8-1-2021

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Comments:
Technical Report: UTEP-CS-21-71

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Abstract Usually, we mostly gauge individual students' skills. However, in the modern world, problems are rarely solved by individuals, it is usually a group effort. So, to make sure that students are successful, we also need to gauge their ability to collaborate. In this paper, we describe when it is possible to gauge the students' ability to collaborate; in situations when such a determination is possible, we explain how exactly we can estimate these abilities.

1 Formulation of the Problem

Gauging ability to collaborate is important. In most classes, we test the students' individual knowledge and the individual ability to apply this knowledge. However, in the modern world, most problems are solved by collaboration, not individually. While the need for collaboration seems to have increased, collaboration itself is not a new phenomenon: many historians believe that the ability to successfully collaborate was the main factor that made our species dominant; see, e.g., [1].

So, to gauge the students' readiness to solve real-life problems, it is important to gauge not only their individual abilities, but also their ability to collaborate, to solve the problems in collaboration with others.

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Gauging ability to collaborate is not easy. A natural way to gauge the ability to collaborate is to combine students into groups, and to assigns tasks to these groups. This way, by grading the result, we can gauge the ability of the group to collaborate. The problem is that it is not easy to translate this information into individual grades:

- If a group has been successful, this does not necessarily mean that all members of this group mastered the art of collaboration. So, if we give everyone from a successful group a very good grade, for some students who have not yet mastered this skill very well, the resulting grade will be undeserved.
- Similarly, if a group has not been very successful, this does not necessarily mean that all members of this group deserve a bad grade on collaboration abilities, a few of them may be better – and so for them, the bad grade based on the project as a whole would also be undeserved.

Remaining problem and what we do in this paper. So, a fair estimation of the students' ability to collaborate is still an important challenge. In this paper, we provide a possible way to solve this challenging problem.

2 How Group Productivity Depends on the Ability to Collaborate

What is given. In other to gauge the students' ability to collaborate, it is important to understand how the group's productivity depends on the students' ability to collaborate. For this purpose, let us introduce natural notations.

For each student i, we will denote:

- this student's individual skills by s_i,
- this student's ability to collaborate by c_i, and
- the amount of effort that the student applied by e_i.

Based on this data, we want to describe the productivity p. In other words, we want to come up with a formula that describes productivity of a group of n people as a function of these inputs:

$$p = p(s_1, \ldots, s_n, c_1, \ldots, c_n, e_1, \ldots, e_n).$$

How to come up with a model: main idea and the resulting formula. To come up with a simple model, we will use only the smallest terms in the Taylor expansion which are consistent the commonsense understanding of the situation.

In general, the first terms in the Taylor expansion are linear terms, so, from the purely mathematical viewpoint, it may seem reasonable to use these terms here as well, i.e., to take

$$p = p_0 + \sum_{i=1}^{n} p_{si} \cdot s_i + \sum_{i=1}^{n} p_{ci} \cdot c_i + \sum_{i=1}^{n} p_{ei} \cdot e_i.$$

However, from the commonsense viewpoint, this formula makes no sense.

First, if no one has any skills, individual or collective, there is no productivity. So, when $s_i = c_i = 0$, we should have $p = 0$. This implies that for all possible values of e_i, we should have $p_0 + \sum_{i=1}^{n} p_{ei} \cdot e_i = 0$. This means that $p_0 = 0$ and $p_{ei} = 0$ for all i.

Similarly, if none of the students applies any effort, there will be no productivity. This implies that $p_{si} = p_{ci} = 0$, so all linear terms should be 0s.

From the commonsense viewpoint, the only possibility to get some productivity is:

- either when at least one student has non-zero individual skills s_i and non-zero effort e_i; the simplest term with this property is the product term $e_i \cdot s_i$;
- or at least two students $i \neq j$ have non-zero ability to collaborate and apply non-zero efforts; the simplest term with this property is $e_i \cdot c_i \cdot e_i \cdot c_j$.

