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Connected Mathematics Learning and Gender Equity in Predominantly Latino/a High Schools: Case of Spatial Reasoning

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CASE OF SPATIAL REASONING

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Dedication

I dedicate this dissertation to all of my family members who believed in me and prayed for me all these years; to my mom, Maria Esperanza Martinez Falcon, my father, Julian Avila Falcon, my children, Jeremy Nathan, Jonathan Ray, Ray Michael, Nayomi Irene, and Michael Raymond. This dissertation also goes to my Tia Sara who passed away and was always proud of my education.

CONNECTED MATHEMATICS LEARNING AND GENDER EQUITY IN PREDOMINATELY LATINO/A HIGH SCHOOLS: CASE OF SPATIAL REASONING

by

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DISSERTATION

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Abstract

This study analyzed interventions used in improving the mathematics achievement in spatial reasoning tasks for females called connectedness. Gender achievement in mathematics has been a controversial topic because of the wide variance in research. Some research has found a difference between the genders in mathematics while others argue there is no difference in mathematical achievement. The Seven Clever Piece Tangrams were used in the mixed method study as the instrument of spatial reasoning tasks. Freshmen participants (N=719) from southwest high schools in a border town participated in one of two groups: control (n=247) and treatment (n=472). Of the participants, 379 were male and 340 were female. The participants were predominately Latino/a (83.6%). Of the two groups, the treatment group received a connectedness intervention based on feminist epistemologies regarding mathematical reasoning, multiple strategies, and social cognition. ANOVA results show the treatment group increased scores more than the control group ($p<0.05$) in spatial reasoning in which we attribute to connectedness activities. Furthermore, females further increased their scores more than males. The findings of the study confirm an achievement disparity between genders and validate the intervention of connectedness as a factor in decreasing gender difference in success in spatial reasoning tasks.

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Chapter 1: Introduction

"Knowledge emerges only through invention and re-invention, through relentless, impatient, continuing, hopeful inquiry human beings pursue in the world, with the world, and with each other."

(Paulo Freire, Pedagogy of the Oppressed, 2005)

1.1 INTRODUCTION

There has been discussion and research arguing whether or not a disparity exists between females and males in regards to mathematical achievement in schools. The two views disagree with research on both sides of the spectrum with various reasons and studies to back up their arguments. Some researchers argue there is an achievement gap (Gluck & Fitting, 2003) and others have argued there is no achievement gap (Hyde & McKinley, 1997; Hyde & Mertz, 2009). Mathematical scores have increased for fourth graders and eighth graders over recent years, but a small gender gap continues to exist (NAEP, 2012). The disparity between the genders in mathematical achievement appears around elementary school and continues through middle school (NAEP, 2012). Others argue there is no difference in mathematical achievement between boys and girls at any grade level (Hyde, Lindberg, Linn, Ellis, & Williams, 2008). However, during the middle school years, females' confidence in mathematics begins to decrease (Good, Ratta, & Dweck, 2012). More specifically, researchers have found a disparity between the genders in spatial reasoning ability (Contreras, Martínez-Molina, & Santacreu, 2012) and mental rotation tasks (Heil, Jansen, Auaiser-Pohl, & Neuburger, 2012). One thing favorable is that the genders take the same amount of mathematics courses and are generally equal in preparation from high school courses to enter the Science, Technology, Engineering, and Mathematics (STEM) careers; however, few females actually do (AAUW, 2010; see also AAUW, 2008).

The proposed research is a mixed method study (Creswell, 2008; 2011; Tashakkori & Teddlie, 2002) conducted through the lens of feminist epistemology (Belenky, Clinchy, Goldberger, & Tarule, 1997; Brister, 2009; Burton, 1995; Duran, 2003; Harding, 1991) and Chicana epistemology (Delgado-Bernal, 1998) to analyze if there is a difference in the spatial reasoning performance between males and females and if an intervention named connectedness statistically decreased the achievement disparity. The majority of the participants were of Latino/a ethnicity. Latinas were the majority of the females (85%) and the males were primarily Latino (75%). This study issues the following research questions:

- Research Question #1: Is there a difference between females and males' performance on spatial reasoning tasks across the elementary, middle school, and high school levels?
- Research Question #2: To what extent does connectedness improve females' performance on spatial reasoning tasks?
- Research Question #3: a) How do male and female students' strategies differ on spatial reasoning tasks after implementation of connectedness and b) how much do male and female students' strategies change on spatial reasoning tasks after implementation of connectedness?

This research analyzed the achievement of not only male and females in spatial reasoning tasks but allows access on how Latino/as also perform on spatial reasoning tasks. Illustrating the significance of achievement gaps between Latina/o students and White students is necessary in creating interventions to a possible achievement disparity in spatial reasoning. With the high numbers of Latina/o students coming to our classrooms and schools, educators across the country need research in this area. The National Council of Mathematics (NCTM) also creates literature for assisting teachers in mathematics and diversity (NCTM, 2009).

There is a lacuna on research analyzing the difference in achievement in mathematical spatial reasoning among Latinas. Research has surely looked at whether there is an achievement difference or not between the genders in spatial reasoning (Gluck & Fitting, 2003; Hyde & McKinley, 1997; Hyde & Mertz, 2009). This study will first look at whether or not achievement dissimilarity exists between males and females. Having found a statistically significant difference in achievement levels between the target groups, this study implemented an intervention in order for females to perform at the same level of their male counterparts in spatial reasoning tasks.

This mixed method nested study also includes sub-studies. The mixed method was chosen in order to collect quantitative data and qualitative data and to determine how one method validates the other. The first segment of the study implements a sub-study. The sub-study will determine whether or not there is an achievement gap between the genders in the elementary, middle, and high school levels. The quantitative segment of the study is comprised of two groups, a control group and a treatment group. The control group will serve as the baseline of the study. The treatment group in this study will receive the intervention of connectedness. This will determine whether or not connectedness is the reason for the growth in scores and the closing of the variance between females and males. After the data is collected, the qualitative portion will allow for further analysis on student thinking involving spatial reasoning tasks by interviewing students.

This study will look at several ideas and theories based on feminist epistemology. Under this theory, the research focused on how girls learn and know what they know. Furthermore, other ideas the research will examine are those of connected knowers. What is connected knowing and how does this study apply it in the context of spatial reasoning and female learning? Second, what does research say about gender differences in mathematics, knowing, and learning? Third, are Latino/as at a disadvantage in schooling and in mathematics? These guiding questions provide a framework for the study.

The organization of this chapter will begin with the background of the researcher, followed by the background of the study, research focus, research aim and objectives, value of the research, and a brief summary of the subsequent chapters.

1.2 BACKGROUND OF THE RESEARCHER

In selecting a topic as my research, my schooling as a Latino student in a predominantly White conservative catholic German ancestry school based in a small north Texas town served as my part of my motivation to transform today's schools and curricula. My motivation came from being able to better serve students who share my culture and be an advocate for female higher achievement in mathematics. My opportunities as an educator in the schools system for almost fifteen years and as a Latino student in a predominantly White school gave me plenty of experience. My experience as a Latino student and the struggles with a school and a curriculum did not explore nor understand my uniqueness of language, culture, and identity. Ogbu (1992) would say my situation is one of a voluntary minority, but my feelings are that of an involuntary minority. Ogbu dissected the ways minorities become minorities; voluntary and involuntary. Voluntary minorities were able to place themselves in a specific place in society from their own volition. Involuntary minorities were those that were placed into a specific society without their consent. A culture of Whiteness and a label of being illiterate in English with a need of special services (due to a second language at home even though English was the dominant language my parents spoke to me), were not only demeaning but also discriminatory. My low capability of reading, writing, and speaking Spanish were ignored by the school as they continued to place me in pull-out programs to improve my spelling, grammar, and English vocabulary, which were already above average. This became a regular program for all Latino/a students. High school also encapsulated White thinking and ideals in curriculum, particularly in mathematics education. Math textbooks were certainly White dominated with the absence of all cultures and their mathematical

ideologies. Never once were cultures of Aztecs, Incas, Africans, or Asian and their contributions to the mathematics of the world ever taught in my school.

My experience as a mathematics educator in the middle school for over fifteen years also gave me insight into how females learn. As an educator in a predominantly Latino/a middle school population in a southwest border town, my experiences as a Latino student enabled me to understand the needs of students who share my culture and strive to understand students from other cultures. What brought me to research females in their achievement in mathematics and more specifically Latinas came from my experience as a teacher. First of all, as a teacher in a predominately low socio-economic status middle school which held at least 93% of Latino/as in the southwest, my first principal was female. She had once accused me in a public faculty meeting of favoring boys over girls. This, of course, struck me in my heart. Although, being the first to admit my imperfectness, never had such an idea or manner been shown in my classroom. My mathematics classroom was one of a critical pedagogy nature (Darder, Baltodano, &Torres, 2009; Freire, 2005; Giroux, 1988, 2011; Kincheloe, 2008; McLaren, 2007; Rossatto, Allen, & Pruyn, 2006) empowering not just females in mathematics achievement but everyone. Not just Latinas, but all females in the classroom excelled just as well as their male counterparts in mathematic assignments and tests. The principal changed her position of my supposedly unfair practices, when the results of the year's state assessment showed how the females outperformed the males in the classroom in percentage of those passing. My curriculum and pedagogy has always been a strong proponent of assisting females in achieving highly in mathematics class. My classroom was not one which favored one gender over the other due to my own male gender. This brings another topic of discussion, my own gender as a male teacher and the influence over students.

Historically, the mathematical sciences have been predominately male dominated. I also argue that my own gender has placed me in the position of unearned privileges. A male researcher conducting a study regarding feminist epistemology and equitable mathematics achievement may be problematic to

some. Some female readers may prefer for another female to tell them how to increase female achievement in mathematics. Society has preferences in who should give them advice in changing their practices or views. As a critical pedagogist, I acknowledge my own insufficiencies in conducting research regarding females. How could I possibly understand what a female is thinking and feeling in a mathematics class? I do not propose to know everything in regards to feminism, feminist epistemology, and feminist thinking. However, I have learned about female thinking and learning from these theories. It has molded my teaching in mathematics to include ideals in promoting both genders. I also speak from years of experience teaching and learning from females in my own math courses. I believe this has given me an insight into the struggles and obstacles they encounter in mathematics. I believe it makes me a qualified candidate in of narrating the females' voice speaking for the hundreds of young ladies I have taught in my career.

I also understand my sense of unearned privileges received over the years. Being born a male has given me unearned sense of status in society; with it also comes unearned sense of privileges. I denounce those privileges in being an advocate for females. I have deconstructed those systems of privileges and try hard every day to prevent them from placing me ahead of females. Much like Delgado-Gaitan (1993), I also understand that as an outsider, I will never completely understand the struggles of females. Despite my best efforts, I will never become an insider.

As a critical pedagogist, my views, work, and research revolve around critical pedagogy. Based on the works of Paulo Freire, critical pedagogy is a teaching philosophy which promotes empowerment, liberation, equality, access, diversity, transformation, reflection, etc. It challenges students and teachers to question structures in society which hold beliefs and practices of domination. Society is not developed by the poor, but by hegemonic forces which create structures in schools, communities, politics, economics, religion, and globalization to ensure their beliefs and practices reign supreme over others. Critical pedagogy is the neutralizing factor which reveals and deconstructs these structures and

hegemonic forces which disenfranchise humans. It questions who, what, where, why, and how these structures and forces came to existence. One of these forces this study will investigate is the differences in mathematical achievement between males and females.

Critical pedagogy is a key component in schooling in order to create a relationship between teaching and learning. It is merely an approach to better understand the world around us. Teachers develop method and strategies which assist students in questioning their world in a concept called critical consciousness. Critical consciousness is the removal of the blinds placed in front of our eyes to oppress and dominate. With these blinds, we do not question. Critical consciousness is the awareness of oppressive structures which exist locally and globally which have been put in place to serve the hegemonic group. Teachers and students together learn how to unlearn what they have learned. Through a process of unlearning, students relearn, reflect, and evaluate society.

Critical pedagogy has not gone without its own critics. Paulo Freire also had his share of criticism (Ladson-Billings, 1997; Weiler, 2001). Freire did not touch upon feminist issues which is an important part of this study. Freire by no means was a perfect human being but always stated for the world to reinvent him.

One reason for studying and researching feminist epistemologies is to better my own understanding of females' thinking and learning in order to create an intervention to assist them in mathematical achievement. Conducting this type of research forces my own understanding of unearned privileges as a male to better create these interventions to assist females in their own understanding of any mathematical context. Furthermore, in no way am I a proponent against males thinking and learning in mathematics. It is within this study, I try to develop an intervention to assist females in opening doors of careers in the mathematical sciences. I am not a proponent of one gender over the other; but in establishing curriculum, teaching and learning techniques which promotes Pedagogy, Equity, Mathematics, Diversity, and Social Justice (PEMDAS) for both genders.

1.3 BACKGROUND OF STUDY

This section will establish the research in a specific context of Latina education, provide reasons in why this study is important, define important terms, provide some necessary background information, statement of the problem, and the theoretical framework of the study.

There has been concern about the achievement of females in mathematics particularly within minority groups. Latina/os are at the bottom of the mathematics achievement list behind their counterparts of Caucasian, Asians, and African Americas (National Council of La Raza, 1999). This has caused much concern among states and school districts across the nation. A Latina/o education fact sheet from the National Council of La Raza (NCLR) reveals some startling statistics. Elliot (2005) states,

> Latinas have dramatically lower education levels than their peers. In 2004, 41.8% of all Latinas age 15 to 64 did not have high school diplomas, compared to 17.1% of White women, 22.7% of Black women, and 15.6% of Asian women. Of those Latinas who did have diplomas, half (54.0%) of them pursued education beyond high school, compared to 65.6% of White women, 58.2% of Black women, and 76.4% of Asian women (p.2).

According to Kohler & Lazarín (2007), Latina/os students increased to 19% of the school population while White students decreased to 58 percent. Su (2009) states 77% of Latina/o students now attend majority non-White schools. In 1994, the elementary and secondary school population of Latino/as reached 12.7% and made up 6.4% of Gifted and Talented (GT) programs while Whites comprised 65.7% of the school population and accounted for 80.2% of GT programs (NCLR, 1999). Among 12-14 year olds, 39% were below modal grade while 30% of Whites were retained (NCLR). Latino/as experience more intense school isolation and a decrease in exposure to White students

(NCLR) . Dropout rates are also very conclusive of problems within cultures. Among 16-24 year olds, the dropout rate for Latino/as was 29.4% in 1996 compared to 7.3% for Whites (NCLR). Curricula do not cater to Latino/as.

The Latino/a population accounted for half of the population growth of the total population in the U.S in the ten years (U.S. Census Bureau, 2010). The Mexican population is still numerically and proportionally the largest Latino/a group in the U.S. According to the U.S Census Bureau (2010), the Latino/a population had a 43% growth since 2000 compared to 4.9% of non-Latino/as. Fifty percent of the Latino/a population reside in the southwest part of the U.S. including California, Texas, and Arizona. New Mexico also contains a large number of Latino/as at 47% of its population (U.S. Census Bureau, 2012).

More recent statistics show how achievement gaps between Latino/as and Whites are continuing to increase. Latino/as have a 53.2% graduation rate compared to 74.9% of White students (Kohler & Lazarín, 2007). Foreign born Latina/os account for 25.3% of all dropouts and 38.4% of this group are born outside of the U.S. (Kohler & Lazarín). Seventy five percent of native born Latino/as complete high school and only 46% of foreign born Latino/as were high school graduates (Kohler & Lazarín, 2007). Enrollments in advanced courses also display a significant gap. Forty seven percent of White students complete advanced mathematic courses while 31.1% of Latino/as complete the courses (Kohler & Lazarín, 2007). Forty five percent of schools offer advanced math courses which Latino/as attend (Kohler & Lazarín, 2007). Ten percent of White students enroll in GT courses while 3% of Latino/as enroll in these courses (Kohler & Lazarín, 2007).

English Language Learners (ELL) also has significant statistics showing their increase of enrollment. Forty five percent of Latina/o children in schools are ELL. There was a 56% increase of ELL enrollment in schools from 1995 to 2005 (Kohler & Lazarín, 2007). Poverty and schools minorities attend vary greatly from their counterparts. Among 4th graders, 49% of Latino/as enroll in schools with

the highest measure of poverty compared to 5% of White students. College degrees also are increasingly favoring one specific racial group. Twelve percent of Latinos enroll in college compared to 69% of white undergraduates. Twelve percent of Latino/as earn a bachelor's degree while 30.5% of Whites (Kohler & Lazarín, 2007). This data is also confirmed in the National Center of Education Statistics (NCES) from U.S. Department of Education (USDE) Institute of Education Sciences (USDE, 2003).

High stakes testing in the state of Texas creates greater achievement gaps between Latino/as and White students. With the state adopting strict guidelines for graduation, Latina/as find it difficult to stay in school and proceed to higher educational institutions. Latina/as choose to dropout of schools with curricula which do not embrace their potential and academic ability.

Latino/as continually to lag behind white students in TAKS testing scores. Latina/o seventh graders passed the math portion of the TAKS test at 74% while White scored 87% (TEA, 2009b). Latino/a students in eleventh grade pass the math portion of the TAKS test with 75% compared to 89% of White students (TEA, 2009b).

Discipline is also a factor which contributes to the downfall of Hispanics in Texas schools**.** Hispanics had 2,203,340 students in Texas schools compared to 1,626,638 of White students (TEA, 2009d), a 1.3 to 1 ratio. Yet, 282,799 Latino/as where suspended out of school compared to 92,689 of White students creating a ratio of 3 to 1. Furthermore, 803,097 Hispanics were placed on in-school suspensions compared to 408,529 of White students (TEA, 2009c). Hispanics nearly double the number of White students with in-school suspensions and yet their student population is not doubled.

The setting for the study was ranked sixth in the top ten places with the highest number and percentage of Latino/as (U.S. Census Bureau, 2010). This provides an excellent location in studying Latino/as while more Latino/a students will be enrolling in schools. Research should be conducted to better understand how to improve teaching and learning of mathematical concepts particularly for the Latino/a population.

The borderland between the Unites States and Mexico is a perfect setting for studying how the Latino/a population learn mathematics. Since the researcher's career has been teaching mathematics on the borderland for the past sixteen years, it seems a great combination of knowledge of the Latino/a population and the region to successfully conduct a study of this nature.

1.3.1 Definition of Terms

In order to prevent any misunderstandings related to this study, a definition of terms is needed in order to assist readers. Glasser and Smith (2008) state,

> Theoretical terms that play key roles in researcher' analyses should be explained clearly enough in print that readers can determine what parts of the examined world are associated with them. Published research should make clear how the researchers have conceptualized and defined their key terms in the social and educations settings that they have studied. Clear meanings for conceptual terms are needed for education research to become an effective form of communication between researchers and readers (p. 344).

In agreement with Glasser and Smith, the definition of terms is necessary in order to explain how the terms will be used in this study.

Connectedness

Connectedness is too complex and contains too many important variables in order to define here. In order to give justice to the definition of connectedness and its characteristics, a chapter has been designated for this task. Chapter two will further discuss the intricacies and definition of what is introduced as the idea of connectedness. Connectedness is constructed of three major ideas which have been developed juxtaposed with feminist development (Miller, 2000): mathematical reasoning, social cognition, and multiple strategies.

Gender

Gender is a complicated term to define. Gender and sex at times have the same equal meaning. However, in this study we tend to use the term gender instead of sex along with Glasser and Smith (2008) when they explain the "biological distinction" would dominate and that the term gender would outweigh sex. They offer a second reason to use gender as it avoids the referral to people's sexual activity much like sex does (Glasser & Smith, 2008). Therefore the term gender prevents the awkwardness of vagueness (Glasser & Smith, 2008). This study will then use the term gender instead of sex. Gender will be defined as the cultural term to refer to women and men as social groups (APA, 1994). Damarin and Erchick (2010) worked on trying to clarify the meaning of gender in the mathematics education research. In a postmodern model, Damarin and Erchick (2010) say gender is better to be seen as a process and that "gender is more a doing than a being" (p. 318). As explained by the APA (1994), "gender is cultural and is the term to use when referring to men and women as social groups" (p.47). This study and the researcher also understands that "gender cannot be addressed in isolation from other social and cultural issues, such as those raised by race, class, religion, sexual orientation, disability, and other forms of difference. In other words, all girls are not alike, nor are all boys" (Ginsberg, Shapiro, & Brown, 2004, p. xviii).

Equitable

Equitable in the sense of this study is trying to create opportunities where both genders can succeed in mathematics. Furthermore my use of equity also promotes the equal treatment towards Pedagogy, Equity, Mathematics, Diversity, and Social Justice (PEMDAS.)

Spatial Reasoning

Spatial reasoning consists of the ability to use spatial tasks such as arranging objects to fit in a required space and place in mathematics activity such as a mental rotation task. Spatial reasoning is the ability to use logic in understanding the physical elements of objects and their partaking in a particular space. According to Cassidy (2007), spatial perception is a task which requires participants to locate horizontal and vertical in a two dimensional display.

Mental Rotation

Mental rotation is the ability to imagine how objects will look when they are rotated in space. For example, students may be asked to think about a rotation of a figure of 90 degrees and choose the result from a display (Halpern, 2000).

Spatial Visualization

Spatial visualization is the ability for students to perform complex, analytical processing of spatial information; for example, imagining a three-dimensional object in a two-dimensional plane.

Difference

In this study we use a difference in explaining the inequality of achievement between the genders in mathematical attainment in particular to spatial reasoning and spatial reasoning tasks. The word gap has a negative connotation in that it suggests “a break in the barrier” according to the Merriam-Webster Dictionary. This study will use achievement differences instead of achievement gap as designated by different researchers in the literature review and elsewhere (Gluck & Fitting, 2003; Hyde & McKinley, 1997; Hyde & Mertz, 2009).

Achievement

Achievement in any particular context can mean several different things. Here, the study will use achievement defined as the increase in student learning displayed in their performance in Pre-Test and post-test scores.

Mathematical Reasoning

Mathematical reasoning is the students’ ability to use mathematics in any form in order to better understand the meaning of the mathematical concept they are learning. This type of reasoning could be quantitative as students use numbers to make sense of the mathematics. Geometrical reasoning also is another form of mathematical reasoning in which students use geometry theories and principles to better understand the world around them. Another form of mathematical reasoning is algebraic reasoning. Algebraic reasoning is the ability of students to use numbers and symbols to create problem solving techniques.

Multiple Strategies

Multiple strategies merely mean the different ways students can solve problems. This may be creating tables, graphs, charts, visuals, pictures, ways in completing a task, etc.

Social Cognition

Social Cognition comes from the ideas of an influential chapter by Patricia H. Miller (2000) titled "The Development of Interconnected Thinking." Miller breaks down the feminist development of interconnected thinking in three segments. One segment is called social cognition. Here, social cognition is contextualized by "people who value social relationships and develop a morality based on caring may develop a heightened sensitivity to the psychological live of others because they need to understand others and to be understood by others" (p. 54). Furthermore, "the acquisition of knowledge involves a 'conversation', a social interaction, a relationship between the knower and the physical or social world" (p.54). Social cognition is the understanding of a person knowing they are a "social situated person- a person embedded in social relationships" (p. 54). It is the "theory-of-mind in society" (p. 54). Second, the person must also participate in the process of recursive thinking, reciprocity, empathy, and perspective talking (Miller, 2000). Third, they must also think about other's mental states, emotionally with themselves and others. And finally, people learn from others; questioning where knowledge comes from, can it be trusted, does it count, is there access, and how do social relationships affect the quality of knowledge (Miller, 2000).

Latino/a

The term Latino/a refers to a person of Cuban, Mexican, Puerto Rican, Dominican, South or Central America, or other Spanish culture or origin regardless of race.

1.3.2 Statement of the Problem

"Gender equity is not a *female* issue but a *human* issue"

(Sanders, Koch, & Urso, 1997, p. 4).

For many years, scholars have debated about whether there is an achievement difference between genders in mathematics. Researchers have also debated whether there is a difference in the achievement among the genders in spatial reasoning. Gender achievement in mathematics has been a controversial topic because of the wide variance in research. Some research has found a difference between the genders in mathematics while others argue there is no difference in mathematical achievement.

Women are still underrepresented in the science, technology, engineering and mathematics fields (National Science Foundation , 2008). The status quo keeps females at a loss since society believes males have the advantage when learning mathematics. However, at first glance, it may seem like the research is revealing something new. Females may have difficulty in learning mathematics or have severe anxiety (Halpern, 2009) because the curriculum is geared toward males. This initial perception fails to take into account that mathematics explained without a context and geared to both genders is difficult. There are reasons girls don't like math (Halpern, 2009). Some suggestions are lack of confidence, lack of skills, differences in problem solving approaches, people orientation preference, no manipulatives, and exact solutions (Halpern, 2009).

Unless we change school curricula, we will continue to have trouble with society's status quo of women and their low percentage of careers in the mathematics and sciences. We need to understand the importance of women's input in the field of mathematics and the sciences. By rethinking our approach to school curricula, teaching, learning, and how females can participate and be successful in mathematics and science, we can place critical pedagogy in order to facilitate critical consciousness and create agents of change (Darder, Baltodano, &Torres, 2009; Duncan-Andrade & Morrell, 2008; Freire, 2005; Giroux, 1988, 2011; Kincheloe, 2008; McLaren, 2007; Rossatto, Allen, & Pruyn, 2006).

With the large range of research within female accomplishment in mathematics and spatial reasoning, this study will claim to see how females achieve in spatial reasoning tasks. This study will also participate in an intervention in order to help the success of females in mathematics.

1.3.3 Theoretical Framework

The theoretical framework of feminist epistemology grounds the proposed research. Feminist epistemology attempts to understand and interpret how females come to know what they know. Damarin (1998) explains the application of feminist empiricism and feminist-standpoint epistemology. In particular, she summarizes various components of feminist standpoints; knowledge is situated by the standpoint of the knower and it is imperative for women to construct knowledge through their lives and experiences. Jacobs and Becker (1997) develop feminist-standpoint epistemology through analysis of different categories of female knowing. One of the categories considered by Jacobs and Becker - *connected knower* -suggest knowing comes from the knowledge through the contact of another's experience. Connected learning builds on the ideals of teaching with intuition, experience, conjecture, generalization, induction, creativity, and content in which females progress. Becker (1995) made the case that women follow a distinct path of thinking from men in a "different voice". In this way, neither way of thinking is dominant, but it may help us understand and improve women's participation in mathematics (Becker, 1995; Boaler, 1997; Burton, 1995).

Feminist epistemology helps us to learn that females are different than males in their ways to know, understand, and interpret knowledge; females are more empathetic than their male counterparts in terms of knowing and learning; females progress in learning when they can connect to other's experiences; females value participation and collaboration to develop their ideas and create a nurturing, unified atmosphere; females are connected knowers and learners; learners which need connections within and outside of mathematics. The conceptual framework for the proposed study based on feminist epistemological perspective is presented in Figure 1.1.

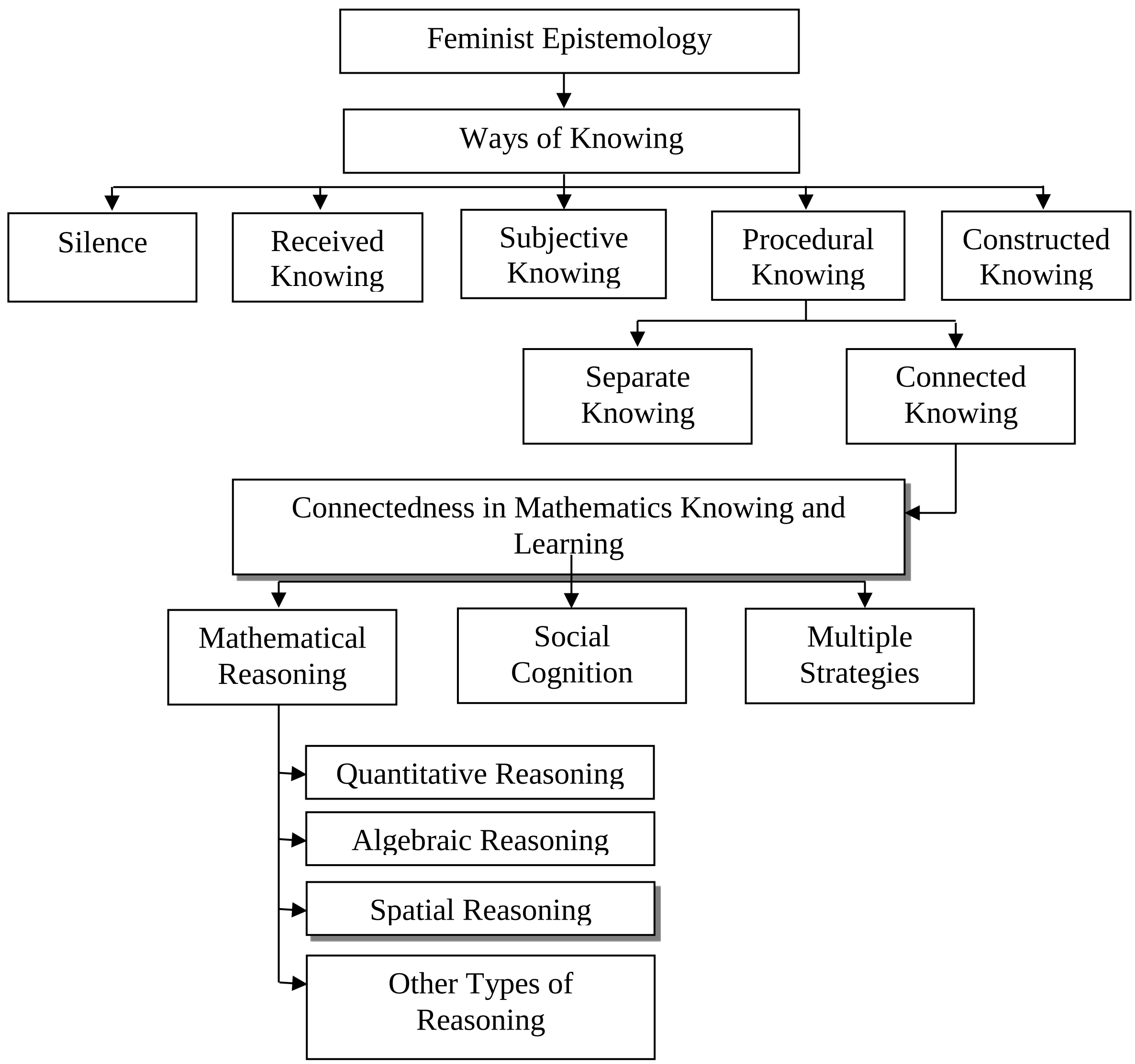


Figure 1.1. Conceptual Framework of the Study.

