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Freedom of Will, Non-Uniqueness of Cauchy Problem, Fractal Processes, Renormalization, Phase Transitions, and Stealth Aircraft

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Abstract We all know that we can make different decisions, decisions that change – at least locally – the state of the world. This is what is known as freedom of will. On the other hand, according to physics, the future state of the world is uniquely pre-determined by its current state, so there is no room for freedom of will. How can we resolve this contradiction? In this paper, we analyze this problem, and we show that many physical phenomena can help resolve this contradiction: fractal character of equations, renormalization, phase transitions, etc. Usually, these phenomena are viewed as somewhat exotic, but our point is if we want physics which is consistent with freedom of will, then these phenomena need to be ubiquitous.

1 Freedom of Will and Physics: A Problem

A problem: reminder. We all know that in some situations, we can make different decisions, and these decisions will change our state and the state of others – i.e., change the state of the world. The possibility to make different decision is known as *freedom of will*. The problem is that this experience is inconsistent with physics.

This inconsistency is very clear in Newtonian mechanics, where the state of the world at any future moment of time is uniquely determined by the current state. Strictly speaking, in Newtonian physics, all our actions are pre-determined – just like all other changes in the state of the world are pre-determined, so freedom of will is an illusion. Interestingly, Einstein himself seriously believed that freedom of will is an illusion: he could not swim but he liked to go yachting alone, and when his

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friends expressed concern about this, assured them that everything is pre-determined and nothing can be changed. For most of us, however, freedom of will is real.

The situation is a little bit less pre-determined in quantum physics, but in quantum physics still, the current state of the world uniquely pre-determines the probabilities of all future state. To be more precise, Schroedinger equations pre-determine the values of the wave function, and the wave function uniquely determined all future probabilities; see, e.g., [1, 3]. So, if we face several similar decisions at future moments of time:

- it is not possible to predict how exactly the state will change every time,
- but the proportion of times in which we make a certain decision is pre-determined by the past state of the world.

This also contradicts to our intuition.

In a nutshell, freedom of will means that we can change the state of the world just by making a mental decision. Many people do claim that they can move things or otherwise change the state of the world by simply using their thoughts, but so far, none of such claims have been confirmed: human thought cannot change the state of even a single particle. So how can we resolve the above contradiction between modern physics and common sense?

What we do in this paper. In this paper, we propose a possible solution to this problem.

2 Possible Solutions

How physical theories are described. Traditionally, physical theories have been described by differential equations, equations that describe how the state's rate of change depends on the current state. Lately, however, most theories are described by describing a functional – known as *action* – for which the actual trajectory of how the system's state changes with time is the one that minimizes action [1, 3]. To be more precise, what is often described is not the action itself but the so-called *Lagrangian* whose integral over space and time forms the action.

In line with this, let us consider how freedom of will can be explained in both these approaches.

Case of differential equations. Let us first consider the case when the physical laws are described in terms of a differential equation $\dot{x} = f(x)$, where x is the state of the world at the moment of time t – as described by states of the particles, states of the fields, etc.

In general, differential equations enables us, given the state $x(t_0)$ at some moment of time, to predict the states $x(t)$ at all future moments of time. The problem of predicting the future state $x(t)$ based on the current state $x(t_0)$ is known as the *Cauchy problem*.

Sometimes, the solution to Cauchy problem is not unique. In mathematics, it is known that for some differential equations, the Cauchy problem has several different solutions. Such situations happen with physically meaningful differential equations as well: see, e.g., [2], where this non-uniqueness is used to explain the "time arrow" – irreversibility of many macro-phenomena.

Non-uniqueness should be ubiquitous. Non-uniqueness helps to resolve the contradiction between physics and freedom of will: we can follow several different trajectories without violating the physical laws.

However, by itself, the current non-uniqueness does not fully resolve this contradiction: namely, we can make decisions at any moment of time. while in general in physics, non-uniqueness is rare, it is limited to few exceptional cases and/or exceptional moments of time. To fully resolve the contradiction, we need to make sure that non-uniqueness is ubiquitous. How can we do it?

Need for non-smooth (fractal) equations. If the right-hand side $f(x)$ of the corresponding physical equation is analytical, then usually, there is a unique solution (at least locally), this was proven already in the 19 century Cauchy himself, the mathematician who first started a systematic study of what we now call the Cauchy problem. This means that, if we want consistency with common sense, we have to consider right-hand sides which are not analytical and probably not even smooth – e.g., fractal.

