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Why 70/100 Is Satisfactory? Why Five Letter Grades? Why Other Academic Conventions?

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Why 70/100 Is Satisfactory? Why Five Letter Grades? Why Other Academic Conventions?

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Abstract Why 70/100 is usually a threshold for a student's satisfactory performance? Why there are usually only five letter grades? Why the usual arrangement of research, teaching, and service is 40-40-20? We show that all these arrangements – and other similar academic arrangements – can be explained by two ideas: the Laplace Indeterminacy Principle and the seven plus minus two law.

1 Why 70/100 Is Satisfactory?

Formulation of the problem. In the standard US teaching arrangement, about 70 points out of 100 means a satisfactory grade – less than that is failing.

A similar proportion works well outside the academic world: e.g., at Google, if you have fulfilled 70% of your annual goals, this is considered to be a satisfactory performance.

Since this arrangement is actively used for a long time, it probably reflects the intuitive idea of a satisfactory learning level. But a natural question remains: how can we explain this empirical fact – that namely 70/100 is the satisfactory threshold?

What is satisfactory: intuitive idea. Crudely speaking, satisfactory means that the amount of the course material that the student know is (significantly) larger than

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the amount of the course material that the student does not know. Equivalently, the amount of the course material that the student does not know is (much) smaller than the amount of the course material that the student knows.

We need to formalize this idea. If we did not have the imprecise words "significantly" and "much", the formalization would be very straightforward: the proportion k of the course material that the student knows should be larger than the proportion $d = 1 - k$ of the course material that the student does not know: $d < k$. This would mean that the threshold would be 50/100. However, while, e.g., 0.51 is larger than $1 - 0.51 = 0.49$, one cannot say that it 0.51 significantly larger than 0.49.

Yes, 0.49 is smaller than 0.51, but, intuitively, 0.49 is not a meaningful representative of numbers which are smaller than 0.51. If you ask a person to name a typical representative of numbers which are smaller than 0.51, it is highly improbable that this person will select a value 0.49. So, what is the typical representative of numbers smaller than a given one?

Analysis of the problem. In general, once we have a number k, what is a typical representative of all the non-negative numbers which are smaller than k?

To answer this question, let us first note that while from the purely mathematical viewpoint, there are infinitely many numbers on the interval $[0,k]$, in practice, there is usually some small amount h such that values whose difference is smaller than h are indistinguishable. For example, for grades scaled from 0 to 100, it is usually 1 point or, sometimes, 0.1 points.

In this case, we have only finitely many possible smaller values: 0, h, $2h$, $3h$, $\ldots$, all the way to the largest value $n \cdot h$, where $n \approx k/h$. For example, if $h = 1$, then for grades smaller than 70, we have 70 different possible values 0, 1, 2, 3, $\ldots$, all the way to 69. For $h = 0.1$, we get possible values 0, 0.1, 0.2, 0.3, $\ldots$, all the way to 69.9.

When we say that some value t is a "typical" representation of all these values, what we mean that this typical value should be kind of close to all possible values, i.e., that we should have $t \approx 0$, $t \approx h$, $t \approx 2h$, $t \approx 3h$, $\ldots$, $t \approx n \cdot h$. In other words, the tuple $(t,t,t,t,\ldots,t)$ formed by the left-hand sides of these approximate equalities should be close to the tuple $(0,h,2h,3h,\ldots,n \cdot h)$ formed by the right-hand sides.

Tuples of real numbers can be naturally represented as points in the corresponding multi-D space, and thus, the distance

$$d((t,t,t,t,\ldots,t),(0,h,2h,3h,\ldots,n \cdot h)) =$$

$$\sqrt{(t-0)^2 + (t-h)^2 + (t-2h)^2 + (t-3h)^2 + \ldots + (t-n \cdot h)^2} \qquad (1)$$

between the corresponding points is the natural measure of closeness between the tuples. The closer the tuples, the more typical is the value t. Thus, we need to select the value t for which the distance (1) is the smallest possible.