1.3 Research Focus

The research focus of this study is on creating a gender equitable secondary mathematics, specifically in spatial reasoning, using connectedness. Connectedness is an integral part of creating an intervention to decrease the difference between mathematical success between the genders. Connectedness will be further introduced and defined in the following chapter.

First, the mathematical achievement among females and males has been widely discussed and argued. This study will try to answer the question if there is such an attainment variance between the genders. This will enhance the current research and validate one of the two opposing sides of the argument. The hypothesis states that there will be a difference in spatial reasoning tasks between the genders. I do not wish to see females disadvantaged in any mathematical concept or context. In this way, my research could then focus on why they are equal in achievement.

Secondly, if there is variance on the achievement of spatial reasoning tasks, then we will assess an intervention called connectedness, which will be further addressed in chapter two.

This study hopes to fill the lacuna on research in gender equity using spatial reasoning tasks. There is plenty of research on White and African American students, but there is limited research on Latino/as. How this study is different from previous studies and how we can fill the lacuna is in two segments. First, the study is different because most of the participants are of Latino/a ethnicity. This gives us an insight into their spatial reasoning ability especially along the border of Mexico and the U.S. As stated before, there is a lack of research analyzing the mathematical spatial reasoning among Latino/a students. This research, although not exactly targeting ELL students, does provide some information. The second portion of the study begins to examine an intervention of connectedness which may enhance the learning capabilities of both genders in particular to Latina females. This intervention can have major implications on the teaching and learning of not only Latino/as but of all ethnicities regardless of gender.

1.4 Research Aim and Objectives

The aim and objectives of the research consisted of three major components. The three major components formed the following research questions:

- Research Question #1: Is there a difference between female and male performance on spatial reasoning tasks across the elementary, middle school, and high school levels?

- Research hypothesis 1, H_1: There will be a difference in achievement between boys and girls across the grade levels.
- Null Hypothesis 1, H_0: There will be no difference in achievement between boys and girls across the grade levels.

- Research Question #2: To what extent does connectedness improve females' performance on spatial reasoning tasks?
  - Research hypothesis 2, H_1: Connectedness will assist both genders in increasing their spatial reasoning abilities and assist females in decreasing the achievement difference.
  - Null Hypothesis 2, H_0: Both genders did not have a change in spatial reasoning tasks after the intervention.
- Research Question #3: a) How do male and female students' strategies differ on spatial reasoning tasks after implementation of connectedness and b) how much do male and female students' strategies change on spatial reasoning tasks after implementation of connectedness?
  - Research hypothesis 3, H_1: Male and female strategy levels will include better understanding of the connection between area and side length on spatial reasoning tasks after connectedness.
  - Null Hypothesis 3, H_0: Male and female strategy levels did not have a change.

This mixed method nested study hopefully determined whether or not there is a difference in the mathematical achievement between the genders in spatial reasoning tasks. The study aimed to solve the problem by collecting and analyzing data pertaining to the mathematical context of spatial reasoning using the Seven Piece Chinese Tangrams and by implementing an intervention called connectedness.

1.6 SIGNIFICANCE OF THE STUDY

The value of this study is multi-purposed. First, this study was created to analyze whether there is an actual achievement variance among Latina females in regards to spatial reasoning compared to males. Since most of the participants are of Latino/a ethnicity, it provides an insight into their achievement in spatial reasoning tasks. With this new research, teachers, administrators, and researchers can understand the epistemology of Latino/as and implement changes in their curricula to serve not just males but females as well. Furthermore, as stated before, the void exists because of the prevalent information of mathematical achievement among Whites and African Americans and the lack of research among interventions for females and spatial reasoning. This study will hope to add to current research on how Latino/as understand and learn mathematics. Lastly, it is also important to note that all genders and ethnicities can benefit from the findings of this research. If the study can find methods on how females' best learn mathematics, then it has contributed important research in the field of teaching, learning, and culture.

Secondly, this study is aimed in introducing an innovative teaching strategy called connectedness. If the study can show connectedness is the reason for females, particularly Latinas, in closing the disparity between genders, then the study is a very important addition to the body of research in mathematics, gender, and ethnicities. This may influence how administrators, teachers, and policy-makers create curriculum and pedagogy changes. It will also influence how researchers study and analyze Latinas' achievement in mathematics.

1.7 DISSERTATION CHAPTERS

This study is sectioned into ten chapters: Introduction, Connectedness, Literature Review, Methodology, Data Collection, Analysis, Findings, Discussion, Implications, and Conclusion. The second chapter of the study is Connectedness. Here we will define in detail what connectedness is, what it is not, where in the study it will be embedded, and how it affects teaching and learning. The third chapter will be comprised of research both classic and recent. The literature review will mainly

illuminate the gap in the inquiry on how females, particularly Latinas, are limited in research in the area of mathematics and spatial reasoning tasks. Chapter four will discuss the methodology. The instruments of the isolated tasks and the connected tasks will be shared. The Seven Clever Piece Chinese Tangrams will be defined along with some historical context of the puzzle. The chapter will contain a sub-study in order to answer the first research question. Is there a true difference in achievement between females and males in the aspect of spatial reasoning? This section will also describe in detail the second sub-study of the intervention of connectedness in order to answer the research questions. Can the intervention of connectedness close the disparity between the two genders in spatial reasoning tasks? Also included in the chapter will be a discussion of where connectedness was placed in each of the intervention activities. Chapter five will illustrate and examine student samples and discuss the data collection techniques. Chapter six describes the analysis procedures. Chapter seven is divided into two parts. The first section contains the quantitative finding of the study and how they are related to the research questions. The second section will contain the qualitative findings of the study. The eighth chapter discusses each research question and whether or not the hypothesis of each question was supported. Chapter nine will define the implications and limitations of the study. The final chapter ten will consist of a conclusion including final thoughts of the researcher and the significance of the study.

Chapter 2: Connectedness

> *"…regarding connectedness, individuals are relational beings who are embedded in social relationships more than they are separated, autonomous, and distanced from others."*
>
> *(Patricia H. Miller & Ellin Kofsky Scholnick, Toward a Feminist Developmental Psychology, 2000)*

Connectedness is a new term which involves many aspects of female learning. Connectedness is not just about female learning; it assists all students to become successful in mathematics. This chapter will define connectedness and its theories arising from feminist epistemology. Feminist epistemology assists in defining connectedness and how it is related to the teaching and learning of female students. Ideas from several authors and researchers will be used to assist in the definition of connectedness. This particular study used connectedness in three major components: mathematical reasoning, multiple strategies, and social cognition. These three components came from the influential chapter from Miller (2000) where she defined the three components as: scientific reasoning, social cognition, and cognitive strategies. The researcher has taken these three components and revised them in order to complement mathematics such as spatial reasoning. The chapter's goal is to assist the reader in becoming more familiar with connectedness and to provide the theoretical framework of the study.

2.1 WHAT IS CONNECTEDNESS?

The National Council of Teacher of Mathematics states that students should connect ideas within mathematics they are learning (NCTM, 2000). Furthermore, Hodgson (1995) says,

> Students interpret classroom activities in light of their existing beliefs and assimilate information into their existing knowledge structures. As a result, each student constructs a kind of "personalized" mathematics. To

> some, mathematics is a collection of isolated rules and facts. However, mathematics can also be perceived as a network of ideas in which each idea is connected to several others (p. 13).

Becker (1995) recalls the efforts of Carol Gilligan's work. Gilligan examined Kolhberg's stages of moral development which was based on an all-male sample and made the case that women follow a different path of thinking from men in a "different voice". In this way, neither way of thinking is dominant, but it may help us understand and improve women's participation in mathematics (Becker, 1995; see also Boaler, 1997; Burton, 1995). Gilligan was one of the first authors to use the terms "separate" and "connected" knowing. Other authors describe separate and connected knowing as well as shown in the following paragraphs.

Gilligan also influenced the work of Belenky, Clinchy, Goldberger, and Tarule (1997). In their book, *Women's Way of Knowing*, the authors studied 135 women for five years in order to explore their experiences, problems as learners and knowers, and their concepts of self and relationship with others. The women ranged from different ethnicities, educational backgrounds, ages, socio-economic status, and family make up. The authors' main goal was to provide women with a "voice." The results of their study found stages of development in women's knowing differ from those of men. The stages were: silence, received knowing, subjective knowing, procedural knowing, and constructed knowing. These stages are not meant to be sequential but the knowing does represent a progression through dependence to autonomy and from uncritical to critical (Becker, 1995).

The silence stage was defined by keeping females in their proper place. This stage is where the female is oppressed; experiences a disconnection, obeys, and is seen but not heard. The women in this stage, tended to accept whatever knowledge of the authority as true. During the received knowing stage, women learn by listening to the authority. In most cases, it would be the teacher's influence on the

student's knowledge. Perhaps "banking education" is another example where the teacher merely deposits knowledge into the student (Freire, 2005). The women take authority figures as the source of truth. The subjective stage or "inner knowledge" is where the women now become conscientious or what Freire (2005) would call critical consciousness or *conscientizaçáo*. They find inner strength within themselves to begin transforming their lives. This stage also contains the quest for self. The women walk away from their past to start anew by listening and watching inward. The third stage of procedural knowledge or the "voice of reason" contains separate and connected knowing. Separate knowers find authorities who are knowledgeable and act as mentors. Separate knowing is impersonal ways of knowing and reason. They are critical thinkers and doubt many ideas. Separate knowers perceive everyone and even themselves as being possibly wrong. Connected knowers, on the other hand, believe trustworthy knowledge comes from experience and not authorities; in addition, they possess empathy in order to learn from other people's knowledge (see also Fiske, 2009). Sharing experiences with others allows us to learn from them. "Connected knowing involves feeling, because it is rooted in relationship; but it also involves thought"(Belenky, Clinchy, Goldberger, & Tarule, 1997, p. 121). I do not project. I receive the other into myself, and I see and feel with the other." Belenky et al. continue,

> One undergraduate said, "When I'm reading a book, I can open my mind to the point where I see what the author was all about, see the *isness* of what he was trying to say." And another said, "You must let the poem pass into you and become part of yourself, rather than something you see outside yourself…" (p. 122).

Table 2.1 displays more words associated with the two types of procedural knowing. The last stage, constructed knowledge, is where knowledge is constructed. The women in this stage tend to contribute to the empowerment in the life of others. They tackle the issues of the day and work best in

trying to resolve them. Furthermore, constructed knowers have been transformed and now possess the consciousness needed to become part of the world rather than isolated from the world. They have become critical thinkers and agents of change.

Table 2.1. Separate vs. Connected Knowing.

Separate Knowing	Connected Knowing
Logic	Intuition
Rigor	Creativity
Abstraction	Hypothesizing
Rationality	Conjecture
Axiomatics	Experience
Certainty	Relativism
Deduction	Induction
Completeness	Incompleteness
Absolute truth	Personal process tied to cultural environment
Power and control	
Algorithmic approach	Contextual
Structure and formality	

Source: Belenky, Clinchy, Goldberger, & Tarule, (1997)

Even though Belenky, Clinchy, Goldberger, & Tarule (1997) influenced many researchers, their research has not come without its critics. There are two main critiques about their study. Hare-Mustin and Marecek (1988) argue that men were not included into the study. Furthermore, the fact that the study suggests all women go through the stages of knowing in sequential order is said to be essentializing.

In her study, Boaler (1997) stated that girls preferred the "connected" way of working. Boaler's study of two schools with different curricula showed an increase in female achievement with the curriculum which characterized a connected way of learning. These girls preferred an "open style of

mathematics which was inquiry based, relative, and experiential appeared to fit with the preferences that Belencky et al. (1986) described as 'connected'" (p. 339).

Clinchy (1996) also describes the intricacies of separate and connected knowing in the realm of feminism. Separate knowing is the adversarial stance toward new ideas where connected knowing and knowers embrace new ideas and look for what is right. She also describes connected knowing as a procedure which is similar to subjectivism showing respect for views different from their own. Connectedness draws from intuition and feelings as knowledge comes from one's own experiences. Morrow and Morrow (1995) also say females become excited about inventing new knowledge when they develop a sense of voice. They also become part of the inner circle of knowers. Make sure structures for small group work give students enough time to share task equitably (Morrow & Morrow, 1995).

Knight, Elfenbein, and Messina (1995) investigate the validation of the Knowing Styles Inventory (KSI). The authors researched whether dimension of connected and separate knowing emerge in a quantitative study and whether the structure of the KSI were similar for females and males in three studies. Their results suggest there is justification for using KSI to examine connected and separate knowing. According to the authors, KSI emerged as an instrument with validity and reliability.

Connected, Equitable Mathematics Classroom (CEMC) is explained by Goodell and Parker (2001) in terms of students, curriculum, and teachers. The authors have defined twelve characteristics of CEMC in regards to their analysis and synthesis of gender differences in mathematics through the three basic categories mentioned. The characteristics range from student access to challenging mathematics curricula, teachers connecting mathematics to the real world, and to the curriculum focusing on social justice. However, the most profound characteristics are that which implement and encourage student confidence, literacy, and development of voice. Teachers should connect mathematics to the real world. Furthermore, "the curriculum is designed within a social and cultural

context, challenges stereotyped, and values the contributions of women and minority groups" (p. 420) and "includes a focuses on issues of social justice and world problems that enables student to challenge social conditions" (p. 421). These are ideas from critical pedagogy (Darder, Baltodano, &Torres, 2009; Freire, 2005; Giroux, 1988, 2011; Kincheloe, 2008; McLaren, 2007; Rossatto, Allen, & Pruyn, 2006) with mathematics (Frankenstein, 1987; Gutstein, 2003a, 2003b, 2005, 2006; Gutstein, Lipman, Hernandez, & de los Reyes, 1997; Lesser & Blake, 2007; Skovsmose, 1994), critical race theory (see Bell, 1992; Bonilla-Silva, 2006; Hooks, 1994; 2003; Ladson-Billings & Tate, 2006; Shapiro & Purpel, 2009; Takaki, 1993; Tatum, 1997; Wise, 2005), and mathematics with feminism and social justice (Spielman, 2008).

Zohar (2006) begins to analyze the relationships between learning and connected knowledge. Connected knowers apply intimacy to what they are learning (Zohar, 2006). They simply have empathy toward "gaining a deep understanding of other people" (p. 1581). In regards to impersonal objects, the connected knower will use empathy in the attempt to share the experience of an idea (Knafo, Zahn-Waxler, Davidov, Hulle, Robinson, & Rhee, 2009; Zohar, 2006). In the field of mathematics, this can be difficult to do.

Zohar (2006) also states understanding for the knower is something acquired. "In terms of ways of knowing, understanding means the creation of connections among concepts, connections between the knower's own concepts and the formal concepts he/she is studying in school, and connections between specific concepts and their context" (p. 1587).

Furthermore, Zohar (2006) then begins to re-examine studies pertaining to females and mathematics and comes up with the conclusion that,

> Girls have a strong need to understand what they learn (i.e., to create interconnections among the concepts they study) rather than to be engaged in rote learning or rules and algorithms for solving

> problems. Girls also prefer learning that involves connections among learners, connections with their own personal knowledge and experiences, and connections with additional contexts. Thus, the findings concerning girls striving for understanding may be explained as findings concerning girls' striving for more connected knowledge (p. 1592).

Becker (1995) also exclaims that "mathematics needs to be taught as a process, not as a universal truth handed down by some disembodied, non-human force. Mathematics knowledge is not a predetermined entity." (p. 168). Students should be able to see mathematics and create it themselves within the world around them.

Another type of thinking in feminist epistemology is interconnected. Miller (2000) describes interconnected thinking in four ways. First, the thinker sees connections between the phenomenon and the context. Second, the knower is connected to the phenomenon. Third, the knower's mental representation is connected within themselves, emotions, body, and actions. And finally, the knower is connected to the context including other knowers. "Knowledge is situated; a view is always from somewhere" (p. 47). Miller also categorizes six characteristics of cognition and cognitive development: contextual-relational reasoning; complex networks of multiple, multidirectional causal connections; reciprocity, connecting, and dialogue between knower and known; emphasis on social aspects of cognition; knowledge as co-constructed with other people; and attention to, and valuing of, diversity. Furthermore, Miller stresses that interconnections are dichotomous.

> This thinking stresses mutual influence over master, harmony over domination, complex models over simple ones, understanding over control, and the whole organism over the action of one part (Keller, 1985). Interconnected knowing is not unscientific thinking, it is different

> scientific thinking: it focuses on function, organization, and development rather than on simple mechanical causes (p.50).

In addition,

> This focus on causal webs, connections, the whole organism, and complexity is also reflected in a rejection of the dichotomies that characterize traditional thinking (e.g. Keller, 1985). Some of these dualistic concepts that organize experience into opposites, and often value on element more than the other, are mind versus body, reason versus emotion, self-versus object, inner versus outer world, thinking versus doing, and organism, versus environment (e.g. Harding, 1986). In most feminist epistemologies, these pairs are seen as connected rather than separated, as parts of a whole rather than independent parts, and as equally important and interactive rather hierarchical (p. 50-51).

Knowing is interconnected because the knower is connected to what is known (Miller, 2000). “That is, certain kinds of knowledge may require the knower to become immersed in that which is to be known, rather than independent of it and distanced from it” (p. 51). She also describes how intimacy and connections occur by using an example from a female scientist.

> One needs to “get inside an idea,” “listen to the material,” and develop a “feeling for the organism” (Keller, 1983). In the words of oncological immunologist Anna Brito, “Most importantly you must identify with what you are doing. If you really want to understand about a tumor, you have got to be a tumor” (Goodfield, 1982, p.226). …geneticist Barbara McClintock said, “I know every plant in the field, I know them intimately,

> and I find it a great pleasure to know them" (Keller, 1983) (Miller, 2000, p. 51).

This is the heart of connectedness. If the study were to define connectedness in a couple of sentences, it would use the previous quote from above. Connectedness is about becoming one with what you are learning. When a student becomes connected to the context and the content of mathematics so deeply, they become that mathematical content, topic, material, number, variable, etc. Connectedness is transforming a dry, unrealistic, distanced, disconnected mathematical number, shape, variable, etc…into the person's state of mind. No longer is the number, shape, or mathematical context an isolated phenomenon but instead it becomes immersed within the learner creating an interaction and relationship between the two. In this relationship, empathy, intimacy, and a connectedness evolve between the knower and the known. This relationship therefore develops cognitively (Piaget, 1971) as a state rather than a procedure (Miller, 2000). The relationship becomes reciprocal advancing past typical relationships between people and objects of mathematics. One must become the triangle. One must become pi. One must become the theorem.

Miller (2000) provides examples of how intimacy and empathy can create a sense of connection as well. "Knowledge gradually emerges from the interaction between the knower and the known. Thus, knowing is an active, constructive process in which neither the known nor the knower dominates" (p. 51). This is what Paulo Freire would say about the connections of teachers and the student, in this case the knower and the known (Freire, 2005; Vygotsky, 1978; 1986). Therefore, dialogue is an epistemological instrument for learning about one's self, others, and the phenomenon (Miller, 2000).

Interconnected thinking is also a social act where there are "relationships, connecting, equality, conversation, dialogue, cooperation, negotiation, acceptance, and intimacy" (Miller, 2000, p. 53). During classwork, females need these types of actions when they are learning in pairs, groups, etc. Group interaction can be strengthened when these characteristics are implemented. Instead of

dominance and separation, the goal of the group is to increase bonding and develop a consensus within the group (Miller, 2000). With respect to this, collaboration is an important aspect.

Miller (2000) further explains the three applications of cognitive development: scientific reasoning, social cognition, and cognitive strategies. First in scientific reasoning, Miller explains how we first “emphasize children’s growing awareness of a web of causal relations among parts and between parts and whole in the physical world, their awareness of the influence of the physical context of the phenomenon studied, and their understanding of dynamic systems” (p. 56). Second, social cognition is: studying the child as a socially situated person; interconnected knowing and acquiring knowledge; reciprocity, empathy, perspective talking, and recursive thinking; thinking about other’s mental states; and finally how the epistemological community influences what children learn (see also Burnett & Blakemore, 2009). It is also important to be aware of the sources of knowledge; whose knowledge is trustworthy, countable, accessible, and how it affects the quality of knowledge.

And third, cognitive strategies are the multiple strategies children use while problem solving. Collaboration over competition can assist in the development of the child’s idea or strategy when their ideas are often nurtured and supported (Miller, 2000; see also Morrow & Morrow, 1995).

Through feminist epistemology, we have shed light on the importance of knowing how females come to know, learn, think, and develop intellectually. Connectedness is a strategy which can be implemented in curricula to improve the accessibility to mathematics for females. We have learned females are different than males in their ability to comprehend and interpret knowledge. We have also learned females are more empathetic than their male counterparts in terms of knowing and learning. Females prosper when they can connect to other’s experiences. We have also learned females participate in groups to develop their ideas and create a nurturing, unified classroom atmosphere. We also have come to understand they are connected learners; learners which need connections between mathematics and the world around them.

Spitzer, White, and Flores (2009) believe helping one student helps all students in their understanding of mathematical concepts. Creating a learning environment to foster math concepts consists of encouragement, respect, value of students' ideas and thinking, taking risks, collaborative work, and reflective thinking. Spitzer et al., (2009) recommended teachers to know students at different levels using multiple representations in connecting concrete math concepts to representational levels then subsequently to abstract levels. Teachers can create mathematical connections of content to life experiences creating a space for relevancy (Joseph, 1993; Spitzer et al.).

In conclusion, connectedness derived its theories from feminist epistemology. Through several researchers such as Miller (2000), Zohar (2006), and specifically Belenky, Clinchy, Goldberger, & Tarule (1996), connectedness has developed through the knowledge of feminist thinking and learning. Connectedness is developing a relationship between the knower and the known. In other words, connectedness assists in establishing relevant relationships between the learner and the mathematical concept. By no means is connectedness just a mathematical intervention. It can be used in conjunction with other topics. However, in this study connectedness has essentially two main parts. The first part of connectedness is the actual connecting of mathematical ideas and concepts rather than isolated ideas separated from each other. Secondly, connectedness creates a deeper knowledge of the mathematical concept such as spatial reasoning by the way of empathy and intimacy. Learners get to know the math topic as if they were they math topic itself; student become immersed into what they are learning.

Further discussion of how connectedness was implemented in the lessons will be given in Chapter 4 Methodology. Chapter 4 contains the how, what, why, and where connectedness resides in the activities. Chapter 3 will discuss previous literature on gender differences, feminist epistemology, spatial reasoning, and ethnicity as to reveal the lacuna.

Chapter 3: Literature Review

This literature review will discuss females and mathematics in order to study the effects of connectedness and connected tasks which improve the spatial reasoning of females. There are several articles which have discussed the mathematical achievement gap between females and males. Some have argued the gap exists in spatial relationships (Linn & Peterson, 1985) and others have argued the gap has closed and is no longer a significant factor (Hyde & McKinley, 1997; Hyde & Mertz, 2009). The main theoretical framework used to evaluate the research will be feminist epistemology. Feminist epistemology is the idea of how females understand and think. We will investigate how females come to know what they know, how they make connections, and analyze what are the factors which inhibit and allow their understanding of mathematical concepts.

The review will also cover important theories and categories which enable us to improve our understanding of thinking and learning as it pertains to females. Under feminist epistemology, we will review literature pertaining to females and the ideas of feminism, gender differences, knowing, connectedness, spatial reasoning, equity, social justice, and ethnicity. This review will introduce these ideas, clarify others, and expand on some for the purpose of our own understanding of women's mathematical thinking.

3.1 GENDER DIFFERENCES

Gurian (2011) takes a look at the brain based theory of gender difference in the influential book *Boys & Girls Learn Differently: A Guide for Teachers and Parents*. Boys tend to use more space at a younger age and even invade the space of girls (Gurian, 2011). Boys tend to experience more processing than girls by looking at the algorithms of mathematics. Furthermore, boys are more active in their learning when involving space oriented to their body movement (Gurian, 2011). This then gives boys an advantage because of the further stimulation of spatial abilities (Gurian, 2001).

It is quite relevant in research the disparities between males and females in mathematics, and their patterns in knowing (see also Baxter-Magolda, 1992). These gender differences and similarities have been discussed quite in length by many researchers. The widely read report from the American Association of United Women (AAUW) began to look how schools were shortchanging girls (AAUW, 1995) and the gender gap as still failing children (AAUW, 1999; Sadker, Sadker, & Zittleman, 2009).

There are ideas of gender differences being prevalent and how these differences in mathematics are declining and even non-existent. Leder (1992) reviews the historical investigations of research involving gender and mathematics. A table summarizes articles from the Journal for Research in Mathematics Education (JRME) from 1978- 1990. Some explanations are considered which account for the gap in gender differences in mathematical learning such as environmental, learner-related, and cognitive variables. This table is a good starting point in reviewing previous literature. Evidence of the gender gap in mathematics is reflected in part by students' performance on standardized exams. For example, in a study of the NAEP from 1990 to 2003, male students outperformed their female peers in grades 8 and 12 (McGraw, Lubienski & Strutchens, 2006).

Other factors also contribute to gender differences. Test anxiety is also a problem for females (Halpern, 2009; James, 2009; see also Fennema & Sherman, 1976). “Test anxiety is a chronic problem for girls in math and it starts early” (James, 2009, p. 55). Girls have a problem retrieving important information when stressed (James, 2009). “They become quiet and upset and the more upset they become, the less they can remember” (p. 55). Sense of belonging can also be a factor.

> Females’ lowered sense of belonging—perhaps in response to their perceptions of their learning environments—can make an academic community an uncomfortable, unwelcoming place to be, causing them to drop out of the domain. When the domain is something as fundamental as mathematics, domain avoidance essentially shuts the door to careers in

science, engineering, and technology (Good, Rattan, & Dweck, 2012, p. 714).

Women may hear some things in the math environment that erode their desire to pursue math in the future (Good, et al., 2012). Some messages may be that women's math ability is a trait and they have less of this ability than men (Good, et al.).

Koontz (1997) focuses on two projects involving gender inclusive curricula. The first project's goal was to encourage girls to pursue mathematics, science, and computer courses in high school. The second project distributed information to teachers about intervention programs. The results show girls who participate in hands-on experiences, role model contacts, and encouragement positively impact their attitudes and course selections in mathematics and science.

Forgasz and Leder (2001) illustrate four of their studies pertaining to girls and mathematics. The studies range from daily experiences of students in math classes to an instrument measuring the extent of how mathematics is stereotyped as a gendered domain. Other findings included whether prediction of students' future involvement in mathematics could be diagnosed from current involvement in mathematics. Guiso, Monte, Sapienza and Zingales (2008) used data from the 2003 Programme for International Student Assessment (PISA) that reports on 276,165 fifteen-year-old students from forty countries who took identical tests in mathematics and reading. Their findings indicate in more gender-equal cultures, the math gender gap disappears and the reading gender gap becomes larger between girls and boys. Girls outperform boys in reading as the math gap decreases in gender-equal countries.

Hyde and Mertz (2009) answer three questions of whether there is a real gender difference in mathematics performance in the general population, among gifted and talented and if there are females who have extreme mathematical talent. Using data from studies researching math achievement, they found the gender gap no longer exists. A meta-analysis of several countries showed there is little difference in mathematical achievement between the genders (Else-Quest, Hyde, & Linn, 2010; Hyde,

Lindberg, Linn, Ellis, & Williams, 2008). In the previous paragraph, the research showed there is a difference in achievement between males and females. Therefore, there is research on both sides of the argument of whether there is a difference in achievement. Some researchers argue that there is a difference and others say there is not.

Studies have shown the gender gap in other countries has significantly narrowed over time. Byrnes explains the variations in mathematic achievement between boys and girls. Some variations include why males perform better in mathematics than girls after the age of fifteen, why girls outperform males in computational skills and the same in test which measure problem solving abilities. Byrnes (2008) proposes a Three Conditions Model which assists in explaining gender differences: both genders have a genuine opportunity to learn, they are willing to take advantages of learning, and are able to take advantage of those opportunities. Therefore, according to Byrnes, gender differences can be explained based on exposure to learn, levels of motivation, and different levels of aptitude in both gender groups.

Research also argues teachers can also influence gender gaps. According to Dee (2007), the teacher's own gender may also have influence on the achievement difference between genders. Teachers have biases toward males and females in the classroom and how they engage them (Dee, 2007). There is also controversial evidence teachers have biases in how they engage both genders in the classroom. Teachers are more likely to praise boys in their comments and merely acknowledge girls in theirs (AAUW, 1992; Kleinfeld, 1998; Lewin, 1998; Sadker and Sadker, 1994; Saltzman, 1994). A teacher's gender is a factor which shaped gender equity in the classroom (Jones & Dindia, 2004). Other reasons state how a student responds to the teacher's gender can also be a factor. A student will have improved performance academically in mathematics when assigned to a same-gender teacher (Dee, 2007). The findings of the study from the National Education Longitudinal Study of 1988 suggest that female teachers have a small positive effect on the test scores of girls in math achievement (Dee, 2007).

