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Sofia Holguin
The University of Texas at El Paso, seholguin2@miners.utep.edu

Vladik Kreinovich
The University of Texas at El Paso, vladik@utep.edu

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Shape of an Egg: Towards a Natural Simple Universal Formula

Sofia Holguin and Vladik Kreinovich

Abstract Eggs of different bird species have different shapes. There exists formulas for describing the usual egg shapes – e.g., the shapes of chicken eggs. However, some egg shapes are more complex. A recent paper proposed a general formula for describing all possible egg shapes; however this formula is purely empirical, it does not have any theoretical foundations. In this paper, we use the theoretical analysis of the problem to provide an alternative – theoretically justified – general formula. Interestingly, the new general formula is easier to compute than the previously proposed one.

1 What Is the Shape of an Egg: Formulation of the Problem

It is important to describe egg shapes. Biologists are interested in describing different species and different individual within these species. An important part of this description is the geometric shape of each biological object. Of course, in addition to the shape, we need to describe many other characteristics related to the object's dynamics.

From this viewpoint, the simplest to describe are bird eggs. First, their shapes are usually symmetric – and do not vary that much as the shapes of living creatures in general. Second, eggs are immobile. In contrast to other creatures their shape does not change – until they hatch. So, to start solving a more general problem of describing shapes of living creatures, a natural first step is to describe shapes of different eggs.

How egg shapes are described now. At present, there are several formulas that describe egg shapes.

Sofia Holguin and Vladik Kreinovich
Department of Computer Science, University of Texas at El Paso, 500 W. University
El Paso, Texas 79968, USA,
e-mail: seholguin2@miners.utep.edu, vladik@utep.edu

All known eggs have an axis of rotation. Usually, in describing the egg shape, this axis is used as an x-axis. Because of this symmetry, to describe the shape of an egg, it is sufficient to describe its projection on the xy-plane. Once we have this projection, all other points can be obtained by rotating this projection around the x-axis.

This projection can, in general, be described by a formula $F(x,y)=0$ for an appropriate function $F(x,y)$. Historically the first formula for the egg shape was produced by Fritz Hügelschäffer, see [1, 3, 4, 5, 6, 7, 9]. According to this formula, the egg shape has the form

$$y^2 = \frac{a_0 + a_1 \cdot x + a_2 \cdot x^2}{b_0 + b_1 \cdot x}. \tag{1}$$

This formula can be further simplified. First, we can use the same trick that is used to find the to the quadratic equation. Namely, by an appropriate selection of the starting point of the x-axis, i.e., by using a new variable $x' = x + x_0$, with $x_0 = a_1/(2a_2)$ we can simplify the numerator $a_0 + a_1 \cdot x + a_2 \cdot x^2$ to the simpler form. Indeed, since

$$a_2 \cdot (x')^2 = a_2 \cdot \left(x + \frac{a_1}{2a_2}\right)^2 = a_2 \cdot \left(x^2 + \frac{a_1}{a_2} + \left(\frac{a_1}{2a_2}\right)^2\right) = a_2 \cdot x^2 + a_1 \cdot x + \frac{a_1^2}{4a_2},$$

we have

$$a_0 + a_1 \cdot x + a_2 \cdot x^2 = a_0' + a_2 \cdot (x')^2, \text{ where } a_0' \stackrel{\text{def}}{=} a_0 - \frac{a_1^2}{4a_2}.$$

Second, we can divide both the numerator and the denominator by $a_2 > 0$, resulting in a simplified formula

$$y^2 = \frac{a_0'' + x^2}{b_0' + b_1' \cdot x}, \text{ where } a_0'' \stackrel{\text{def}}{=} \frac{a_0'}{a_2}, b_0' \stackrel{\text{def}}{=} \frac{b_0}{a_2}, \text{ and } b_1' \stackrel{\text{def}}{=} \frac{b_1}{a_2}. \tag{2}$$

For practical purposes, it turned out to be convenient to use a different formula for the exact same dependence:

$$y^2 = \frac{B^2}{4} \cdot \frac{L^2 - 4x^2}{L^2 + 8w \cdot x + 4w^2}, \tag{3}$$

for some parameters B, L, and w.

It turns out that some eggs are not well-described by these formulas. To describe such unusual egg shapes, [3] proposed a different formula

$$y^2 =$$

$$\frac{B^2}{4} \cdot \frac{(L^2 - 4x^2) \cdot L}{2(L - 2w) \cdot x^2 + (L^2 + 8L \cdot w - 4w^2) \cdot x + 2L \cdot w^2 + L^2 \cdot w + L^3}. \tag{4}$$

It is therefore desirable to come up with a single formula for the egg shapes, that would explain both the usual shapes (2)-(3) and the non-standard shapes (4).