A non-negative expression (1) is the smallest if and only if its square

$$d^2((t,t,t,t,\ldots,t),(0,h,2h,3h,\ldots,n \cdot h)) =$$

$$(t-0)^2+(t-h)^2+(t-2h)^2+(t-3h)^2+\ldots+(t-n\cdot h)^2 \qquad (2)$$

is the smallest. Differentiating this expression with respect to the unknown t and equating the resulting derivative to 0, we conclude that

$$2\cdot(t-0)+2\cdot(t-h)+2\cdot(t-2h)+2\cdot(t-3h)+\ldots+2\cdot(t-n\cdot h)=0. \qquad (3)$$

Dividing both sides of this equality by 2 and moving all free terms to the right-hand side, we get

$$(n+1)\cdot t=0+h+2h+3h+\ldots+n\cdot h=(0+1+2+3+\ldots+n)\cdot h. \qquad (4)$$

It is known that

$$0+1+2+3+\ldots+n=\frac{n\cdot(n+1)}{2},$$

hence the equality (4) takes the form

$$(n+1)\cdot t=\frac{n\cdot(n+1)}{2}\cdot h,$$

and thus,

$$t=\frac{n\cdot h}{2}.$$

Since $n\cdot h\approx k$ – and the difference between these two value is of order h, i.e., negligible, we conclude that $t\approx k/2$.

In other words, among all the values which are smaller than k, the typical value is

$$t=\frac{k}{2}. \qquad (5)$$

Comment. In the above argument, we implicitly assumed that all possible values 0, h, $2h$, $3h$, ..., are equally possible. This assumption makes sense – since we have no reason to assume that some of these values are more probable than others. Such an argument is known as *Laplace Indeterminacy Principle*. It is a particular case of a very successful more general argument of this type known as the Maximum Entropy Approach; see, e.g., [2].

Resulting formalization leads to approximately 70/100 threshold for Satisfactory. Let us apply the above description (5) to our problem. Our description of satisfactory is that the proportion $d=1-k$ of the course material that a student does not know is much smaller than the proportion k of the course material that the student knows. It is reasonable to select, as a threshold for this property, a "typical" smaller-than-k value, i.e., $k/2$.

The condition that $1-k=k/2$ leads to $k=2/3=0.66\ldots$, i.e., indeed to approximately 70%.

2 Why 40-40-20 Proportion for Research, Teaching, and Service: First Explanation

Formulation of the problem. In many universities, it is recommended that faculty spend 40% of their time on researchj, 40% on teaching, and 20% on service. Again, the fact that this arrangement is widely accepted means that it corresponds to the intuitive ideas and is empirically reasonable. How can we explain this empirical fact?

Intuitive idea. Intuitively, the idea is that we should spend equal time on research and teaching, and less time on service.

Let us formalize this idea. The proportion r of time spent on research should be equal to the proportion t of time spent on teaching, and should be larger that the proportion of time s spent on service. Equality is straightforward: $r = t$. In line with the above general description, it is reasonable to formalize the fact that the proportion s is smaller than the proportion $r = t$ as $s = r/2$.

This formalization leads exactly to the 40-40-20 arrangement. Let us show that the above formalization explains the above arrangement. Indeed, from $r+t+s = 1$, $t = r$, and $s = r/2$, we conclude that $2r + r/2 = 2.5r = 1$, hence $r = 0.4$. Thus, $t = r = 0.4$ and $s = r/2 = 0.2$, which is exactly the current arrangement.

3 Why 40-40-20 Proportion for Research, Teaching, and Service: Second Explanation

Seven plus minus two law. Our second explanation is based on the well-known "seven plus minus two" law (see, e.g., [3, 4]), according to which we naturally divide everything into 7 ± 2 clusters – into how many depends on the person. Because of this, a person who divides everything into 9 clusters will not pay serious attention to 1/9-th of the time, a person who divides everything into 5 clusters will not pay serious attention to any activity that takes less than 1/5-th of the overall time, etc.