Furthermore, a female teacher reduces the test scores of boys by almost 0.05 standard deviations. Another finding suggested that female teachers reduce the achievement of both boys and girls in mathematics; however this could be because of less resources assigned to female math teachers and that they may often be assigned to lower-achieving classes (Dee, 2007).

3.2 FEMINIST EPISTEMOLOGY

Feminist epistemology is the study of how females learn and think. Damarin (1995) explains the application of feminist empiricism and feminist-standpoint epistemology. In particular, the article summarizes various components of feminist standpoints; for example, knowledge is always situated by the standpoint of the knower and it is imperative for women to construct knowledge through their lives and experiences. Bergin (2002) explains the theories of Michael Welbourne in communicating the importance of testimony when pertaining to the transfer of knowledge from one individual to another. The author argues for an alternate definition of testimony pertaining more to a primary means of human knowledge acquisition within difference to privilege and differences in world views. The conveyance of knowledge begins with the speaker of knowledge which then transmits the knowledge to the listener. From a feminist epistemology, testimony more accurately reflects the situations of knowledge through communication.

Brister (2009) defines "feminist epistemology" as the epistemology informed by feminist concerns, analyses, and categories. She also explores the relation between feminist epistemology and the problem of philosophical skepticism. Jacobs and Becker (1997) further develop on five categories of female knowing. Of the five categories, *connected knowers* suggest knowing comes from the knowledge through the contact of another's experience. Connected learning builds on the ideals of teaching with intuition, experience, conjecture, generalization, induction, creativity, and content in which females develop. Ideals of feminist pedagogy increase the opportunity for all learners to achieve.

In the areas of knowledge, Harding (1993) describes the subject or agent of knowledge as,

> First, this subject of knowledge is culturally and historically disembodied or invisible because knowledge is by definition universal… Empiricism insists that scientific knowledge has no particular historical subject. Second, in this respect, the subject of scientific knowledge is different in kind from the objects whose properties scientific knowledge describes and explain, because the latter are determinate in space and time. Third, though the subject of knowledge for empiricists is transhistorical, knowledge is initially produced ("discovered") by individuals and groups of individuals….Fourth, the subject is homogenous and unitary, because knowledge must be consistent and coherent (p. 63).

Knowledge is universal; hence, it is neither male nor female dominated. Apple (2000) is more concerned with whose "official" knowledge we are teaching in school curricula (see also Apple, 2009). Burton (1995) also says knowing mathematics is a function of who is claiming to know, how that knowledge is represented, how it was achieved, and the connections between various knowledge. Arnot, David, & Weiner (1999) describe the mathematics curriculum as one which diminished girls' good performance in the early years by creating mathematic curricula which were proponents of rule-following and rote-learning.

Other researchers have also analyzed the connection between mathematics and feminist epistemology (see Damarin, 2008). Burton (1995) suggests mathematics heavily relys on texts with inert information. Patterns in teaching are based on competition and focused on the individual learner (Burton, 1995). As a solution, Ruskai (1996) found in her own teachings that females particularly responded well in cooperative learning environments. Bell and Norwood (2007) also agree females also work well in cooperative groups where a consensus is built and ideas are developed. She also recommends treating people as individuals and to bring up the confidence level of all students.

Additionally, Ruskai (1996) states, "…what works for most people in one group doesn't necessarily work for everybody" (p. 439).

3.3 Spatial Reasoning

Previous research also has sought to locate the source of the gender gap among specific domains of mathematical reasoning. One such domain is spatial ability, which some research has found to be related to mathematical competencies (Clements, Battista, Sarama, & Swaminathan, 1997). Fenemma and Sherman (1977) studied several variables including mathematics achievement, spatial visualization, attitude, motivation, usefulness, and courses. The third cognitive variable studied, spatial visualization, showed males tended to score higher than females. 598 females and 644 males in grades 9-12 were tested. The results show males do perform better than females in spatial visualization in two of the four high schools. Therefore, the authors argue the small difference in spatial visualization were contrary to their expectations and consistent with growing skepticism.

Connections between spatial cognition abilities and mathematical performance have also been researched. Royer and Garofoli (2005) suggested spatial cognition would be less important in class related performance than in assessment performance. Royer and Garofoli (2005) also suggest that the reason for male dominance in mathematic assessments over females is because of this hypothesis; however, there are no male advantages in classroom performance in spatial cognition. Other studies have analyzed the gender differences in mathematics performance in the SAT-M (Nutall, Casey, & Pezaris, 2005). Specifically, Nutall et al. have researched the mental rotations ability of females and found a difference in gender achievement. However after further investigation, scores for both genders were amended for their performance in mentally rotating images. They suggest that "mental rotation ability is a critical factor contributing to gender differences on the math SATs among higher-ability high school and college students" (p. 126). Their study also looked at specific questions where males dominate. Nutall et al. found that by the 8th grade, females' lower math scores were a result of poor

spatial-mechanical skills. Furthermore, their analysis on block-building skills and math achievement resulted in a positive correlation between the two. Students who perform better in the middle grades in block- building tend to do better in later math achievement in high school. While research has demonstrated gender differences in spatial ability, these differences appear to attenuate with practice (Baenninger & Newcombe, 1989) and modifications of the task (Robert & Chevreir, 2003).

Other research suggests there is no significant difference in spatial ability. Within their meta-analysis of several studies of these types of spatial ability, Hyde and McKinley (1997) found that there is no suggestion of gender differences.

> Depending upon the nature of the test, the gender difference can range in magnitude from near zero (as in the Embedded Figures Tests) to large (in three-dimension mental rotation). In addition, a meta-analysis of research on the efficacy of programs for improving spatial ability found that scores on spatial ability tests can be improved by training (Baenninger & Newcombe, 1989). It follows that even these gender differences are not necessarily immutable (p. 37).

Another meta-analysis was analyzed from Hyde and Lindberg (2007) when they researched spatial performance. What they found were researchers such as Linn and Peterson whose findings suggest that males outperformed females in spatial visualization, spatial reasoning, and mental rotation. Hyde et.al also found that Voyer, Voyer, & Bryden found a significant difference in mental rotation in their meta-analysis favoring males. Others have also claimed that visual-spatial ability depends on the type of test used (Halpern, 2000). Furthermore, research has shown differences between females and males in the strategies employed in spatial reasoning tasks (Gluck & Fitting, 2003). This type of research has further influenced my own research by determining whether or not such differences are evident.

3.4 EQUITY/SOCIAL JUSTICE

Mathematics is not ordinarily juxtaposed with equity and social justice. However, in this case since the research is about gender equity, it contains a social justice component as well. Social justice, in the context of the study, is developed in order to make the playing field of mathematics education fair. Furthermore, minority groups have been consistently doing poorly in schools along with their performance on high stakes testing. This section will discuss schooling as a deficit way of thinking along with accountability and how mathematics can become a tool for social justice.

Accountability with the passing of the No Child Left Behind Act (NCLB) is creating extraordinary tension on administrators, teachers, and students. Valencia (1997) deficit thinking ideology is based on how schools blame the victim, use oppression, educability, and heterodoxy to provide a negative education for Latina/o students. Valenzuela (1997) states "…the Texas system of educational accountability has failed- and will continue to fail- Latina/o and other minority youth and their communities" (p. 1).

In "*Standardized or Sterilized? Differing Perspectives on the Effects of High-Stakes Testing in West Texas*," Hampton (2004) states how administrators and teachers create a test and drill program focusing on a sterile and narrow curriculum driven by state assessments. Hampton also argues how policy makers far removed from the classroom are making policies which punish teachers, students, and schools. Cultural diversity is replaced by a narrow test driven curriculum (Hampton, 2004). Hampton (2004) argues that schools become prevalent with monotony and ultimately become standardized and sterilized. We can also become familiar with how other minority group research can benefit Latina/o students.

Research on African-American students and mathematics can also teach us about how to diversify for Latina/o students in instruction and mathematics. In "*Still Not Saved: The Power of Mathematics to Liberate the Oppressed*," Leonard (2009) argues how standardized tests disadvantage

students of color (see also Rothstein, 2007). He found that literature and studies of underachievement in Black students were plentiful. Leonard also uses his experiences as a student and a math teacher to engage in research to improve African American students' way of learning and understanding mathematics and successes. Leonard (2009) also found that socio-economic status alone does not predict academic performance. Anyon (2005) and Rothstein (2007) argue differently by stating how socio-economic status does affect achievement in school and how social class influences a student's education. Leonard, a 4th grade teacher, uses a project, The City, where students learn to read and write their own world using mathematics (Leonard, 2009). He also creates a learning environment with the project which students collaborate with parents and community members in building smaller replica houses, businesses, organizations and facilities. Learning mathematics is meaningful and authentic by building and creating mathematics identity and mathematics socialization (Leonard, 2009). Leonard also argues that "everyone, regardless of race, ethnicity, or gender does mathematics on a regular basis" (p. 324). Mathematical power calls into question actions which disenfranchises while privileging others (Leonard, 2009). "Critical mathematics literacy can be used to critique existing hierarchies and social structures that create barriers and limit Black children's opportunity to learn" (Leonard, 2009, p. 326). Leonard becomes very transformative in his thinking and ability to understand the power and emancipatory potential knowledge can instill in students from all minorities. Martin (2000) studied African American students and their narratives as they begin to utilize their mathematical identities and its importance. Students stated how mathematics can invoke individual agency which leads to a promising future to assist them in achieving their goals (Martin, 2000).

Spitzer, White, and Flores (2009) and their statement of how mathematics prepares students for the workplace is problematic. A hidden curriculum of protecting the status quo certainly is detrimental to the upward mobility of minorities, in this case, Latina/os. Preparing students for the workplace is preparing students to enter the same exact social class. Their suggestion indicates a paradigm of white

curriculum to oppress minorities in providing them with educations to provide society a working force for a capitalistic society. No longer must we accept the status quo and the dehumanizing banking approach to education (Freire, 2005). Mathematics can illuminate problems in society, unveil injustices, seek out inequalities, and provide data and statistical information to support such ideals. Mathematics can implement culture, identity, agency, and transformation consciousness so we can question patterns, reasoning, data, structures, ideals, economics, policies, government programs, and issues of social class, race, and gender. Reasons for research and changing the current mathematical paradigm are important if we are to bridge achievement gaps and restructure the education of Latina/os.

3.5 ETHNICITY

This section will introduce the context of ethnicity as an important factor in the study. It will discuss how Latino/as are the fastest growing population in the U.S. as well as data on their performance in high stakes testing. It will compare statistics of high school completion among Latino/as and Whites along with how curricula favor one over the other.

Between 2000 and 2010, the Latino child growth was the greatest among all ethnic and racial groups under the age of 18 (NCLR, 2011). Latinos represented 23% of the total child population in 2010 (NCLR). There is startling data about Latino/as and their education. According to the Texas Education Agency (2011a), 70.8 percent Latino/as complete high school compared to 88.8 percent of White students. For grades seven and eight, the dropout rates are 0.2 percent for Latino/as and 0.1 percent for White students. In grades nine through twelve, 3 percent of Latino/as dropout compared to 1.1 percent of white students. Latino/as do not believe in a system which devalues them. Schools become deficient in their ability to address needs of Latino/a students (Valencia, 1997) and the importance of culture.

With the implementation of the high stakes testing Texas Assessment of Knowledge and Skills (TAKS) tests, Latino/as achievement continues to lag behind their white student counterparts. Seventy

four percent of Latino/as in seventh grade pass the TAKS Math portion compared to 87 percent of White students. Seventy five percent of Latino/as in eleventh grade pass the Texas Assessment of Knowledge and Skills (TAKS) Math portion compared to 89 percent of white students (TEA, 2009). With the new implementation of the State of Texas Assessment of Academic Readiness (STAAR) in 2012 and its more rigorous testing, tests results have been low. Sixty six percent of Latino/as pass the ninth grade mathematics test in 2011 compared to 83% of Whites and 87% Latino/a pass the math graduation requirement compared to 94% Whites (TEA, 2011b).

High stake testing, with the passing of the NCLB as Ghani likes to say “No Behind Left on the Child” (Abdel Ghani Setra, personal communication, November, 2009) across the nation, is putting unprecedented strain on administrators, teachers, and students. Valencia (1997) states how high stakes testing produces obstacles for Latina/o success in schools. His ideology is based on how Latina/o students fail because of deficit thinking schools possess as they blame victims, use oppression, educability and heterodoxy. Valencia is a critic of this type of thinking. Valenzuela (2005) states “…the Texas system of educational accountability has failed- and will continue to fail- Latina/o and other minority youth and their communities” (p. 1). Valenzuela (2005) continues,

> The very notion of a mainstream, standardized education experience implies a system disregard of children’s personal, cultural, and community –based identities. Rather than providing children with an empowering sense of how their lives can connect productively to the world that they inherit, a test-centric curriculum compelled by long arm of the state through standardized , high-stakes testing reduces children’s worth to their test scores. (p.4)

Valenzuela and Valencia are arguing against a onetime high stakes testing assessment in order to understand the knowledge of students regardless of their culture. These types of tests are incapable of

truly assessing what a student knows or does not know much like textbooks are incapable of teaching everything students need (Apple, 2004). Students come with all types of background and student experiences (see Chiarelott, 2006; Hodgkinson, 2007; Cushner, McClelland & Safford, 2003; Darling-Hammond & Friedlander, 2008; Villegas & Lucas, 2002) which teachers ignore to value and understand in improving their own pedagogy and the education of Latina/o students. Hampton (2005) also states how schools become monotonous with test driven curricula which ultimately sterilize students by narrowing their education. High stakes testing and NCLB pushes educators and schools to goals of test scores and positive ratings such as exceed expectations and recognized status rather than educating students in the interests of the public.

This mathematics is certainly a drill and kill pedagogy which is irrelevant, unrealistic, and subtractive (Valenzuela, 1999). We explain things mathematically in explaining systems and structures in our world. Mathematics is a tool in the interpretation of the world's gifts to humans and its mathematical beauty in nature. Mathematics instruction and knowledge should empower students. Mathematics should illuminate students to question realities. Mathematics should question structure and systems which recreate social class and the status quo. Math should marry up with social justice in identifying statistics and math context which uphold hegemonic ideals from the dominant group (Gutstein, 2006). Math should question the actual math context students are learning now. Whose math are we learning? What purpose does mathematics serve? Who does it serve? How does it funnel me into a specific class? How does math recreate society? Which math would serve a better purpose for society at large? What is math preparing me for? Is mathematics of today preparing me to enter the workforce of society, if so why?

According to Gay (2007), education should teach reality and be sensitive to how different ethnic groups learn. Teachers can assist connections by problem posing (Freire, 2005), self-learning, peer learning, real situations, authentic assessments, active learning, appropriate context, construction of

knowledge and service learning (Chiarelott, 2007). Math curricula must question realities so students gain critical consciousness (Freire, 2005; McLaren, 2007) in structures within societies which oppress them. Curricula should engage students in promoting inquiry, discovery, risk taking, rigor, and creativity (Darling-Hammond & Friedlander, 2008; Greene, 2007; Noddings, 2007; Sternberg & Lubart, 2007).

Curriculum must be sensitive to the diversity of students. Grande (2004) also argues to create pedagogy that is emancipatory (Freire, 2005; McLaren, 1994). Villegas and Lucas (2002) above all promote a culturally responsive teaching. They define culturally responsive teaching by having pre-service teachers use reflective writings, simulation/games, family histories, sociocultural factors, personal histories and histories of diverse groups, service learning, school visits, and practice (Villegas & Lucas, 2002). In a mathematical context, all of these are applicable in any classroom.

A culturally responsive mathematics education; for example, ethnomathematics, is one which values students' diversities. We can learn from ethnomathematics and the mathematics of each of our student's cultures, backgrounds, and experiences (Chiarelott, 2007; Cushner, McClelland & Safford, 2007; Darling-Hammond & Friedlander, 2007; Hodgkinson, 2007; Villegas & Lucas, 2002). D'Ambrosio (1997) states how the term "ethno" includes "all culturally identifiable groups with their jargons, codes, symbols, myths, and even specific ways of reasoning and inferring" (p.17). One form of ethnomathematics is street mathematics. Nunes, Schliemann and Carraher (1993) refer to street mathematics as the informal mathematics students learn from outside of school as an answer to the need of mathematical situations they encounter (Ascher, 1991; Powell & Frankenstein, 1997). Ethnomathematics can provide teachers and students a new perspective on learning mathematics from other cultures. Autoethnomathematics is how teachers and students discover and analyze their own street mathematics to uncover hidden treasures of knowledge and mathematical applications. Math curricula can now change paradigms children have to one with a potential of creating constructive and

critical mathematics to empower themselves in transforming society and opposing injustices of the world. Schools and teachers who implement such curricula value and acknowledge students' cultures and diversities. More research can assist in teachers achieving practical applications of culturally diverse mathematics (Greer, Mukhopadhyay, Powell, & Nelson-Barber, 2009).

In conclusion, the literature review consists of many ideals of whether there is an achievement disparity among the genders. The gender difference in mathematical performance consists of a wide aspect of research. Some researchers argue there is a gender difference in mathematical performance. Others have argued there is no difference in achievement between females and males. Throughout their arguments, there has been little discussion about the performance specifically about Latino/as. Although there is plentiful data on how Latino/as lag behind Whites in high stakes testing, there is no element geared toward how Latino/as perform on specific mathematical tasks. But more specifically, a lacuna exists within the research of Latino/as achievement in spatial reasoning. There is plentiful of research on other ethnic groups but none on Latino/as. As more and more Latino/as enter the classrooms, more research can assist in improving teaching and learning for minority students.

Chapter 4: Methodology

This chapter will discuss the methodology of the mixed method study conducted to research the spatial reasoning tasks of Latino/as. The mixed method approach was felt to be the appropriate structure for conducting the study because of the two components of quantitative and qualitative characteristics. The quantitative portion of the study allowed for the comparison of two groups and their achievement in spatial reasoning tasks. This enabled opportunities to answer the research questions. The qualitative portion of the study was needed in order to better understand students' spatial reasoning strategies.

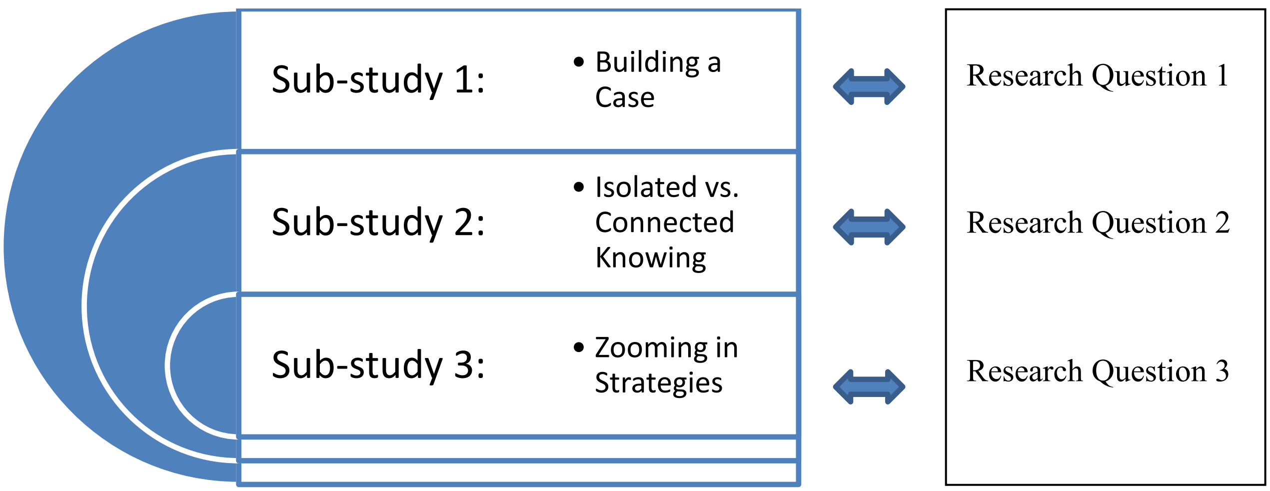


Figure 4.1. Illustration of the Nested Study.

The study is comprised of three sub-studies as seen in Figure 4.1. The first study was initiated in order to determine whether there was a difference in spatial reasoning achievement between males and females to answer the following research question:

- Research Question #1: Is there a difference between female and male performance on spatial reasoning tasks across the elementary, middle school, and high school levels?
    - Research hypothesis 1, H_1: There will be a difference in achievement between boys and girls across the grade levels.

- Null Hypothesis 1, H_0: There will be no difference in achievement between boys and girls across the grade levels.

The second sub-study was developed to provide a control group baseline of spatial reasoning achievement levels and a treatment group to answer the following research questions

- Research Question #2: To what extent does connectedness improve females' performance on spatial reasoning tasks?
    - Research hypothesis 2, H_1: Connectedness will assist both genders in increasing their spatial reasoning abilities and assist females in decreasing the achievement difference.
    - Null hypothesis 2, H_0: Both genders did not have a change in spatial reasoning tasks after the intervention.

The third sub-study was created in order to research the strategic competence of both the treatment and control group in the following research question.

- Research Question #3: a) How do male and female students' strategies differ on spatial reasoning tasks after implementation of connectedness and b) how much do male and female students' strategies change on spatial reasoning tasks after implementation of connectedness?
    - Research hypothesis 3, H_1: Male and female strategy levels will include better understanding of the connection between area and side length on spatial reasoning tasks after connectedness.
    - Null hypothesis 3, H_0: Male and female strategy levels did not have a change.

The control group study was designed so participants did not receive connectedness. The treatment group received the connectedness intervention which is described further in the chapter.

This chapter is organized into four sections. The first section of the study is the setting which will deliberate the significance of the borderland. The second section is the instrument section which will further explore the historical significance of Tangrams and area dissection. The third section of the chapter will discuss the first sub-study which researched the achievement disparity between the genders. The final section will discuss the second sub-study of isolated tasks vs. connectedness tasks. This section will also reveal the components of the connected tasks of: the Pre-Test, research tangrams, warm-up, learning about tangrams, side lengths and areas, core activity, level three strategy, reflection, and the post-test.

4.1 SETTING

The setting of the study and sub-study occurred in eleven southwest schools near the border of Texas and Mexico from 2009-2012. The border city where the study was conducted comprised of 82.2% Latino/a and 17.8% non-Latino/a (U.S. Census, 2010). The economic parts of the city are industry, military, and the influence of consumerism from Mexico. The schools ranged from elementary to high school and were predominately Latino/a. They varied from low socio-economic areas of the city to middle class and upper class parts of town. This provided a wide range of socio-economic statuses. The two school districts encompassed the eleven schools.

The first district has been nationally and state-wide recognized for its participation in dual language programs and for the programs implemented in teaching Latino/a students in non-dual language. It is the second largest school district in the border city serving 44,729 students in a culturally diverse setting. Almost 92% of the students are Latino/a, 80.9% are economically disadvantaged, and 23.9% are Limited English Proficient (LEP). The district has 62 campuses, and employs 8,000 administrators, teachers and support staff. It is one of the largest districts in the state.

The same district has many honors and awards. The district was named a national Broad Prize finalist for two consecutive years in 2010 and 2011. The Broad Prize is awarded to urban school districts that demonstrate the greatest overall performance and improvement in student achievement while reducing achievement gaps among low-income and minority students. As a two year finalist, the

district has received a total of $460,000 in scholarship money for graduating seniors. In 2010 and 2011, the district earned the State Education Agency rating as a Recognized (the second highest standard) district based on student scores from state standardized tests. In 2011, 12 schools were rated as Exemplary (the highest state standard) and 20 were rated as Recognized. In addition, ten campuses have been designated as National Blue Ribbon Schools, an accolade which recognizes public and non-public elementary, middle, and high schools where students achieve at very high levels and/or where the achievement gap among ethnicities is narrowing.

The second district is the largest district in the city. With more than 64,000 students in 94 campuses, the district also is the 10th largest district in Texas and the 61st largest district in the United States. It also is the city's largest employer with nearly 9,000 employees and has an annual operating budget of $461 million. Organized in 1883, the district is not only large, but also one rich in history. In 2008, the district celebrated its 125th anniversary. The demographics are the same as the first district which also varies in socio-economic status. The district enrollment states 82.6% of the student population are Latino/a and 54.2% are tracked as At-Risk (see Oakes, 1986 for tracking). Of the total student population 71.1% are economically disadvantaged, 25% are English Language Learners (ELL), and 2.4% are immigrants.

4.2 INSTRUMENTS

4.2.1 Tangrams and Area Dissection: An Epistemological and Didactical Perception

Tangrams have a history as far back as the Chinese and from Archimedes himself. The Seven Clever Piece Chinese Tangrams are comprised of seven pieces comprising of a parallelogram, a square, and five isosceles triangles of various sizes. Archimedes' puzzle is similar but instead is constructed by fourteen pieces in creating a square. Tangrams and the Archimedes puzzle can be essential in investigating properties of algebra and geometry specifically area dissection, measurement, length, area, ratios, spatial reasoning, and spatial visualization. This section will examine the history of area

dissection, epistemology, and didactics of tangrams and area dissection in order to better understand the pedagogy implications of tangrams in the mathematics classroom.

History and Area Dissection

The word "tangram" may have come from the Tanka people who were traders who played this puzzle and passed it on to sailors (Coombs, Penna, & Schimschock, n.d.) The word might come from an obscure English word "tramgram" which means puzzle or trinket (Coombs et.al, n.d.) Fancy tangram sets made of ivory, jade, and other fine materials soon appeared. Others were made from fired clay, or wood (Coombs et.al, n.d.)

The oldest known mathematical puzzle dates from Archimedes, more than two millennia ago. It is, in fact, a dissection puzzle show in Figure 1 which appears in a treaty known today as Archimedes' *Stomachion* (or *Ostomachion*, or *Syntemachion*), contained in a *Palimpsest* written over by an anonymous medieval scribe compiling prayers. (The word palimpsest literally meant, in Greek, "scraped again").

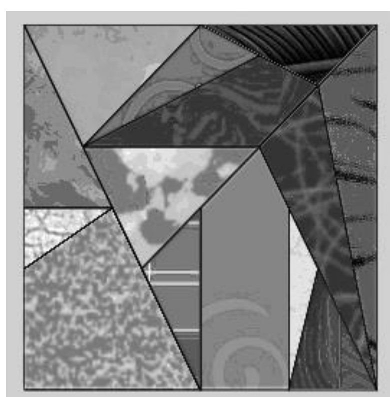


Illustration 4.1. Archimedes' dissection puzzle.

Apparently, Archimedes described in detail, piece by piece, a dissection of a square into 14 pieces and asked how many different arrangements are possible (Illustration 4.2).

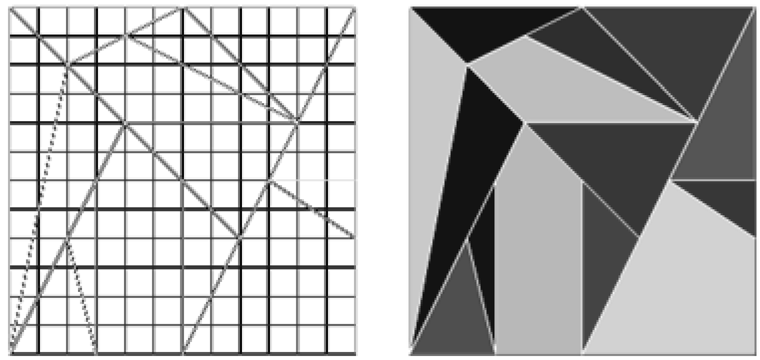


Illustration 4.2. Archimedes' puzzle in a grid.

This problem of computational geometry was only recently solved—as it happens, by a mathematician with a doctorate in mathematics from Cornell University, Bill Cutler. Cutler showed that there are a total of 17,152 solutions, but some can be considered equivalent if a rotation or a reflection is performed. Cutler showed that Archimedes' puzzle has exactly 536 truly distinct solutions as follows in Illustration 4.3.

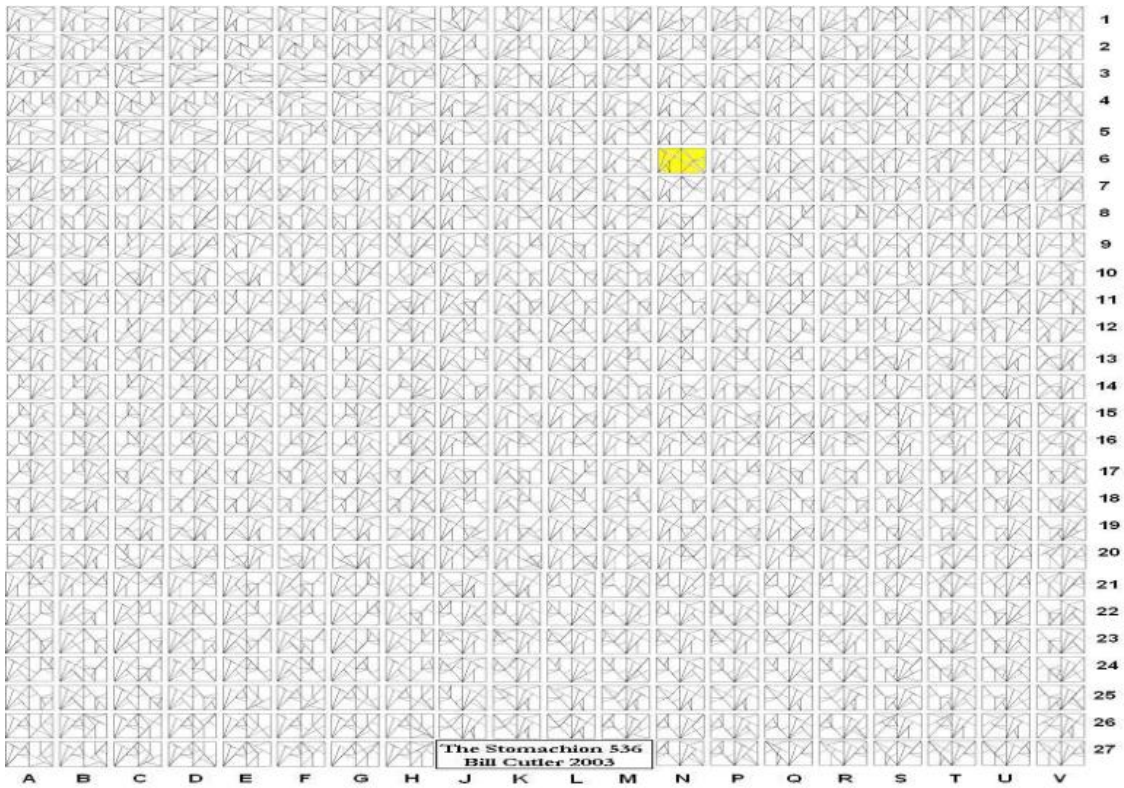


Illustration 4.3. Cutler's solutions.

