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Spiral Arms Around a Star: Geometric Explanation

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Abstract Recently, astronomers discovered spiral arms around a star. While their shape is similar to the shape of the spiral arms in the galaxies, however, because of the different scale, galaxy-related physical explanations of galactic spirals cannot be directly applied to explaining star-size spiral arms. In this paper, we show that, in contrast to more specific physical explanation, more general symmetry-based geometric explanations of galactic spiral can explain spiral arms around a star.

1 Formulation of the problem

Spiral shapes are ubiquitous in astronomy. Spiral arms are typical in galaxies. For example, our own Galaxy consists largely of such arms.

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A recent discovery and a related challenge. Recently, very similar spiral arms were discovered around a star; see, e.g., [3]. How can we explain the appearance of spiral arms around a star?

Why is this a challenge? There are many physical theories that explain the appearance of logarithmic spiral arms in galaxies. However, because of the different scale, galaxy-related explanations cannot be directly applied to explaining spiral arms around a star.

What we do in this paper. In this paper, we show that while specific *physical* explanations of galaxy spiral arms cannot be applied to explain spiral arms around a star, a more general symmetry-based *geometric* explanation can be applied to the newly discieverd phenomenon.

2 Analysis of the problem and the resulting explanation

How are spiral arms in galaxies explained? For galaxies, several dozen different physical theories have been proposed that explain the same spiral shape. All these theories, based on completely different physics, successfully explain the same shape.

This led researchers to conclude that this shape must have a simple geometric explanation. Such an explanation has indeed been proposed; see, e.g., [1, 2].

Let us recall this explanation.

Initial symmetries and inevitability of symmetry violations. The distribution of matter close to the Big Bang was practically uniform and homogeneous. Thus, this distribution was invariant with respect to shifts, rotations, and scalings.

However, such a distribution is unstable. If at some point, density increases, then matter will be attracted to this point, and the disturbance will increase.

Thus, the original symmetry will be violated.

Which symmetry violations are more probable? According to statistical physics, it is more probable to go from a symmetric state to a state where some symmetries are preserved. For example, typically, when heating, the matter does go directly: from a highly symmetric crystal state to the completely asymmetric gas state. It first goes through intermediate liquid state, where some symmetries are preserved.

The more symmetries are preserved, the more probable the transition. From this viewpoint, it is most probable that matters forms a state with the largest group of symmetries.

Resulting shapes consists of orbits of symmetry groups. If there is a disturbance at some point a, and the situation is invariant with respect to some transformation g, this means that there is a disturbance at the point $g(a)$ as well.

So, with each point a, the resulting shape contains all the points

$$G(a) = \{g(a) : g \in G\}.$$

The set $G(a)$ is known as an *orbit* of the group G. For example, if G is the group of all rotations around a point, then $G(a)$ is the sphere containing a.

Original symmetry group. In the beginning, we have the following 1-dimensional families of basic transformations:

- three families of shifts $x \mapsto x + a$ – in all three dimensions,
- three families of rotations $x \mapsto Rx$ – around all three axes, and
- one family of scalings $x \mapsto \lambda \cdot x$.

We can combine them, so we get a 7-dimensional symmetry group.

What is the next shape? What is the shape with the largest symmetry?

If we have all three shifts, then from each point, we can get to every other point. In this case, the whole 3-D space is the shape. Thus, for perturbation shapes, we can have at most two families of shifts.

If we apply two families of shifts to a point, we get a plane. The plane also has:

- one family of rotations inside the plane, and
- one family of scalings.

Thus, the plane has a 4-dimensional symmetry group.

If we have one family of shifts, we get a straight line. It also has rotations around it and scaling. So, a straight line has a 3-dimensional symmetry group.

If we have no shifts, but all rotations, then we get a sphere. A sphere has a 3-dimensional symmetry group.

So far, the most symmetric shape is the plane. A detailed analysis of all possible symmetry groups confirms this.

So, the most probable first perturbation shape is a plane. This is in accordance with astrophysics, where such a proto-galaxy shape is called a pancake.

What next after a pancake? The planar shape is still not stable. So, the symmetry group decreases further.

From the 2-D shape of a plane, we go to a symmetric 1-D shape. The generic form of a 1-D group is exactly logarithmic spiral. In polar coordinates, it has the form $r = a \cdot \exp(k \cdot \theta)$. For every two its points (r, θ) and (r', θ'), there is spiral's symmetry transforming (r, θ) and (r', θ'):

- first, we rotate by $\delta = \theta' - \theta$,
- then, we scale $r \mapsto \exp(k \cdot \delta) \cdot r$.

This is what we observe in galaxies.

What is after spirals? What will happen next? The spiral is also unstable. So, we go from the continuous 1-D symmetry group to a discrete one.

In the resulting shape, we have points whose distances from a central point form a geometric progression $r_n = r_0 \cdot q^n$. Interestingly, this is exactly the Titius-Bode formula describing planets' distances from the Sun.

The only exception to this formula is that after Earth and Mars, this formula gets a distance to the asteroid belt between Mars and Jupiter. Astrophysicists believe that there was a proto-planet there torn apart by Jupiter's gravity, and asteroids are its remainder.

Resulting explanation of spiral arms around a star. A general geometric analysis shows that at some point, we should reach a spiral shape. This explains the observed spiral arms around a star.

This also explains why such arms are rare and were never observed before. Indeed, in our Solar system – and in other places – we have moved to the next stage, of planets following Titius-Bode law.

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