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Shall We Be Foxes or Hedgehogs: What Is the Best Balance for Research?

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Shall We Be Foxes or Hedgehogs: What Is the Best Balance for Research?

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Abstract Some researchers have few main ideas that they apply to many different problems – they are called hedgehogs. Other researchers have many ideas but apply them to fewer problems – they are called foxes. Both approaches have their advantages and disadvantages. What is the best balance between these two approaches? In this paper, we provide general recommendations about this balance. Specifically, we conclude that the optimal productivity is when the time spent on generating new ideas is equal to the time spent on understanding new applications. So, if for a researcher, understanding a new problem is much easier than generating a new idea, this researcher should generate fewer ideas – i.e., be a hedgehog. Vice versa, if for a researcher, generating a new idea is easier than understanding a new problem, it is more productive for this person to generate many ideas – i.e., to be a fox. For researchers for whom these times are of the same order, we provide explicit formulas for the optimal research strategy.

1 Foxes and Hedgehogs

Foxes and hedgehogs: a positive viewpoint. In his famous essay [1], Isaiah Berlin, an American philosopher, divide all the thinkers into:

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- *hedgehogs*, who have one main idea (or a few main ideas) and apply it (them) to several problems, and
- *foxes*, who have many different ideas.

Some great thinkers were hedgehogs (Freud and Zadeh come to mind right away), some – like Aristotle – were foxes. At first glance, it looks like both types of thinkers could reach great results. But each of these two types has its limitations.

Foxes: a negative viewpoint. At first glance, what can be wrong with having many interesting ideas, with always learning many interesting ideas? Well, the problem is that you may spread yourself too thin.

For example, in mathematical logic, Georg Kreisel was one of the most productive authors, publishing many papers with interesting ideas; see, e.g., [3, 4]. This did not bother hedgehogs, but several foxes – eagerly interested in learning new ideas – complained that they have no time to do their own research: they have to read all new papers by Kreisel.

Hedgehogs: a negative viewpoint. Lotfi Zadeh, himself clearly a hedgehogs, liked to emphasize what can go wrong with this approach, by reminding us of the saying that if all you have is a hammer, then everything starts looking like a nail.

We have seen many examples of this in politics, when an originally successful idea gets used everywhere; in popular medicine, where successful medicines like antibiotics gets too overused, etc. In Russia, where several of us are from, we had a silly joke showing this problem. A young man wants to become a writer, so he is taking an entrance exam to the writer's program.
– What can you say about Tolstoy's War and Peace?
– Never read it.
– ??? Did not you say that you want to become a writer?
– Yes, but I want to be a writer, not a reader.
In science, some hedgehogs become such writers-not-readers: they may have a had a great idea, but later on, their reluctance to adopt new ideas makes them not very productive. This even happened to great Einstein, who started as a fox – e.g., his Nobel prize was for photo-effect, not for relativity – but who spent several not-very-productive last decades on a single not-very-successful idea of a unified field theory.

There should be a balance. Since both extremes can be counterproductive, there should be a balance between these two extremes, a balance that leads to the maximal possible productivity.

What is this balance? In this paper, we provide a simple model of the situation, and we use this model to provide recommendations on the best balance.

2 Let Us Model the Situation

We need to generate new ideas. The whole idea of research is to solve problems that no one was able to solve before. This means that the existing ideas are not enough to solve the corresponding problem – you need to have a new idea, or at least a new twist on an existing idea.

Generating ideas: notations. Let us assume that a researcher spend time t_I on developing a new idea (or a new twist on a new idea). Then, if during a certain period of time T_0, the researcher comes up with I ideas, then overall, during this period, this researcher spends time $T_I = t_i \cdot I$ on coming up with new ideas.

Fox and hedgehog. For a hedgehog, $I \approx 1$, while for a fox, the number of new ideas I is much larger than 1: $I \gg 1$.

