5-1-2022

How Can the Opposite to a True Theory Be Also True? A Similar Talmudic Discussion Helps Make This Famous Bohr's Statement Logically Consistent

Miroslav Svitek
Czech Technical University in Prague, svitek@fd.cvut.cz

Vladik Kreinovich
The University of Texas at El Paso, vladik@utep.edu

Comments:

Technical Report: UTEP-CS-22-57

How Can the Opposite to a True Theory Be Also True? A Similar Talmudic Discussion Helps Make This Famous Bohr's Statement Logically Consistent

Miroslav Svítek and Vladik Kreinovich

Abstract In his famous saying, the Nobelist physicist Niels Bohr claimed that the sign of a deep theory is that while this theory is true, its opposite is also true. While this statement makes heuristic sense, it does not seem to make sense from a logical viewpoint, since, in logic, the opposite to true is false. In this paper, we show how a similar Talmudic discussion can help come up with an interpretation in which Bohr's statement becomes logically consistent.

1 Formulation of the Problem

Bohr's statement: reminder. In [1], the famous physicist Niels Bohr cited "the old saying of the two kinds of truth. To the one kind belong statements so simple and clear that the opposite assertion obviously could not be defended. The other kind, the so-called 'deep truths', are statements in which the opposite also contains deep truth."

On the intuitive level, this makes sense. This statement summarizes many Bohr's ideas about quantum physics. For example:

- in some cases, it makes sense to consider elementary particles like electrons, protons, or photons, as point-wise particles;
- on the other hand, in some other cases, it makes sense to consider elementary particles not as point-wise particles but as waves.

Miroslav Svítek
Faculty of Transportation Sciences, Czech Technical University in Prague, Konviktska 20
CZ-110 00 Prague 1, Czech Republic, e-mail: svitek@fd.cvut.cz

Vladik Kreinovich
University of Texas at El Paso, 500 W. University, El Paso, Texas 79968, USA
e-mail: vladik@utep.edu

However, from the purely logical viewpoint, Bohr's statement does not make sense. The problem is that from the viewpoint of logic, Bohr's statement does not seem to make sense, since in logic, the opposite to a true statement is false.

What we plan to do about it. The above Bohr's statement is very famous. There was a lot of discussion of this statements.

Some of the discussants mentioned that similar statements were made in the past, for example, a similar statement was made in the Talmud [3], where the corresponding statement has been accompanied by a more detailed discussion. In this paper, we show that the ideas from this more-detailed discussion can help to make Bohr's statement logically consistent.

2 Analysis of the Problem and the Resulting Solution

What the Talmudic discussion says. The corresponding Talmudic statement is related to a contradiction between the two statements made by the respected Rabbis: "Make for yourself a heart of many rooms, and enter into it the words of Beit Shammai and the words of Beit Hillel, the words of those who declare a matter impure, and those who declare it pure."

How we can interpret this discussion. An important point is that this discussion does not say that we should simply add both contradictory statements to our reasoning – this would cause a contradictions. What this discussion seems to say is that we need to keep these abstract statements separate – "in separate rooms":

- We should not consider these two statements together when we argue about abstract things, like whether some matter is pure or not – otherwise we will get a contradiction. So, when we try to decide how to act, we should not combine arguments of both sides.
- However, it seems to make sense to accept both:
  - the practical advice coming from the arguments of the first school and
  - the practical advice coming from the arguments of the second school.

Let us describe this interpretation in precise terms. We have a general set of statements. Some statements from this set are about practical consequences; we will call them *practical*.

Intuitively, if two statements A and B are about practical consequences, then their logical combinations $A \,\&\, B$, $A \vee B$, and $\neg A$ are also about practical consequences. Thus, the set of practical statements must be closed under such combinations.

We have a statement a which is *abstract* – i.e., which is not equivalent to any practical statement. What Talmudic discussion seems to recommend is to consider *both* practical consequences of the statement a *and* practical consequences of its negation $\neg a$. Can this lead to a logically consistent set of recommendations?

Let us describe this setting in precise terms.

Definition.