Since we decided to limit ourselves to the smallest non-zero terms – which is usually called the first approximation – we thus conclude that the desired expression for p should be a linear combination of terms $e_i \cdot s_i$ and $e_i \cdot c_i \cdot e_j \cdot c_j$, i.e., we should have

$$p = \sum_{i=1}^{n} a_i \cdot e_i \cdot s_i + \sum_{i<j} b_{ij} \cdot e_i \cdot c_i \cdot e_j \cdot c_j, \quad (1)$$

for some coefficients $a_i > 0$ and $b_{ij} > 0$.

A priori, we have no reasons to believe that some student's skills affect the resulting productivity in different ways. Thus, all the coefficients a_i should be equal to each other: $a_1 = \ldots = a_n$. Let us denote the common value of a_i by a. Similarly, all the coefficients b_{ij} corresponding to different pairs (i, j) should be equal to each other. Let us denote their common value by b. Then, the formula (1) takes the following simplified form

$$p = a \cdot \sum_{i=1}^{n} e_i \cdot s_i + b \cdot \sum_{i<j} e_i \cdot c_i \cdot e_j \cdot c_j. \quad (2)$$

Let us simplify this formula. According to the formula (2), the only way the value s_i enters the formula is via the product $e_i \cdot s_i$. There is no way to separate these two quantities – and this makes sense: if a student does not even try, how can we determine whether this student has the skills? So, the only thing that we can observe are not "hidden" skills s_i, but the actually applied skills $\widetilde{s}_i \stackrel{\text{def}}{=} e_i \cdot s_i$. Similarly, we cannot observe the hidden ability to collaborate, we can only observe the product $\widetilde{c}_i \stackrel{\text{def}}{=} e_i \cdot c_i$. In terms of these "actual" variables, the formula (2) takes the following simplified form

$$p = a \cdot \sum_{i=1}^{n} \widetilde{s}_i + b \cdot \sum_{i<j} \widetilde{c}_i \cdot \widetilde{c}_j. \qquad (3)$$

Finally, to make this formula even simpler, we can re-scale the student-characterizing parameters $\widetilde{s}_i$ and $\widetilde{c}_i$ into $S_i \stackrel{\text{def}}{=} a \cdot \widetilde{s}_i$ and $C_i \stackrel{\text{def}}{=} \sqrt{b} \cdot \widetilde{c}_i$. In terms of these re-scaled values, the formula (3) gets the following form:

Final description of our model. The productivity of a group has the form

$$p = \sum_{i=1}^{n} S_i + \sum_{i<j} C_i \cdot C_j, \qquad (5)$$

where:

- the value S_i describes the individual skills of the i-th student, and
- the value C_i describe the ability of the i-th student to collaborate.

What we want. Based on the observed productivity values p corresponding to different groups – including "groups" consisting of only one student – we want to reconstruct the values C_i (and, of course, the values S_i as well).

3 Analysis of the Problem: When We Can Determine the Values C_i (and How) and When We Cannot

Simplest case: two students. Let us start with the simplest case of two students. In this case, we do not have much of a choice:

- we can give both students individual assignments, and thus, by observing the resulting productivity $p_i = S_i$, find their individual skills S_i, and
- we can also give them a joint assignment, and observe the joint productivity

$$p_{12} = S_1 + S_2 + C_1 \cdot C_2.$$

Based on the result of the joint assignment, we get the value $S_1 + S_2 + C_1 \cdot C_2$. Once we know S_1 and S_2, we can therefore determine the product $C_1 \cdot C_2$. However, based only on the product, we cannot determine individual numbers C_1 and C_2.

This impossibility makes perfect mathematical sense: we only have three possible measurement results p_1, p_2, and p_{12}, so we only have three equations for four unknowns S_1, S_2, C_1, and C_2 – not enough to uniquely determine all the desired quantities S_i and C_i.