Resulting explanation. The main objectives of a university are teaching and research, service is clearly not that important – but we still want people to do service, otherwise the university will not function smoothly – serve on committees, develop curricula, etc. We do not want faculty to spend too much time on service, but we want them to take it seriously.

Thus, it is reasonable to select for the service, the smallest possible proportion that would still be taken seriously by everyone, no matter whether they divide everything into 5 or into 9 clusters. Thus, we need the smallest number which is larger than all the corresponding thresholds $1/9$, $1/8$, $1/7$, $1/6$, and $1/5$. One can easily see that this smallest non-negligible number is exactly $1/5 = 20\%$, which is exactly how much time is allocated to service.

If we consider research and reaching to be equally important, then the remaining time $1 - 0.2 = 0.8$ should be equally divided between these two activities, into two equal parts of 40% and 40%. So, we indeed get an explanation for the 40-40-20 arrangement.

4 Why 50-30-20 Proportion for Research Universities: Two Explanations

What we want to be explained. In many research universities, the usual proportion is different: 50% for research, 30% for teaching, and 20% for service. How can we explain this arrangement?

First explanation. The main idea behind this arrangement is that a faculty should spend less time on teaching than on research, and less time on service than on teaching. In our notation, this means that we should have $s < t$ and $t < r$.

According to our formalization, this implies that $t = r/2$ and $s = t/2$ (hence $s = r/4$). Thus, the condition that $r + t + s = 1$ implies that

$$r + r/2 + r/4 = (7/4) \cdot r = 1,$$

hence $r = 4/7 \approx 0.57$, $t = r/2 = 2/7 \approx 0.29$, and $s = t/2 = 1/7 \approx 0.14$. The resulting 57-29-14 arrangement is indeed close to 50-30-20.

Second explanation. Let us see what seven plus minus two law implies in this situation. For service, we still want to the smallest non-negligible proportion, i.e., 20%. The difference from the previous case is that instead of allocating equal time to research and teaching, we allocate more time to research.

Teaching is important, so a reasonable idea is to allocate to teaching the largest possible time for which the difference between teaching and research time should be significant to everybody. As we have mention, the smallest non-negligible difference is 20%. So, we have $t + r = 1 - 0.2 = 0.8$ and $r - t = 0.2$. This implies exactly $r = 0.5$, $t = 0.3$, and $s = 0.2$ – exactly the 50-30-20 arrangement.

5 Why Five Letter Grades

What we want to explain. In the US system, number of points is transformed into one of five “letter grades” – A (excellent), B (good), C (satisfactory), D (sometimes passable), and F (fail). Letter grades are usually the only thing that does into the student’s transcript.

In Russia – where two of us are from – we have a different system, but also 5 main grades. Why five?

Comment. At our university, periodically, faculty raise the need to have a more specific scale, with the possibility to have $A-$, $B+$, and other combination of grades. However, every time, a significant proportion of faculty objects, and the motion does not pass.

In Russia, we had such an plus-minus option, we could even have two pluses like $5++$ for a really outstanding performance, and $3--$ for an almost failing one. However, these pluses and minuses did not go into an official transcript and were not taken into account when computing the average grade.

Natural explanation. We want the difference between letter grades to be clearly understood by everyone, irrespective of whether they divide everything into 5, 7, or 9 clusters. This means that we must have no more than 5 grades – otherwise, if we had 6 or more letter grades, the difference between some of these grades would not be clear to those who divided everything into 5 clusters. This explains why we normally use 5 letter grades.

Comment. A similar fact is true for musical scales. Traditionally, many cultures had different scales, some have 5 notes (*pentatonic scales*), the traditional Western scale has 7 notes – which corresponds to the most frequent number of 7 clusters, and practically all the scales have between 5 and 9 notes – in full agreement with the seven plus minus two law.

6 Why Excellent Is Usually Close to 90

Idea. Excellent means that there may be some minor faults in the student's knowledge of the course material, but overall, no one should be able to notice any major fault, irrespective of whether this person divides everything into 5 or 9 clusters.