Understanding problems: notations. To be able to solve a problem, it is important to spend some time understanding this problem. This is not easy – especially if this problem is from an area which is different from the researcher's main area of expertise. Let us denote the average time needed to understand a problem by t_P, and the number of different problems the researcher learns during the period T_0 by P. Then overall, during this period, the researcher spends time $T_P = t_P \cdot P$ on learning new problems.

We need to apply these ideas. The whole purpose of coming up with new ideas is to solve problems – and the whole purpose of learning a problem is to try to solve it. If one idea is not working on a problem, a reasonable approach is to apply a different idea. Some problems are solved, most are not – unless we are dealing with a genius who solves all the problems, and such geniuses are rare. In general, a researcher applies all his/her ideas to all the problems that he/she tries to solve – otherwise, what is the purpose of learning a new problem if you do not try to solve it by using all ideas you have?

Let t_0 denote the time that it takes, on average, to try one idea on one problem. Then, to try each of I ideas on each of P problems, we need time $t_0 \cdot I \cdot P$.

Resulting constraint. The overall time that a researcher spends on inventing ideas, learning the problems, and trying ideas on problems cannot exceed T_0. Thus, we have the following constraint:

$$t_I \cdot I + t_P \cdot P + t_0 \cdot I \cdot P \leq T_0. \tag{1}$$

What do we want? The main objective of research is to solve problems. The more problems we solve altogether, the more successful we are in our research efforts. From this viewpoint, we should therefore aim for maximizing the number of solved problems.

How many problems can we solve this way? A priori, we do not know which idea will work on which problem. So, it is natural to assume that for each pairs of an idea and a problem, there is the same probability that this particular idea will solve this particular problem. This assumption is known as Laplace Indeterminacy Principle; see, e.g., [2]. Let p_0 denote this joint probability. This probability means that out of all $I \cdot P$ pairs, the proportion of those that lead to solution is equal to p_0. Thus, the overall number of problems solved by a researcher is equal to

$$p_0 \cdot I \cdot P. \tag{2}$$

So, we arrive at the following optimization problem.

Resulting optimization problem. Let us assume that we are planning for time period T_0. For a given researcher, we know:

- the average time t_I that it takes this researcher to come up with a new idea or a new twist on an idea;
- the average time t_P that it takes this researcher to understand a new problem;
- the average time t_0 that it takes this researcher to apply an idea to a problem; and
- the probability p_0 that a randomly selected idea will solve a randomly selected problem.

We want to find the number of ideas I and the number of problems P that maximize the expected number (2) of solved problems under constraint (1).

Let us now solve this problem.

3 Let Us Solve the Resulting Optimization Problem and Thus Find the Optimal Balance Between Fox and Hedgehog Strategies

First simplification. If in the constraint (1), we have a strict inequality, this would mean that we can increase either I or P (or both) without violating the constraint and thus, increase the value of the objective function (2). Thus, the maximum of the objective function is attained when in the constraint (1), we have equality, i.e., when

$$t_I \cdot I + t_P \cdot P + t_0 \cdot I \cdot P = T_0. \tag{3}$$

So, we have a problem of optimizing the objective function (2) under the constraint (3).

Second simplification. In terms of T_I and T_P, we have

$$I = \frac{T_I}{t_I}, \;\; P = \frac{T_P}{t_P}, \text{ and thus, } t_0 \cdot I \cdot P = c \cdot T_I \cdot T_P, \tag{4}$$

where we denoted

$$c \stackrel{\text{def}}{=} \frac{t_0}{t_I \cdot t_P}. \tag{5}$$

In these terms, the constraint (3) takes the form

$$T_I + T_P + c \cdot T_I \cdot T_P = T_0, \tag{6}$$

and the objective function (2) takes the form

$$p_0 \cdot I \cdot P = c_0 \cdot T_I \cdot T_P, \text{ where } c_0 \stackrel{\text{def}}{=} \frac{p_0}{t_I \cdot t_P}. \tag{7}$$

So, the problem becomes: to maximize the expression (7) under the constraint (6).