We can further examine Archimedes' puzzle by finding the areas of each individual piece and analyzing a possible pattern (Illustration 4.4).

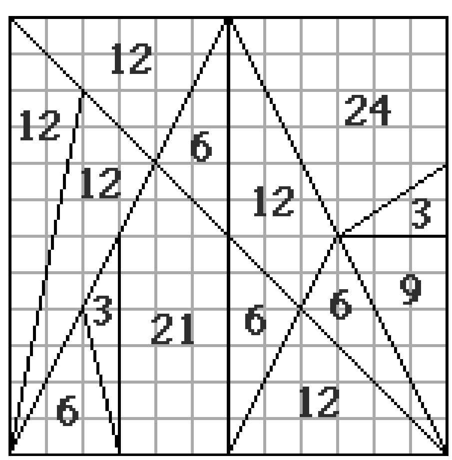


Illustration 4.4. Dissection puzzle with areas.

If each piece of the puzzle is simplified by the common divisor 3, we obtain the following picture in Illustration 4.5.

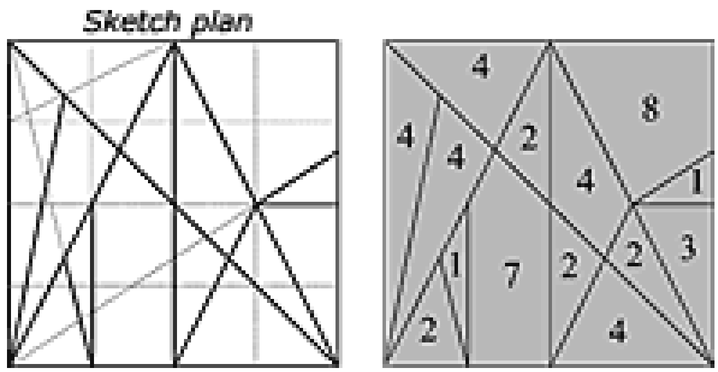


Illustration 4.5. Areas simplified.

This makes obvious that the area of each piece in Archimedes' puzzle is a multiple of 1/48 of the area of the whole square. A separation of the pieces and a convenient rearrangement shows that simple fractions of areas can be constructed using the pieces of the *Stomachion* in Illustration 4.6.

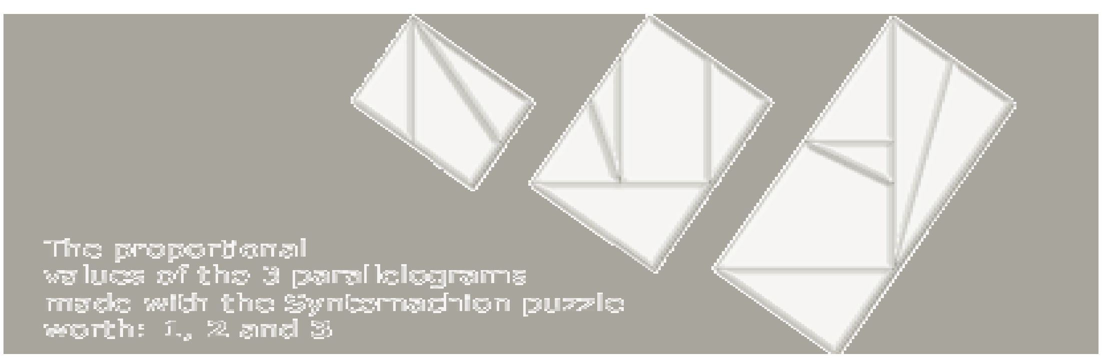


Illustration 4.6. *Stomachion* constructions.

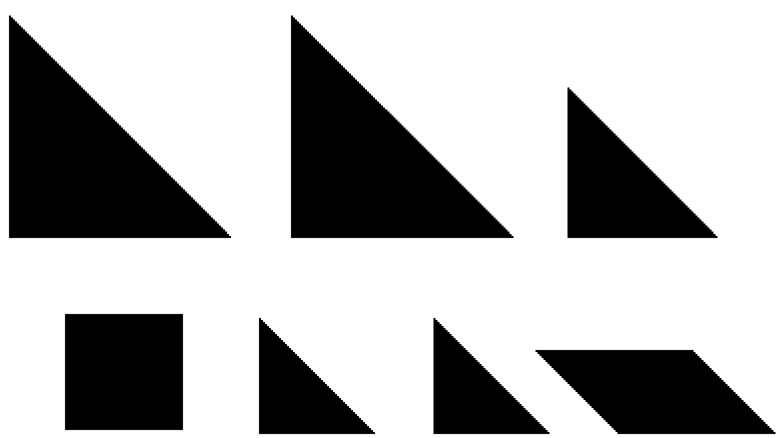
Patterns

Patterns in Archimedes' puzzle are relevant. Other math historical figures have also contributed to the epistemology of patterns such as Apollonius, Sir Isaac Newton, and John Wallace.

Measurement

Tangrams and the Archimedes puzzle contain several mathematical concepts. One of the main concepts is geometry, although some algebra is also present. The puzzles consist of ratios, symmetry, side lengths, areas, and diagonals to name a few. Probabilities and reflections can also develop a deeper understanding of the intricacies in the puzzles. Consider the following task: construct a square using 7 Tangram pieces (Illustration 4.7).

Illustration 4.7. The seven clever piece chinese tangrams.



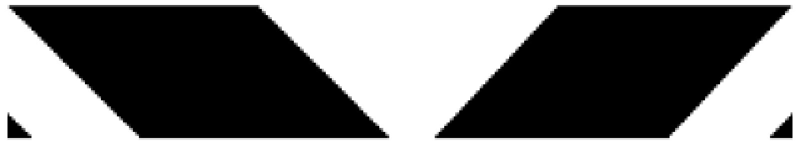
The Seven Clever Piece Tangrams were named by the Chinese. The Tangrams consist of two large isosceles triangles, one medium triangle, two small isosceles, a square, and a parallelogram. This task

requires spatial reasoning skills including but not limited to geometric transformations such as translation, rotation, and symmetry, understanding properties of shapes, non-standard units, as well as understanding forward and backward relationships between area and side lengths (Tchoshanov, 2011). The completed square using all seven pieces is shown in Illustration 4.8.

Illustration 4.8. The completed square.

The reasons the researcher used Tangrams are multifaceted. Tangrams allow for several mathematical concepts. First, a student must be able to understand similarity and congruence. They must also understand which pieces of the tangrams are related to each other in size and shape. Also another important aspect of creating the square is reflection. Some pieces; for example, vary when reflected such as a parallelogram. If the parallelogram is rotated 360°, then the exact same shape is recreated. However, if the parallelogram is reflected, then the piece itself changes in formation. If other pieces are reflected such as the triangle and the square, then the piece is identical as before after rotations. However, the parallelogram takes on a different shape when reflected, whether it leans to the left or to the right (Illustration 4.9). If the parallelogram is leaning to the right, students must reflect the figure to lean toward the left in order to fit into the square.

Illustration 4.9. Parallelogram reflected.



The Seven Piece Clever Chinese Tangrams and Archimedes' puzzle can assist in developing and/or increasing achievement in geometrical spatial reasoning and visualization along with other mathematical concepts. The Tangrams and the puzzle are not limited to geometry because other concepts such as patterns and ratios can also be developed. As area dissection is an important aspect to the current mathematics curriculum of today, we can use Tangrams and Archimedes' puzzle as instruments for implementing hands-on activities. Students will have opportunities to understand the historical implications of Tangrams and Archimedes and the connections to their current curriculum.

Endnotes

Illustration 4.1. Source: http://en.wikipedia.org/wiki/Image:Stomachion.JPG

Illustration 4.2. Source: http://mathworld.wolfram.com/Stomachion.html

Illustration 4.3. See the online comment on the Mathematical Association of America web site and a subsequent article in the national press.

Illustration 4.4. Source: http://www.gamepuzzles.com/536solt.htm

Illustration 4.5. Source: http://www.archimedes-lab.org/latin_ostomachion.html

Illustration 4.6. Source: http://www.barbecuejoe.com/stomachion.htm

4.3 STUDY I: ACHIEVEMENT DISPARITY BETWEEN GENDERS?

- Research Question #1: Is there a difference between female and male performance on spatial reasoning tasks across the elementary, middle school, and high school levels?

- Research hypothesis 1, H_1: There will be a difference in achievement between boys and girls across the grade levels.
- Null Hypothesis 1, H_0: There will be no difference in achievement between boys and girls across the grade levels.

This study determined whether or not there is an achievement disparity between the genders across the elementary, middle school, and high school levels (N=589). The elementary school consisted of n=66 participants, the middle school n=187 participants, and the high school n=336 students. The study was administered with the assistance of participating teachers (N=8) from a Master's class at the local university. Each teacher participated in the study in order to give them experience in administering the study themselves. Afterwards, each teacher went to their prospective schools and administered the study and collected the data. The data from each teacher participant was then aggregated and analyzed to determine the possibility of an achievement gap between the genders along the different levels of schooling.

4.3.1 The Study

A Pre-Test was given to the 589 participants across the grade levels of elementary to high school as described above. The Pre-Test used the Seven Clever Piece Chinese Tangrams in order to analyze whether or not there is an achievement disparity among the genders. The study took place in just one class period in the prospective schools. The Master teachers administered the study within the same month because of the limited supplies of the Tangrams needed. The activity consisted of solely a pretest in spatial reasoning.

Participants

In the first study, to determine whether or not there is an achievement disparity between the genders, the participants came from several high schools, middle schools, and elementary schools in the border region. A total of N=589 connectedness students were selected in a cluster random sampling. Of these, 336 students were from the high school level with 165 females and 171 males. The middle school

level consisted of 187 participants of which 90 were female and 97 male. The elementary school had less participants consisting of 66 total students with 50% females and males. The second connectedness study consisted of different participants from various high schools. The total of number of participants were N=719 where 340 were female and 379 male. Of the females, 85% were Latina compared to 82% Latino males. These students were also selected in a cluster random sample. Even though ethnicity data was not collected in the first study of N=589, with the demographics of the other schools and the border city, it is safe to say the majority were also Latino/a.

Warm-up

First, the teachers distributed the Tangrams in the pre-packaged packets of the seven pieces and a warm-up worksheet (see Appendix A.) Second, the students participated in a warm-up session to get familiar with the pieces. The teacher then would tell the students to fill out the required information on the paper. The information the students needed to complete were: ID number, gender, ethnicity, date, and teacher's name. The students were then instructed by the teacher to create any kind of figure using all seven pieces; for example, a house, a cat, a car, etc. The teacher displayed a timer using an overhead projector or a LCD projector. The teacher instructed the students they would have up to ten minutes to create a figure. Once the students placed the pieces in a strange figure, they were instructed to record the time and to copy the outline of the figure using a pencil. Afterwards, when all students completed the outline, the teacher then instructed the students to exchange their papers with a partner. The teacher then told the students to try and recreate the figure their partner had outlined using all seven Tangrams. Once again, the teacher displayed the clock timer for students to record the time it took them to recreate their partner's figure. The timing of creating and recreating the figures allowed for two things. First, it gave students some experience with having to finish an activity within a designated time frame. Second, it would prepare them for having to complete the Pre-Test within a timeframe. This warm-up activity gave students time to become familiar with the pieces and some play time fun.

Pre-Test 7 Pieces

The next step in the study was to administer the Pre-Test. Students were again instructed to complete the required information on the top of the worksheet (see Appendix B) and reminded to record the time needed in order to complete the activity. The teacher briefly explained the instructions of the activity. The students merely had to create a square using all seven Tangrams. When the students completed the square, the teacher explained to trace the border of the square and the location of each piece in the square then to cover their square with another sheet of paper. The teacher then explained to the students to work individually and that they had ten minutes to complete the activity. The teacher also explained to the students this activity was not for a grade and students had the right not to participate. Once the timer started, the students could then begin to work on the activity. When the activity and timer began, the teacher walked around the room in order to monitor students. At the conclusion of the ten minutes, the teacher would then collect the worksheets and score them. If a student completed the task within the ten minutes, a score of 1 was given. If a student did not complete the task within the timeframe, then a score of 0 was given.

Analysis

All eight teachers individually completed the data collection form (see Appendix D) and brought them to the Master's class. The professor at the university and I then compiled all the data and began the analysis. We recorded the number of students and created percentages among those who completed the Pre-Test and those who did not. Then students and scores were separated by gender and school level. A table and a graph were created to further examine the student scores by gender and school level to determine whether there was a difference in achievement between the genders in spatial reasoning.

Table 4.1. Spatial Task Completion Results by Gender and School

		Pre-Test
Elementary School	Females N=33	3%
	Males N=33	8%

Middle School	Females N=90	17%
	Males N=97	24%
High School	Females N=165	62%
	Males N=171	75%

Table 4.1 is the table from the data of all eight teachers divided among the three school levels. In addition, Figure 4.2 gives another perspective of the data.

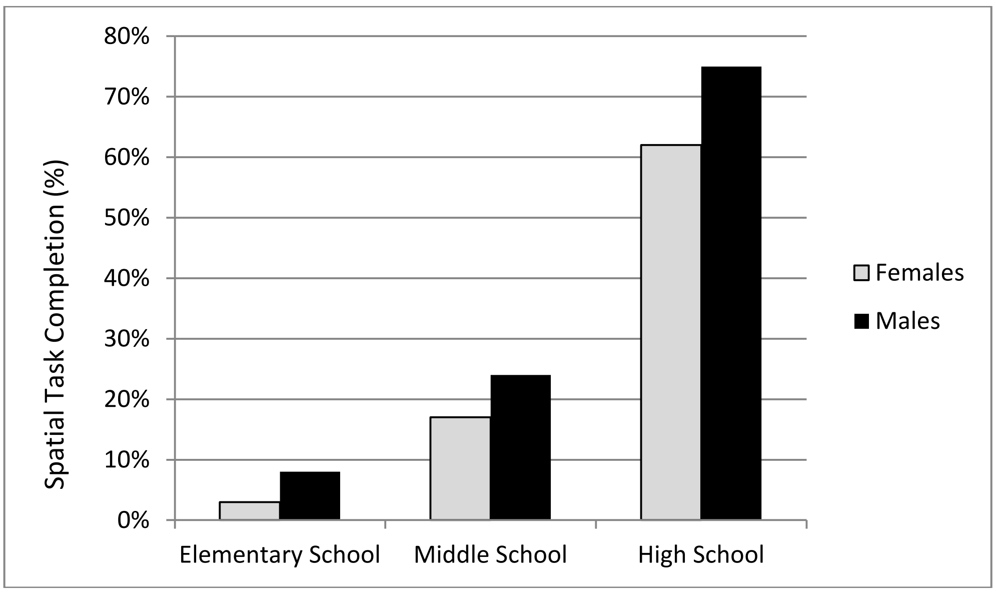


Figure 4.2. Spatial Task Completion Results by Gender and School.

Table 4.2 gives the descriptive statistics of both genders.

Table 4.2. Descriptive Statistics of Spatial Task Completion of Both Genders.

	Females	Males
Mean	0.635	0.7513
SE	0.0878	0.0629

Median	0.685	0.765
SD	0.2483	0.1778
SV	0.0617	0.0316
Kurtosis	-0.2307	0.4176
Skewness	-0.3458	-0.531
Confid Level(95%)	0.2076	0.1486

Findings

According to the study of 589 school participants varying from elementary school through high school, our findings suggest and confirm previous research that an achievement disparity does exist between the two genders in spatial reasoning tasks (Contreras, Martínez-Molina, & Santacreu, 2012; Gluck & Fitting, 2003; James, 2009; McGraw, Lubienski & Strutchens, 2006; NAEP, 2012; Sadker, Sadker, & Zittleman, 2009.) Male students compared to females students in the elementary school overall had a higher completion rate of the spatial task activity by 5%. In the middle school the males had higher rate of completion of 7% over females. The trend continued in the high school level where male students completed the activity of spatial reasoning with a 7% percent higher rate than females. The data suggests the disparity in achievement among the genders in spatial reasoning tasks almost stays constant from elementary to high school. Furthermore, the data suggested the difference in achievement is only 7% percent in the middle school and high school levels. This percentage difference is maintained throughout the grade levels. Females are not too far behind their male counterparts. This percentage can easily be reduced to nothing if an intervention is administered.

In regards to research question #1 and according to the study, the null hypothesis, $H_{0,}$ is rejected. Because the null hypothesis is rejected, the study can then proceed into further research in answering the rest of the research questions.

4.4 STUDY II: ISOLATED VS. CONNECTED SPATIAL REASONING TASKS

The second study, Study II, was designed in order to answer the following research questions:

- Research Question #2: To what extent does connectedness improve females' performance on spatial reasoning tasks?

- Research hypothesis 2, H_1: Connectedness will assist both genders in increasing their spatial reasoning abilities and assist females in decreasing the achievement difference.
- Null Hypothesis 2, H_0: Both genders did not have a change in spatial reasoning tasks after the intervention.

To set up the study, it was necessary to create a control group and a treatment group to decide whether the null hypothesis will be rejected or not. Study II was divided into two segments: isolated and connected spatial reasoning tasks. The isolated portion of the study was the control group and the connected segment of the study was the treatment group. The control group merely took a Pre-Test and a post-test without any intervention in between. The treatment received the intervention of connectedness in between the Pre-Test and post-test.

Setting

Both the isolated and connectedness studies were completed at various high schools during the duration of a year.

4.4.1 Isolated Spatial Reasoning Tasks

This segment of the chapter will discuss the Isolated Spatial Reasoning Tasks. This sub-study is isolated because there is an absence of a treatment intervention. The control group in the isolated task will only participate in a Pre-Test and post-test. This allows a baseline for later analysis between the control group and the treatment group. The isolated spatial reasoning tasks were completed during a period of six weeks. It took approximately six weeks for the researcher to collect data on the several class periods.

Participants

Participants for the isolated spatial reasoning study were selected using a cluster random sampling. Several high schools in the border region participated in the study. Of the participants (N=247), 126 were male and 121 were female. Of the males, 82.5% were Latino compared to 83.5% Latinas.

Methodology

Several high school teachers were recruited to assist in the research. The teachers were trained by the researcher and provided the research materials for their classrooms. All classes were first given the Pre-Test and two weeks later the Post-Test was administered during the same class period.

Pre-Test 7 Pieces

Each student was given a set of tangram pieces and the Pre-Test. Students were told to complete the information on the top of the worksheet (see Appendix C) and prompted to record the time needed in order to complete the activity. The Pre-Test asked the students to create a square using all seven Tangram pieces within ten minutes. As soon as the students completed the square, the researchers explained to trace the border of the square and the location of each piece in the square then to cover their square with another sheet of paper. The researchers told the participants to work individually stating they had only ten minutes to complete the activity. The researchers walked around the room and monitored the students. This was important in order to keep the study as valid and reliable as possible. The researchers frequently informed the students of time remaining throughout the activity. Once the time was up, the researchers then collected all samples. Lastly, the researchers assessed the samples. If a student completed the task within the ten minutes, a score of 1 was given. If a student did not complete the task within the timeframe, then a score of 0 was given.

Post-Test 5 Pieces

Two weeks after the Pre-Test, the researchers revisited the classes and administered the Post-Test (see Appendix L.) Here, the instructions and methodology of the activity were exactly the same as the Pre-Test. The only difference was that students had to create a square now using only five of the seven pieces. The students who completed the activity in ten minutes were given a score of 1 and those who did not complete the activity were given a score of 0.

Theory

The theory behind creating a square using only five pieces is multifaceted. First, students must analyze and choose which five pieces of the seven they are going to use to construct the square. Second,

students should understand the connection between area and side lengths to create the square. Although some students will use trial and error, we want them to conceptualize the connection between what area the square must be and which side lengths can construct such a square. Further details of how the students conceptualize the areas and side lengths of the square will be explained during the Table and Core Activities.

4.4.2 Connected Spatial Reasoning Tasks

This particular sub-study is called Connected Spatial Reasoning Tasks because this study is aimed toward the treatment group. The treatment group will receive specific lessons with connectedness ideas unlike the control group, Isolated Tasks. This segment of the chapter will discuss the participants, Pre-Test, and the connectedness activities. The connectedness activities are: Research Tangrams, Learning about Tangrams, Side Length and Area Activity, Core Activity, Level Three Strategy, Collaborating on Strategy Levels, and a Reflection.

Participants

Participants for this portion of the study were students of the researchers. The researcher acknowledges how using your own students can become problematic (Clark & McCann, 2005). However, the researchers did reiterate in class the Internal Review Board (IRB) process and participation was voluntary with no grades being affected. Of the treatment group (N= 472), 253 were male and 219 were female. Of the males, 81.8% were Latino and of the females, 86.3% were Latina. This group of students participated in all seven portions of the study: Pre-Test, Research, Table activity, Core activity, Level Three Strategy, Reflections, and the Post-test.

Pre-Test

The Pre-Test is the same exact test given to the previous study participants. The method for administering the Pre-Test was kept the exact same. This particular Pre-Test began to look at the strategies students used. A question at the end of the Pre-Test asked students to state what strategy they used in the activity.

Research Tangrams

One idea of connectedness is the ability to become more familiar with a mathematical content. Knowing is interconnected because the knower is connected to what is known (Miller, 2000). "That is, certain kinds of knowledge may require the knower to become immersed in that which is to be known, rather than independent of it and distanced from it" (p. 51) (Boaler, 1997; Zohar, 2006.) In order to become immersed into Tangrams and ideas of its mathematical content as connectedness suggests, the first activity with students was to require them to conduct some background research. The research allowed students to create their own understanding of Tangrams rather than from the researcher. Students were recommended to use the Library, internet, books, etc. The activity asked some questions to assist in facilitating the research (see Appendix E):

- What is a tangram?
- Where did they come from?
- Find some history of Tangrams.
- What are some literature books that talk about Tangrams?
- What areas of mathematics do you think Tangrams can be included?
- What can you learn with Tangrams?

The following day students shared with their own group the research they had conducted. Group leaders were then asked to share common ideas found within the group. With having some understanding of the connection to mathematics and some history, students continued with the next Tangram activity, the Warm-Up.

Warm-up

The students were given the warm-up activity and instructed in the same manner as before.

Learning about the Tangram Pieces

In this activity, the students worked in groups of three or four. This activity allowed the students to analyze each of the seven Tangrams (see Appendix F.) First, the participants looked at each piece and gave the name of the shape. Second, the students will determine which pieces are congruent, show the congruent pieces, and explain why they are congruent. Third, the participants analyzed the pieces for similarity, showed which pieces are similar, and explained why they are similar. Figure 4.4 displays the table showing the six categories of information. As seen in the table, students will first sketch the Tangram pieces (pieces are illustrated for the reader.) The next step was to name the tangram pieces and find the area of the Tangram pieces. Finding the area of the Tangram pieces is a very important step because it will allowed students to engage in analyzing the side lengths in order to determine the area. Students then listed the side lengths of the Tangram pieces. Furthermore, this gave the students the opportunity to see all the different side lengths. The activity then asked for the number of pieces which are congruent to the Tangram piece they are analyzing. This step will assist the students in finding the final step which is to find the total area of the congruent pieces. Once all of the 5 sets of pieces were found, the students completed the bottom portion of the table which requested them to list the different side lengths of the Tangram pieces. This enabled students to see the various side lengths in which they will be using. The final step in the activity will be to find the total area of all the Tangrams together.

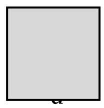
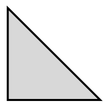
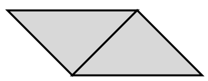
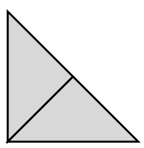
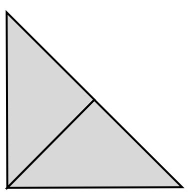
#	Sketch the Tangram Piece	Name the Tangram Piece	Area of the Tangram Piece	Side Lengths of the Tangram Piece	Number of Congruent Pieces	Area of Congruent Pieces
1.		*Square*	*1*	*a =1*	*1*	*1*
2.						
3.						
4.						
5.						
		List different side lengths of the Tangram pieces:		***Total area of all 7 pieces:***		

Figure 4.3. Learning About the Tangram Pieces Table.

Side Length and Areas

The students then participated in an activity which promoted connectedness in Side Length and Areas (see Appendix G.) Just allowing students to write down and observe the concept of the Tangram pieces is not enough. Connectedness provided students opportunities to connect the ideas of side length

in relationship with area. Here, the students were asked to complete the table in Figure 4.5. A portion of the table is pre- filled in order to promote analysis

Table 4.3. Table of Side Lengths and Areas.

Side Length a=	1		2	
Process	1^2	$(\sqrt{2})^2$		
Area of Square	1 $unit^2$			8 $unit^2$

of the process in creating side lengths from areas and vice versa. With the larger sized Tangram pieces as well as the student pieces, groups were then asked to complete the table according to side length, process, and area of the square created. A poster was then created by the researcher which listed the areas of squares that can be developed with the different side lengths which students can refer to in the following activities. More connectedness is provided in the following Core Activity.

Core Activity

The Core Activity also required students to work in groups of 5 or less. This activity promoted connectedness by having students build on each other's input and promoted students to support diverse thinking (Belenky, Clinchy, Goldberger, & Tarule, 1997; Miller, 2000). Students were also encouraged to reach a consensus in their group as the purpose of the group was about cooperation not competition (Belenky, Clinchy, Goldberger, & Tarule, 1997; Miller, 2000). Each group got one set of Tangram pieces with each student choosing a Tangram they wanted to represent. In this activity, there are congruent Tangrams so a student would be both congruent pieces. The students then "became a

Tangram" in order to promote empathy and intimacy with the Tangrams (Belenky, Clinchy, Goldberger, & Tarule, 1997; Boaler, 1997; Miller, 2000; Zohar, 2006). The researcher then had students volunteer to hold the larger Tangrams. First the researcher picked two figures. Students then answered some questions guided by the researcher: What would this figure say to this figure? How are they related? How are they different? The researcher would then pick two more figures and ask the same questions. Students were then probed to come up with questions to ask.

The next part of the activity was to complete the Core Activity (see Appendix H.) The students had to create and sketch squares with various numbers of pieces: for example, make a square with one piece. The students then developed squares with two pieces, three pieces, four pieces, and six pieces. Students had to complete the activity with some restrictions; they could only touch the piece in which they chose "to become." Students also had to analyze whether some arrangements of creating squares could be produced in more than just one way. They had to determine what the different arrangements were and why the square could be made in more than one method.

Level Three Strategy

This activity is based on the idea of developing the level three strategy (see Appendix I.) The reason behind the activity is for students to see the connection between area and finding a compatible side length. The activity takes the students step by step in analyzing areas and possible side lengths along with Tangrams which have the specific characteristic side length to create such a square. Some of the questions from the activity ask students to name the possible areas given the different side lengths. A series of questions ask the participants if squares with specific areas can be constructed. Hopefully students see the areas of 1,2,4,8 as only possible areas of squares. Based on the Side Length and Areas activity, students can deconstruct the possible side lengths of those areas as 1, $\sqrt{2}$, 2, and $2\sqrt{2}$. Connectedness in this activity is more about students becoming familiar with the connection of area and side length and constructing their own knowledge (Clinchy, 1996; Knight, Elfenbein, and Messina, 1995) rather than a banking approach (Freire, 2005).

Collaborating on Strategy Levels

The next activity consisted of constructing levels of strategies with the students (see Appendix J.) In order for students to become more familiar with advancing their knowledge of the connection between area and side length, levels of strategies were developed. The researcher merely facilitated the activity while the students developed the levels of strategies themselves in a whole group discussion. First, the students began to brainstorm the different strategies students used in completing the squares with three and four pieces in the previous activity and placed them in order based on difficulty and strategy. A connectedness piece of the activity asked students to dialogue and engage with the class (Miller, 2000.) The lowest level the students developed was trial and error; for example. Other levels were developed by students in conjunction with the researcher as they constructed their own knowledge rather than being told what the levels were (Miller, 2000.) The next activity required students to participate in metacognition (Boaler, 2005; Carr, Alexander, Folds-Bennett, 1994; Carr, 2009; Hacker, Dunlosky, & Graesser, 2009).

Reflection

In this activity, students were required to reflect on the process of why an arrangement of six Tangrams could never create a square (see Appendix K.) The researcher asked the participants to think aloud and write down their thoughts. In the same activity, the students also had to think aloud about their strategies in constructing the squares with 1, 2, 3, and 4 Tangrams and record their thinking. This also is a connectedness idea. Connectedness promotes reflection opportunities which allow students to think and reflect metacognitively (Miller, 2000). After this activity, the researcher then reviewed the activities and main ideas of area and side length before the post-test.

