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Why People Overestimate Small Probabilities?

David Amparan and Vladik Kreinovich

Abstract It is a known empirical fact that people overestimate small probabilities. This fact seems to be inconsistent with the fact that we humans are the product of billions years of improving evolution – and that we therefore perceive the world as accurately as possible. In this paper, we provide a possible explanation for this seeming contradiction.

1 Formulation of the problem

People overestimate small probabilities. It is known that people routinely overestimate small probabilities when making decisions. They overestimate the probability of rare side effects – and thus, refuse to take important vaccinations.

Experiments performed by the Nobelist Daniel Kahneman and his team show that indeed, most people overestimate small probabilities; see, e.g., [1] (see also [2, 3]).

But why? This is a fact, but how can we explain this fact from the biological viewpoint?

At first glance, the more adequately we understand the situation, the more adequate decision we can make. So why did evolution preserve this clearly biased perception of small probabilities?

What we do in this paper. In this paper, we provide a possible answer to this “why”-question.

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2 How do we know probabilities?

To provide an explanation, let us recall how we learn the probabilities.

Probabilities are estimates based on our experience. If we saw some event \( n \) times out of \( N \), then we estimate the probability as the ratio \( \frac{n}{N} \).

Of course, this is only an approximate estimate. If we flip a perfectly symmetric coin 10 times, we may get \( n = 5 \) heads, but we may also get 6 or 4 or 7.

3 Which outcomes are possible?

If an event has probability \( p \), how many times out of \( N \) can it occur? If the actual probability is \( p \), then out of \( N \) tries:

- the event happens on average in \( \mu \equiv p \cdot N \) times, and
- the variance of number of events is equal to \( \sigma^2 = N \cdot p \cdot (1 - p) \).

For small \( p \), we have \( 1 - p \approx 1 \), so \( \sigma^2 \approx \mu \) and thus, \( \mu \approx \sigma^2 \).

Usually:

- if we have a distribution with a known mean and standard deviation,
- we conclude – with high confidence – that the actual value is somewhere between \( \mu - k \cdot \sigma \) and \( \mu + k \cdot \sigma \); see, e.g., [4].

Here, \( k = 2, 3, 6, \ldots \) depending on the desired level of confidence.

4 So what can we conclude about the probability?

Suppose that some event occurred \( n \) time out of \( N \). So, the only information that we can conclude about its probability \( p \) is that \( \mu - k \cdot \sigma \leq n \leq \mu + k \cdot \sigma \).

Since \( \mu = \sigma^2 \), equivalently,

\[
\sigma^2 - k \cdot \sigma \leq n \leq \sigma^2 + k \cdot \sigma, \tag{1}
\]

where \( p = \sigma^2 / N \).

- The first of the two inequalities (1) is the inequality \( \sigma^2 - k \cdot \sigma \leq n \), i.e., equivalently, \( \sigma^2 - k \cdot \sigma - n \leq 0 \). Due to the known properties of quadratic functions, this inequality means that the non-negative value \( \sigma \) is between the roots of the corresponding quadratic equation \( \sigma^2 - k \cdot \sigma - n = 0 \), i.e., that

  \[
  \frac{k - \sqrt{k^2 + 4n}}{2} \leq \sigma \leq \frac{k + \sqrt{k^2 + 4n}}{2}.
  \]
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The left-hand side expression is always non-positive, so the left inequality is always satisfied. Thus, to satisfy the first of the two inequalities (1), it is sufficient to have

$$\sigma \leq \frac{k + \sqrt{k^2 + 4n}}{2}. \quad (2)$$

- The second of the two inequalities (1) is the inequality $n \leq \sigma^2 + k \cdot \sigma$, i.e., equivalently, $\sigma^2 + k \cdot \sigma - n \geq 0$. Due to the known properties of quadratic functions, this inequality means that the non-negative value $\sigma$ is either smaller than the smaller root of the corresponding quadratic equation $\sigma^2 + k \cdot \sigma - n = 0$ or larger than the larger of the roots, i.e., that

$$\sigma \leq -\frac{k - \sqrt{k^2 + 4n}}{2} \text{ or } -\frac{k + \sqrt{k^2 + 4n}}{2} \leq \sigma.$$

The expression $-\frac{k - \sqrt{k^2 + 4n}}{2}$ is always non-positive, so the first inequality is never satisfied. Thus, to satisfy the second of the two inequalities (1), it is sufficient to have

$$-\frac{k + \sqrt{k^2 + 4n}}{2} \leq \sigma. \quad (3)$$

By combining the inequalities (2) and (3), we conclude that

$$\frac{\sqrt{k^2 + 4n} - k}{2} \leq \sigma \leq \frac{\sqrt{k^2 + 4n} + k}{2},$$

so

$$p \overset{\text{def}}{=} \frac{2n + k^2 - k \cdot \sqrt{k^2 + 4n}}{2N} \leq p \leq \bar{p} \overset{\text{def}}{=} \frac{2n + k^2 + k \cdot \sqrt{k^2 + 4n}}{2N}.$$

5 Which probability value should we select?

We know that

$$p \overset{\text{def}}{=} \frac{2n + k^2 - k \cdot \sqrt{k^2 + 4n}}{2N} \leq p \leq \bar{p} \overset{\text{def}}{=} \frac{2n + k^2 + k \cdot \sqrt{k^2 + 4n}}{2N}.$$

We have no reason to consider one of the values from the interval $[p, \bar{p}]$ as more probable. So, it makes sense to consider all these values equally possible.

In this case, a natural idea is to select the average of these values, i.e., the mid-point

$$\frac{p + \bar{p}}{2} = \frac{n}{N} + \frac{k^2}{2N}.$$

This value is always larger than the frequency $n/N$ – which is the usual (and unbiased) estimate of the actual probability.
This provides a possible explanation of why we, in general, overestimate the values of small probabilities.

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