Next simplest case – three students: analysis. In the case when we have three students:

- we can give all students individual assignments, and thus, by observing the resulting productivity $p_i = S_i$, find their individual skills S_i, and

- we can group them into pairs $\{1,2\}$, $\{2,3\}$, and $\{1,3\}$, and observe the joint productivities

$$p_{12} = S_1 + S_2 + C_1 \cdot C_2,\ p_{23} = S_2 + S_3 + C_2 \cdot C_3, \text{ and } p_{13} = S_1 + S_3 + C_1 \cdot C_3.$$

Based on the results of these assignments, we can find the products

$$P_{12} \stackrel{\text{def}}{=} C_1 \cdot C_2 = p_{12} - p_1 - p_2,$$

$$P_{23} \stackrel{\text{def}}{=} C_2 \cdot C_3 = p_{23} - p_2 - p_3, \text{ and}$$

$$P_{13} \stackrel{\text{def}}{=} C_1 \cdot C_3 = p_{13} - p_1 - p_3.$$

The product $P_{12} \cdot P_{23} \cdot P_{13}$ of all three products is equal to $(C_1 \cdot C_2 \cdot C_3)^2$, thus $C_1 \cdot C_2 \cdot C_3 = \sqrt{P_{12} \cdot P_{23} \cdot P_{13}}$. By dividing this product by the known expression for $C_2 \cdot C_3 = P_{23}$, we conclude that

$$C_1 = \frac{C_1 \cdot C_2 \cdot C_3}{C_2 \cdot C_3} = \frac{\sqrt{P_{12} \cdot P_{23} \cdot P_{13}}}{P_{23}} = \sqrt{\frac{P_{12} \cdot P_{13}}{P_{23}}}.$$

Similarly, we can determine all three values C_i. Thus, we arrive at the following method.

Case of three students: how to determine the values C_i describing the students' ability to collaborate. We give each student an individual assignment, and observe the resulting productivity $p_i = S_i$. This way, we determine the values S_i.

We then give each pair of students a group assignment and thus determine the corresponding group productivities p_{12}, p_{23}, and p_{13}. Based on these values, we compute $P_{ij} = p_{ij} - p_i - p_j$, and then compute

$$C_1 = \sqrt{\frac{P_{12} \cdot P_{13}}{P_{23}}};\ \ C_2 = \sqrt{\frac{P_{12} \cdot P_{23}}{P_{13}}};\ \ C_3 = \sqrt{\frac{P_{13} \cdot P_{23}}{P_{12}}}.$$

Case of three students: possible alternative methods. For each student i, we need to determine 2 values S_i and C_i. So, for 3 students, we need to determine $3 \cdot 2 = 6$ parameters. For this, we need to perform 6 experiments – which is exactly what the above method does.

In addition to these 6 experiments, we could also make a group of all 3 students, so overall, we have 7 possible experiments, corresponding to groups

$$\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \text{ and } \{1,2,3\}.$$

Let us show that any 6 of these experiments enable us to uniquely determine all the desired values S_i and C_i.

Indeed, in the above method, we omitted the $\{1,2,3\}$ experiment. What if we omit one the individual-measuring experiments? Without losing generality, let us assume that we miss experiment $\{1\}$. In this case, we get

$$S_2 = p_2,\ S_3 = p_3, \text{ and } C_2 \cdot C_3 = p_{23} - p_2 - p_3.$$

We also know the values

$$p_{12} = S_1 + S_2 + C_1 \cdot C_2,$$

$$p_{13} = S_1 + S_3 + C_1 \cdot C_3, \text{ and}$$

$$p_{123} = S_1 + S_2 + S_3 + C_1 \cdot C_2 + C_2 \cdot C_3 + C_1 \cdot C_3.$$

In this case,

$$p_{12} + p_{23} + p_{13} = 2 \cdot (S_1 + S_2 + S_3) + C_1 \cdot C_2 + C_2 \cdot C_3 + C_1 \cdot C_3$$

and thus,

$$p_{12} + p_{23} + p_{13} - p_{123} = S_1 + S_2 + S_3.$$

Since we know S_2 and S_3, we can therefore determine S_1 as the difference

$$S_1 = (p_{12} + p_{23} + p_{13} - p_{123}) - p_2 - p_3.$$

Once we know S_1, we can determine all the values C_i as above.