Post-Test

The post-test instructions were given exactly the same as the Pre-Test (see Appendix L.) The only difference was that students had to create a square now using only five of the seven pieces. The students who completed the activity in ten minutes were given a score of 1 and those who did not complete the activity were given a score of 0.

4.5 SUB-STUDY III: STRATEGIC COMPETENCE

This study was created in order to research the following questions.

- Research Question #3: a) How do male and female students' strategies differ on spatial reasoning tasks after implementation of connectedness and b) how much do male and female students' strategies change on spatial reasoning tasks after implementation of connectedness?
    - Research hypothesis 3, H_1: Male and female strategy levels will include better understanding of the connection between area and side length on spatial reasoning tasks after connectedness.
    - Null hypothesis 3, H_0: Male and female strategy levels did not have a change.

Interviews

In the qualitative portion of the study (Denzin & Lincoln, 2005), students were interviewed to gain a better understanding of how connectedness assisted in their achievement (see Appendix M.) Six students were selected based on the performance on strategy levels. Two participants, one male and one female were chosen who increased in strategy level, and the final four participants, two male and two female, were chosen whose strategy stayed the same. The following questions were asked:

1. What tangram piece did you pick and how did you feel being that piece?
2. (Remind them of strategies and levels) What strategy do you feel comfortable with and why?
3. (Task on flip of parallelogram.) Without touching the pieces, how would you complete the square? Be specific in describing your actions using geometric transformations (e.g. translation, rotation, reflection.)
4. Now show me: How much time did it take for the student to flip the parallelogram piece?
5. Describe what the Level 3 strategy was? What is challenging for you to understand the Level 3 strategy, why or why not?

In conclusion, this chapter discussed four main components of the study. The first component was the setting followed by a historical didactical perspective of Tangrams and area dissection. The third component was based on the first sub-study of the research. This study looked at the control group as a baseline of spatial reasoning task achievement. The final section of the chapter discussed the main portion of the research which is connectedness. Connectedness was embedded in several activities: researching the tangrams, learning about the pieces, connection of area and side length, core activity, strategy development of level three, collaboration, and a reflection. The next chapter five will now take a look at the process of data collection followed by student examples of responses in the activities of both sub-studies.

Chapter 5: Results and Findings

5.1 Introduction

This chapter will discuss the results and findings of the mixed method study. The reason for using both terms results and findings is to include both the quantitative and qualitative methodologies. Quantitative studies use the term results and findings tend to be related to qualitative research (Calabrese, 2009). For review, Figure 4.1 illustrates the sub-studies and how they are connected to the research questions.

Through the theoretical framework based upon feminist epistemology and how females prefer to learn, activities were created to ensure three main concepts. These concepts are: mathematical reasoning, social cognition, and multiple strategies. The various activities have these three concepts embedded within them. Mathematical reasoning allows for students to see connections of mathematics through quantitative reasoning, algebraic thinking, geometric thinking, spatial reasoning, as well as others. Social cognition is the concept allowing students to collaborate with each other creating understanding and harmony among the group. It also nurtures an environment where the student becomes a socially situated person who shows empathy and intimacy with others and the mathematical concept they are learning. With this in mind, activities were created to establish the three main concepts. The activities were established in order to assist the student in recognizing the relationship and reverse relationship between area and side length. By reverse relationship, the student finds the side length of a square given the area of the square.

5.2 RESULTS

5.2.1 Results for Research Question 1

The first research question was developed by the need to build a case. Building a case requires data which can be analyzed to determine if there is a difference between the genders' performance on spatial reasoning skills. The following question therefore was established:

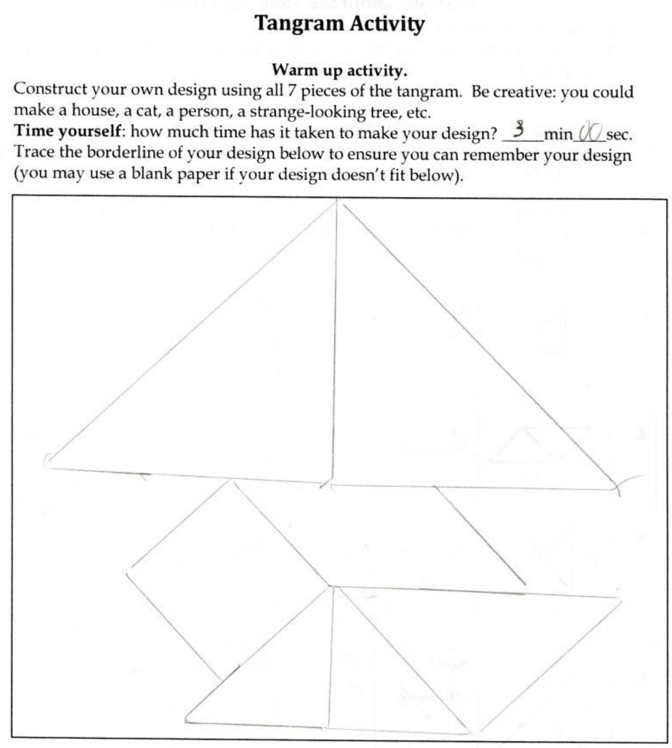
- Research Question #1: Is there a difference between female and male performance on spatial reasoning tasks across the elementary, middle school, and high school levels?
  - Research hypothesis 1, H_1: There will be a difference in achievement between boys and girls across the grade levels.
  - Null Hypothesis 1, H_0: There will be no difference in achievement between boys and girls across the grade levels.

If the null hypothesis holds true, then basically there is no dissertation. But before the results are discussed, it is important to revisit the methodology.

Students were selected from schools ranging from elementary to high school in a southwest town near the border between U.S. and Mexico. A total of 589 students were selected through a cluster random sampling. Of these, 336 students were from the high school level with 171 males and 165 females. The middle school level consisted of 187 students of which 97 were male and 90 female. The elementary school consisted of 66 total students with exactly half being females and the other half males. Even though ethnicity data was not collected of the first study of N=589, with the demographics of the other schools and the border city, it is safe to say the majority was Latino/a.

First, the students were given a warm-up where they simply constructed a figure with the seven tangram pieces and traced the borderline. Next, students exchanged papers with a partner who had to try to recreate the figure with the tangrams. Illustration 5.1 shows an example.

Illustration 5.1. Student sample of warm-up.



Next, the students participated in the Pre-Test which required them to create a square with all seven tangram pieces. Students were given a score of 1 if they were able to complete the square within the time limit of fifteen minutes. If students did not complete the square within the timeframe, a score of 0 was then given. The results of the research are given below in Table 5.1.

Table 5.1. Spatial Task Completion Results by Gender and School.

		Pre-Test
Elementary School	Females N=33	3%
	Males N=33	8%
Middle School	Females N=90	17%
	Males N=97	24%
High School	Females N=165	62%
	Males N=171	75%

Table 5.2 is the table from the data of all teachers divided among the three school levels. In addition, Figure 5.2 gives another perspective of the data.

According to the data, there is a difference in achievement between the genders in spatial reasoning tasks. We then revisit the hypothesis and find it to be true. The average (M) indicates that males have a higher completion rate in creating a square with seven pieces at M=0.75 compared to M=0.635 for females with a standard deviation (SD) of 0.1778 and 0.2483 and with a standard error (SE) of 0.0629 and 0.0878 respectively. Furthermore, the sample variation (SV) is quite low indicating a good sample with 0.0316 for males and 0.0617 for females.

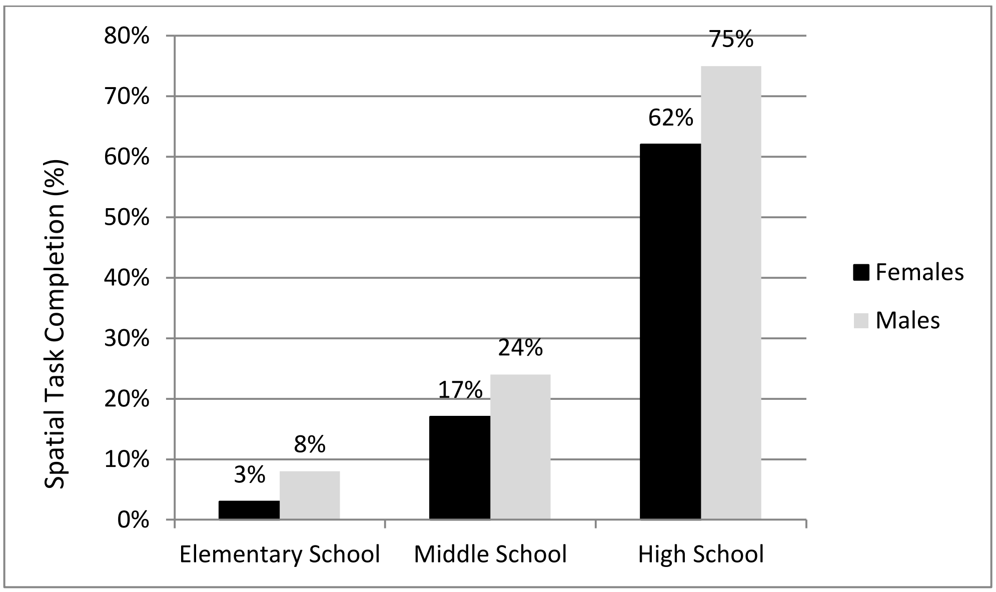


Figure 5.1. Pre-Test Spatial Task Completion Results by Gender and School.

Table 5.2 gives the descriptive statistics of both genders.

Table 5.2. Descriptive Statistics of Spatial Task Completion of Both Genders.

	Females	Males
Mean	0.635	0.7513
SE	0.0878	0.0629
Median	0.685	0.765
SD	0.2483	0.1778
SV	0.0617	0.0316
Kurtosis	-0.2307	0.4176
Skewness	-0.3458	-0.531
Confid Level(95%)	0.2076	0.1486

This research confirms prior research indicating a general difference in achievement between the two genders (see Contreras, Martínez-Molina, & Santacreu, 2012; Gluck & Fitting, 2003; James, 2009; McGraw, Lubienski & Strutchens, 2006; NAEP, 2012; Sadker, Sadker, & Zittleman, 2009.) The data suggests that the difference between the genders does not necessarily increase or decrease along the grade levels. In elementary school, males scored higher in spatial reasoning with a completion rate of eight percent compared to females who had a three percentage completion rate. With the difference only being five percentage points, the difference is not extremely large. In middle school, the difference increases up to seven percentage points where males have a 24% completion rate compared to 17% for girls. Although, the increase in completion rate is evident, the difference still exists. Furthermore if we examine the completion rates of students in high school, the results are increase but the difference still exists. In high school, males again outscored the females with 75% and 62% respectively. Again, although completion rates have increased for both genders, the difference in spatial reasoning remains the same at seven percent.

5.2.2 Results for Research Question 2

The second research question bases itself from the first research question. If there is no difference in female and male achievement in spatial reasoning, then the research is finished. However, since there is an almost consistent difference among the genders, the second research question then

comes into play. The second research question bases itself around the idea of connectedness as an intervention. Therefore we have as the second research question as:

- Research Question #2: To what extent does connectedness improve females' performance on spatial reasoning tasks?
    - Research hypothesis 2, H_1: Connectedness will assist both genders in increasing their spatial reasoning abilities and assist females in decreasing the achievement difference.
    - Null Hypothesis 2, H_0: Both genders did not have a change in spatial reasoning tasks after the intervention.

Two groups were created to research the impact of connectedness. The first group, the control group, merely was given the Pre-Test and the Post-Test. Of the control group (N=247), 126 were males and 121 were females. The participants were cluster randomly selected from various southwest high schools. The Pre-Test and the Post-Test were given two weeks apart in order to give some time in between tests. The results of the control groups' time averages are given in Table 5.3. Males decreased their time by 38 seconds

Table 5.3. Control group average task completion times (sec.).

	Pre-Test	Post-Test
Males	765.125	727.225
Females	760.748	754.152

in the Post-Test compared to the females decrease of only 6 seconds. Figure 5.3 also shows the same information in a chart. The horizontal lines represent little change in task completion times between the two tests.

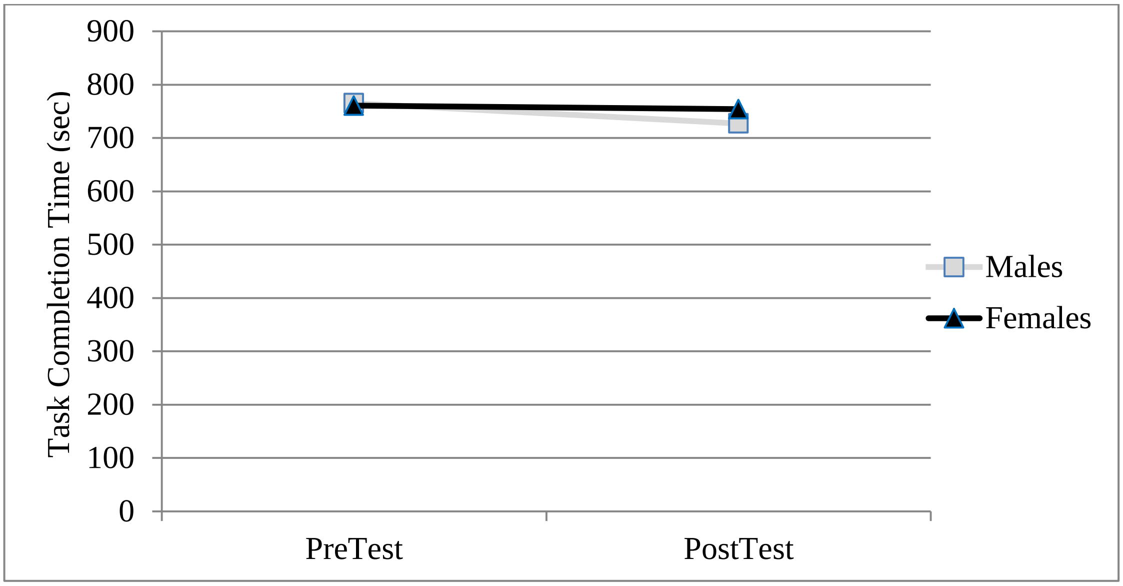


Figure 5.2. Control Group Average Task Completion Times.

Furthermore, the results of the control groups' task score averages are shown in Table 5.4 and Figure 5.4.

Table 5.4. Task completion average scores.

	Pretest	Post-Test
Males	0.319	0.444
Females	0.359	0.393

Males increased their average score by a total of 0.13 points compared to the females with just a 0.04 increase.

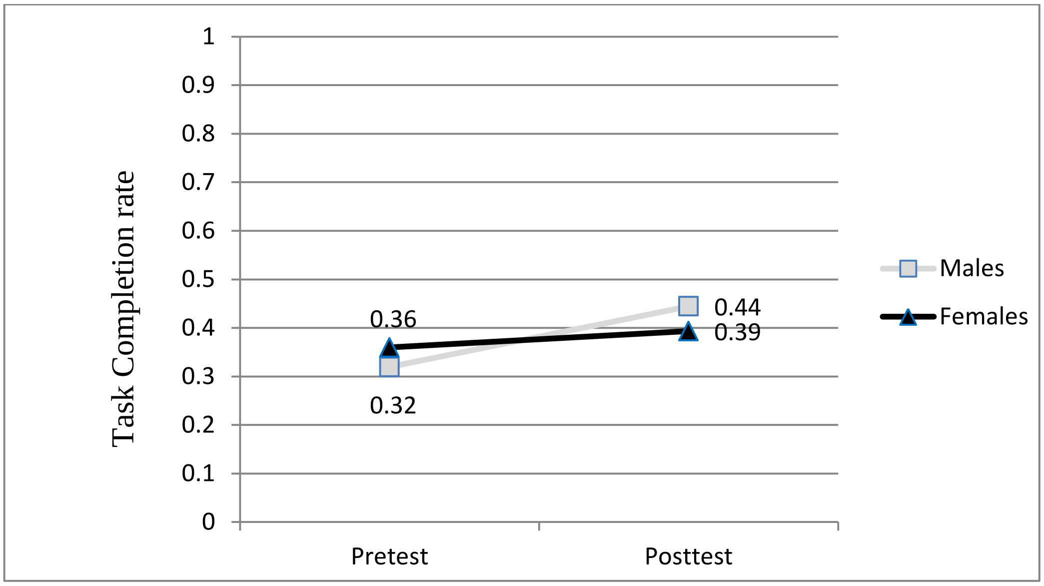


Figure 5.3. Task Completion Average Scores.

The second group contained the treatment group who received activities with connectedness. Remember, connectedness is based on three concepts of: mathematical reasoning, social cognition, and multiple strategies. The treatment group also was given a Pre-Test and a Post-Test as measure to quantify the difference in scores. The participants (N= 472) were cluster randomly selected from several southwest high schools. The reason behind the disparity in numbers between the treatment and the control group was based on the length of time for the connected activities. Administration would only allow for one classroom at a time to participate in the activities at any one time and less during state testing months. Therefore, for each treatment group the set of activities lasted approximately six days.

The treatment group participants for this portion of the study were students of the researchers. Of the 472 participants, 219 were female and 253 were male. Of the females, 86.3% were of Latina descent and of the males 81.8% were Latino. This group of students participated in all seven portions of

the study: Pre-Test, Research, Table activity, Core activity, Level Three Strategy, Reflections, and the Post-test.

The treatment group participated in many activities. Other than the Warm-up, Pre-Test and the Post-Test, students started with researching the tangrams. Illustration 5.2 shows a student sample of their research. After researching the tangrams, students shared their findings with the classroom.

Illustration 5.2. Student sample of research.

Get to know Tangrams better by using resources such as the Internet, Library, etc…, to research about them.

1) What is a tangram?

a tangram are seven flat shapes.

2) Where did they come from?

They came from China

3) Find some history of Tangrams.

Tangrams were brought to america in 1815

4) What are some literature books that talk about Tangrams?

"The Tangram Book", "Greatest Puzzle of all Time", and the "Eighth book of Tan" are books that talk about tangrams

5) What areas of mathematics do you think tangrams can be included?

Tangrams can be used in Geometry

6) What can you learn with tangrams?

Tangrams can be used as puzzles but a game that test your skills and knowledge

The next activity was created in order for students to become familiar with the tangram pieces. Illustration 5.3 shows a student sample of the activity. Here, students were asked to participate in mathematical reasoning while looking at how the tangrams are congruent or

Illustration 5.3. Student sample of mathematical reasoning.

#	Sketch the Tangram Piece	Name the Tangram Piece	Area of the Tangram Piece	Side Lengths of the Tangram Piece	Number of Congruent Pieces	Area of Congruent Pieces
1.	a	Square	1	$a = 1$	1	1
2.		small right Isoceles Triangle	1/2	$1, 1, \sqrt{2}$	2	1
3.		parallelogram	1	$1, 1, \sqrt{2}, \sqrt{2}$	1	1
4.		medium right Isoceles triangle	1	2	1	1
5.	2, $2\sqrt{2}$, 2	large right Isoceles triangle	2	$2, 2, 2\sqrt{2}$	2	4
		List different side lengths of the Tangram pieces:		$1, \sqrt{2}, 2, 2\cdot\sqrt{2}$	Total area of all 7 pieces:	8

similar. This activity allowed for students to analyze several components of each tangram. The activity was completed in a whole classroom discussion. First, students were asked to sketch the tangram piece and also give the piece a name. Second, the students found the area of each piece by using the length of the square as one unit. Third, students then found the lengths of all sides by either using the unit length or by other methods such as the Pythagorean Theorem in order to find hypotenuses. Once the side lengths were developed, students recorded them in the table. Fourth, students then had to determine how many congruent pieces there were compared to the tangram they were evaluating. Finally, students

then added up all the areas of the congruent tangram pieces and also recorded their responses in the table. This activity was created in order for students to see that if all of the tangram areas were combined a total area of 8 units^2 would be revealed. This new development will then be used in later activities. Students also recorded all the possible side lengths of the tangrams which are 1, $\sqrt{2}$, 2, and $2\sqrt{2}$. This new information will also be needed in future activities. Illustration 5.4 shows the tangrams in larger size for students to review with.

Illustration 5.4. Classroom tangrams.

The next step in the mathematical reasoning was to analyze the relationship between side lengths and areas of squares. Illustration 5.5 shows a student sample.

Illustration 5.5. Student sample of side length and area.

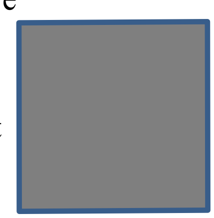
Side Length a=	1	$\sqrt{2}$	2	$2\sqrt{2}$
Process	1^2 (1x1)	$(\sqrt{2})^2$	2^2	$(2\sqrt{2})^2$
Area of Square	1 unit^2	2	4	8 unit^2

7pieces

Here the idea is to have students fill out the table corresponding to a specific side length, process, and area of the square. As you can see, the student was able to find the four possible areas of squares given all possible side lengths of the tangrams. For example, if a tangram has a side length of two units, then we can simply square the side length to find the area. In this case, the area would be four square units and so forth. The key point here is for the student to establish the four possible side lengths of 1, 2, 4, and 8 given the side lengths of 1, $\sqrt{2}$, 2, and $2\sqrt{2}$. Here are the side lengths and the corresponding tangram figures (figures not drawn to scale.)

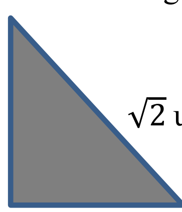
Square

1 unit



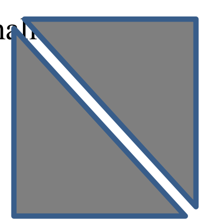
Small triangle

1 unit



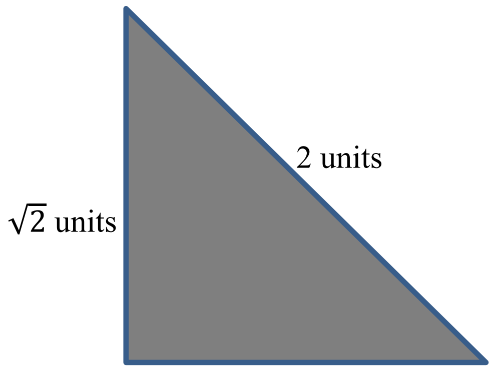
$\sqrt{2}$ units

The square has a side length of 1 unit and the area is 1 unit^2. The small triangle has two side lengths from the leg and the hypotenuse of 1 unit and $\sqrt{2}$, respectively. The area of the small triangle is $\frac{bh}{2}$ giving us $\frac{1 \times 1}{2}$ and an area of ½ units. If the small triangle was converted into a square, then the area would consist of twice the area which is 1 unit. The 1 unit then becomes a possible area of a square created b ngram and the sma

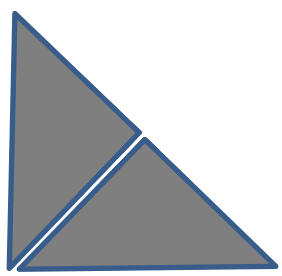


Next, the medium triangle has the following side lengths.

Medium triangle

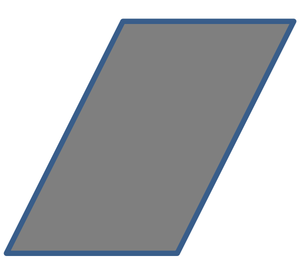


We then use the previous formula to find the area of a triangle and get $\frac{\sqrt{2}\times\sqrt{2}}{2}$ for an area of the medium triangle of 1 unit. We can also see that two small triangles each with an area of ½ units create the medium triangle.

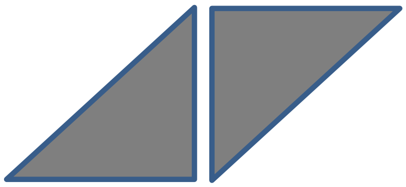


In order to find the area of a square created by the medium triangle, we again use the side length of $\sqrt{2}$. Areas of squares are given by the formula $A = s^2$. In this case the side length is $\sqrt{2}$ and inserted into the formula gives us $A = (\sqrt{2})^2$ for an area of 2 units^2. So, currently we have established the areas of 1 unit^2 and 2 units^2.

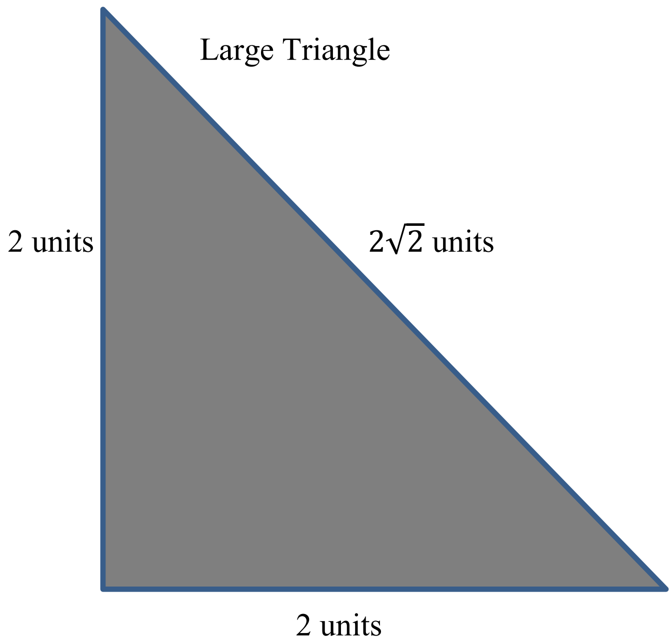
The next tangram is the parallelogram. Here, we are not able to construct a square from the parallelogram, but are still able to find the area of the parallelogram.



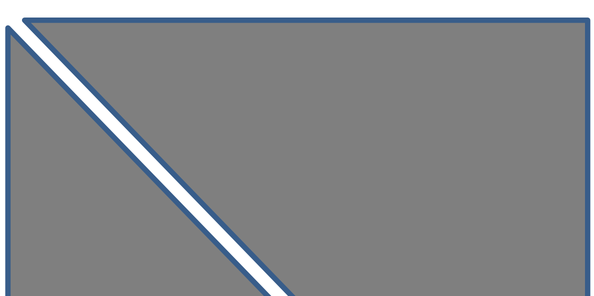
The area of a parallelogram is given by the formula of $A = bh$. We know the height is 1 unit and the base is 1 unit. If we insert the numbers into the formula we get $A = 1 \times 1$ to give us 1 unit^2. We can also create the parallelogram with two small triangles.



The last figure, the large triangle, provides us with two new side lengths of 2 units and $2\sqrt{2}$ units. Again, if we choose to use the formula for areas of triangles we get $\frac{2\times 2}{2}$ for an area of the large triangle of 2 units^2.

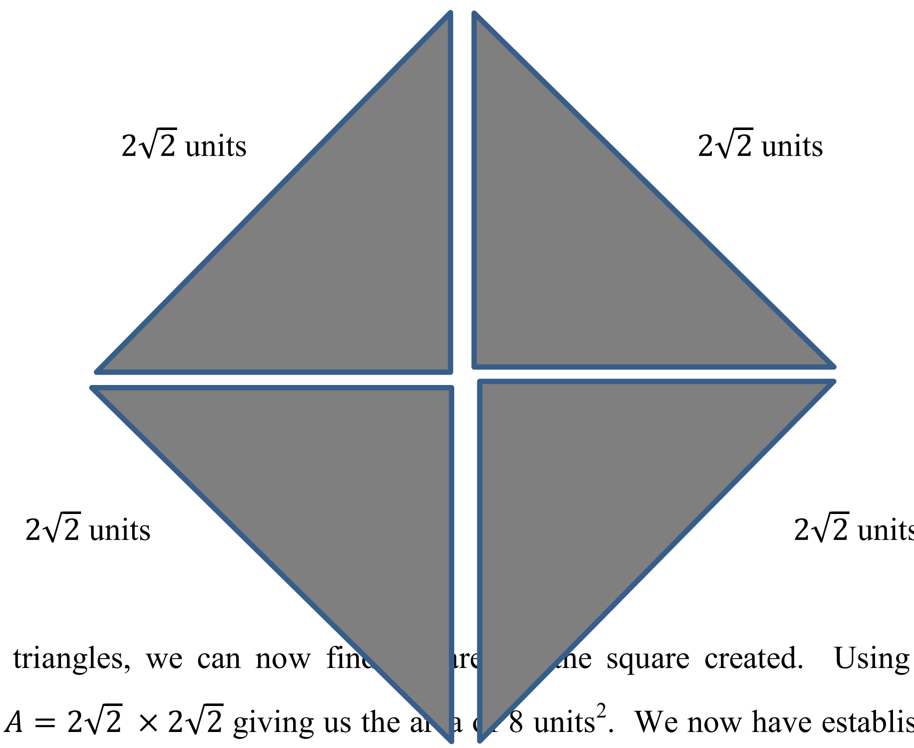


Now we can create two squares from the given lengths. First, we will create a square with a side length of 2.



2 units

Areas of squares are given by the formula $A = s^2$. We then insert 2 units into the formula to get $A = 2 \times 2$ for an area of 4 units^2. We now have established three possible areas of squares with 1, 2, and 4 units^2. The second side length given from the hypotenuse of the larger triangle of $2\sqrt{2}$ units is a little trickier to create a square; however, very possible using four large triangles.



