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How Probable is a Revolution? A Natural ReLU-Like Formula that Fits the Historical Data

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Abstract In his recent book “Principles for Dealing with the Changing World Order”, Ray Dalio considered many historical crisis situations, and came up with several data points showing how the probability of a revolution or a civil war depends on the number of economic red flags. In this paper, we provide a simple empirical formula that is consistent with these data points.

1 Formulation of the Problem

Economic crises often lead to internal conflicts. On many historical occasions, economic problems such as high inequality, high debt level, high deficit, high inflation, and slow (or absent) growth led to internal conflicts: revolutions and civil wars.

However, the internal conflict is not pre-determined by the state of the economy: sometimes an economic crisis causes a revolution, and sometimes a similar crisis is resolved peacefully.

What we would like to predict. There are many factors besides economy that affect human behavior. So, if we only consider the state of the economy, we cannot exactly predict if this state will cause a revolution or not. At best, we can predict the probability of a revolution.

What information we can use to make this prediction. Clearly, the more severe the economic crisis, the more probable is the revolution. A natural measure of severity is the proportion of economic “red flags”; for example:

- if we only have high inequality, but inflation remains in check and the economy continues growing, then the probability of a revolution is low;

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- on the other hand, in situations when high inequality is accompanied by high inflation and slow growth, revolutions becomes much more probable.

Empirical data. In Chapter 5 of his book [1], Ray Dalio describes the results of his statistical analysis of the problem. Specifically, he considered all known economic crises:

- he grouped them into groups depending on the proportion of red flags, and
- for each group, he counted the proportion of cases in which the economic crisis caused internal strife.

His results are as follows:

- when the proportion of red flags is smaller than 40%, then the probability of a revolution is 12%;
- when the proportion of red flags is between 40% and 60%, the probability of a revolution is 11%;
- when the proportion of red flags is between 60% and 80%, the probability of a revolution is 17%; and
- when the proportion of red flags is larger than 80%, the probability of a revolution is 30%.

What we do in this paper. In this paper, we provide a simple natural formula that is consistent with this empirical data.

2 Analysis of the Problem and the Resulting Formula

What happens when there are few red flags. It is natural to start our analysis with the first of the above groups, i.e., with the cases in which the proportion of red flags was reasonably low. In such cases, we observe an interesting phenomenon – that even we increase the proportion of red flags, the probability of a revolution does not increase. Actually, it is looks like the probability even decreases – from 12% to 11% – but this small decrease is not statistically significant and is, thus, probably caused by the fact that the sample size is not large.

It looks like in cases there are very few red flags, the probability of a revolution remains at the same level (small but still the same), all the way from 0% to 60% of red flags. The fact that revolutions happen even in good economic situations, when there are few – or even none – economic red flags, is in good accordance with our above comment that while economy is important, it is not the only factor affecting human behavior.

Natural conclusion: need to analyze the excess probability. Since it looks like there is some probability of a revolution even when economy is in good shape, what we can predict based on the proportion of red flags is the *increase* of this probability, i.e., the difference between:

- the probability corresponding to a given proportion of red flags and
- the economy-independence probability of a revolution.

The largest value of the proportion-of-red-flags at which the probability of a revolution still does not increase is 60%. At this level, the probability of a revolution is 11%. Thus, to study how economy affects the probability of a revolution, we need to compute the excess probability, i.e., to subtract 11% from all the empirical probabilities. As a result, we get the following data:

- when the proportion of red flags is smaller than 60%, then the excess probability of a revolution is $11-11=0\%$;
- when the proportion of red flags is between 60% and 80%, the excess probability of a revolution is $17-11=6\%$; and
- when the proportion of red flags is larger than 80%, the excess probability of a revolution is $30-11=19\%$.

Preparing for numerical interpolation. We know three values, and we want to extend these values to a formula that would describe the probability of a revolution y as a function of proportion of red flags x. In mathematics, such an extension is known as *interpolation*.

Usual interpolation formulas for the dependence $y=f(x)$ assume that we know, in several cases, the value of x and the corresponding value of y. In our case:

- we know the exact values of y,
- however, for x, we only know intervals.