Let us use Lagrange multiplier method. Since the constraint has the form of equality, we can use the Lagrange multiplier method to solve the corresponding constrained optimization problem. Namely, for some λ, the original constrained optimization problem is equivalent to the unconstrained problem of optimizing the expression

$$c_0 \cdot T_I \cdot T_P + \lambda \cdot (T_I + T_P + c \cdot T_I \cdot T_P - T_0). \tag{8}$$

For an unconstrained optimization problem, maximum is attained when all the partial derivatives are equal to 0.

Differentiation the expression (8) with respect to T_I and equating the derivative to 0, we conclude that

$$c_0 \cdot T_P + \lambda + \lambda \cdot c \cdot T_P = 0,$$

hence

$$T_P \cdot (c_0 + \lambda \cdot c) = -\lambda,$$

and

$$T_P = -\frac{\lambda}{c_0 + \lambda \cdot c}. \tag{9}$$

Similarly, differentiation the expression (8) with respect to T_P and equating the derivative to 0, we conclude that

$$c_0 \cdot T_I + \lambda + \lambda \cdot c \cdot T_I = 0,$$

hence

$$T_I \cdot (c_0 + \lambda \cdot c) = -\lambda,$$

and

$$T_I = -\frac{\lambda}{c_0 + \lambda \cdot c}. \tag{10}$$

First conclusion. By comparing the expressions (9) and (10), we conclude that we have

$$T_I = T_P, \tag{11}$$

i.e., that *the time spent on inventing new ideas should be equal to the time spent on learning new problems.*

So fox or hedgehog? From (11), we conclude that

$$I = \frac{t_P}{t_I} \cdot P. \tag{12}$$

So:

- For researchers for whom $t_P \ll t_I$, i.e., for whom it is much easier to understand a new problem than to come up with a new idea, it is better to generate fewer ideas but apply them to many problems – in other words, to be a hedgehog.
- On the other hand, for researchers for whom $t_I \ll t_P$, i.e., for whom it is much easier to come up with a new idea that to understand a new problem, it is better to generate many ideas but apply them to fewer problems – in other words, to be a fox.

For the cases when the times t_I and t_P are of the same order, the formula (12) provides the desired optimal balance.

So what are the optimal values of P **and** I**?** In the optimal case, when $T_I = T_P$, the constraint (6) takes the form

$$2T_I + c \cdot T_I^2 = T_0. \tag{13}$$

By solving this quadratic equation, we get

$$T_I = T_P = \frac{\sqrt{1 + c \cdot T_0} - 1}{c}, \tag{14}$$

where c is determined by the formula (5). Thus,

$$I = \frac{T_I}{t_I} = \frac{t_P}{t_0} \cdot \left(\sqrt{1 + \frac{t_0}{t_I \cdot t_P} \cdot T_0} - 1 \right) \tag{15}$$

and

$$P = \frac{T_P}{t_P} = \frac{t_I}{t_0} \cdot \left(\sqrt{1 + \frac{t_0}{t_I \cdot t_P} \cdot T_0} - 1 \right). \tag{16}$$

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References

1. I. Berlin, *The Hedgehog and the Fox: An Essay on Tolstoy's View of History*, Princeton University Press, Princeton, New Jersey, 2013.
2. E. T. Jaynes and G. L. Bretthorst, *Probability Theory: The Logic of Science*, Cambridge University Press, Cambridge, UK, 2003.
3. P. Odifreddi (ed.), *Kreiseliana: About and Around Georg Kreisel*, A. K. Peters/CRC Press, Natick, Massachusetts, 1996.
4. P. Weingartner and H.-P. Leeb, *Kreisel's Interests: On the Foundations of Logic and Mathematics*, College Publications, Rickmansworth, UK, 2020.