $c \stackrel{\text{def}}{=} f(1)$.

Since we must have $f(g) < g$, we should have $c < 1$. Thus, we arrive at the following conclusion.

Resulting recommendation. In the above absence-of-information case, the only fair scale-invariant scheme is for all agents to select a certain percentage c of their original request – e.g., 70% or 60% – and to decrease this percentage c until we find a solution for which each agent can get this percentage of his/her original request.

Comments. Once this value c is found, the gain $g_i = u_i - u_i(S)$ will be equal to $g_i = c \cdot (\overline{u}_i - u_i(S))$ and thus, the utility of the i-th agent will be equal to

$$u_i = u_i(S) + c \cdot (\overline{u}_i - u_i(S)) = c \cdot \overline{u}_i + (1 - c) \cdot \underline{u}_i.$$

In this problem, the smallest utility $\underline{u}_i$ the agent can end up with is the status-quo value $u_i(S)$. In these terms, the resulting gain has the form

$$u_i = c \cdot \overline{u}_i + (1 - c) \cdot \underline{u}_i.$$

Interestingly, this expression coincides with another expression for decision theory: Hurwicz formula that describe the equivalent utility of a situation in which all we know is that the actual utility will be in the interval $[\underline{u}_i, \overline{u}_i]$; see, e.g., [3, 4, 5]. The formulas are similar, but there is an important difference between these two situations:

- in the Hurwicz situation, the coefficient c depends on each user, it describe how optimistic the user is;
- in contrast, in our situation, the coefficient c is determined by the group as a whole, it describe how much the group can achieve.

The above solution is also similar to the usual solution to the bankruptcy problem, when everyone gets a certain percentage $c \cdot m_i$ of the amount it is owed – e.g., 20 cents the dollar, and the coefficient c is the largest for which such solution is possible.

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