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What Is a Reasonable Way to Make Predictions?

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Abstract Predictions are usually based on what is called laws of nature: many times, we observe the same relation between the states at different moments of time, and we conclude that the same relation will occur in the future. The more times the relation repeats, the more confident we are that the same phenomenon will be repeated again. This is how Newton's laws and other laws came into being. This is what is called inductive reasoning. However, there are other reasonable approaches. For example, assume that a person speeds and is not caught. This may be repeated two times, three times – but here, the more times this phenomenon is repeated, the more confident we become that next this, he/she will be caught. Let us call this anti-inductive reasoning. So which of the two approaches shall we use? This is an example of a question that we study in this paper.

1 Formulation of the Problem

1.1 Making predictions is important

One of the main objectives of science is to predict what will happen in the future. Another important objective is to make the future more beneficial. This objective also requires predicting how different strategies will affect the future of the world. Prediction is one of the main objectives of science. So in long run, this is one of the main objectives of all the tools that science uses – including AI tools. So, to make these tools more efficient, it is important to understand:

- how we make predictions, and
- how we *should* make predictions.

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1.2 At first glance, the answer to these questions is straightforward

Predictions are usually based on what is called *laws of nature*:

- many times, we observe the same relation between the states at different moments of time, and
- we conclude that the same relation will occur in the future.

The more times the relation repeats, the more confident we are that the phenomenon will repeat again. This is how Newton's laws and other laws came into being. This is what is called *inductive reasoning*; see, e.g., [1, 2, 3].

Comment. It is important not to confuse:

- inductive reasoning – where we make a prediction based on a finite number of observations, and
- *mathematical induction*, when we prove a statement $\forall n\,P(n)$ by proving that $P(0)$ is true and for every n, $P(n)$ implies $P(n+1)$.

1.3 Situation is not so simple

However, there are other reasonable approaches. For example, assume that a person speeds and is not caught. This may be repeated two times, three times. Here:

- the more times this phenomenon is repeated,
- the more confident we become that next time, he will be caught.

This is why gamblers continue to gamble after losing. This is why entrepreneurs try again after failing several times. Let us call this *anti-inductive reasoning;* see, e.g., [3].

So which of the two approaches shall we use?

1.4 This should be decided by an experiment

We are accustomed to the fact that everything is decided by experiments. So, a natural way to select one of these two approaches is to compare them with the experimental data.

But how do we decide, based on this data, which approach is better? For this decision:

- a scientist will use inductive reasoning, while
- another person will use anti-inductive reasoning.

What will happen? This is one of the questions that we analyze in this paper.

2 Analysis of the Problem on a Simplified Case

2.1 Simplified case: a description

For simplicity, let us fix some natural number n, and consider the following simplified versions of the two approaches.

- The first approach is that:
  - if something repeats n times (or its negation repeats n times),
  - we predict that this will happen the next time.
- The second approach is that if something happens n times, the opposite will happen the next time.

2.2 Case study

Suppose that we have a phenomenon – e.g., Sun rising in the morning – that holds for $2n+1$ moments of time.

In the first approach:

- After the first n cases, we predict that the Sun will rise again – and it does.
- We do a similar prediction for moment $n+2$ – and again, our prediction turns out to be correct.
- For n moments in a row, predictions based on our reasoning are correct.
- So, by applying inductive reasoning to these n cases, we conclude that inductive reasoning is a valid approach.

But what if we use the second approach?

- We predict that at the moment $n+1$, the Sun will *not* rise – but it rises!
- This repeats n times, so n times, are predictions are wrong.
- We are then applying the same anti-induction to select the approach.
- Since our approach failed n times, we conclude that next time, it will work.

2.3 Surprising conclusion

So, in this case, no matter how many experiments we perform:

- the proponents of both approaches will remain convinced
- that their approach will work the next time around.

2.4 What we discuss in this paper

In this paper, we describe this situation in precise terms.

This is still the beginning of this research. We will present more challenges than results. However, we will formulate these important challenges in precise terms, so these challenges become:

- not just vague philosophical ideas,
- but precisely formulated mathematical questions.

3 General Case

3.1 Let us describe the situation in precise terms

We want to predict whether some property P will hold. To make this prediction, we used previous observations. Let us assume that we observed similar situations N times. For each i from 1 to N, we define s_i as follows:

- if the property P was satisfied in the i-th observation, we take $s_i = T$;
- if the property P was not satisfied in the i-th observation, we take $s_i = F$;
- if it is unknown whether P was satisfied, we take $s_i = U$.

The set of all such sequences will be denoted by $\{T,F,U\}^*$.

Prediction rule $M(s)$ means that, for each such sequence s, we predict:

- either that P will be satisfied at the next moment of time: $M(s) = T$,
- or that P will not be satisfied: $M(s) = F$,
- or we do not have enough data for predictions: $M(s) = U$.