- *By a* signature Σ*, we mean a finite list of:*
  - *variable types* $t_1, t_2, \ldots,$
  - *constants* $c_1, c_2, \ldots,$ *of different types,*
  - *predicate symbols* $P_1(x_1, \ldots, x_{n_1})$, $P_2(x_1, x_2, \ldots, x_{n_2}), \ldots,$ *and function symbols* $f_1(x_1, \ldots, x_{m_1})$, $f_2(x_1, \ldots, x_{m_2})$, $\ldots,$ *where* x_i *are variables of given types;*
- *By a* formula *in the signature* Σ*, we mean any formula obtained by using logical connectives* $\&$, $\vee$, $\neg$, $\rightarrow$, $\leftrightarrow$ *and quantifies* $\exists x$ *and* $\forall x$ *over variables of each type.*
- *We say that a formula is* closed *if each variable is covered by some quantifier. Closed formulas will also be called* statements.
- *By a* theory T*, we mean a finite list of statements.*
- *We say that a statement* f *is* true *if it follows from statements from* T.
- *We say that a statement* f *is* independent *if neither this statement not its negation are true.*
- *We say that a theory* T *is* consistent *if whenever it implies a statement* s*, it does not imply the opposite statement* $\neg s$.
- *Let* σ *be a subset of* Σ *– that includes only some types, predicates and function symbols from* Σ*. Statements using only variable types, constants, predicates, and function from the signature* σ *will be called* practical.
- *We say that a statement* a *is* abstract *if this statement is not equivalent to any practical statement.*

Proposition. *Let* T *be a consistent theory, and let* a *be an abstract independent statement. Then, if you add to the set of all true practical statements:*

- *all the practical statements that follow from* a*,* and
- *all the practical statements that follow from* $\neg a$*,*

the resulting set of statements will still be consistent.

Discussion. So, if we add practical consequences of an abstract statement and of its negation, we will indeed get no contradiction. In this sense, Bohr's statement is indeed logically consistent.

Proof. The above proposition – and its proof – extend a similar results proved in our earlier paper [2].

Let us prove this proposition by contradiction. Let us assume that the enlarged set is inconsistent, i.e., that it contain both a practical statement p and its negation $\neg p$. These two statements p and $\neg p$ cannot both follow from a – otherwise, that would mean that a implies contradiction and thus, a is not true – while we assumed that a is independent.

Similarly, the two statements p and $\neg p$ cannot both follow from $\neg a$. Thus, one of these two statements p and $\neg p$ follows from a, and another one from $\neg a$. If $\neg p$ follows from a, then we can name $p' \stackrel{\text{def}}{=} \neg p$, and have $p = \neg p'$. So, without losing generality, we can conclude that a implies p and $\neg a$ implies $\neg p$.

From the fact that $\neg a$ implies $\neg p$, it follows that p implies a. Thus, a implies p and p implies a, i.e., the statements a and p are equivalent. However, we assumed that the statement a is abstract, so it cannot be equivalent to any practical statement. This contradiction shows that the enlarged set cannot be inconsistent – i.e., that the enlarged set is consistent.

The proposition is proven.

3 Acknolwledgments

This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and HRD-1834620 and HRD-2034030 (CAHSI Includes), and by the AT&T Fellowship in Information Technology.

It was also supported by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478, and by a grant from the Hungarian National Research, Development and Innovation Office (NRDI).

References

1. N. Bohr, "Discussion with Einstein on epistemological problems in atomic physics", In: P. A. Schlipp (ed.), *Albert Einstein: Philosopher–Scientist*, Open Court, Peru, Illinois, 1949, pp. 201–241; reprinted in: J. Kalckar, *Niels Bohr Collected Works, Vol. 7, Foundations of Quantum Physics II (1933–1958)*, Elsevier, Amsterdam, The Netherlands, 1996, pp. 339–381.
2. H. T. Nguyen and V. Kreinovich, "Classical-logic analogue of a fuzzy 'paradox' ", *Proceedings of the 1996 IEEE International Conference on Fuzzy Systems*, New Orleans, September 8–11, 1996, Vol. 1, pp. 428–431.
3. Tosefta Sotah 7:12, https://www.sefaria.org/Tosefta_Sotah.7.12