Formulation of the problem. The following empirical general formula was proposed in [3]:

$$y = \pm\frac{B}{2}\sqrt{\frac{L^2 - 4\cdot x^2}{L^2 + 8\cdot w\cdot x + 4\cdot w^2}}\times$$

$$\left(1 - \frac{\sqrt{5.5L^2 + 11Lw + 4w^2}\cdot(\sqrt{3}BL - 2D_{L/4}\sqrt{L^2 + 2wL + 4w^2})}{\sqrt{3}BL\cdot(\sqrt{5.5L^2 + 11Lw + 4w^2} - 2\sqrt{L^2 + 2wL + 4w^2})}\times\right.$$

$$\left.\left(1 - \sqrt{\frac{L\cdot(L^2 + 8wx + 4\cdot w^2)}{2(L-2w)\cdot x^2 + (L^2 + 8Lw - 4w^2)\cdot x + 2Lw^2 + L^2w + L^3}}\right)\right).$$

However, this formula is purely empirical. It is desirable to come up with a general formula that has some theoretical foundations.

This is what we do in this paper.

2 Towards a General Formula

Main idea. In general, any shape can be described by a formula $P(x,y) = 0$ for an appropriate function $P(x,y)$. We have no a priori information about this shape. So, to get a good first approximation to the actual shape, let us use the usual trick that physicists use in such situations: expand the unknown function $P(x,y)$ in Taylor series and keep only the first few terms in this expansion; see, e.g., [2, 8]. The first non-trivial terms provide a reasonable first approximation; if we take more terms into account, we can get a more accurate description.

Details. In general, if we expand the expression $P(x,y)$ in Taylor series in terms of y, we get the following equation for the egg's shape:

$$P_0(x) + y\cdot P_1(x) + y^2\cdot P_2(x) + \ldots = 0, \tag{5}$$

for some functions $P_i(x)$.

As we have mentioned, the shape of an egg is symmetric with respect to rotations around an appropriate line. So, if we take this line as the x-axis, we conclude that with each point (x,y), the shape also contains a point $(x,-y)$ which is obtained by rotating by 180 degree around this axis. Thus, once the equation (5) is satisfied for some x and y, the same equation must be satisfied if we plug in $-y$ instead of y. Substituting $-y$ instead of y into the formula (5), we get:

$$P_0(x) - y\cdot P_1(x) + y^2\cdot P_2(x) + \ldots = 0. \tag{6}$$

Adding (5) and (6) and dividing the result by 2, we get a simpler equation:

$$P_0(x) + y^2 \cdot P_2(x) + \ldots = 0. \tag{7}$$

We want to find the smallest powers of y that lead to some description of the shape. According to the formula (7), the smallest power of y that can keep and still get some shape if the quadratic term – proportional to y^2. If we only keep terms up to quadratic tn terms of y, the equation (7) takes the following form:

$$P_0(x) + y^2 \cdot P_2(x) = 0, \tag{8}$$

i.e., equivalently,

$$y^2 = \frac{-P_0(x)}{P_2(x)}. \tag{9}$$

Since we kept only terms which are no more than quadratic in y, it makes sense to also keep terms which are no more than quadratic in x as well. In other words, instead of the general functions $-P_0(x)$ and $P_2(x)$, let us keep only up-to-quadratic terms in the corresponding Taylor series:

- instead of a general formula $P_0(x)$, we only consider a quadratic expression

$$-P_0(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2,$$

- and, instead of a general formula $P_2(x)$, we only consider a quadratic expression

$$P_2(x) = b_0 + b_1 \cdot x + b_2 \cdot x^2.$$

In this case, the equation (9) takes the form

$$y^2 = \frac{a_0 + a_1 \cdot x + a_2 \cdot x^2}{b_0 + b_1 \cdot x + b_2 \cdot x^2}. \tag{10}$$

Similarly to the transition from the formula (1) to the formula (2), we can simplify this formula by introducing a new variable $x' = x + x_0$ for which the numerator of the formula (10) no longer contains a linear term. In terms of this new variable, the formula (1) takes the simplified form given in the next section.

3 General Formula

Resulting formula.

$$y^2 = \frac{a'_0 + a'_2 \cdot (x')^2}{b'_0 + b'_1 \cdot x' + b'_2 \cdot (x')^2}. \tag{11}$$

Previous formulas are particular cases of this general formula. Let us show that the previous formulas (2)-(3) and (4) are particular cases of this formula. Indeed:

- the standard formula (2)-(3) corresponds to the case when $b' = 0$, i.e., when there is no quadratic term in the denominator;
- the formula (4) fits exactly into this formula.

Our general formula has exactly as many parameters as needed. The complex general formula proposed in [3] contain 4 parameters. At first glance, one may get an impression that our formula (11) has 5 parameters: a'_0, a'_2, b'_0, b'_1, and b'_2. However, we can always divide both numerator and denominator by a'_0 without changing the value of the expression. In this case, we get an equivalent expression with exactly four parameters:

$$y^2 = \frac{1 + a''_2 \cdot (x')^2}{b''_0 + b''_1 \cdot x' + b''_2 \cdot (x')^2}, \tag{12}$$

where $a''_2 \stackrel{\text{def}}{=} a'_2/a'_0$ and $b''_i \stackrel{\text{def}}{=} b'_i/a'_0$. Thus, our formula (11) is not too general – it has exactly as many parameters as the previously proposed general formula.

In what sense is our general formula better. First, our formula is theoretically justified. Second, as one can see, it is easier to compute than the previously proposed general formula.

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