What if we omit one of the paired experiment? Without losing generality, let us assume that we miss experiment $\{2,3\}$. In this case, we have all the values $S_i = p_i$, and we also have

$$p_{12} = S_1 + S_2 + C_1 \cdot C_2,$$

$$p_{13} = S_1 + S_3 + C_1 \cdot C_3, \text{ and}$$

$$p_{123} = S_1 + S_2 + S_3 + C_1 \cdot C_2 + C_2 \cdot C_3 + C_1 \cdot C_3.$$

Thus, we can determine $C_1 \cdot C_2 = p_{12} - p_1 - p_2$, $C_1 \cdot C_3 = p_{13} - p_1 - p_3$, and

$$C_1 \cdot C_2 + C_2 \cdot C_3 + C_1 \cdot C_3 = p_{123} - p_1 - p_2 - p_3.$$

Thus, we can find the remaining value $C_2 \cdot C_3$ as

$$C_2 \cdot C_3 = (C_1 \cdot C_2 + C_2 \cdot C_3 + C_1 \cdot C_3) - C_1 \cdot C_2 - C_1 \cdot C_3 =$$

$$(p_{123} - p_1 - p_2 - p_3) - (p_{12} - p_1 - p_2) - (p_{13} - p_1 - p_3) =$$

$$p_{123} - p_{12} - p_{13} + p_1.$$

Once we know $C_2 \cdot C_3$, we can determine the values C_i as above.

General case. In the general case, we can divide students into groups of 3 and follow one of the above procedures for each triple.

Caution. As we have mentioned, to determine $2n$ unknowns S_i and C_i, we need to have at least $2n$ results – i.e., we need to perform at least $2n$ measurements. It is important to notice that the very fact that we have performed $2n$ measurements does not necessarily mean that we can uniquely determine all $2n$ values.

An important counterexample is when all the groups have the same size k. Let us show that in this case, the unique determination is not possible. Indeed, let us show that in this case, the same observations p_g corresponding to different k-element groups $g \subset \{1,\ldots,n\}$ are consistent not only with the actual values C_i but also with modified values $C'_i = C_i + \delta$. Indeed, for each i and j, we have

$$C'_i \cdot C'_j = (C_i + \delta) \cdot (C_j + \delta) = C_i \cdot C_j + \delta \cdot C_i + \delta \cdot C_j + \delta^2.$$

Thus, if we add up these products for all $(k-1) \cdot k/2$ pairs $i, j \in g$, we get

$$\sum_{i,j \in g,\ i<j} C'_i \cdot C'_j = \sum_{i,j \in g,\ i<j} C_i \cdot C_j + (k-1) \cdot \delta \cdot \sum_{i \in g} C_i + \frac{(k-1) \cdot k}{2} \cdot \delta^2.$$

Thus, we have

$$\sum_{i,j \in g,\ i<j} C'_i \cdot C'_j = \sum_{i,j \in g,\ i<j} C_i \cdot C_j + \sum_{i \in g} \delta_i,$$

where we denoted

$$\delta_i \stackrel{\text{def}}{=} (k-1) \cdot \delta \cdot C_i + \frac{k-1}{2} \cdot \delta^2.$$

Therefore,

$$\sum_{i,j \in g,\ i<j} C_i \cdot C_j = \sum_{i,j \in g,\ i<j} C'_i \cdot C'_j - \sum_{i \in g} \delta_i,$$

and thus, for each k-element group g, we have

$$p_g = \sum_{i \in g} S_i + \sum_{i,j \in g,\ i<j} C_i \cdot C_j = \sum_{i \in g} S_i + \sum_{i,j \in g,\ i<j} C'_i \cdot C'_j - \sum_{i \in g} \delta_i,$$

i.e.,

$$p_g = \sum_{i \in g} S'_i + \sum_{i,j \in g,\ i<j} C'_i \cdot C'_j,$$

where we denoted $S'_i \stackrel{\text{def}}{=} S_i - \delta_i$.

Thus, indeed, the same observations p_g are consistent not only with the actual values S_i and C_i, but also with different values S'_i and $C'_i = C_i + \delta$. Thus, *to uniquely determine the values S_i and C_i, we need to have groups of different sizes.*

Acknowledgments

This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and HRD-1834620 and HRD-2034030 (CAHSI Includes), and by the AT&T Fellowship in Information Technology.

It was also supported by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478, and by a grant from the Hungarian National Research, Development and Innovation Office (NRDI).

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