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Fault Detection in a Smart Electric Grid: Geometric Analysis

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Fault Detection in a Smart Electric Grid: Geometric Analysis

Hector Reyes, Dillon Trinh, and Vladik Kreinovich

Abstract The main idea behind a smart grid is to equip the grid with a dense lattice of sensors monitoring the state of the grid. If there is a fault, the sensors closer to the fault will detect larger deviations from the normal readings that sensors that are farther away. In this paper, we show that this fact can be used to locate the fault with high accuracy.

1 What is a smart electric grid

The main idea is to set up a lattice of sensors that would monitor the electric grid; see, e.g., [1]. Based on the measurement results provided by the sensors:

- we would get a good picture of the current state of the grid, and
- we would be able to effectively control it.

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2 How the grid of sensor can detect faults

Each sensor measures characteristics of the electric current at its location. Each fault affects all the sensors, some more, some less.

By observing the changes in the sensor signals, we can detect the existence of the fault. We can also get some information of the fault's location.

Sensors closer to the fault's location will detect a stronger change in their measurements results than sensors which are further away. Thus, by comparing the measurement results of the two sensors, we can decide whether the fault is:

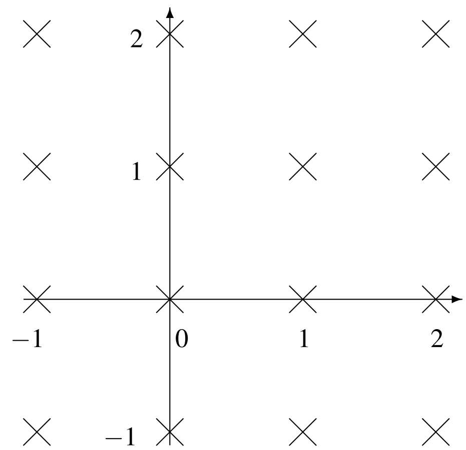
- closer to the first sensor or
- closer to the second sensor.

3 Let us describe this situation in precise terms

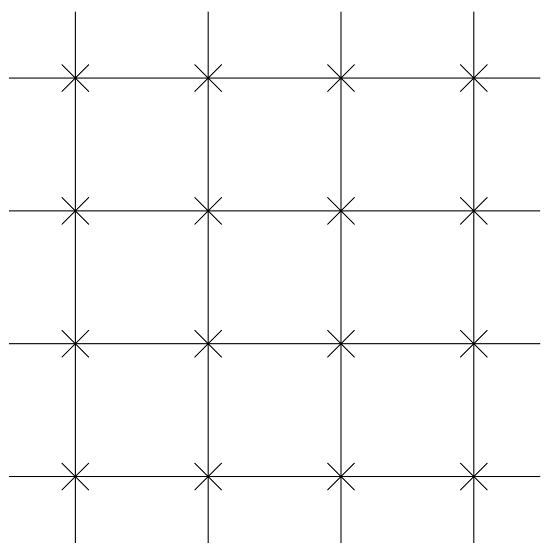
Let us consider the case when the sensors form a (potentially infinite) rectangular lattice. For simplicity of analysis, let us select a coordinate system in which:

- the location of one the sensors is the starting point $(0,0)$, and
- the distance between the closest sensors is used as a measuring unit.

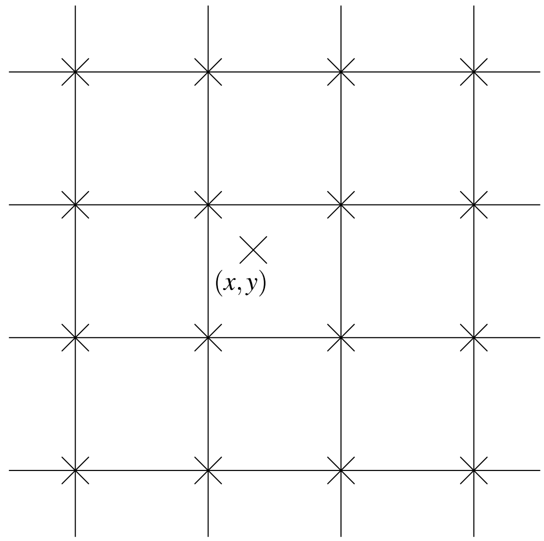
In this coordinate system, sensors are located at all the points (a,b) with integer coordinates.



These sensors divide the plane into squares $[a, a+1] \times [b, b+1]$.