So, a prediction rule M is a mapping $M : \{T,F,U\}^* \mapsto \{T,F,U\}$.

3.2 Prediction rule must be fair

A priori, we have no reason to prefer P or its negation $\neg P$. So, we should make the same prediction whether we consider P or $\neg P$. We call this absence of a priori preference *fairness*. So, for observations of $\neg P$, we should get the same conclusion.

Let us describe fairness in precise terms.

- For each observation $s = (s_1,\ldots,s_n)$ of P, the observation of $\neg P$ is $\neg s = (\neg s_1,\ldots,\neg s_n)$, where $\neg u \stackrel{\text{def}}{=} u$.
- Similarly, prediction $M(s)$ for P means predicting $\neg M(s)$ for $\neg P$.

Thus, fairness means that $M(\neg s) = \neg M(s)$ for all s.

3.3 Meta-analysis: using prediction rule to select prediction rule

Let $s = (s_1, \ldots, s_N)$ be a sequence of observations.

- Based on s_1, we form a prediction $M(s_1)$ for s_2.
- Based on (s_1, s_2), we form a prediction $M(s_1, s_2)$ for s_3.
- In general, based on $(s_1, \ldots, s_i)$, we form a prediction $M(s_1, \ldots, s_i)$ for s_{i+1}.
- Finally, based on $(s_1, \ldots, s_{N-1})$, we form a prediction $M(s_1, \ldots, s_{N-1})$ for s_N.

For each i, we check whether predictions were correct:

- if s_{i+1} and/or $M(s_1, \ldots, s_i)$ are unknown, we take $c_i = U$;
- otherwise, we take $c_i = T$ if $M(s_1, \ldots, s_i) = s_{i+1}$ and $c_i = F$ if $M(s_1, \ldots, s_i) \neq s_{i+1}$.

This way, we get a sequence $c = (c_1, \ldots, c_{N-1})$ of truth values describing how well prediction rule M worked. We can now apply the rule M to the sequence c to predict whether M will work the next time.

- If $M(c) = F$, this means that our own induction rule requires us to reject this rule. So, if $M(c) = F$, we say that M is *inconsistent* with the observations s.
- Otherwise, we say that M is *consistent* with s.

3.4 Induction vs. anti-induction revisited

Induction rule M_I means that:

- if the last n elements of s are T, then $M_I(s) = T$;
- if the last n elements of s are F, we take $M_I(s) = F$;
- otherwise, $M_I(s) = U$.

Anti-induction rule M_A means that:

- if the last n elements of s are T, then $M_A(s) = F$;
- if the last n elements of s are false (F), we take $M_A(s) = T$;
- otherwise, we take $M_A(s) = U$.

Here, for all s, we have $M_A(s) = \neg M_I(s)$, i.e., $M_A = \neg M_I$.

3.5 General result

We had an example of a sequence s with which both M_I and M_A were consistent. What happens in the general case? Was it a weird example or is it a general phenomenon?

We prove that this is a general phenomenon.

Proposition 1. *For each fair prediction rule M and for each sequence s:*

M is consistent with s $\Leftrightarrow$ $\neg M$ is consistent with s.

Proof. Let us denote sequences c corresponding to M and $\neg M$ by c^M and $c^{\neg M}$. Here, $c_i^M = T$ if $M(s_1,\ldots,s_i) = s_{i+1}$. This is exactly when we have $\neg M(s_1,\ldots,s_i) \neq s_{i+1}$, i.e., when $c_i^{\neg M} = F$. Thus, $c_i^{\neg M} = \neg c_i^M$.

Based on the sequence $c^{\neg M}$, the rule $\neg M$ will predict

$$\neg M\left(c^{\neg M}\right) = \neg M\left(\neg c^M\right).$$

Since M is fair, we have $M\left(\neg c^M\right) = \neg M\left(c^M\right)$. Thus:

$$\neg M\left(c^{\neg M}\right) = \neg M\left(\neg c^M\right) = \neg\neg M\left(c^M\right) = M\left(c^M\right).$$

So, indeed, M and $\neg M$ are consistent or inconsistent simultaneously.

The proposition is proven.

4 Rules must be falsifiable

4.1 An example where a reasonable prediction rule is inconsistent

The fact that both induction and anti-induction rules are consistent with the same observations makes one think that maybe all reasonable rules are always consistent with all the observations. That would be bad, because if something cannot be disproved by experiment, this does not sound very scientific.

Let us show that this is not the case.

Indeed, a natural rule M_m is to go by majority:

- if in s, we had more T than F, we predict T;
- if in s, we had more F than T, we predict F;
- otherwise, we predict U.

