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3-1-2022

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Technical Report: UTEP-CS-22-40

Recommended Citation

Kosheleva, Olga; Kreinovich, Vladik; and Lopez, Louis Ray, "How to Make Quantum Ideas Less Counter-Intuitive: A Simple Analysis of Measurement Uncertainty Can Help" (2022). *Departmental Technical Reports (CS)*. 1679.
https://scholarworks.utep.edu/cs_techrep/1679

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How to Make Quantum Ideas Less Counter-Intuitive: A Simple Analysis of Measurement Uncertainty Can Help

Olga Kosheleva, Vladik Kreinovich, and Louis Ray Lopez

Abstract Our intuition about physics is based on macro-scale phenomena, phenomena which are well described by non-quantum physics. As a result, many quantum ideas sound counter-intuitive – and this slows down students' learning of quantum physics. In this paper, we show that a simple analysis of measurement uncertainty can make many of the quantum ideas much less counter-intuitive and thus, much easier to accept and understand.

1 Formulation of the Problem

Many quantum ideas are counter-intuitive. Many ideas of quantum physics are inconsistent with our usual physics intuition; see, e.g., [1, 2]. This counter-intuitive character of quantum ideas is an additional obstacle for students learning about these effects.

Problem: how can we make these ideas less counter-intuitive? To enhance the teaching of quantum ideas, it is desirable to come up with ways to make quantum ideas less counter-intuitive.

What we do in this paper. In this paper, we propose some ways to make quantum ideas less counter-intuitive. Specifically, in Section 2, we overview the main

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counter-intuitive ideas of quantum physics. In Section 3, we explain how a simple analysis of measurement uncertainty can help.

2 Main Counter-Intuitive Ideas of Quantum Physics

Quantum – discrete – character of measurement results. In non-quantum physics, most physical quantities can take any real values – or at least any real values within a certain range (e.g., possible values of speed are bounded by the speed of light). The change of values with time is smooth. It may be abrupt – as in phase transitions – but still continuous.

In contrast, in quantum physics, for most quantities, only a discrete set of values is possible. Correspondingly, changes are by discontinuous "jumps": at one moment of time, we have one energy value, at the next moment of time, we may measure a different energy value, with no intermediate values.

Non-determinism. We are accustomed to the fact that non-quantum physics is deterministic: if we know the exact current state, we can uniquely predict all future observations and all future measurement results. This does not mean, of course, that we can always predict everything: e.g., it is difficult to predict tomorrow's weather, because it depends on many today's values that we do not know. However, the more sensors we place to measure today's temperature, wind speed, and humidity at different locations (and at different heights), the more accurate our predictions – in line with the belief that if we knew where every atom was now, we could, theoretically, predict the weather exactly.

In contrast, in quantum physics, even if we know the exact current state, we cannot uniquely predict the results of future measurements. (At best, we can predict the probabilities of future measurement results.)

This non-determinism is mostly observed on the micro-level. Quantum effects are sometimes observed on the macro-level – e.g., with lasers – but mostly, they are observed on the micro-level.

There is no non-determinism in consequent measurement of the same quantity. In quantum physics, we cannot predict the result of a measurement: in the same state, we may get different results. However, in quantum physics, if after measuring the value of a quantity, we measure the same quantity again, we get the exact same result.

Entanglement. According to relativity theory, all speeds are bounded by the speed of light. As a result, an event happening a large distance from here cannot immediately affect the result of the local experiment.

However, in quantum physics, we can prepare a pair of particle in an "entangled" state. Then, we separate these two particles at a large distance from each other, and perform measurement on each of these particles.

In this arrangement, we cannot predict the result of each measurement, but, surprisingly, if we know the result of one of these measurements, we can make some prediction about the result of the second one as well.

This fact – known as Einstein-Podolsky-Rosen paradox – was experimentally observed. It has been shown that, strictly speaking, it does not violate relativity theory: namely, this phenomenon cannot be used to communicate signals faster than the speed of light. However, this phenomenon still sounds very counter-intuitive.

3 A Simple Analysis of Measurement Uncertainty and How It Can Help

A simple analysis of measurement uncertainty. Our information about values of physical quantities comes from measurements. Measurements are never absolutely accurate – and even if they were accurate, we would not be able to produce all infinitely many digits needed to represent the exact value, we have to stop at some point. For example, if we use decimal fractions, we cannot even exactly represent the value $1/3 = 0.33\ldots$, we have to stop at some point and use an approximate value

$$1/3 \approx 0.33\ldots 3.$$

In general, there is the smallest possible value h that we can represent, and all other values that we can represent are integer multiples of this value: 0, h, $2h$, $\ldots$, $k \cdot h$, $\ldots$, all the way to the largest value $L = N \cdot h$ that we can measure and represent.

This immediately makes discreteness and "jumps" less counter-intuitive. The first trivial consequence of the above analysis is that the discrete character of measurement results – and the abrupt transitions between measurement results – become natural.

How are these measurement results related to the actual (unknown) value of the corresponding quantity? In the ideal situation, when the measurement instruments are very accurate, and the only reason for discreteness is the need to have a finitely long representation, the result of the measurement comes from rounding the actual values – just like $0.33\ldots 3$ is the result of rounding 1/3, and $0.66\ldots 67$ is the result of rounding 2/3. So:

- the measurement result 0 means that the actual value is somewhere between $-0.5h$ and $0.5h$;
- the measurement result h means that the actual value is somewhere between $0.5h$ and $1.5h$; and
- in general, the measurement result $k \cdot h$ means that the actual value is somewhere between $(k - 1/2) \cdot h$ and $(k + 1/2) \cdot h$.