Let us show that non-smooth right-hand sides indeed lead to non-uniqueness. Indeed, let us consider the simplest possible case when the state of the world is described by a single variable x, and the equation takes the form $\dot{x} = x^\alpha$, with 0 initial condition $x(t_0) = 0$. In this case, one possibility is to have $x(t) = 0$ for all t. On the other hand, there are many other possibilities: e.g., we can have a solution which is equal to 0 until some moment $t_1 \geq t_0$, and then switch to $x(t) = C \cdot (t - t_1)^{1/(1-\alpha)}$ for some constant C.

So, introducing fractal-ness into the equations can help resolve the freedom-of-will-vs-physics problem.

Discussion. The need for non-smoothness is well known in some areas of physics. For example, if we limit ourselves to infinitely smooth solutions of aerodynamics equations, then we arrive at the conclusion that Lord Kelvin made in the late 19 century – that human-carrying heavier-than-air flying machines are not possible – a conclusion that was experimentally disproved by the appearance of airplanes [1].

Renormalization: another reason for non-uniqueness and another opening for freedom of will. In some cases, a solution to a differential equation is locally unique, but after some time, it leads to a physically meaningless infinite value of some physical quantity. For example, a general solution to an equation $\dot{x} = x^2$ is $x(t) = 1/(C - t)$ for some constant C, so for $t = C$, we get an infinite value.

This phenomenon is not purely mathematical, it is a well-known physical phenomenon. The simplest example of such a phenomenon is an attempt to compute the overall energy of an electron's electric field [1]. According to relativity theory, since the electron is an elementary particle and not a combination of several independent sub-particles, it must be a point-wise particle: otherwise, due to the fact that all communication speeds are limited by the speed of light, states of the different spatial parts of the electron at the same moment of time cannot affect each other and would thus serve as such independent sub-particles. For a point-wise particle, the electric field is proportion to r^{-2}, where r is the distance to the electron, and the energy density of the field is proportional to the square of the field, i.e., to r^{-4}. Thus, the overall energy of the electron's electric field is equal to the integral of this energy density r^{-4} over the whole space – and one can check, by using radial coordinates, that this integral is infinite.

So, the overall energy of the electron – which is the sum of its rest-mass energy $m_0 \cdot c^2$ and the overall energy of its electric field – is supposed to be infinite, while we know that it is finite and very small. Of course, our analysis ignored quantum effects, but if we take quantum effects into account, the result remains infinite.

How is this problem resolved now? The usual way – known as *renormalization* – is to consider, for each $\varepsilon > 0$, a model in which an electron has a finite radius $\varepsilon > 0$. For this model, the overall energy of the electric field is finite. Within this model, the rest mass $m_0(\varepsilon)$ is then selected in such a way that the overall energy of the electron – the sum of the rest-mass energy $m(\varepsilon) \cdot c^2$ and the overall energy of the electric field – becomes equal to the observed value. For each quantity of interest, as a prediction, we then take the limit of predictions in different models when ε tends to 0.

For each $\varepsilon > 0$, we have uniqueness, but there is no guarantee that the corresponding predictions will tend to some limit. In situations when the sequence of predictions corresponding to different ε does not converge, we do not have a definite prediction – which also opens room for freedom of will.

What about the optimization approach. In the optimization approach, non-uniqueness appears when we have two or more trajectories or states with the exact same smallest possible value of the objective function. This phenomenon is known in physics: e.g., during the phase transition such as melting, at some point, both the solid and the liquid states have the same value of the objective function. There is also a related phenomenon of unstable equilibrium, when, e.g., the smallest push can move body on top of a rotation-invariant mountain downhill – but it is difficult to predict in which direction it will move. So maybe we *can* test the ability of people to use their thoughts to change the state of the world – by testing this ability on such unstable equilibrium situations.

An additional feature of such phase transitions is non-smoothness. Non-smoothness (and even discontinuity) is typical for optimization problems, where it is known as a "bang-bang control". For example, the stealthiest shape of an aircraft is when its surface is not smooth, but is formed by several planar parts. For a smooth surface, there are always parts the shape that reflect the radar's signal back to the radar, but for such a piece-wise planar shape, there are only a few reflected directions, and the probability that one of them goes back to the radar is close to 0.

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