What happens if we apply this rule to a periodic sequence $s = (T,F,T,F,\ldots)$ for which:

- we have $s_{2k} = F$ for all k, and
- we have $s_{2k+1} = T$ for all k.

Here:

- For even $i = 2k$, we have equally many Ts and Fs, so $M(s_1,\ldots,s_i) = U$, thus $c_i = U$.
- For odd $i = 2k+1$, we have more Ts than Fs, so $M(s_1,\ldots,s_i) = T$. For $i = 2k+1$, we have $s_{i+1} = s_{2k+2} = F$, so $c_{2k+1} = F$.

So, the sequence c_i has only Fs and Us. Thus $M(c) = F$.

In other words, the majority rule is inconsistent with this sequence.

4.2 A problem with simple induction

Let us show that, somewhat unexpectedly, the simple induction M_I – as described in the previous sections – cannot be falsified and is, thus, not a very scientific approach.

Proposition 2. *For $n > 1$, no sequence s is inconsistent with the prediction rule M_I.*

Proof. The only way to show that the observation sequence s is inconsistent with M_I is when the corresponding sequence c contains n false values in a row, i.e., if n times in a row, the prediction rule M_I did not work: $c_N = \ldots = c_{N+n-1} = F$.

When it did not work the first time, this means that we have

$$s_{N+1} \neq M^I(s_{N-n+1}, \ldots, s_N).$$

By definition of the simple induction rule M^I, this can happen in two possible situations:

- either we have $s_{N+1} = F$ and $s_{N-n+1} = \ldots = s_N = T$,
- or we have $s_{N+1} = T$ and $s_{N-n+1} = \ldots = s_N = F$.

Let us first consider the first situation. In this case, at the moment $N+1$, the last n values of the sequence s are:

- several (namely, $n-1$) T-values $s_{N-n+2} = \ldots = s_N = T$,
- followed by an F-value $s_{N+1} = F$.

In this situation, the simple induction rule M^I does not predict anything at all, so we have $c_{N+1} = U$ ("unknown"), and we cannot have $c_{N+1} = F$.

Similarly, in the second situation, the last n values of the sequence s are:

- several (namely, $n-1$) F-values $s_{N-n+2} = \ldots = s_N = T$,
- followed by a T-value $s_{N+1} = F$.

In this situation, the simple induction rule M^I also does not predict anything at all, so we have $c_{N+1} = U$ ("unknown"), and we cannot have $c_{N+1} = F$.

In both situations, we cannot have $c_N = c_{N+1} = \ldots = F$ and thus, the simple prediction rule indeed cannot be falsified. The proposition is proven.

The situation is not better for the simple anti-induction principle M_A either:

Corollary. *For $n > 1$, no sequence s is inconsistent with M_A.*

Proof. This immediately follows from Proposition 2 if we take into account Proposition 1, according to which any sequence s is consistent with the prediction rule M_I if and only if it is consistent with its negation $M_A = \neg M_I$.

5 Conclusions and future work

5.1 Predictions: naive idea

How do we make predictions? At first glance, the situation sounds straightforward: if we observe some phenomenon sufficiently many (n) times, then we naturally conclude that the same phenomenon (e.g., rising of the sun) will happen again the next time. This argument is known as inductive reasoning.

5.2 What we show: situation is more complex that it may appear

However, in principle, we can consider the opposite rule: if something happens sufficiently many times, then we expect that the opposite will happen the next time. For example, if someone was speeding many times and never got taught, we expect that he/she will eventually get caught by the police.

So, which principle should we use for prediction: inductive reasoning or the above-described "anti-inductive" reasoning? A natural idea is to use the same principle to select the prediction principle itself. For example, if we believe in inductive reasoning, then if this principle led to good predictions n times, we expect it to be working the next time as well. Similarly, if we believe in anti-inductive reasoning, then:

- if this principle does not lead to good predictions n times in a row, we expect it to be working next time – and,
- vice versa, if anti-indiction reasoning led to good predictions n times in a row, we expect this principle to fail next time.

This seems to provide an experimental way to test which principle better suits the observations.

Somewhat unexpectedly, we show that it is not possible to experimentally distinguish between the two principle: each sequence of observations which is consistent with induction is also consistent with anti-induction, and vice versa. Moreover, we show that neither of these two principle can be falsified at all – so both principles are dubious from the scientific viewpoint, according to which scientific laws and techniques must be, in principle, falsifiable by experiments.

5.3 Future work

The above results are just the beginning. We need to analyze more realistic formulations of the induction rule this way, as well as other possible rules. We need some experiments: what will happen if we apply different rules to different sequences of observations? From the more theoretical viewpoint: can we algorithmically (and feasibly algorithmically) check whether a given prediction rule is falsifiable?

We hope that our work will inspire others analyze to these important methodological questions.

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