So, to apply the usual interpolation techniques, we need to select a single point within each of these intervals.

Which point should we select? In statistical analysis, for each random variable X, the natural idea is to select a value x for which the mean square difference

$$E[(X-x)^2]$$

is the smallest possible. It is known (see, e.g., [5]) that this value is equal to the mean $x=E[X]$ of the corresponding random variable.

In our case, we only know that the value x belongs to some interval $[\underline{x},\overline{x}]$, we do not know what is the probability od different values from this interval. Since we have no reason to believe that some values x from this interval are more probable than others, it makes sense to assume that all these probabilities are equal, i.e., that we have a uniform distribution on this interval. This argument – widely used in statistical analysis – is known as Laplace Indeterminacy Principle; it is an important particular case of the general Maximum Entropy approach; see, e.g., [4].

For the uniform distribution on an interval, its mean value is this interval's midpoint. Thus, for interpolation purposes, we will replace each interval by its midpoint:

- the interval $[60,80]$ will be replaced by its midpoint

$$\frac{60+80}{2}=70, \text{ and}$$

- the interval $[80,100]$ will be replaced by its midpoint

$$\frac{80+100}{2} = 90.$$

After this replacement, we get the following conclusion:

- when the proportion of red flags is $x_1 = 60\%$, then the excess probability of a revolution is $y_1 = 0\%$;
- when the proportion of red flags is $x_2 = 70\%$, the excess probability of a revolution is $y_2 = 6\%$; and
- when the proportion of red flags is $x_3 = 90\%$, the excess probability of a revolution is $y_3 = 19\%$.

Actual interpolation. Interpolation usually starts with trying the simplest possible dependence – i.e., linear one $y = a + bx$, when the difference in y is proportion to a difference in x: $y_i - y_j = a \cdot (x_i - x_j)$ for all i and j, i.e., when the ratio

$$\frac{y_i - y_j}{x_i - x_j}$$

is the same for all pairs $i \neq j$.

In our case, we have

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{70-60} = \frac{6}{10} = 0.6$$

and

$$\frac{y_3 - y_1}{x_3 - x_1} = \frac{19-0}{90-60} = \frac{19}{30} \approx 0.63.$$

These two ratios are indeed close. If we take into account that, as we have mentioned earlier, a 1% difference between y-values is below statistical significance level, and that 19 could as well be 18 – for which

$$\frac{y_3 - y_1}{x_3 - x_1} = \frac{18-0}{90-60} = \frac{19}{30} = 0.6,$$

we get a perfect fit. Thus, we conclude that for $x > 60\%$, the dependence of y on x is linear: $y = 0.6 \cdot (x - 60)$. So, we arrive at the following formula.

Resulting formula. When we know the proportion x of red flags, we expect the excess probability y of the revolution to be equal to:

- $y = 0$ when $x \leq 60$, and
- $y = 0.6 \cdot (x - 60)$ for $x \geq 60$.

These two cases can be describe by a single formula

$$y = \max(0, 0.6 \cdot (x - 60)). \quad (1)$$

To get the full probability p of the revolution, we need to add the non-economic 11% level to y, and get

$$y = 11 + \max(0, 0.6 \cdot (x - 60)). \quad (2)$$

Comment. The formula (1) is very similar to the Rectified Linear Unit (ReLU) function – an activation function used in deep learning, which is, at present, the most effective machine learning tool; see, e.g., [3].

Why this formula? So far, we have provided purely mathematical/computational justification for this formula. How can we explain it from the viewpoint of economy and human behavior?

A natural explanation of the first part – when the probability of a resolution does not increase with x – is that in this situations, just like in physics, there is static friction: systems do not drastically change all the time, one needs to reach a certain threshold level of external forces to make the system drastically change.

Linear dependence after that is also easy to explain: as in physics (see, e.g., [2, 6]), most real-life dependencies are smooth and even analytical, and in the first approximation, any analytical dependence can be described by a linear function.

Caution. Of course, these are preliminary results, we should not trust too much formulas based on a few points.

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