With the four large triangles, we can now find the area of the square created. Using the formula of $A = s^2$, we have $A = 2\sqrt{2} \times 2\sqrt{2}$ giving us the area of 8 units^2. We now have established the only

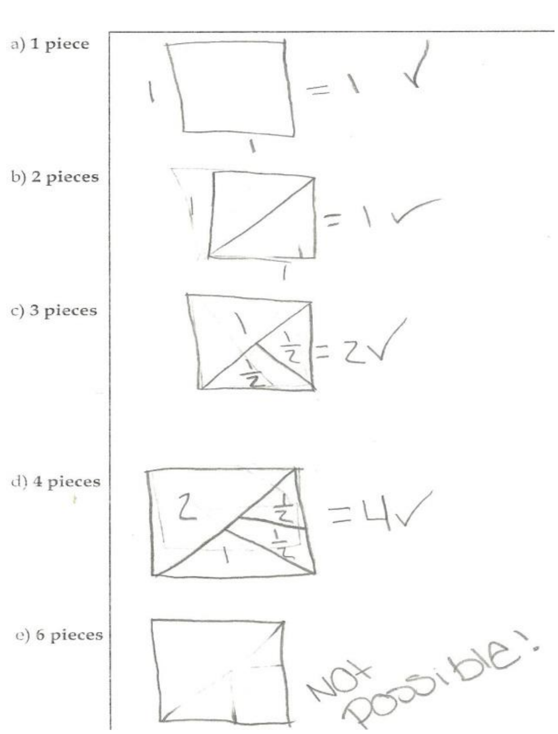
four possible areas of 1, 2, 4, and 8 units^2 by creating squares with the various side lengths of tangrams. Table 5.6 shows the completed table.

Table 5.6. Side Lengths and Areas.

Side Length a=	1	$\sqrt{2}$	2	$2\sqrt{2}$
Process	1^2	$(\sqrt{2})^2$	$(2)^2$	$(2\sqrt{2})$
Area of Square A=	1 unit^2	2 units^2	4 units^2	8 units^2

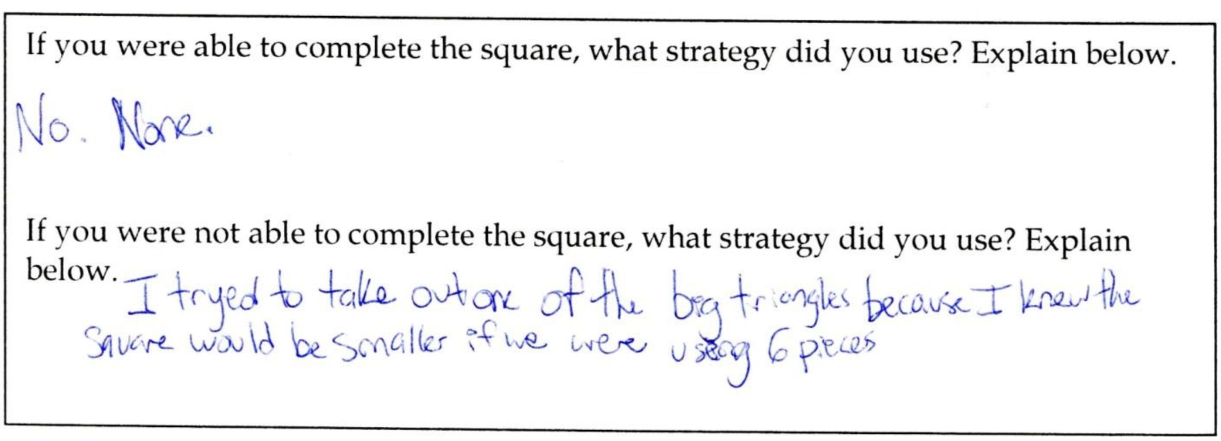
The next set of activities dealt with constructing squares of various tangram pieces. This core activity especially promoted the connectedness concept of social cognition. The students were required to work in groups of five. Five was chosen as the total members of the group so each student could represent a tangram piece (since two of the tangram pieces each have a congruent partner.) The directions for the activity asked participants to share their ideas, build on each other's input, support diverse thinking, and reach a consensus so everyone agrees on the construction of the squares. The teacher stressed the idea of groups as an instrument of cooperation and not competition. Furthermore, each group received a set of the seven piece Chinese tangrams. Each member of the group was then required to choose a tangram piece they wanted to represent. Two people in the group will be two congruent pieces each. A stipulation to the activity was students could only touch their piece in order to construct squares with the group. Illustration 5.6 shows a student sample of constructing squares.

Illustration 5.6. Student sample of core activity.



Students then were asked which squares had more than one way to be constructed with the same number of tangram pieces. For example, there are three ways to construct a square with four tangrams. Participants were also asked why a square could not be constructed with six tangram pieces. Illustration 5.7 shows a student response. The student went further and labeled the areas of each tangram piece to find the total area for the square they created.

Illustration 5.7. Student response.



If you were able to complete the square, what strategy did you use? Explain below.

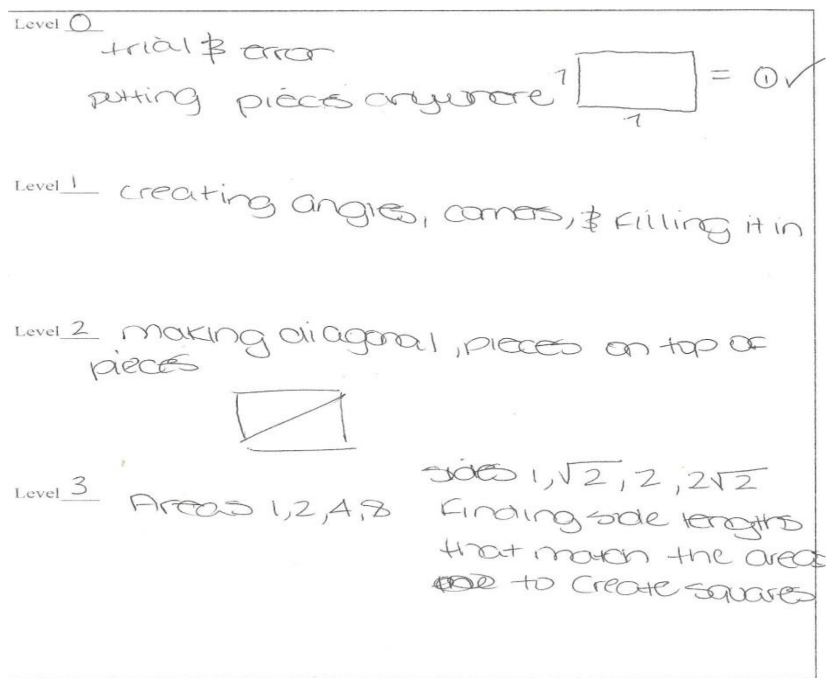
No. None.

If you were not able to complete the square, what strategy did you use? Explain below.

I tryed to take out one of the big triangles because I knew the square would be smaller if we were using 6 pieces

The next activity's goal was to have students' establish levels of strategies in order to complete the squares. Through a whole group class discussion, the participants developed a series of strategies such as trial and error and creating corners first to create the squares. Illustration 5.8 shows an example of a student's work in developing the strategies.

Illustration 5.8. Developing strategies.



The final activity in the connectedness strand was for students to write a reflection on why a tangram with six pieces is not possible. We specifically asked students to think aloud and write their thoughts down. Illustration 5.9 shows an example of student work. This student felt it was necessary to label all of the areas for the level three strategy. They also included the various possible side lengths corresponding to the areas.

Illustration 5.9. Student reflection.

You cannot make a square with 6 pieces because the areas possible 1, 2, 4, 8. 7 pieces create an area of 8 and any less would not equal one of the areas listed

The purpose behind the reflection piece was for students to realize that a square is not possible to create with six pieces. If all seven tangram pieces added up together equal 8 units^2, taking a piece away decreases the area. For example, the smallest piece (the small triangle) has an area of ½ units^2. Taking away that area would leave a possible area of 7 ½ units^2. Students can immediately determine 7 ½ units^2 is not one of the possible square areas of 1, 2, 4, and 8 units^2 we developed in earlier activities. Furthermore, if the largest tangram piece area (large triangle) is taken away, then students would subtract the area of 2 units^2 to get a left over area of 6 units^2. The left over area is also not one of the possible areas of 1, 2, 4, and 8 units^2. Students then came to the conclusion if they take away either the smaller tangram or the largest tangram, neither one of them gets close to the next largest possible area of 4 units^2 because an area of 8 units^2 is given by all seven tangrams.

Next, we asked the participants to think aloud about the strategies they used in order to create the squares and write down their thoughts. Illustration 5.10 shows an example of a student reflection. The student states they had used three of the strategy levels of connecting angles, trial and error, and finding side lengths that match the area.

Illustration 5.10. Student reflection.

attempted to connect all angles
trial and error
finding side lengths that match the square areas to create squares

Lastly, the students reviewed with the teacher in a whole class discussion on the possible areas, the side lengths, and strategy levels. Afterwards, students were then given the Post-Test which required participants to create a square with only five tangram pieces. The students timed themselves on completing the square and also wrote a reflection on the strategy they used. For review, students were given a score of one if they completed the square and a zero if they did not complete the square. Times were collected and analyzed along with the completion rate. Table 5.7 displays the task completion average times.

Table 5.7. Task completion average times.

	Pre-Test (min.)	Post-Test (min.)
Females	13.3	6.58
Males	10.18	5.01

On the Pre-Test, task completion average times were high for both groups. Females took longer to create a square with a seven tangram pieces than did the males. The difference between the genders in Pre-Test times was approximately 3.11 minutes. However, in the Post-Test, both genders decreased their times compared to the Pre-Test. Females decreased 6.7 minutes compared to the males' decrease of 5.1 minutes. Figure 5.5 displays the data in a graph.

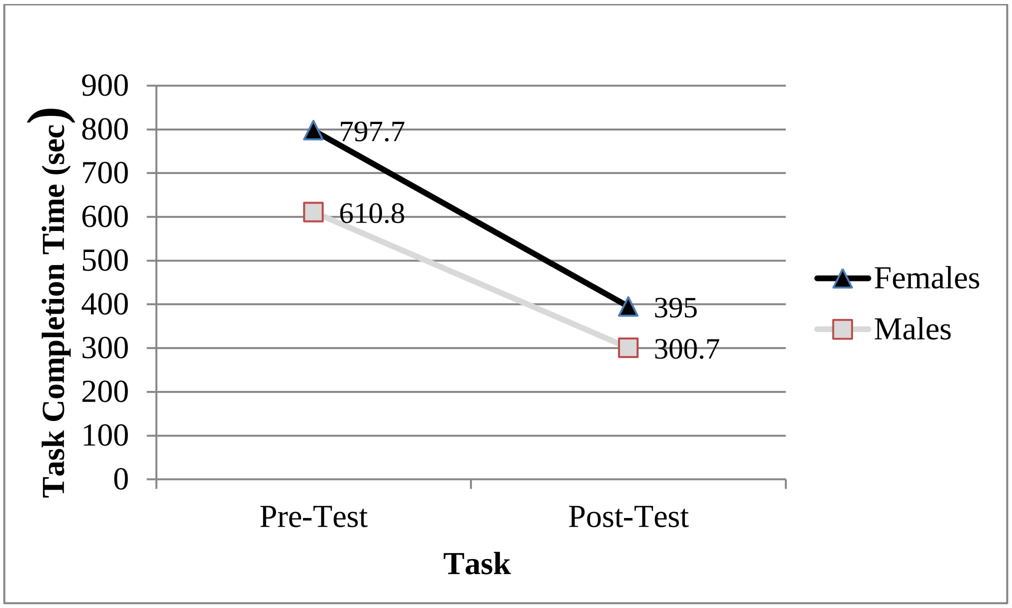


Figure 5.4. Task Completion Time.

Females decreased their time more than did the males. As mentioned before, the difference in gender times was 3.11 minutes for the Pre-Test. Similarly, the difference in Post-Test scores between the genders fell to 1.57 minutes. Females decreased the difference by 1.54 minutes, cutting the difference by almost 100%. Table 5.8 provides a chart for the task completion rate of the participants separated by gender.

Table 5.8. Task completion rate.

	Pre-Test	**Post-Test**
Females	62%	86%
Males	75%	91%

Females and males scored a 62% and 75% task completion rate, respectively. The difference between the genders was 13% with the males again completing the Post-Test at a higher rate than females. The males increased their score between tests by 16%. Females, on the other hand, increased their score by 24%. The following Figure 5.6 shows the results in a table. As mentioned before, the difference in gender completion

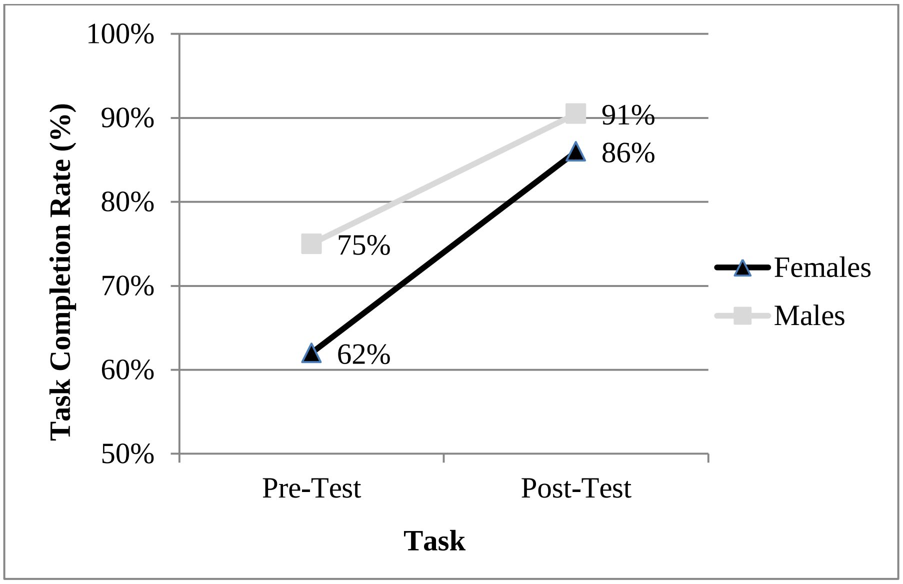


Figure 5.5. Task Completion Rate.

gender times in the Pre-Test was 13%. After the Post-Test, females decreased the difference between the genders to 5%. Females decreased the difference in completion rates by more than 100%.

Another mode of analysis is through the mean scores between the control and the treatment group. Table 5.9 displays the mean scores for both groups.

Table 5.9. Mean scores for pre and post-tests by group.

	Pre-Test	Post-Test
Control (n= 247)	0.42	0.54
Treatment (n= 472)	0.44	0.82

The control group's mean Pre-Test score was 0.417 compared to the Treatment group score of 0.436. Here there is no significant difference in mean Pre-Test scores between the two groups with a difference of 0.19 on a scale from 0 to 1. Remember, students were given a score of 0 if they did not

complete the task and a score of 1 if they completed the task. However, there is an increase in the difference in mean score in the Post-Test. The control group Post-Test mean score was 0.543 compared to the treatment group of 0.818. Figure 5.6 shows the data in a line graph.

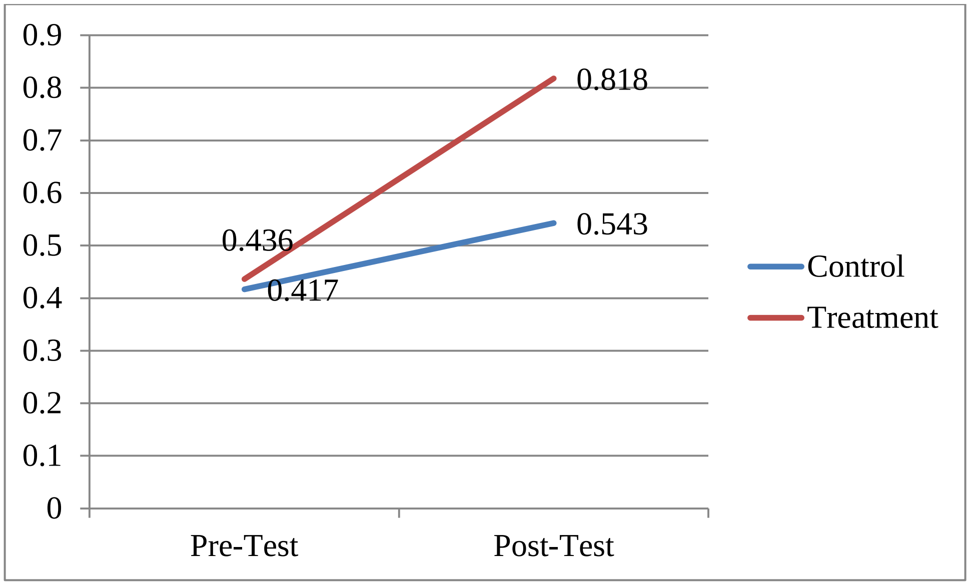


Figure 5.6. Mean scores by group.

Further analysis in Table 5.10 is developed by investigating the mean scores by group through gender.

Table 5.10. Means scores by group and gender.

		Mean	SE
Male	Control	0.1429	0.677
	Treatment	0.32	0.55
Females	Control	0.107	0.68
	Treatment	0.452	0.583

In order to study the main effects of the data, Analysis of Variance (ANOVA) was used to determine whether or not the difference between the genders in interaction effect from Pre-Test to Post-Test was statistically significant. Table 5.11 displays the two way ANOVA analysis. The interaction effect between the genders and between the Pre and Post is included.

Table 5.11. Two-way ANOVA Tests of Between-Subjects Effects.

Dependent Variable: Gain

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	24.445[a]	3	8.148	20.813	.000
Intercept	43.251	1	43.251	110.473	.000
Gender	.987	1	.987	2.521	.113
Treatment	21.626	1	21.626	55.238	.000
Gender * Treatment	2.223	1	2.223	5.679	.017
Error	274.054	715	.392		
Total	343.000	719			
Corrected Total	298.499	718			

The analysis of variance indicates two major findings:

1) there is a statistically significant differences between the gain of females vs. males in the treatment group that used connectedness as an intervention;

2) most importantly, there is a statistically significant difference between gains of females and males with regard to the interaction of two independent variables: gender and treatment ($F(1, 719)=5.679$, $p< .05$, $\eta^2 = 0.772$.

5.2.3 Results for Research Question 3

The third research question is comprised of two parts. For review, here is the third research question:

- Research Question #3: a) How do male and female students' strategies differ on spatial reasoning tasks after implementation of connectedness and b) how much do male and female students' strategies change on spatial reasoning tasks after implementation of connectedness?

- Research hypothesis 3, H_1: Male and female strategy levels will include better understanding of the connection between area and side length on spatial reasoning tasks after connectedness.
- Null Hypothesis 3, H_0: Male and female strategy levels did not have a change.

As discussed earlier, this particular research question has both quantitative and qualitative components. The first part of the research question discusses how do strategies differ which requires a qualitative aspect of research. The second part of the question asks how much do strategies change referring to a quantitative feature of research. We will discuss the second portion of the research question first in quantifying how much the strategies have changed per gender after the implementation of connectedness.

During the activity “Development of Strategic Competence (see Appendix I),” the treatment group analyzed the different strategies used to create squares with different number of tangram pieces. The researcher did not provide the levels of strategic competence to the students, the participants created the levels by themselves. First, the researcher asked the participants to name all the different strategies they used to create the squares. Secondly, the research asked the students to rank them in order from easiest to those which required more thinking. In a consensus, the participants developed three basic levels. Table 5.12 shows the student developed strategy levels. Level 1 consists of students moving pieces around using trial and error. Some

Table 5.12. Strategy levels.

Level 1	• Trial and error • Moving pieces around
Level 2	• Basic properties of sides and angles • Geometric transformations • Properties of diagonals and congruency
Level 3	• Inverse connectedness between area and side length

students wanted to use the term "random trial" when they were not actually randomly choosing pieces. The researcher also had made the same mistake. This was also the control groups' main strategy. In Level 2, the participants categorized the strategies of creating corners first or creating right 90° angles would be next. Students also decided placing tangram pieces on top of others to see their congruency would also be included. If students used diagonals, they believed this would also be in the second level. Level 3 was created by using inverse connectedness between area and side length. Students who were able to find a side length given the area were seen as using reverse connectedness. For example, if the combined area of the tangram pieces was four square units, then the students deducted the side length, by using the square root as in the table "Side Lengths and Areas (Appendix G)." Table 5.13 shows how the strategy levels changed for females and males. Not all participants were asked about Pre-Test and Post-Test strategy levels. This data was collected after some initial analysis. The researcher came to the conclusion that strategy level analysis would be important to include. Only 67 participants were asked about their strategy levels since this occurred late in the data collection process. Figure 5.7 displays the data for males.

Table 5.13. Pre-Test and Post-Test strategy levels (N=67).

Pre-Test	Level 1	Level 2	Level 3

Male (n=39)	26	10	3
Female (n=28)	18	8	2
Post-Test	Level 1	Level 2	Level 3
Male	24	6	9
Female	6	9	13

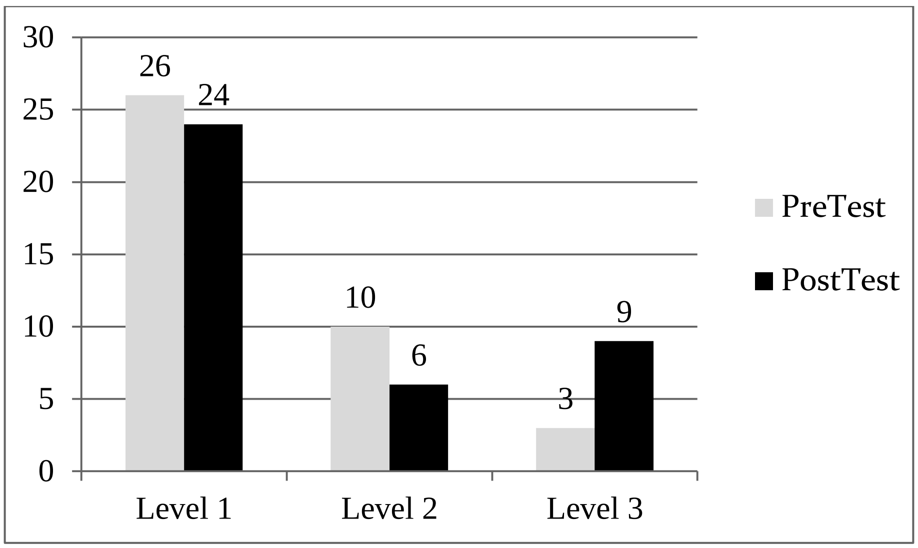


Figure 5.6. Strategy Levels Males Use (n=39).

As the data shows, the males increased their strategic competence. Figure 5.8 shows the strategic competence for females. As shown in the chart, females increased their level three

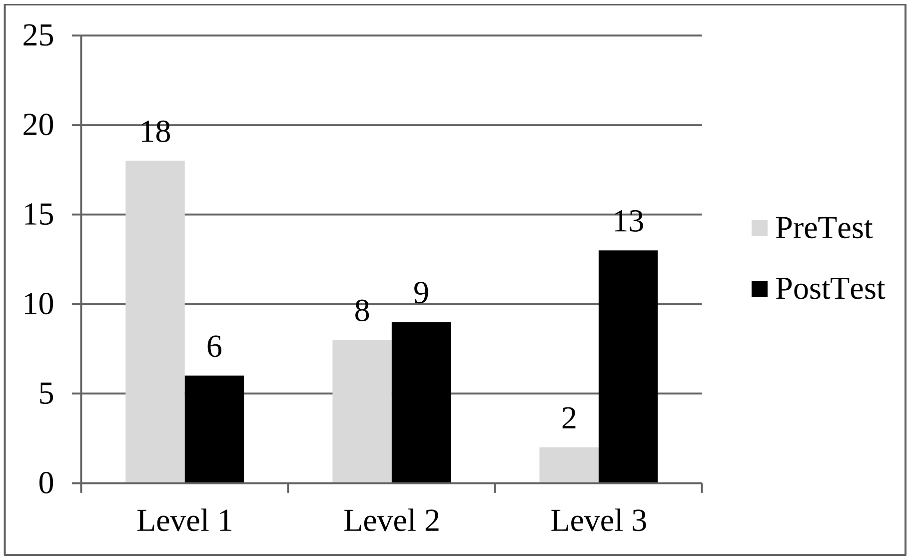


Figure 5.7. Strategy Levels Females Use (n=28).

strategy during the intervention of connectedness. To further investigate the effects of connectedness on the strategy levels of females and males, Chi-Square analysis was

Table 5.14. Chi-square analysis for females.

Females (n=28)	L1	L2	L3	Totals
PreTest	18	8	2	28
Post-Test	6	9	13	28
TOTAL	24	17	15	56
Chi-Square	14.125	DF=2	P-value 0.001<0.01	

implemented. Table 5.14 reveals the analysis for females. The Chi-Square p-value is statistically significant with $p<0.001$ at a p-value level of $p<0.01$. For the males, Table 5.15 displays the information. The Chi-Square value does not show a statistically significant change in the male strategic competence with a p-value of 0.13 compared to $p<0.05$. Chi-Square test

Table 5.15. Chi-square analysis for males.

Males (n=39)	L1	L2	L3	Totals
PreTest	26	10	3	39
Post-Test	24	6	9	39
TOTAL	50	16	12	78
Chi-Square	4.08	DF=2	p-value=0.13>0.05	

results show that it is most likely that female students' improvement in strategic competence compare to male students is due to the connectedness treatment.

As mentioned earlier, the border city where the study was conducted comprised of 82.2% Latino/a and 17.8% non-Latino/a (U.S. Census, 2010). With the participants dominantly Latino/a, there was limited data in other sub-groups such as Whites and African Americans to sufficiently perform further analysis such ANOVA, and Chi-Square to investigate statistically significance among and within ethnic groups. The results would have been skewed and non-representative among ethnic groups other than Latino/a. General ANOVA is most accurate in experimental data when the sample sizes are equal (Gravetter & Wallnau, 2013). Now we will investigate the analysis from the first part of the third research question in the findings section.

5.3 FINDINGS

As discussed earlier, the findings section is developed by qualitative research as results are more geared to quantitative research. Here, this section of the study moves toward the qualitative portion of the study. The reason behind this mixed method study is to look at another avenue towards answering our third research question.

5.3.1 Findings for Research Question 3

- Research Question #3: a) How do male and female students' strategies differ on spatial reasoning tasks after implementation of connectedness and b) how much do male and female students' strategies change on spatial reasoning tasks after implementation of connectedness?
    - Research hypothesis 3, H_1: Male and female strategy levels will include better understanding of the connection between area and side length on spatial reasoning tasks after connectedness.
    - Null Hypothesis 3, H_0: Male and female strategy levels did not have a change.

The first part of the research question required considering how qualitative research can assist in developing a process to answer this specific question. Two separate groups were interviewed. The first group was interviewed in the fall semester while the second group was interviewed during the spring semester. In between both interviews, adjustments were made in order to perfect the interviews resulting in the analysis of both groups separately. The first sets of interviews were short; however, showing some results. The second set of interviews allowed for more in depth questions resulting in diverse results.

Sampling was purposive (Lincoln & Guba, 1985). Students were selected based on certain criteria. In the first group, six students were selected based on the performance on strategy levels. Two participants, one male and one female were chosen who increased in strategy level, and the final four participants, two male and two female, were chosen whose strategy stayed the same. The only requirement was to have an equal amount of females and males participate in the study to promote an equal chance of voice. Much like Reyes (2007), "One of my coding systems of domain analysis consisted of color-coding particular words and expressions found in the interview transcripts." This is how the analysis was basically done. The following questions were asked:

1. What tangram piece did you pick and how did you feel being that piece?
2. What strategy do you feel comfortable with and why?
3. (Task on flip of parallelogram.) Without touching the pieces, how would you complete the square? Be specific in describing your actions using geometric transformations (e.g. translation, rotation, reflection.)
4. Now show me: How much time did it take for the student to flip the parallelogram piece?
5. Describe what the Level 3 strategy was? What is challenging for you to understand the Level 3 strategy, why or why not?

This first section will analyze students whose strategy levels stayed the same then afterwards those whose strategic competence improved. Four students were selected to participate in the interviews whose strategy level stayed the same. The four students were Rita, Rafael, Nena, and Juan (pseudonyms.) Rita was a female freshman student who strategic competence stayed at Level 1 using corners and angles to construct the various squares in the Pre-Test and Post-Test.

R: What tangram piece did you pick and how did you feel being that piece?

Rita: The big triangle….it helps fit all the pieces.

R: What strategy do you feel comfortable with and why?

Rita: Level 1…I like working with corners first.

Rita decided to choose the big triangle as the one she would represent because “it helps fit all the pieces.” Here we see Rita choosing a tangram piece which will work well with other pieces. She feels the big triangle is important in this way.

R: Without touching the pieces, how would you complete the square? Be specific in describing your actions using geometric transformations (e.g. translation, rotation, reflection.)

Rita: Get the parallelogram and put it in…translate it.

R: Now show me: (How much time did it take for the student to flip the parallelogram piece?)

(Rita did not get it at first then flipped the parallelogram.) 10 sec.

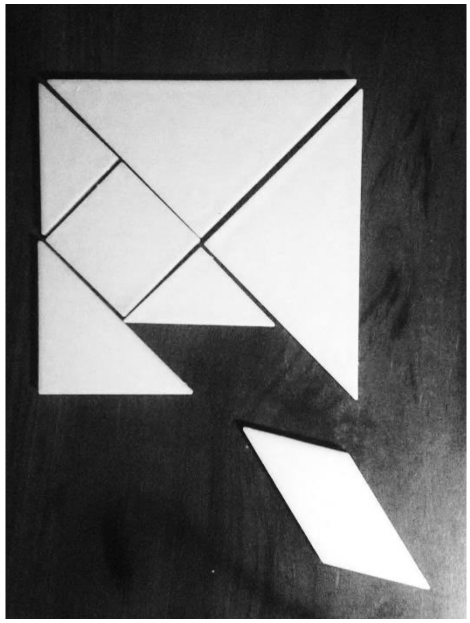
This task required a mental reflection of the parallelogram piece. Rita had difficulty visualizing how to place the parallelogram to complete the square. Illustration 5.11 shows how the task was set up. This set up of the square required the student to reflect the parallelogram in order to complete the square. Rita tried to place the parallelogram as is but could not complete the square at first chance. Then she realized she had to reflect the parallelogram in order to complete the square. Rita took ten seconds to complete the square.

