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Why Self-Esteem Helps to Solve Problems: An Algorithmic Explanation

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Abstract It is known that self-esteem helps solve problems. From the algorithmic viewpoint, this seems like a mystery: a boost in self-esteem does not provide us with new algorithms, does not provide us with ability to compute faster – but somehow, with the same algorithmic tools and the same ability to perform the corresponding computations, students become better problem solvers. In this paper, we provide an algorithmic explanation for this surprising empirical phenomenon.

1 Formulation of the Problem

Self-esteem helps to solve problem: a well-known phenomenon. It is known that good self-esteem helps to solve problems; see, e.g., [1, 2, 3, 4, 7, 8, 9, 10, 11, 12]. There is a correlation between students' academic performance and their general self-esteem, there is an even higher correlation between the student's academic performance and their discipline-related self-esteem.

It is also known that helping students to boost their self-esteem makes them, on average, better problem solvers.

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Why is this a problem. The above phenomenon is such a commonly known fact that people do not even realize that from the algorithmic viewpoint, this does not seem to make sense. Indeed:

- From the algorithmic viewpoint, what we need to solve a problem is an appropriate algorithm and a sufficient amount of computation time.
- However, self-esteem does not mean we know new ways of solving the problem, it does not mean that we have gained additional time.

So why does it help?

What we do in this paper. In this paper, we provide a possible algorithmic explanation for this phenomenon.

2 Our Explanation

Clarification of the phenomenon: it is about problems with unique solution. The correlation between self-esteem and academic performance is usually at the level of straightforward problems – like arithmetic – where there is exactly one correct solution.

This does not mean that this phenomenon is limited to simple arithmetic; for example:

- the derivative of a function is also uniquely determined by this function, and
- most differential equations with given initial conditions have a unique solution.

The reason for this uniqueness limitation is clear:

- in such situations, we have an objective characteristic of the academic performance – e.g., the number of correct answers
- while in more vague situations – e.g., writing an essay about a book – criteria are much more subjective and thus, difficult to exactly quantify.

How uniqueness helps: reminder. Interestingly, uniqueness *does* help to find solutions to well-defined problems: namely, there is a general computational result that uniqueness implies computability; see, e.g., [5, 6] and references therein. Let us formulate this result in precise terms.

For real-life well-defined problems, solutions usually can be described by a finite list of numbers. Let us give two examples.

- If we are looking for a shape of a building or an airplane, this shape is usually described by splines, i.e., by several glued-together pieces each of which is characterized by a polynomial equation of a given degree. Thus, to describe the shape, it is sufficient to describe the coefficients of all these polynomial equations.
- If we are looking for the best way to invest money, then a solution means describing, for each possible way of investing (different stocks, different bonds, etc.) the exact amount of money that we should invest this way.

In all these cases, finding the desired solution means finding the values of the corresponding numerical quantities $x_1,\ldots,x_n$.

For each real-life quantity, there are usually limitations on possible ranges; examples:

- The amount of money invested in a given stock must be greater than or equal to 0, and cannot exceed the overall amount of money that we need to invest.
- Similarly, speed in general is limited by the speed of light, and the speed of a robot is limited by the power of its motors.

There may be other general limitations on these values x_i. For example, the building code may require that each building is able to withstand the most powerful winds that happen in this geographic area.

To properly describe such general limitations, it is important to take into account that in most real-life problems, the values x_i can only be implemented with some accuracy $\varepsilon > 0$. In other words, what we will have in reality when we, e.g., design the building, is some values $\widetilde{x}_i$ which are ε-close to x_i, i.e., for which $|\widetilde{x}_i - x_i| \le \varepsilon$. We want to be able to make sure that for all such ε-close values $\widetilde{x}_i$, the resulting design will satisfy the desired general limitation – e.g., withstand the most powerful winds that happen in this geographic area.

For each ε, we can list all possible approximations of this type. For example, if we consider values between 0 and 10 with accuracy 0.1, we can consider values 0, 0.1, 0.2, ..., 9.9, and 10.0. In general, we can similarly take values 0, ε, 2ε, ... By combining such values and selecting only the combinations that satisfy the desired constraints, we will have a finite list of possible tuples $(x_1,\ldots,x_n)$ such that every possible tuple is ε-close to one of the tuples from this list. A set for which we can algorithmically find such a list for each ε is known as *constructively compact*.

We are interested in finding the values x_i which are possible – in the sense that they satisfy the general constraints – and which satisfy the specific constraints corresponding to this particular problem. These constraints are usually described by a system of equations and inequalities

$$f_1(x_1,\ldots,x_n) = 0,\ldots, f_m(x_1,\ldots,x_m) = 0,$$

$$g_1(x_1,\ldots,x_n) \ge 0,\ldots, g_p(x_1,\ldots,x_n) \ge 0, \tag{1}$$

with computable functions $f_i(x_1,\ldots,x_n)$ and $g_j(x_1,\ldots,x_n)$. For example:

- we may be given the overall budget – a constraint of equality type, and
- within this budget, we want to make sure that the building has enough space to adequately host at least a pre-determined number of inhabitants – a constraint of an inequality type.

The first version of the uniqueness-implies-computability result is as follows:

- In general, no algorithm is possible that would always, given computable functions f_i and g_j on a constructive compact set C for which the system (1) has a solution, return this solution.

- There does exist a general algorithm that, given computable functions f_i and g_j on a constructive compact set C for which the system (1) has a *unique* solution, always returns this solution.

Comment about optimization. In many practical situations, the corresponding system (1) has many solutions. In this case, we are looking for the *best* solution, i.e., for the solution for which some objective function $h(x_1,\ldots,x_n)$ attains its largest (or its smallest) value. For example, we may want to minimize cost, or minimize the pollution, or maximize the number of passengers on an airplane.

For such problems, uniqueness also helps:

- In general, no algorithm is possible that would always, given a computable function h on a constructive compact set C, return the tuple $(x_1,\ldots,x_n)\in C$ at which this function attains its largest possible value
- There does exist a general algorithm that, given a computable function h on a constructive compact set C for which the maximum of h is attained at only one tuple, always returns this tuple.

Resulting explanation. For many problems given to K-12 students, there is exactly one solution.

- In these terms, self-esteem means that a student is confident that he/she can come up with a solution. By virtue of the above algorithmic result, this means that the student can, in principle, simply apply the general uniqueness-implies-computability algorithm and find the solution – even when this student is not yet fully able to apply techniques studied in class.
- On the other hand, without a good self-esteem, the student is not confident that he/she will come up with a solution. In such situation of non-uniqueness, no general algorithm is possible, so a student who is still struggling with the class material is, in general, not able to solve the corresponding problem.

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