Non-determinism becomes less counter-intuitive. Let us consider the following simple situation. We have an inertial body traveling at the same speed in the same direction. We measure the distance d that it travels in the first second, and, based on this measurement, we want to predict the distance D that this body will travel in 2 seconds. From the purely mathematical viewpoint, if we ignore measurement uncertainty, the answer is trivial: $D = 2d$.

What will happen if we take measurement uncertainty into account? Suppose that the measurement of the 1-second distance led to the value $k \cdot h$. This means that the actual (unknown) distance is located somewhere in the interval

$$[(k-1/2)\cdot h, (k+1/2)\cdot h].$$

In this case, one can easily see that the value $D = 2d$ is located in the interval $[(2k-1)\cdot h, (2k+1)\cdot h]$. If we round different values from this interval, we get three possible values: $(2k-1)\cdot h$, $2k\cdot h$, and $(2k+1)\cdot h$. Thus, even when we know that the original value was $k \cdot h$, we cannot uniquely predict the result of the next measurement: it can be one of the three different value.

In other words, we have non-determinism – exactly as in quantum physics.

This non-determinism is mostly observed on the micro-level. On the macro-level, when the values k are much larger than 1 (which is usually denoted by $1 \ll k$), the non-determinism is relatively small: the relative error in predicting $\widetilde{D} = 2k \cdot h$ where the actual value is $D = (2k-1)\cdot h$ is equal to

$$\frac{\widetilde{D}-D}{D} = \frac{h}{2k\cdot h} = \frac{1}{2k} \ll 1.$$

On the other hand, for small values, e.g., for $k = 1$, the resulting relative error is $1/(2k) = 50\%$ – a very large uncertainty.

There is no non-determinism in consequent measurement of the same quantity. We cannot predict what will be the result of measuring D, but if we measure it again right away, we will, of course, get the same result as last time – just like in quantum physics.

Entanglement. Let us assume that we have two particles 1 and 2 at the same location. Measurement shows that both their electric charges are 0s, and that the overall charge of both particles is 0. Then, we separate these particles and place them in an electric field where the force acting on a particle with charge q is equal to $4q$.

Let us see what will happen in this situation if we take measurement uncertainty into account. The fact that the measured values of both charge q_1 and q_2 is 0 means that the actual values of each of these charges is located in the interval $[-0.5h, 0.5h]$. The fact that the measurement of the overall charge $q_1 + q_2$ result in 0 value means that this overall charge is also located in the same interval $[-0.5h, 0.5h]$.

For each particle i, the only thing we can conclude about the actual value of the force $F_i = 4q_i$ is that this value located in the interval $[-2h, 2h]$. So, in principle, possible measured values of the force are $-2h$, $-h$, 0, h, and $2h$. The fact that the overall charge was measured as 0 does not restrict possible values of the measured force; indeed, for the first particle:

- it could be that $q_1 = -0.5h$ and $q_2 = 0$; in this case, the observed value of the force F_1 is $-2h$;
- it could be that $q_1 = -0.3h$ and $q_2 = 0$; in this case, the observed value of the force F_1 is $-h$;
- it could be that $q_1 = 0$ and $q_2 = 0$; in this case, the observed value of the force F_1 is 0;
- it could be that $q_1 = 0.3h$ and $q_2 = 0$; in this case, the observed value of the force F_1 is h;
- it could be that $q_1 = 0.5h$ and $q_2 = 0$; in this case, the observed value of the force F_1 is $2h$.

Similarly, all five values are possible for the measured value of the force F_2.

However, this does not mean that all possible combinations of measured force values are possible. Indeed, while it is possible that the measured value of F_1 is $-2h$, and it is possible that the measured value of F_2 is $-2h$, it is *not* possible that both these measured values are equal to $-2h$. Indeed:

- such situation would mean that $F_1 \in [-2.5h, -1.5h]$ and $F_2 \in [-2.5h, -1.5h]$, thus $F_1 + F_2 \in [-5h, -3h]$, and
$$F_1 + F_2 \le -3h;$$
- however, we know that $F_1 + F_2 = 4 \cdot (q_1 + q_2)$, and since $q_1 + q_2 \in [-0.5h, 0.5h]$, we get $F_1 + F_2 \in [-2h, 2h]$, and
$$-2h \le F_1 + F_2.$$

The two resulting inequalities cannot be both true, so this case is indeed impossible.

Thus, if we measure the force F_1 acting on the first particle and get the measurement result $-2h$, this would mean that we can immediately predict which values of the force measured at a faraway particle are possible and which are not possible – exactly as in the quantum entanglement situation.

Warning. While, as we have shown, the analysis of measurement uncertainty helps make quantum ideas less counter-intuitive, it is important to remember that the corresponding mathematics of quantum physics is very different from the mathematics of measurement uncertainty.

On the qualitative level, yes, there is some similarity, and it may help the students, but on the quantitative level quantum physics is completely different.

Comment. In our analysis, we used simple examples of arithmetic operations on uncertain numbers. It may be beneficial to provkide a general description of such situations: if we have the measurement results $i \cdot h$ and $j \cdot h$, then for each arithmetic operation $\odot$ – be it addition, subtraction, multiplication, or division – we can define $(i \cdot h) \odot (j \cdot h)$ as the set of all roundings of all the values $a \odot b$, where

$$a \in [(i-1/2)\cdot h, (i+1/2)\cdot h] \text{ and } b \in [(j-1/2)\cdot h, (j+1/2)\cdot h].$$

Acknowledgments

This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and HRD-1834620 and HRD-2034030 (CAHSI Includes), and by the AT&T Fellowship in Information Technology.

It was also supported by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478, and by a grant from the Hungarian National Research, Development and Innovation Office (NRDI).

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