R: Describe what the Level 3 strategy was? What is challenging for you to understand the Level 3 strategy, why or why not?

Rita: Angles…square roots.

Here Rita is also having difficulty understanding the level three strategy. With a short answer, it is evident Rita does not know the level three strategy well.

Illustration 5.11. Complete the square task.



The next student, Rafael, also stayed the same in strategic level competence.

R: What tangram piece did you pick and how did you feel being that piece?

Ra: Parallelogram….it looks cool. It was okay.

Rafael is starting to connect to the piece. “It looks cool,” he says.

R: What strategy do you feel comfortable with and why?

Ra: Level 1…trial and error…I’m good at it.

Here, Rafael is showing what he feels comfortable with the trial and error strategy.

R: Without touching the pieces, how would you complete the square? Be specific in describing your actions using geometric transformations (e.g. translation, rotation, reflection.)

Ra: Get the parallelogram in the space after you flip it upside down.

R: Now show me: How much time did it take for the student to flip the parallelogram piece?

Rafael visualized that in order to complete the square; the parallelogram must be flipped upside down and then inserted into the square. Rafael was able to complete the square within three seconds.

R: Describe what the Level 3 strategy was? What is challenging for you to understand the Level 3 strategy, why or why not?

Ra: Using measurement areas of a certain number…the side length is a certain number.

In the description of the Level 3 strategy, Rafael understands measurement and side length are important. He fails to describe the essence of the Level 3 strategy as knowing the area and deducting the side length.

The next female student, Nena, also had strategy levels which stayed the same.

R: What tangram piece did you pick and how did you feel being that piece?

N: A square…it's more fun.

Nena is becoming empathetic to the tangram square piece. "It's more fun," she explains indicating the square has a personality.

R: What strategy do you feel comfortable with and why?

N: Level 1. The angles are easier…you get more visual to make the square.

Nena also chooses Level 1 as the strategy level she is more comfortable with. She specifically says that it enables her to visualize the square.

R: Without touching the pieces, how would you complete the square? Be specific in describing your actions using geometric transformations (e.g. translation, rotation, reflection.)

N: Get the parallelogram and just translate it.

R: Now show me: How much time did it take for the student to flip the parallelogram piece?

Nena was unable to see how the parallelogram needed a reflection before inserting it into the empty space to complete the square. She had difficulty in the mental reflection of the piece. Nena took eight seconds to complete the square.

R: Describe what the Level 3 strategy was? What is challenging for you to understand the Level 3 strategy, why or why not?

N: Using side lengths to create a square.

Nena has some idea the Level 3 strategy requires side lengths. However, she has not mentioned how the areas of squares are directly related to the side lengths of the tangram pieces.

The next student, Juan, also was asked the same questions. Juan's strategies from Pre-Test to Post-Test also stayed the same.

R: What tangram piece did you pick and how did you feel being that piece?

J: Small triangle…goes more with this…easier to place.

Juan has chosen the small triangle to represent because it is "easier to place." Juan also has some feelings of empathy because he realizes the small triangle is easier to place in relation to the other tangram pieces.

R: What strategy do you feel comfortable with and why?

J: (Level) two…see where they fit…1 is the actual answer.

First Juan chooses Level 2 as the most comfortable but then changes his mind to Level 1 (trial and error.)

R: Without touching the pieces, how would you complete the square? Be specific in describing your actions using geometric transformations (e.g. translation, rotation, reflection.)

J: Translate it in and flip the parallelogram.

R: Now show me: (How much time did it take for the student to flip the parallelogram piece?)

Juan understands the mental reflection task he has to complete before translating the parallelogram into the empty space of the completed square. Juan took three seconds to complete the task.

R: Describe what the Level 3 strategy was? What is challenging for you to understand the Level 3 strategy, why or why not?

J: Its square roots.

Juan also has difficulty defining the Level 3 strategy and its connection to area and side lengths. Although he has mentioned a key component of the strategy, square roots, he does not elaborate on how there is such a relationship with the area of the square.

All four of the students Juan, Nena, Rafael, and Rita all had their strategy levels the same after the Post-Test. However, there are patterns in their responses to the questions. It is quite evident all the participants so far had difficulty recalling what the Level 3 strategy. This could be because the researcher did not stress it enough during the intervention activities. Table 5.16 displays some important patterns. All the students have empathy to the pieces they are representing. The pieces have become human almost. The piece “helps”, is “fun”, “easy”, and “cool.” Furthermore, if we look at the mental rotation task, the students also have patterns in common. Table 5.17 shows the mental rotation task responses.

Table 5.16. Empathy in strategy levels.

Female Responses	Male Responses

Same Strategy Level	Rita: The big triangle….*it helps fit all the pieces.*	Juan: Small triangle…goes more with this…*easier to place*.
	Nena: A square…it's more *fun*.	Rafael: Parallelogram….it looks *cool*. It was okay.

Table 5.17. Mental reflection task responses.

	Female Responses	Male Responses
Same Strategy Level	Rita: Get the parallelogram and *put it in…translate it.*	Juan: *Translate it in and flip the parallelogram*.
	Nena: Get the parallelogram *and just translate it.*	Rafael: Get the parallelogram in *the space* after you *flip it upside down.*

Both Rita and Nena had difficulty with the mental rotation task. Both were unable to complete the square at the first try because they merely wanted to translate the parallelogram into the empty space. Their mental rotation of the parallelogram did not happen until they saw that the piece would not fit. After manipulating the tangrams the females saw how to reflect the parallelogram in order to complete the square. On the other hand, the males were able to visualize the mental rotation of the parallelogram before translating the piece into the empty space. The male students used phrases such as "flip it" to indicate the mental rotation. Now we will take a look at the students whose strategy levels improved after the connectedness intervention.

Two students were also selected to participate in the interviews. These participants were also selected to interview based on their improvement of strategy level. Lupe was a student who improved her strategy level and answered the interview questions in this way.

R: What tangram piece did you pick and how did you feel being that piece?

L: Parallelogram…because he's different.

Lupe is also begun to empathize with the parallelogram by stating that "he's different." Lupe has humanized the tangram piece and made it personal to show that the piece is different from other pieces.

R: What strategy do you feel comfortable with and why?

L: One level. I get it better….three, you think too much and feel confused.

Even though Lupe is more comfortable with the first level, she used level three in the Post-Test to create the tangram square of five pieces. Lupe is also convinced the Level 3 strategy requires more thinking and can cause some confusion. Next, Lupe was also challenged in the mental reflection task.

R: Without touching the pieces, how would you complete the square? Be specific in describing your actions using geometric transformations (e.g. translation, rotation, reflection.)

L: Put the parallelogram in there by translation and flip it.

R: Now show me.

Even though in the description Lupe reversed the order of translation and reflection, she was able to demonstrate the reflection of the parallelogram first and then the translation. Lupe took three seconds to complete the square.

> R: Describe what the Level 3 strategy was? What is challenging for you to understand the Level 3 strategy, why or why not?
>
> L: Areas…figure out which ones have that area.

Lupe does relay the important information of finding areas, but fails to mention side lengths in conjunction with them. Lupe shows that even though Level 3 is difficult to describe, she uses the level in completing the Post-Test.

The last male student, Adrian, also improved in strategic competence. Adrian answered the interview questions in this way.

> R: What tangram piece did you pick and how did you feel being that piece?
>
> A: I would be the middle triangle.
>
> R: Why?
>
> A: I was the middle child. The triangle is not too big or too small….and because it's a…type of triangle.
>
> R: Isosceles?
>
> A: Yes

Adrian also empathizes with the middle triangle as he connects it to his childhood experiences as the middle child. He creates a personal connection with the tangram piece.

> R: What strategy do you feel comfortable with and why?
>
> A: One
>
> R: Why?
>
> A: My mind works that way. It's the easiest.

Just as Lupe did, Adrian also chose a level in which he felt more comfortable with even though he used the Level 3 strategy in creating the five piece tangram square in the Post-Test.

> R: Without touching the pieces, how would you complete the square? Be specific in describing your actions using geometric transformations (e.g. translation, rotation, reflection.)
>
> A: I would take the parallelogram…adjust it …and flip it then slide it in.
>
> R: Translation?
>
> A: Yes
>
> R: Now show me: How much time did it take for the student to flip the parallelogram piece?

Adrian took five seconds to complete the square understanding the mental reflection task.

> R: Describe what the Level 3 strategy was?
>
> A: Finding area…make the side length match.

Adrian was able to describe the Level 3 strategy by connecting the given area and finding a side length which matched.

> R: What is challenging for you to understand the Level 3 strategy, why or why not?
>
> A: It was difficult to understand…because of the different numbers.
>
> R: Did I explain it well?
>
> A: Yes….I don't grasp numbers very well…equations I can do.

Adrian was honest in explaining how the Level 3 strategy was difficult to understand. Again, he was able to use the Level 3 strategy in completing the Post-Test task of creating a square with five tangram pieces. Table 5.18 displays all the students' responses towards empathy. Some of the students humanize the tangram pieces by using phrases and descriptions used for people. Table 5.19 displays the information for the mental reflection task.

Table 5.18. Empathy table all students.

	Female Responses	**Male Responses**
<u>Same Strategy Level</u>	**Rita**: The big triangle….*it helps fit all the pieces.*	**Juan**: Small triangle…goes more with this…*easier to place*.
	Nena: A square…it's more *fun*.	**Rafael**: Parallelogram….it looks *cool*. It was okay.
<u>Improved</u>	**Lupe**: Parallelogram…because *he's*	**Adrian**: I would be the middle triangle. *I was*

<u>Strategy Level</u>	different.	*the middle child*. The triangle is not too big or too small….and because it's a…type of triangle.

When all the students are placed by gender and their times are averaged, males tended to complete the square in 3.67 seconds compared to the females' average time of 7 seconds. All students used the phrases of translation; however, there is a shortage of the word reflection. Instead the students used the common term of "flip." This indicated their familiarity with the mental reflection task without actually using the proper term.

Table 5.19. Mental reflection tasks all students.

	Female Responses	Male Responses
<u>Same Strategy Level</u>	Rita: Get the parallelogram and *put it in…translate it.*	Juan: *Translate it in and flip the parallelogram.*
	Nena: Get the parallelogram and just *translate it.*	Rafael: Get the parallelogram *in the space after you flip it upside down.*
<u>Improved Strategy Level</u>	Lupe: Put the parallelogram in there *by translation and flip it.*	Adrian: I would take the parallelogram…*adjust it …and flip it then*

		slide it in.
Avg. time to complete square	7 seconds	3.67 seconds

After the initial analysis, it was felt that another set of interviews were necessary in order to investigate communication and empathy among the groups even further. The second group of interviews was conducted in the spring semester with a second group of participants. Four females and four males were selected by the researcher randomly. The following questions were asked:

1. How did you feel when the group validated your ideas while making the tangrams?
2. How did you feel when the group did not accept your ideas?
3. Did you try to understand other's points of views when they arranged the tangrams in your group and how?
4. Did you accept the ideas of others in the group and how did you feel about it?

These set of questions, unlike the first set of interviews, asked about how the participants were able to communicate and relate with their groups. When asked about how they felt when their group validated their ideas about constructing tangrams, almost all of the students responded by stating they felt good. These girls responded by:

> R: How did you feel when the group validated your ideas while making the tangrams?
>
> Suzie: Um, I guess it did make me feel good that they liked my ideas.
>
> Myra: I would feel good because they agreed and at they would help me if I was doing something wrong.
>
> Michelle: I felt good because we worked as a team. Me and Silvia had pretty much the same idea and we were helping the rest of our group.

Susanna: (pause) mmm… ok because my friends help me to do the work together.

Only one of the boys interviewed responded with good feelings.

Dante: I felt I felt pretty good… mmm… (pause) I just felt good basically.

AL: Well, it would've felt good if I had given them ideas.

Hector: It was alright.

Sam: I felt like we're all thinking the same way even though the strategies might be different, they all lead you into the same result. So like one of the strategies was trial and error and then the other one was making a frame, which is when you make a frame it's your trying something and then if it fails you just make another frame, which is the same thing as trial and error. So everybody said the same strategies only in a different way, so I felt that we were all thinking the same way.

This indicated that females were more sensitive to how the group responded to their ideas more than the boys. Most of the boys responded with very short answers such as "I don't know" and were uncomfortable talking about how others validated them or not. The females were more talkative on this question as they saw a connection to emotions and mathematics perhaps for the first time.

The second question asked about how the participants felt if their group did not validate their ideas. The girls responded by:

R: How did you feel when the group did not accept your ideas?

Suzie: And if it didn't work we would always try something else.

Mayra: Yes, it was fine. I would just try whatever they suggested we try instead. I was OK with that.

Michelle: The group did accept my ideas. The rest of the team didn't really contribute anything.

Susanna: I actually felt it was better, because it made me feel like it was only me thinking that way and many times that helps me to motivate myself to think even harder and go think outside the box, and I think that … It's good.

Five out of the seven girls responded in kind when their group did not validate their ideas. This showed their ability to take criticism or suggestions lightly. The girls showed they were willing to communicate with their group in order to work together. The boys on the other hand again did not speak about emotions as much as the females.

Dante: Well… didn´t accept my ideas (pause) I was ok with it I mean because we have our own opinions some ideas will work and some ideas might… so I mean be based on other ideas.

AL: That didn't happen.

Hector: They did accept my ideas.

Sam: Well everybody has different ideas so if once somebody shots, shoots down my ideas, it's not the end of the world. So maybe my idea wasn't as good as it should have been or they didn't see it how I saw it. How I saw it was a better idea, but it didn't in the end. We still solved the puzzle.

The boys also responded in kind to their group's non-validation of their ideas. Dante and Sam expand on how validation is a workable environment in the group. They are open-minded to others' criticism of their ideas.

The first two questions of the interviews focused on how the group understood and validated the participant's ideas. The next two questions then changed the focus of the interview towards the understanding of other student's feelings in the group.

R: Did you try to understand other´s points of views when they arranged the tangrams in your group and how?

Suzie: Try it. I would go ahead and try what they suggested.

Mayra: Yes I did because they showed me how to do it in a different way.

Michelle: Yea I really tried to understand other people's point of view but since no one else was really helping, me and Sylvia did all the work.

Susanna: Yes I did, um well most of us were constantly just looking at each other and how like the strategies we're using, so I think that's another factor that helped us.

The females were receptive of other student's point of views as they tried to understand their strategies and suggestions. In contrast, the boys tried to understand the other participant's point of views but were more concerned about the technicalities of creating the squares as in "trying to get the job done."

Dante: I tried to understand them by looking at what they were doing mmm… what the tangram pieces to form a square.

AL: Because it was worth a shot to try it out and see if it worked.

Hector: There wasn't really a point of view we just all tried to move the pieces around to make the squares.

Sam: Hum some of the strategies were hum, well first of all I didn't try trial and error first so I was like, so I tried to make the frame always, so when they started saying trial and error I was I could see it happening because there could only be so many….so many possibilities that you can make so if you try a certain combination with a certain shapes then if they don't work then you can eliminate those move on, move forward.

The boys tended to concentrate on the task of creating the squares more than the emotional connection of understanding what their group members were saying. Boys were more concerned with getting the squares completed where the girls were focused on understanding and being sensitive to other's contributions to the group.

The final question asked about the acceptance of the group's ideas.

R: Did you accept the ideas of others in the group and how did you feel about it?

Suzie: I did it ok because mmm.. they were trying to show me and made me learn something new.

Michelle: Yes it did and it felt ok.

Susanna: I did. I actually implied them to my strategies too and it helped me a lot actually.

The boys responded by saying:

Dante: I actually did accept a few amm… how I felt… I felt it was a good thing that I did because it actually helped me out with the tangram pieces.

AL: Well the point was to see if you could solve the puzzle so if I could get all the help I wanted so I felt like alright it didn't, it didn't…it didn't.

Hector: I tried to see other people's point of view but it didn't make me feel anything.

Sam: I did accept them and I felt that there's more than one way to solve the thing so by accepting the ideas and seeing how they worked it I could add to how I was working it and it made it easier to solve the puzzle.

In Hector's case, although he tried to see other participants' point of view, there was no emotional connection. However, Sam and Dante felt that accepting other student's ideas actually helped them.

5.4 SUMMARY

This chapter was based on the results and findings from the connectedness treatment group and the control group. The treatment group received a connectedness intervention which comprised of several intervention activities developed and based on feminist epistemology and three main ideas of connectedness: mathematical reasoning, social cognition, and multiple strategies. With the structure of

a mixed method nested study, three main components of the study were researched with three research questions. The first question of the research dealt with the existence of a difference in achievement among females and males in spatial reasoning. In this case, the Seven Chinese Clever Piece tangrams were used for the research. The results have shown there is a difference in achievement among males and females within all three levels of education: elementary, middle school, and high school. Male scores were higher than those of females. With these results, the researcher was able to proceed with the next set of research questions within the nested study.

The second research question researched how connectedness intervention strategy would affect the performance of females and males. Two groups were created by a cluster random selection. The first group, the control group, merely participated in a Pre-Test and the Post-Test. Their results show the scores were basically the same for both genders in both tests with no improvement in scores. This became the baseline for comparing the second group, the connectedness group. The connectedness group participated in several activities which contained specific strategic contexts of feminist epistemology and connectedness theories. The results showed females decreased the difference in spatial reasoning compared to males with a statistically significant test. Females improved more than males in task completion times and task completion rates.

The third question researched how much and how females and males' strategic competence improved from the Pre-Test to the Post-Test. Results show the females' increase in strategic competence was statistically significant. Although the males did also increase in their strategy levels, their increase was not statistically significant. The second part of the third research question analyzed student responses to an interview about their strategy levels. Two groups of students, those whose strategy levels stayed the same in between tests and those who increased in strategy levels, were selected. All students seem to have empathized with their tangram pieces by implementing humanization characteristics to the pieces. The results also showed students still had difficulty explaining and understanding what the Level 3 strategy was.

The final chapter will bring a closure to the study by examining again the purpose of the study, connectedness, literature review, theoretical framework, and the research questions. Furthermore, the

final chapter will look at the study's weaknesses, strengths, limitations, implications, and cases for future research.

Chapter 6: Interpretations and Recommendations

6.1 INTRODUCTION

In this final chapter, several keys ideas will be revised and discussed further. We first begin to look back to Chapter 1 and revisit key aspects of the research such as the purpose of the statement, the literature review, and the methodology. Furthermore, the theoretical framework will be discussed.

The final chapter contains several sections. First we will summarize the results from the study and then discuss the outcomes of the research according to each research question. Second, a summary statement will be included. Third, implications for policy, practice, and theory will be conveyed. Fourth, recommendations will be discussed. Fifth, limitations of the study will be explained. Finally, implications for further research will allow us to look beyond the study.

If we recall from the beginning chapter of the study, the purpose of the study was to determine whether or not there is a real difference in mathematics achievement between females and males. Some research had indicated there is a gender difference in math achievement while some have argued there is no difference in achievement. This also became part of the statement of the problem of females, in this case Latinas, performing low in mathematics. Latina/os are one of the lowest performing ethnic groups in high stakes testing. Latina/os are the fastest growing population in the Unites States and are expected to be the majority in Texas in the future. The Latina/o population accounted for half of the population growth from 2000 to 2010 in the United States. The Mexican population is proportionally the largest Latina/o group in the U.S. Latina/o education suffers when only 53.2% graduate from high school and 12% enroll in college.

Females have several obstacles to overcome when learning mathematics. Society believes mathematics favors males over females. Males dominate science, technology, engineering, and mathematic careers and fields. Females also have severe anxiety when learning mathematics as well.

Curricula are geared for the type of learning and thinking which favors males. These male dominated curricula do not use manipulatives and expect exact solutions to problems which again favor males over females. With the understanding of feminist epistemology, curricula can enhance females' learning and thinking about mathematics.

Feminist epistemology was the theoretical framework for this nested mixed method study. It allowed the researcher to understand how females preferred to learn and think about mathematics. With several ways of knowing, this study concentrated on procedural knowing, in particular, connected knowing. With connected knowing, three strands of mathematical knowing and learning were developed. These three strands of knowing were: mathematical reasoning, social cognition, and multiple strategies. Mathematical reasoning is several types of reasoning such as quantitative, algebraic, and spatial. Social cognition is the space where females became aware of the content and establish communication over domination in group settings, for example. Multiple strategies allowed for the numerous approaches to problem solving. Some of these strategies can be tables, pictures, and the strategic competence levels of creating squares with the Seven Chinese Clever Piece Tangrams. With the purpose of the study and the statement of the problem clearly defined, research questions were then developed.

Three research questions arose from the purpose and statement of the problem. The research questions consisted of three major components: spatial reasoning differences, connectedness, and strategy levels. From each component, a research question was developed. A literature review helped to locate a lacuna in the research and assisted in the development of connectedness.

Connectedness is a new idea to research which was developed by three major components of mathematical reasoning, social cognition, and multiple strategies. The three components were derived from Miller (2000) who developed three applications of cognitive development in feminist epistemology as scientific reasoning, social cognition, and cognitive strategies. Taking these three

works, the researcher revised the scientific reasoning into mathematical reasoning and cognitive strategies into multiple strategies. Through the literature review, the researcher learned how connectedness can be implemented into the curriculum to improve females' learning of mathematics. With the help of Miller (2000), Zohar (2006), and specifically Belenky, Clinchy, Goldberger, & Tarule (1996), connectedness has developed through the knowledge of feminist thinking and learning. Connectedness allowed learners to know mathematics as if they were the math topic itself. The students became immersed into the learning process. It also created a deeper understanding of math topics through empathy and intimacy within subjects and concepts. Basically, connectedness established an applicable relationship between the student learner and the concepts of mathematics.

Furthermore, the literature review revealed how males and females are treated differently. Females have been shortchanged by the school system (AAUW, 1999; Sadker, Sadker, & Zittleman, 2009). Women often hear things in the math community that corrodes their desire to pursue math in the future (Good, Rattan, & Dweck, 2012). Some communications are that women's math ability is a trait and they have less of this ability than men (Good, et al.). According to Dee (2007), a teacher's gender also has influence on the achievement difference between females and males. Teachers are biased toward males and females in the classroom and how they engage each one (Dee, 2007). Teachers have biases in how they involve both females and males in the classroom. Teachers are more probable to compliment boys with their comments and merely acknowledge girls in theirs (AAUW, 1992; Sadker and Sadker, 1994; Saltzman, 1994; Kleinfeld, 1998; Lewin, 1998; Dee, 2007). A teacher's gender also influenced gender equity in the classroom (Dee, 2007). Next, we will briefly look at the methodology of the research.

The methodology of the study was designed in a nested study format. Three sub-studies were developed based on the three components of the research questions (see Figure 4.1). Participants were selected by a cluster random sample in a southwest part of the U.S. The schools ranged from

elementary to high school and were predominately Latino/a. Two school districts encompassed the eleven schools. The Seven Clever Piece Chinese Tangrams are comprised of a parallelogram, a square, and five isosceles triangles of various sizes and were used as the main research instrument.

6.2 Discussion of Results

This section will include a discussion of the results from the three research questions. Each research question's outcomes will be discussed separately as a single result and then collaboratively.

6.2.1 Discussion of Research Question 1

- Research Question #1: Is there a difference between the female and male performance on spatial reasoning tasks across the elementary, middle school, and high school levels?

The first research question and part of the study required building a case. This case was constructed by a Pre-Test given to 589 participants across the grade levels of elementary to high school; the Pre-Test required students to create a square using all seven tangram pieces. The results show that there is a difference in performance on spatial reasoning tasks between females and males along all three segments of school: elementary, middle school, and high school. This research confirms prior research indicating a general difference in achievement between the two genders (Contreras, Martínez-Molina, & Santacreu, 2012; Gluck & Fitting, 2003; James, 2009; McGraw, Lubienski & Strutchens, 2006; NAEP, 2012; Sadker, Sadker, & Zittleman, 2009.) Of the participants, 66 came from elementary, 187 from middle school, and 336 from high school. This is a large group of participants to determine whether there is a difference in achievement between females and males in the area of spatial reasoning. The researcher, based on experience of teaching mathematics for sixteen years, presumed a slight difference in achievement in spatial reasoning between females and males. The study's teacher participants (N=8) assisted in the collecting of the data. Each teacher administered the Pre-Test in the same manner. The results of the data were aggregated and analyzed. As mentioned before in Chapter 4,

if the first part of the study finds there is no achievement difference between females and males, then the research is concluded. However, in this case, a steady difference in achievement was found across school levels.

Most of the participants in the first sub-study were of Latina/o descent based on the population of the southwest border city with 82.2% Latina/o (U.S. Census, 2010). One of the two school districts which participated in the study had almost 92% Latina population while the other district had 82.6% of the females as Latina. Based on these numbers we can assume that most of the data collected and analyzed has a strong input from the Latina population. When we talk about females, we must remember that a great portion of them are of Latina descent. Even though this study is about a gender difference of mathematical spatial reasoning, we must not forget the influence of ethnicity in the results of the study. There is something to be said about how Latinas learn and comprehend difficult mathematical concepts such as spatial reasoning and mental reflection tasks.

The results from the first research question did reject the null hypothesis. The research question required students to create a square from all Seven Clever Chinese pieces. From the spatial task completion results of the Pre-Test, males scored higher in spatial reasoning tasks. In elementary school (n=66), 3% of females and 8% of males completed the square. This shows a 5% variance in achievement in spatial reasoning. In middle school (n=187), a higher percentage of completion rates were shown by both females and males. For females, the completion rate of the Pre-Test was 17% and 24% for males. The difference in success has increased from 5% in the elementary level to 7% in the middle school between the genders. In the high school freshmen level (n=336), the difference in achievement still existed. The variance between the females and males was 7% as females and males increased their completion of the square to 62% and 75%, respectively. Furthermore, the gap in the completion of the squares still exists at the high school level at a 7% difference between the genders.

What we see is the continuation in the dissimilarity in achievement between the two genders as males outperform females in spatial reasoning tasks across the three school levels.

6.2.2 Discussion of Research Question 2

- Research Question #2: To what extent does connectedness improve females' performance on spatial reasoning tasks?

The second sub-study required participants to be in one of two groups; treatment or control. The control group did the same as the prior participants but also took a Post-Test with no intervention in between. The treatment group (also known as the connectedness group) partook in a series of intervention activities based on connectedness ideas. Some of the intervention activities were conducting research, analyzing the tangram shapes, creating tables of square lengths and areas, strategy levels, and constructing squares with various numbers of tangram pieces. Both groups participated in the Pre-Test and a Post-Test which required the students to create a square using five of the seven tangram pieces.

Results for the control group (also known as the isolated tasks group) showed no improvement for females in spatial reasoning. Of the control group (N=161), 72 were males and 89 were females. The participants were also cluster randomly selected from a southwest high school in the border region. Seven high school freshmen classrooms were selected to participate in the isolated tasks. As mentioned before, no improvement was shown by the females in between the Pre-Test and the Post-Test. Remember, the control group merely participated in these two activities. There was no intervention activities taught to them. This became our baseline for comparison to the treatment group.

For review, the females in the control group decreased their completion time from Pre-Test to Post-Test by only six seconds signifying no learning achievement in the spatial reasoning tasks. In comparison, the males in the control group decreased their time in the Post-Test by 38 seconds showing some sign of achievement; however, not much. The fact that even though no intervention activities

were in place, the males still outperformed the females by reducing their time 32 seconds more than the females. It can suggest that males can adjust very quickly to spatial reasoning requirements of the Post-Test. Remember, the Post-Test asks participants to create a square with only five of the seven tangram pieces. The participants had a timeframe of fifteen minutes to complete the five square tangrams Post-Test. It may also suggest that males tend to perform better under timed circumstances than females. These results agree with Gurian (2011) who suggest males tend to use more space at a younger age to where they even invade space of others. Gurian (2011) also stated boys are more active in their learning when involving space oriented to their body movement giving them an advantage because of the stimulation of spatial abilities.

Several reasons may explain the disparity in achievement. Test anxiety could also been a problem for females in addition to the timed Pre and Post-Tests. This statement agrees with Halpern (2009), James (2009), and Fennema & Sherman (1976) when they state that females suffer test anxiety. Females tend to have test anxiety early in the grade levels and have problems retrieving important information when stressed (James, 2009). The NAEP also found that girls often fall below males in grades 8 and 12 (McGraw, Lubienski & Strutchens, 2006). Furthermore, AAUW also reported how the gender gap is still present (AAUW, 1999; Sadker, Sadker, & Zittleman, 2009).

Another reason could be the teacher's gender. The teacher's gender, in this case male, could have possibly hurt the females as agreeing to Dee (2007). Even though, the researcher equally treated the females and males equally the same, there may be other factors unknown to the researcher that affected females. Females may be intimidated by a male teacher, perhaps. This is certainly out of the researcher's control except to make all students feel equally important and capable of completing the tasks. Finally, Good, Rattan, and Dweck (2012) claim to say society could erode the desire of females' by stating that math ability is a trait more suited for males. Now we will discuss the results of the connected group, the second group, also known as the treatment group.

For the connected group, results show statistically significant increases in females' performance in spatial reasoning with a higher improvement than males. For this study, 83 freshmen students answered the question of whether they had used tangrams before. Of the number of students, 30 of the 47 boys have never used tangrams before. Of the 36 females, only 21 had never used tangrams before. Of the total participants, 61.4% had never used tangrams before. This information was simply used to better understand the participants and their experience with tangrams. The other participants had not been asked to determine their tangram usage because the researcher had inserted the tangram usage question too late for others to answer.