Resulting explanation. To be un-noticeable to a person who divides everything into c clusters, the proportion d of the course material that the student does not know should be smaller than $1/c$ – the smallest amount seriously recognizable by this person. Thus, excellent knowledge means that the part d that the student does not know should be smaller than all possible values 1/5, 1/6, 1/7, 1/8, and 1/9. This is equivalent to requiring that $d < 1/9$ and that $k = 1 - d > 8/9 \approx 0.89$. This is indeed very close to the usual 90/100 threshold for "excellent" (A).

7 How to Allocate Grades to Tests, Homeworks, etc.

Idea. The overall grade comes form adding grades for different tests, assignments, etc. Let us use the above ideas to decide how many points out of 100 to allocate to each test, to the final exam, to different assignments, etc. We will illustrate this idea on two examples.

First example: a regular undergraduate class. We have three tests (also known as midterm exams), homeworks, and a final exam. Intuitively, we should assign similar number of points $t_1 = t_2 = t_3$ to each of the three tests, and approximately the same number of points to the homeworks $h \approx t_i$, but definitely the final exam is more important, so the number of points f allocated to the final exam should be larger:

$$t_i < f.$$

Similarly to what we did earlier, we interpret $t_i < f$ as $t_i = f/2$, i.e., as $f = 2t_i$. Thus, the fact that the sum of all the points is 100 means that

$$t_1 + t_2 + t_3 + h + f = 4t_i + 2t_i = 6t_i = 100.$$

This implies that $t_1 = t_2 = t_3 = h = 100/6 \approx 17$ and $f = 2 \cdot (100/6) \approx 33$.

It is usually more convenient to use round numbers of points, i.e., numbers divisible by 5. For 17, the closest such value is 15, and for 33, it is 35. However, if we take $t_1 = t_2 = t_3 = h = 15$ and $f = 35$, the overall maximum grade is $4 \cdot 15 + 35 = 95 < 100$. To make it 100, we need to increase one of the allocations by 5. Which one we increase? We want to keep all tests equally important, so we cannot increase one of these allocations, we should increase either h of f. Which one?

- If we increase h from 15 to 20, the difference between the new value 20 and the original value ≈ 17 is ≈ 3.
- If we increase f from 35 to 40, the difference between the new value 40 and the original value ≈ 33 is ≈ 7.

So, the smallest deviation from the original arrangement is when we increase h. Thus, we arrive at the following arrangement – that many of our faculty actually use in such situations:

- each of the three tests is worth 15 points,
- all the homeworks are worth 20 points, and
- the final exam is worth 35 points.

Second example: a regular graduate class. We have three tests, homeworks, a project, and a final exam. This time, all three tests and homeworks are equally important just as in the previous example, a project is more important than any of them, and the final exam is the most important. So, we still have $t_1 = t_2 = t_3 = h$. Since the project is more important, we allocate the number of points to it which is larger than t_i. According to our arrangement, this means $t_i = p/2$, i.e., $p = 2t_i$. Similarly, the condition that $p < f$ leads to $p = f/2$, i.e., to $f = 2p$ and thus, to $f = 4t_i$. The condition that these allocations add up to 100 leads to

$$4t_i + p + f = 4t_i + 2t_i + 4t_i = 10t_i = 100,$$

i.e., to $t_i = 10$. So, $p = 2t_i = 20$ and $f = 4t_i = 40$. Thus, in this case:

- each of the three tests is worth 10 points,

- all the homeworks are worth 10 points,
- the project is worth 20 points, and
- the final exam is worth 40 points.

This is close to the arrangement that we came up with empirically.

What if we have a different number of tests. In the undergraduate case, if we have T tests, then;

- each of the tests is worth $100/(T+3)$ points,
- all the homeworks are worth $100/(T+3)$ points, and
- the final exam is worth $200/(T+3)$ points.

In the graduate case, if we have T tests, then:

- each of the three tests is worth $100/(T+7)$ points,
- all the homeworks are worth $100/(T+7)$ points,
- the project is worth $200/(T+7)$ points, and
- the final exam is worth $400/(T+7)$ points.

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