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How to Gauge the Quality of a Multi-Class Classification When Ground Truth Is Known with Uncertainty

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Abstract The usual formulas for gauging the quality of a classification method assume that we know the ground truth, i.e., that for several objects, we know for sure to which class they belong. In practice, we often only know this with some degree of certainty. In this paper, we explain how to take this uncertainty into account when gauging the quality of a classification method.

1 Formulation of the Problem

Traditional methods of gauging the quality of a classification method assume that we know the ground truth. In other words, we assume that for some elements, we know, with certainty, to which class they belong. E.g., in medical diagnostics, we assume that for some patients, we know, with absolute certainty, what was the correct diagnosis.

In real life, however, we are rarely absolutely certain. Usually, there is some degree of uncertainty, some of the "known" classification may turn out to be wrong. Because of this, the values $\widetilde{v}$ of the quality measures that we get when we assume the known classifications to be absolutely true are, in general, different from the ideal values v – that we would have gotten if we knew the actual ground truth. How can we gauge the resulting uncertainty in v?

In the previous papers, this problem was analyzed for the case of 2-class ("yes"-"no") classification; see, e.g., [1]. In this paper, we start extending these ideas and results to the general multi-class case. Specifically, we analyze the uncertainty in accuracy.

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2 Notations: Traditional Approach

Let us introduce the notations needed to describe the traditional methods – that assume that we know the ground truth.

- Let C denote the number of possible classes.
- Classes will be denoted by numbers $c = 1, 2, \ldots, C$.
- Let N be the number of objects whose classification we know.
- Let P_c denote the set of all the objects in the c-th class.
- Let S_c bethe set of all objects that the tested method classifies as belonging to the c-th class.
- By $|S|$, we denote the number of elements in the set S.

The accuracy A is defined as the proportion of correctly classified objects:

$$A = \frac{M}{N}, \text{ where } M \stackrel{\text{def}}{=} \sum_{c=1}^{C} |P_c \cap S_c|.$$

3 Realistic Approach: Formulation of the Problem

In practice, experts are not 100% sure about their classification.

- We have the number $\widetilde{N}$ of objects about which experts provided opinions.
- We know the sets $\widetilde{P}_c$ of all objects that experts classified to the i-th class.
- For each object i, we know the expert's probability p_i that his/her classification of this object is correct.

Based on the expert opinions, we compute the accuracy as

$$\widetilde{A} = \frac{\sum_{c=1}^{C} |\widetilde{P}_c \cap S_c|}{\widetilde{N}}.$$

An important question is: how close is this estimate to the actual accuracy A?

4 Our Solution

Let $\xi(i)$ be 0 or 1 depending on whether the expert's classification of the i-th object is correct. Then:

- with probability p_i, we have $\xi(i) = 1$, and
- with the remaining probability $1 - p_i$, we have $\xi(i) = 0$.

Thus, the mean value and the variance of these variables are

$$E[\xi(i)] = p_i \text{ and } V[\xi(i)] = p_i \cdot (1 - p_i).$$

In these terms, $A = \dfrac{M}{N}$, where:

$$N = \sum_{i=1}^{\widetilde{N}} \xi(i) \text{ and } M = \sum_{c=1}^{C} |P_c \cap S_c| = \sum_{i \in \bigcup\limits_{c=1}^{C} E_c \cap S_c} \xi(i).$$

For large $\widetilde{N}$, a linear combination of a large number of relatively small independent random variables is, in effect, normally distributed. This follows from the Central Limit Theorem; see, e.g., [2]. Thus, both N and M are normally distributed. We can therefore find the distribution of A as the ratio of two random variables M/N with a joint normal distribution.

A joint normal distribution is uniquely determined by its means, variances, and covariance. Here:

$$E[N] = \sum_{i=1}^{\widetilde{N}} p_i, \;\; V[N] = \sum_{i=1}^{\widetilde{N}} p_i \cdot (1 - p_i),$$

$$E[M] = \sum_{i \in \bigcup\limits_{c=1}^{C} E_c \cap S_c} p_i, \;\; V[M] = \sum_{i \in \bigcup\limits_{c=1}^{C} E_c \cap S_c} p_i \cdot (1 - p_i).$$

Here, $N - M$ and M contain different variables and are, thus, independent. Similarly, $(N - E[N]) - (M - E[M])$ and $M - E[M]$ are also independent, with mean 0. Thus:

$$E[((N - E[N]) - (M - E[M])) \cdot (M - E[M])] =$$

$$E[(N - E[N]) - (M - E[M])] \cdot E[(M - E[M])] = 0.$$

Hence, for the covariance, we get

$$C(N,M) \stackrel{\text{def}}{=} E[(N - E[N]) \cdot (M - E[M])] =$$

$$E[((N - E[N]) - (M - E[M])) \cdot (M - E[M])] + E[(M - E[M])^2] = V[M].$$

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