The total number of connected participants was N=336 with 171 males and 165 females. The group was evaluated by their task completion times and their task completion rates. On the Pre-Test participants were required to create a square with all seven pieces. The females' average task completion time was 13.29 minutes and the males' average completion time was 10.18 minutes. The difference in the times was more than 3 minutes favoring the males. Females took longer in completing the Pre-Test.

The Pre-Test completion rate also favored the males. According to the results, 62% of the females were able to complete the seven square tangram assessments. On the other hand, ¾ of the males were able to complete the square within the time limit of fifteen minutes. This was a 13% difference in completion rates for the Pre-Test. This also confirms the results from the first research question of whether there is an achievement difference between males and females. The results also confirm what other researchers have said about an existing disparity in achievement among females and males in spatial reasoning. However, the connectedness activities stemmed from the theoretical framework of feminist epistemology brought about some amazing results.

With such a reasonable difference in the achievement in spatial reasoning between females and males, connectedness showed a significant increase in both genders' Post-Test scores. For the Post-

Test, the participants had to create a square with only five of the seven tangram pieces. This had two reasons. First, the participants would have to choose which of the five pieces they would have to use. Second, the square must be of a different requirement than the Pre-Test so students would not have familiarity with the creation of a square with seven pieces again. In the Post-Test, the males lowered their Post-Test average completion time by five minutes starting at 10.18 minutes to 5.01 minutes reducing the time by 100%. The males' completion rate for the Post-Test increased from 75% to 91% with an increase of 21.33% more males completing the five square tangram assessments. What is rather astonishing was the growth in spatial reasoning for females.

The females did an excellent job in increasing their completion times and rates on the Post-Test. The females were three minutes behind the males in the Pre-Test completion average times and 13% behind in the completion rate. The Post-Test completion average time for the females was 6.58 minutes compared to the Pre-Test average time of 13.29 minutes. The females reduced their task completion average time by 6.71 minutes which is 101.97% change from the Post-Test. In the Post-Test, the females shortened the time difference between the genders in half. The time difference in the Pre-Test was 3.11 minutes but changed to 1.57 minutes in the Post-Test. This is good news. Furthermore, the Post-Test completion rates also decreased for females.

In the Post-Test assessment, females also decreased their scores significantly on the task completion rate. The females increased their completion rate of the tangram square by 24% from 62% on the Pre-Test to 86% on the Post-Test which is a 38.7% percent increase change. The males, on the other hand, increased their score as well from 75% to a 91% completion rate signaling a 21.3% percent increase change. The females outperformed the males in percent increase change. The females' percent increase was 38.7% compared to the males 21.3% increase change. The females also decreased the difference in completion rates. In the Pre-Test, the females were 13% behind the males in the Pre-Test completion rates. In the Post-Test, the females decreased the difference in completion rates to 5%; this

is a 160% decrease change. The interaction effect from Pre-Test to Post-Test was statistically significant using two-way ANOVA. The researcher contributes this significance to connectedness.

Connectedness assisted females and males increase their spatial reasoning skills. Spitzer, White, and Flores (2009) believed helping one student helps all students in their understanding of mathematical concepts. Especially for females, connectedness assisted them in several factors. First, it helped females gain spatial reasoning skills by increasing their completion times and their completion rates on pre and Post-Tests. Secondly, females not only increased their scores and times, but managed to beat males on percent change for both the completion times and completion rates. Connectedness benefits both genders, but assists females at a greater level.

Connectedness originated from feminist epistemology. Girls learn differently than males. With the curriculum dominated by a male's way of thinking, connectedness has provided an alternative. The current curriculum can be a cut and dry practice where girls are often left behind. According to Sadker and Sadker (1995), girls are second-class educational citizens.

> Sitting in the same classroom, reading the same textbook, listening to the same teacher, boys and girls receive very different educations. From grade school through graduate school female students are more likely to be invisible members of classrooms. Teachers interact with males more frequently, ask them better questions, and give them more precise and helpful feedback (p. 1).

Now enters connectedness in order to restore some equality in the curriculum which benefits both females and males.

Connectedness stems from three basic principles of feminist epistemology. If you recall from Chapter 2 (Connectedness), males prefer to learn by separate knowing and females favor connected knowing. Connected knowing involves intuition, creativity, experience, induction, relativism, and

context (Belenky, Clinchy, Goldberger, & Tarule, 1997). Boaler (1997) also stated how girls prefer the connected way of knowing. Curriculum should be designed to challenge stereotypes within a social and cultural context (Goodell & Parker, 2001). We must also understand connected knowers love to apply intimacy to what they are learning (Knafo, Zahn-Waxler, Davidov, Hulle, Robinson, & Rhee, 2009; Zohar, 2006). Miller (2000) also understands how connected knowers must be immersed in opportunities to become intimate within the concepts of what they are learning. Additionally, Zohar (2006) also reiterated that connections must be created between concepts, the knower's perceptions, and formal observations. All of these researchers assisted in the creation of connectedness and its three theories.

Connectedness stems from three main ideas. These ideas are mathematical reasoning, social cognition, and multiple strategies. These three main concepts are by no means isolated. They inter-connect and intra-connect throughout the intervention activities. The three models provided the framework for developing the connected activities for the treatment group. First, mathematical reasoning provides females opportunities to take content, such as spatial reasoning, and break down the components. Females were allowed to use quantitative reasoning, algebraic and geometrical thinking and spatial reasoning. Within the intervention activities, mathematical reasoning was introduced so participants could look at each tangram piece and distinguish congruency and similarity of pieces. It also allowed the students to observe how side lengths of squares are directly related to the areas of squares. Secondly, social cognition involves developing a socially situated person. Social cognition involves mutual influence between students, harmony, understanding, collaboration, reciprocity, empathy, and an emotional relationship with others. An example is in the intervention activities, students chose a tangram piece to represent and become familiar with. Rather than trying to know all the pieces intimately, students chose only one. In groups, participants also constructed squares comprised of various number of tangram pieces. The members of the group shared ideas, built on each

other's input, supported diverse thinking, and reached a consensus in the construction of squares. This promoted social cognition by including the idea of the group as a focus for cooperation and not competition. Furthermore, the group collaborated and identified strategies they used to construct squares of various sizes. And finally, multiple strategies involved inter-subject and intra-subject connections, multiple representation, and different methods to solve problems. The students constructed levels of strategic competence and assigned them to an order of difficulty based on their complexity. Participants reflected on their construction of a square with six tangram pieces then thought aloud about why it is not possible and wrote their examinations. Now we will discuss further results and findings from the third research question.

6.2.3 Discussion of Research Question 3

- Research Question #3: a) How do male and female students' strategies differ on spatial reasoning tasks after implementation of connectedness and b) how much do male and female students' strategies change on spatial reasoning tasks after implementation of connectedness?

The last component of the research analyzed students' strategies in creating squares in the Pre-Test and Post-Test activities. The last research question about levels of strategic competence contained two parts. The first section of the research question dealt with the qualitative portion of the study as the second section provided the quantitative segment. The qualitative portion of the mixed method study conducted interviews and analyzed how differently males and females performed on strategy levels. Patterns arose from the interviews such as how students showed empathy towards the tangram pieces. Another was how students established human characteristics to identify with the tangram pieces such as "him", "cool", and "like me." Additionally, the qualitative portion of the research question also included the mental reflection tasks.

Mental rotation tasks were used to focus on measures which also showed gender differences (Nutall, Casey, & Pezaris, 2005). Our mental reflection tasks revealed males were able to complete the

activity faster than females. In this case, the researcher wanted to know how females would compare to males in a mental reflection task. The mental reflection task asked to students to complete the seven tangram piece square which had six of the pieces properly placed. The last piece, in this situation the parallelogram, was missing. The researcher deliberately placed the parallelogram away from the square and reflected (flipped) it in a wrong position. Students had to recognize where the parallelogram fit in addition to realizing a reflection was needed to complete the square. In order to finish the square, the participants were required to reflect the parallelogram and use translation to place it into the proper position. Two of the females selected to participate in the mental reflection task were unable to complete the square on the first attempt. Three males and one female, on the other hand, were able to complete the square on the first try. The average time for males was 3.67 seconds and 7 seconds for females.

The quantitative segment of the research question investigated how much different the genders performed on strategy levels. Data was collected during the Pre-Test and the Post-Test. Females' increase in strategy level in designing squares from Pre-Test to Post-Test was statistically significant but the males' increase was not statistically significant. This shows that connectedness did assist both genders; however, females more extensively. Next, we will analyze the significance of the study and new questions that arose from the research.

6.3 SUMMARY STATEMENT

This study represents a unique concept of connectedness which has not been involved with any type of research conducted. This study is significant by the new application of connectedness into equitable curriculum design and pedagogy. It also is distinctive in the collaboration between connectedness, spatial reasoning, spatial ability, and strategic competence involving the Seven Clever Chinese Tangrams. Designing intervention activities involving area, geometry, spatial reasoning juxtaposed with connectedness is the first of its kind. This type of intervention can improve how we

teach females and advance our paradigms about developing curriculum which validates both genders. It is also important to understand how this study speculates new inquiry.

In creating new research, this study can influence future studies and methodologies. In this investigation, new ideas were developed and some implemented. But due to time constraints, some of the exploration could not be advanced. For example, the researcher would have liked to study the connection between the performance of Latinas and ethnicity further. This will be discussed later on in the chapter.

Even though the investigation answered the research questions, other inquiries arose from the study. Some new questions were developed and implemented. For example, the mental reflection tasks were inserted late into the research to better understand the metacognition of females and males in spatial reasoning. This gave the study another perspective to analyze the difference in achievement between the genders. Yet, other questions arose from the study. If this study was conducted elsewhere along the border of the U.S and Mexico would the results be similar or different? What else could the researcher do in order to analyze the achievement of females and connectedness within the realm of the intervention activities? This brings us to our next section of implications for further research.

6.4 IMPLICATIONS FOR FURTHER RESEARCH

More research on connectedness and its effects on decreasing the achievement disparity between males and females in mathematics are warranted. Studies are needed to develop and re-develop connectedness within other areas of mathematical concepts. Connectedness is much like an iceberg where we see only the tip. The potential connectedness strategies have, developed from feminist epistemologists such as Patricia Miller is far beneath the ocean. There is a need to improve the methodology used in this study. Since the timeline was restrictive, other researchers may be able to use connectedness in their studies to improve female cognition of other difficult mathematical concepts such as spatial reasoning. With connectedness as a new method in educating females, there is always room

for improvement and progress. The researcher only hopes that this study makes a significant impact on the academic performance of females.

This study instructs further research. There are many topics that can juxtapose connectedness and research. Within this study, more exploration could be done on algebraic and geometrical reasoning. For example, a researcher could create a qualitative study analyzing how student participation in connectedness affects their self-efficacy, determination, disposition, and attitudes towards mathematics. Research on ethnicity and connectedness with the addition of culture can also be worthy of a study or two. Connectedness could also be implemented within other content areas.

Research on connectedness could also include other topics. How does connectedness improve the quality of literacy and bi-literacy for both genders? Furthermore, does connectedness increase female achievement in grade levels other than freshmen students? Studies could be developed which mimic this research at the elementary, middle, high school, college, and adult education levels to see if the results hold true. Another angle at recreating this study could be at other geographic locations. How would connectedness in Mexico, Chile, Canada, and Valenzuela assist females there? There are a limitless number of connectedness studies possible. However, feminist epistemology is an essential piece.

In order for connectedness to be implemented in other studies, it is crucial that feminist epistemology be a part of the theoretical framework. Since connectedness was developed within the framework of feminist epistemology, it would help future researchers to become familiar with how females prefer to learn. This step is very crucial. Understanding connectedness is to understand feminist epistemology and cognition. Female's preference in learning and thinking guide connectedness. The three main concepts of connectedness involve mathematical reasoning, social cognition, and multiple strategies. Connectedness research will evolve and change according to what you want to study. Connectedness by no means is written in concrete. Much like we developed

connectedness through Miller's three concepts of scientific reasoning, social cognition, and multiple representations, this study's methodology and framework will be redeveloped in the next steps in this type of research by someone else. Other research could look at the connections to other theory. For example, the strategic competence levels developed by the study can be compared to Van Hiele's levels of development in geometry.

6.5 IMPLICATIONS FOR FURTHER PRACTICE AND RECOMMENDATIONS

This research definitely impacted the learning and the achievement of females who participated in this study. The results show that females have decreased the variance in achievement compared to males in spatial reasoning. Furthermore, females also outperformed males in positive percent change after interacting with connectedness. As a result, we can change the way we teach females in the classroom in order to create gender equitable connected mathematics learning.

The achievement of females in mathematics, particularly within minority subgroups, is a concern. The Mexican population is still numerically and proportionally the largest Latino/a group in the U.S. Latina/os are at the lowermost end of mathematics achievement behind Caucasian, Asians, and African Americas (National Council of La Raza, 1999). The Latino/a population accounted for half of the total population growth in the U.S in the last ten years (U.S. Census Bureau, 2010). According to the U.S. Census Bureau (2010), the Latino/a population had a 43% increase since 2000 compared to 4.9% of non-Latino/as. This has serious implications on how we teach Latina/o youth. Connectedness can assist Latino/as decrease the achievement variance between themselves and other ethnic groups.

Connectedness has proven itself to be a solution to the achievement disparity between genders. It may also assist Latina/os in decreasing the academic attainment discrepancy amongst ethnic groups. This is purely speculation of course; however, based on the results of this study connectedness can assist in spatial reasoning and possibly other areas as well.

The outcomes of this study can contribute to the pedagogy and curriculum we teach in educational institutions. If we know how connectedness assists females and possibly ethnic groups, districts across the state and nation should take heed. Connectedness has real world impact. We have uncovered a diamond in the rough. We have developed, researched, and verified a solution to the nation's educational dilemma on how to reach underserved and underprivileged female students. Not only did connectedness support females in closing the achievement variance with males, it also aided males by increasing their spatial reasoning as well. Connectedness is not only a female solution, but a human solution.

There are also some recommendations for specific actions to be taken. First, educators should be very aware of the attention we place on females and males. Are we paying more attention to males than females? Are we using pedagogy and curriculum which validates both genders or just one of them? Second, we need to educate other teachers about feminist epistemology and pedagogy through workshops and writing curriculum textbooks which promote feminine ways of thinking and learning. Third, we need to promote connectedness as a solution to closing the achievement disparity between the genders in mathematics. This may be either through publications and/or presentations at conferences. And finally, we should apply for federal and state grants to conduct more research on how we can promote feminine success in mathematics as well as in other STEM areas (Good, Rattan, & Dweck, 2012).

Locally, Project ACE (ACtion for Equity) is part of the National Collaborative Girls Project engaging K-12 teachers in the University of Texas at El Paso's partner districts, university faculty from the Colleges of Education and Engineering, and the University of Texas School of Public Health to move forward an agenda to address four major barriers to Hispanic women's and girls (Grades K -20) participation in higher education, decision to specialize in science, technology, engineering, and mathematics (STEM) programs of study, and enter health, science, technology careers.

6.6 CURRICULUM AND POLICY SUGGESTIONS

From years of teaching experience, the mathematics curriculum had been one sided. The mathematics curriculum of today favors males over females. Textbooks promote learning styles for the individual rather than promoting the individual as a part of a community of learners. We can no longer promote individual scholarship in mathematics because we are creating an injustice to students who learn differently. Mathematics has been structured to be an isolated and individualistic curriculum which leaves females behind. There are no careers in society where an individual works by themselves exclusively; yet schools promote the effort and work of the individual. Society is concerned with competition and the survival of the fittest that we must, "pick ourselves up from the bootstraps" and continue on. We place great expectations on individual goals that in return, we forget the collective.

Mathematics curriculum needs to understand that students are a community of learners. We should promote ideas of connectedness throughout the curriculum. It is true of the saying, "It takes a village to raise a child." The village must not just educate one individual but the entire clan promoting diversity, equity, accessibility, and social justice. School policies must change toward purchasing and implementing mathematics curriculum which encourages gender equity, communication, and collaboration.

6.7 RESULTS TO THEORY

This study used feminist and Chicana epistemology as its' theoretical framework. Feminist epistemology stemmed from the influential work of Belenky, Clinchy, Goldberger, and Tarule (1997) and Chicana epistemology from the research of Delgado-Bernal (1998). Connectedness was developed by the very influential Patricia Miller (2000) who began with three main concepts of scientific reasoning, social cognition, and multiple representations. This research redesigned these three main

concepts into mathematical reasoning, social cognition, and multiple strategies which became the framework for connectedness. Other theories were reflected by the research's outcomes.

Anxiety during the tests also reflected theories from feminist and Chicana epistemology. Female students tended to stress out more than males during the tests as mentioned before by Halpern (2009), James (2009), and Fennema & Sherman (1976). More specifically, the researcher did see where boys used more space than girls much like Gurian (2011) suggested. The boys were more competitive as the girls were just trying to finish the tasks within the time limit. Furthermore, girls preferred group work, cooperation, and consensus building (Bell & Norwood, 2007).

As mentioned in other previous chapters, there has been wide discussion on whether or not there is an achievement difference between females and males in mathematics. This research looked specifically at spatial, algebraic, and geometrical reasoning. Within these three topics, the results have been related to existing theory. Even though there is research stating there is no achievement disparity between the genders, this research agrees with those who state there is variance in success in spatial reasoning between females and males. The results agree with the theories stating there is an achievement variance like those of Gluck and Fitting (2003), NAEP (2012), Contreras, Martínez-Molina, & Santacreu, (2012) and a difference in mental rotation tasks (Heil, Jansen, Auaiser-Pohl, & Neuburger, 2012.) Specifically, the outcomes of the study have stated these differences in attainment in spatial reasoning, algebraic reasoning, and geometrical reasoning are real even throughout the various grade levels agreeing with AAUW (1999; 2010) and Sadker, Sadker, & Zittleman (2009). It's no surprise to the researcher there was a difference in success. As the achievement gap between Latina/os and Whites continues to increase (Kohler & Lazarín, 2007), the results of the study have shown there is a solution. This solution is connectedness. There are no current theories that implement connectedness; however, there are definite possibilities.

These various theories then helped define the methodology. With the knowledge of connectedness, the intervention activities were developed through the framework of feminist epistemology. The participants did research on the tangrams they used. This was based on how females prefer to know about a subject rather than told about it. Other parts of the connected intervention activities such as the "becoming a tangram" came from Miller (2000), Zohar (2006), and Knafo, Zahn-Waxler, Davidov, Hulle, Robinson, & Rhee (2009) in trying to get participants to become intimate and empathetic with the tangrams.

Miller (2000) also influenced the methodology by informing us of how females prefer to learn. Interconnected thinking is also a social act where there are "relationships, connecting, equality, conversation, dialogue, cooperation, negotiation, acceptance, and intimacy" (Miller, 2000, p. 53). During group work, females need these types of actions when they are learning in pairs, groups, etc. Instead of dominance and separation, the goal of the group is to increase bonding and develop a consensus within the group (Miller, 2000). Our intervention activities were framed around this paradigm and included all of its points. Next, we will look at some of the limitations of the study.

6.8 LIMITATIONS

First of all, in no way does this research implicate that the study holds true for all Latina and Latinos across the country. This study was specifically designed for this particular area. Completion of the study in different areas of the country could provide different results. By no means does the researcher indicate these results hold true for all Latina/os. However, it is safe to say the sample sizes can be representative of the population in this southwest border town. If this study were replicated at another geographical area of the U.S., different results may transpire.

There were limitations to the study. First, the study was held in two main districts in the southwest region of the U.S. and Mexico border. Perhaps other districts could have been included in the research had the time limit to conduct the study was extended for a longer period of time. This would

have enabled a better understanding of females along the entire border. Other towns near the border could also have been sites to conduct the study. Secondly, if the opportunity had arisen to conduct the study in Mexico, the researcher would have done that as well. However, with the recent rise in violence across the border, this would not have been practical, but hopefully someday.

6.9 CONCLUSION

In conclusion, this study was an attempt to understand females, in this case Latinas, and their learning of spatial reasoning in order to promote gender equity. Based upon how Latinas are among one of the lowest performing ethnic subgroups in mathematics, this research's main purpose was to test an intervention strategy called connectedness. For the purposes of this study, spatial reasoning was selected in conjunction with the Seven Clever Chinese Tangrams in order to investigate connectedness as an intervention theory. This study has shown connectedness is a solution to increasing the achievement of females in spatial reasoning tasks. Situated in a border town next to Mexico, this study had compelling percentages of Latina/os participants.

Within the statement of the problem of the study, females are left behind in schools. Females achieve equally in early grades with males, but begin to fall behind in middle school and further behind in high school. So this research analyzed three main components. Is there a difference in achievement, how does connectedness improve spatial reasoning skills, and will strategies change? One of the salient features of this dissertation comes from the large number of students who participated in the study. Furthermore, the design of the intervention activities was original and stemmed from feminist epistemology theory. Another highlight of the dissertation was the outcomes of the research.

The results of the study were outstanding. Each of the research question's outcomes agreed with our hypotheses and provided new results in this field of inquiry. The first research question allowed us to examine whether or not there existed an achievement disparity between females and males in spatial

reasoning. Our results indicated a difference in achievement between the genders throughout the grade levels consisting of elementary through high school. This allowed the researcher to state a claim of an existing achievement disparity between females and males. This result had major implications because it revealed a pattern throughout the grade levels that females do not perform as well as males in spatial reasoning. This allowed the researcher to continue to implement the intervention of connectedness.

With the applied intervention of connectedness, the results again were proven to be statistically significant. The second research question analyzed whether females can perform equally as well as males on spatial reasoning tasks with connectedness. The outcomes of the research showed that connectedness was the main factor in increasing test scores for both genders. Additionally, female Post-Test scores outgained those of males in percentage of increase. Although the females did not quite catch up to the males, the results showed significant improvement in their spatial reasoning. These results were eye opening. The results proved connectedness was the factor in closing the difference in achievement between females and males. Another significant result of the study was the increase in spatial reasoning competence.

Strategic proficiency was also a component of the research. The outcomes of the study showed females increased their strategic competence much greater than males. Even though both genders increased their strategic levels, females outperformed males in this category. Competence levels were created by the students in order to assess how they improved in their ability in creating squares. Furthermore, females and males became empathetic with the tangram pieces. During the interviews, both genders implied they humanized the tangrams by transforming them from unrelated pieces of tangrams to those with feelings and having human characteristics.

Ultimately, the goal of the research was to analyze whether a connectedness intervention could impact the learning and achievement of females in order to create a more gender equitable environment. This study did just that. The research has made an important contribution to spatial reasoning in

conjunction with feminist epistemology and how we know females prefer to learn. With this new found knowledge and scholarship in academia, this existing information can provide newer and accessible curricula which acknowledge and validate all students regardless of gender.

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Appendix

Appendix A Tangram Activity

What is a tangram?
Tangram is an old Chinese puzzle known as “seven clever pieces”.

Warm up activity.
Construct your own design using all 7 pieces of the tangram. Be creative: you could make a house, a cat, a person, a strange-looking tree, etc.
Time yourself: how much time has it taken to make your design? _____min____sec.
Trace the borderline of your design below to ensure you can remember your design (you may use a blank paper if your design doesn’t fit below).

Exchange your sketch with your classmate so you could recreate her design and he/she can recreate yours. Time yourself again: how much time has it taken to recreate it? _____min____sec.

Appendix B Pre-Test

Construct a square using all 7 pieces. Work individually to ensure accuracy.
Time yourself: how much time has it taken to make the square? _______min ______sec.
Sketch you square below (not only the borderline but also location of each piece in the square).

Appendix C Pre-Test

Construct a square using all 7 pieces. Work individually to ensure accuracy.
Time yourself: how much time has it taken to make the square? _______min ______sec.
Sketch you square below (not only the borderline but also location of each piece in the square).

What strategy have you used to construct the square? Explain below.

Have you ever worked with tangrams before? (Circle one) YES NO

Appendix D Data Collection Form

Survey Number	Grade Level	Gender*	Ethnicity**	Warm-Up***		Pre-Test***		Post-Test***	
				Time	Score	Time	Score	Time	Score
Total		Females (N=___)							
		Males (N=___)							
Average= Total/N		Females							
		Males							

*** Use 1 - for Male and 2 - for Female**
**** Use 1 - for White, 2 - for African American, 3 - for Hispanic, 4 - for Asian/ Pacific Islander, 5 – for American Indian, 6 – for Other**
***** Under Time record time spent on task in seconds, under Score use 1 – if task was completed within 15 min time limit, and use 0 – if task was NOT completed during the 15 min time limit.**

Appendix E Research Tangrams

Get to know Tangrams better by using resources such as the Internet, Library, etc…, to research about them.

- What is a tangram?

- Where did they come from?

- Find some history of tangrams.

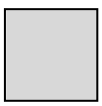
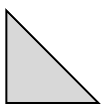
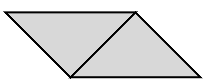
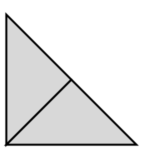
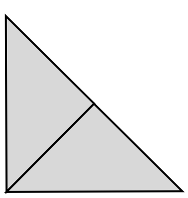
- What are some literature books that talk about tangrams?

- What areas of mathematics do you think tangrams can be included?

- What can you learn with tangrams?

Appendix F Learning about the Tangram Pieces

1) Look at each piece and tell what shape it is.
2) Are some pieces congruent? Show congruent pieces and explain why they are congruent.
3) Are some pieces similar? Show similar pieces and explain why they are similar.

#	*Sketch the Tangram Piece*	*Name the Tangram Piece*	*Area of the Tangram Piece*	*Side Lengths of the Tangram Piece*	*Number of Congruent Pieces*	*Area of Congruent Pieces*
1.	 *a*	*Square*	*1*	*a =1*	*1*	*1*
2.						
3.						
4.						
5.						
		List different side lengths of the Tangram pieces:			***Total area of all 7 pieces:***	

Appendix G Side lengths and Areas

Complete the table below using the previous activity to fill out the possible side lengths and the corresponding square areas.

Side Length a=	1		2	
Process	1^2	$(\sqrt{2})^2$		
Area of Square	1 unit^2			8 unit^2

Appendix H Core Activities

Construct squares using 1, 2, 3, 4, and 6 pieces. You will work in small groups of 5 to share your ideas, build on others input, support diverse thinking, and reach a consensus so everyone agrees on the construction of the squares. Small groups are about cooperation not competition. Each group will get a set of Tangrams. Pick a piece you want to represent. Two people in the group will be two congruent pieces (the two small isosceles triangles or the two large isosceles triangles.) Sketch your squares below* (including interior lines):

a) **1 piece**

b) **2 pieces**

c) **3 pieces**

d) **4 pieces**

e) **6 pieces**

*For some arrangements there is more than one way to make a square. What are they and why?

Appendix I Level Three Strategy

1. What possible areas do we have altogether considering different side lengths (hint: refer to previous tables)?

2. Can a square with an area of 1 be constructed with the given number of Tangram pieces?

3. Can a square with an area of 2 be constructed with the given number of Tangram pieces?

4. Can a square with an area of 4 be constructed with the given number of Tangram pieces?

5. Can a square with an area of 8 be constructed with the given number of Tangram pieces?

6. If we identified an area of a possible square to be constructed (steps 2-5), what would be a side length of this new square?

7. Identify pieces that have or could make the side length discovered in step 6.

Appendix J Collaborating on Strategy Levels

What are the different strategies your group used in creating a square with 3 and 4 pieces? As a class, let's co-construct some levels using dialogue and engagement and categorize these strategies.

Level ____

Level____

Level____

Level____

Appendix K Reflection

An arrangement with six pieces in constructing a square is not possible. Think aloud and write your thoughts below about why it is not possible.

Think aloud about strategies that you have used to construct squares and write your thoughts below.

Review the Pre-Test in a whole group discussion with your teacher.

Appendix L Post-Test

Construct a square using **5 pieces**. Work individually for accuracy.
Time yourself: how much time has it taken to make the square? ______min_____sec.
Sketch you square below (not only the borderline but also location of each piece in the square).

What strategy have you used to construct the square? Explain below.

Appendix M Interview Questions

1. What tangram piece did you pick and how did you feel being that piece?
2. (Remind them of strategies and levels) What strategy do you feel comfortable with and why?
3. (Task on flip of parallelogram.) Without touching the pieces, how would you complete the square? Be specific in describing your actions using geometric transformations (e.g. translation, rotation, reflection.)
4. Now show me: How much time did it take for the student to flip the parallelogram piece?
5. Describe what the Level 3 strategy was? What is challenging for you to understand the Level 3 strategy, why or why not?

Curriculum Vita

Raymond Falcon earned an Associate of Science degree from South Plains College in 1993. He received his Bachelor of Interdisciplinary Studies from the University of Texas at El Paso (UTEP) in 1996. He received his Master of Arts in Teaching in 2009 from UTEP. In 2009, he joined the doctoral program in Teaching, Learning, and Culture at UTEP.

While pursuing his degree, he worked as a mathematics teacher in the Ysleta Independent School District in secondary schools. He also worked for the College of Education's Project Success and as a research assistant in the department of Teacher Education.

He has presented research at conferences including the American Educational Research Association (AERA) in New Orleans, San Diego, and Denver. Dr. Falcon has published five articles in the Education Resources Information Center (ERIC) since 2009. He has signed a contract to be employed by the University of Saint Joseph in West Hartford, Connecticut as an assistant professor of mathematics education in the teacher education department in the fall of 2013.

He will be teaching critical race theory, critical pedagogy, diversity, and mathematics education. In the future, he will embark in organizing math education conferences on the border. He will teach pre-service educators and conduct research in mathematics in association with social justice. I want to collaborate with professors to conduct research on diversity, teaching, and mathematics and publish research to communicate the fascinating theories in our diverse and unique placement on the border.

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This dissertation was typed by Raymond Falcon.