Each spatial location (x, y) is in one of these squares:

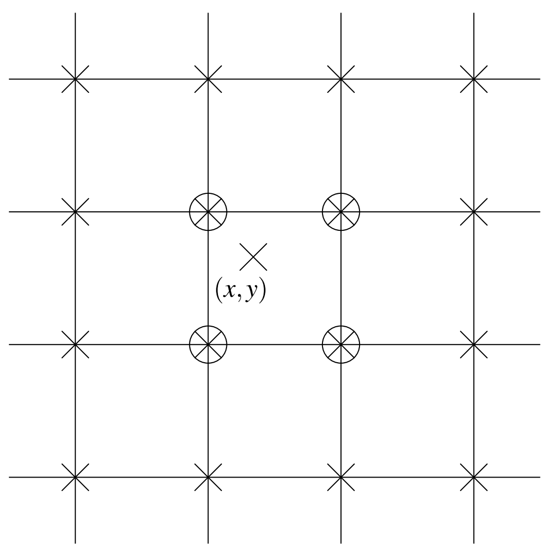


One can easily check that:

- for each spatial location within a square,
- the vertices (a,b), $(a,b+1)$, $(a+1,b)$, and $(a+1,b+1)$ of this square are the closest grid points.

Thus:

- by finding the 4 sensors at which the disturbance signal is the strongest,
- we can find the square that contains the location of the fault.



4 Research question

Can we determine the location of the fault more accurately than “somewhere in the square”?

5 Our answer

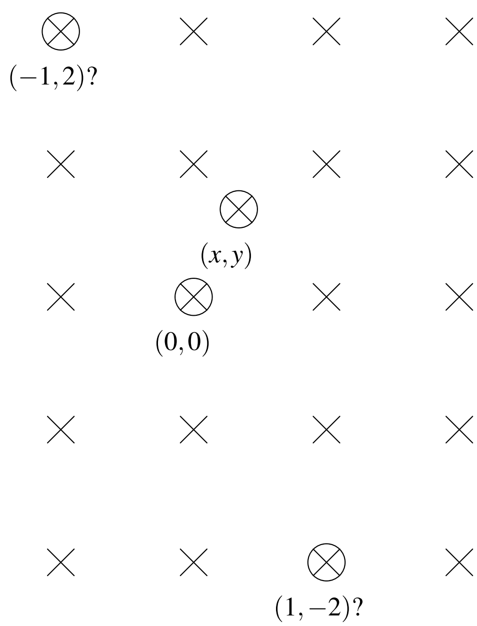
We show that, in principle:

- by using the lattice of sensors,
- we can locate the fault with any desired accuracy.

Indeed, without losing generality, let us assume that the square containing the fault is the square $[0,1] \times [0,1]$. In other words, we know that the coordinates (x,y) of the fault satisfy the inequalities $0 \le x \le 1$ and $0 \le y \le 1$.

For each pair of positive integers (p,q), we can check whether

- the sensor at $(p,-q)$ gets a stronger signal than
- the sensor at $(-p,q)$.



The first sensor’s signal is stronger if and only if:

- the squared distance $d^2(f,s_1) = (x-p)^2 + (y-(-q))^2$ between the fault f and the first sensor s_1 is smaller than

- the squared distance $d^2(f,s_2) = (x-(-p))^2 + (y-q)^2$ to the second sensor.

One can check that $d^2(f,s_1) < d^2(f,s_2)$ if and only if $q \cdot y < p \cdot x$, i.e., if and only if

$$\frac{y}{x} < \frac{p}{q}.$$

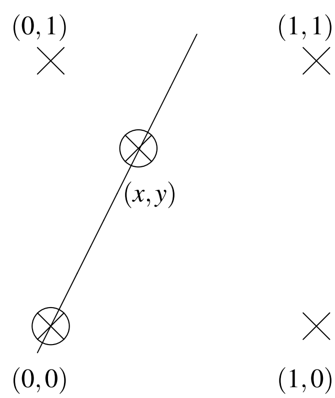
A real number can be uniquely determined if we know:

- which rational numbers p/q are smaller than this number and
- which are larger.

Thus:

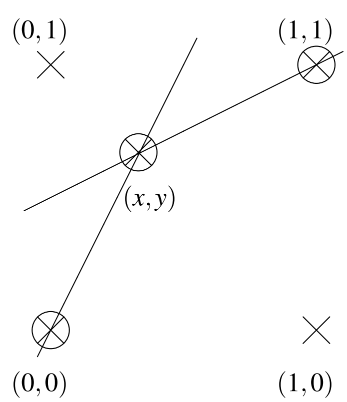
- by comparing signals from different sensors,
- we can determine the ratio $r \stackrel{\text{def}}{=} y/x$ with any given accuracy.

Hence, we can determine the line $y = r \cdot x$ going through $(0,0)$ that contains the fault:



Similarly, we can find a straight line going through the point $(1,1)$ that contains the fault. Thus:

- the fault's location can be uniquely determined
- as the intersection of these two straight lines.



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