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Why Pre-Teaching: A Geometric Explanation

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WHY PRE-TEACHING: A GEOMETRIC EXPLANATION

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Abstract Traditionally, subjects are taught in sequential order: e.g., first, students study algebra, then they use the knowledge of algebra to study the basis ideas of calculus. In this traditional scheme, teachers usually do not explain any calculus ideas before students are ready – since they believe that this would only confuse students. However, lately, empirical evidence has shows that, contrary to this common belief, pre-teaching – when students get a brief introduction to the forthcoming new topic before this topic starts – helps students learn. In this paper, we provide a geometric explanation for this unexpected empitical phenomenon.

Keywords*: Pre-teaching, abstract to a paper, geometric model of learning.*

FORMULATION OF THE PROBLEM

Traditional approach to teaching. Traditionally, students learn the material in sequential order. For example, when learning mathematics, they first learn arithmetic, then they learn algebra, then they learn the basics of calculus. When the students are still struggling with arithmetic, we do not explain to them that algebra can help (and, by the way, many elementary-school problems become easier if we use algebra). Similarly, when students are still still learning algebra, we do not teach them elements of calculus.

Why traditional approach. The main reason why teaching is done this sequential way is that teachers believe that introducing even the basics of the new topic when the students are still stuggling with the previous topic will only confuse the students.

And indeed such a confusion does happen. For example, elementary school students, when struggling with complex arithmetic problems, often ask their parents and/or their older siblings for help. These helpers already know algebra, so for them, a natural way to solve many of these problems is to use algebra, to denote the unknowns by x and y and to solve the resulting equations or systems or equations. However, when they explain their solution to the students, many students get confused even more.

Unexpected success of pre-teaching. While unscheduled accidental introduction of a new topic is indeed usually counter-productive, recent experience of many teachers has shown that a planned brief introduction of the main ideas of the next topic -- before the student started studying thos topic deeply – actually drastically improves the effectiveness of teaching.

But why? The empirical success of this strategy – known as *pre-teaching* – raises the following natural question: why is this strategy successful?

What we do in this paper. In this paper, we provide a simple geometric explanation of the success of pre-teaching.

GEOMETRIC MODEL AND THE RESULTING EXPLANATION

Geometric model. The overall objective of studying all the topics of a given subject is to help the students move from the original state s – in which they do not have any knowledge in any of these topics – to the desired final state S, in which they have at least satisfactory (and ideally perfect) knowledge of all these topics.

Each student's state of knowledge can be characterized by how well the student knows all these topics. We can gauge the student's knowledge of each topic by a number – e.g., by this student's grade on the corresponding test. Thus, the state of knowledge of each student at each moment of time can be characterized by the tuple of the corresponding grades (x, y, …). This tuple can be naturally represented as a point in the multi-D space whose dimension d is equal to the number of topics.

In these geometric terms, the original state – when the students do not have any knowledge about any of the topics – is the state (0, 0, …).

The exact numerical description of the desired final state depends on the scale. Our analysis and our conclusion work for any grading scale. So, without losing generality, let us use the usual US scale, in which perfect knowledge is described by 100 points. In these terms, the desired final state is the state (100, 100, …).

Our objective is to bring the students from the initial state (0, 0, …) to the desired state (100, 100, …), and to bring them as efficiently as possible – i.e., by the shortest possible path.

TRADITIONAL APPROACH VS. PRE-TEACHING: GEOMETRIC ANALYSIS

Let us describe both traditional and pre-teaching approached in terms of our geometric model.

Traditional approach. In the traditional approach, we first achieve the student's perfect knowledge in the first topic. In geometric terms, this means that we go from the original state (0, 0, …) to the state (100, 0, …). In this state, the student has perfect knowledge (100 points) of the first topic and no knowledge of all other topics.

After that, we start teaching the second topic and go from the state (100, 0, …) to the state (100, 100, 0, …). Next, we teach the third topic, etc. On each stage, i.e., for each of these topics, the length of the path between the starting and the final states of this stage is equal to 100:

- On the first stage, this is the length of the straight line path between the points (0, 0, …) and (100, 0, …).
- On the second state, this is the length of the straight line path between the points (100, 0, …) and (100, 100, 0, …), etc.

We have d topics, so we have d stages. On each stage, the length of the corresponding path is equal to 100. Thus, the overall length of the learning path corresponding to the traditional approach is equal to 100 * d.

But is this the shortest distance? From the geometric viewpoint, the shortest path between the points (0, 0, …) and (100, 100, …) is the straight line connecting the two points, i.e., the line formed by the points (a, a, …) where the value a goes from 0 to 100. The length of this shortest path is equal to 100 * sqrt(p), where sqrt(p) denotes the square root. This length is much shorter than the value 100 * p corresponding to the traditional teaching. Even in the simplest case, when we only have two topics, we get a 30% decrease in path length – i.e., equivalently, a 30% decrease in time needed to teach both topics. In general, the more topics we have, the larger the decrease.

So, a natural conclusion is that to teach more effectively, we need to go from the traditional approach to a more efficient approach -- which is closer to the shortest path.

What does efficiency mean in this case? The shortest path means that as we increase the value of the first coordinate – i.e., as student learn the first topic – we do not keep the value of ths second coordinate (i.e., the student's level of knowledge in the second topic) at 0. Instead, while the students are still studying the first topic, we should already introduce, as early as possible, elements of the following topics.

This is exactly what pre-teaching is doing. This is exactly what pre-teaching is about: introducting concepts from the next topic(s) while students are still studying the material from a previous topic. In this sense, our model indeed explains why pre-teaching is effective.

ADDITIONAL ARGUMENTS IN FAVOR OF PRE-TEACHING AND OTHER APPLICATIONS OF THE CORRESPONDING GENERAL IDEA

Addional arguments on favor of pre-teaching. How do we all learn to talk? Peoplea all around us talk. Yes, sometimes parents explicitly teach us the basics first, but they do not limit the words that they use when talking in front of their kids to these basics. This way, while we are still learning the basics, we also get exposure to more complex constructions. We learn these constructions only later, but the original exposure probably helps.

How students learn a foreign language? The traditional approach is to first learn the pronunciation, then learn the basic words and constructions, etc. However, the usual language advice to folks who just came to the US is to watch TV: not special educational programs where the language is limited, but regular TV programs where all kinds of words and constructions are used – and in general, to immerse into the language culture, to communicate as much as possible.

For mathematics, there is a famous example of Sofia Kovalevsky: when she was a child, her father, a teacher of mathematics, did not have enough money to buy fancy wallpaper for her room, so he used pages from an old disacreded calculus book as wallpaper. Clearly, as a small child, she did not understand the formulas with integrals and derivatives but, as she mentioned in her memoirs, some understanding did occur, since the memories of these formulas made it easier for her later one to study calculus.

Other applications of the general idea. The above geometric idea can be used beyond school teaching. First, it can be applied to adult teaching as well. We professionals learn all the time – mostly by reading research papers. It is interesting to mention that until some time in the 20 century, the only way to learn from the paper was to read it from the beginning to the end. Now, all papers are required to come with abstracts – i.e., in effect, with pre-teaching. After the abstract, comes Introduction, which is a more detailed dcescription of the material, and only then, the main part.

On a different topic: religious people want to be both successful in life and reach spiritual heights. A traditional approach is to live normal life and then go to a church and pray, but it turns out that a more efficient approach is to combine these activities – e.g., before starting to pray, to get oneself into a spiritual mood, i.e., in effect, to pray that your prayers will be successful.

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