7-1-2021

Why Quantum Techniques Are a Good First Approximation to Economic Phenomena, and What Next

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Comments:
Technical Report: UTEP-CS-21-68

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Abstract

Somewhat surprisingly, several formulas of quantum physics – the physics of micro-world – provide a good first approximation to many social phenomena, in particular, to many economic phenomena, phenomena which are very far from micro-physics. In this paper, we provide three possible explanations for this surprising fact. First, we show that several formulas from quantum physics actually provide a good first-approximation description for many phenomena in general, not only to the phenomena of micro-physics. Second, we show that some quantum formulas represent the fastest way to compute nonlinear dependencies and thus, naturally appear when we look for easily computable models; in this aspect, there is a very strong similarity between quantum techniques and neural networks. Third, due to numerous practical applications of micro-phenomena, many problems related to quantum equations have been solved; so, when we use quantum techniques to describe social phenomena, we can utilize the numerous existing solutions – which would not have been the case if we use other nonlinear techniques for which not many solutions are known.

All this provides an explanation of why quantum techniques work reasonably well in economics. However, of course, economics is different from quantum world, quantum equations only provide a first approximation to economic situations. In this paper, we use the ideas behind our explanations to speculate on what should be the next – not-exactly-quantum – approximation to social and economic phenomena.

1 Formulation of the Problem

1.1 In general, different levels are described by different equations

Many processes in our world occur at different scales:

- cosmological processes describe what is happening on the level of Universe as a whole,
- astrophysics describes what is happening on the level of Galaxies and stars,
- Earth sciences describe what is happening on the level of a planet,
- macrophysics, biology, and social sciences describes what is happening to our-size ("macro") level, and
- finally, microphysics – mostly quantum physics – describes what is happening on the level of molecules, atoms, and elementary particles.

Of course, there are some similarities between different levels – after all, all theses processes obey general laws of physics (see, e.g., [12, 31]) – but still, the differences between different levels are usually much larger than these similarities. As a result, different level usually use different techniques, different methodologies.

When people naively try to apply equations and ideas from one level to other levels, they rarely succeed. Naive 19 century attempts to describe electrons orbiting nuclei in a similar way as planets orbiting Earth immediately led to paradoxical conclusions – e.g., that, due to tidal forces (enhanced by electric charges), all electrons will fall on their nuclei after a few seconds. So, quantum physics was invented to avoid this paradox.

Similarly, attempts to naively apply Newtonian physics to the world as a whole lead to paradoxes – e.g., if we assume that the stars are uniformly distributed in the Universe, then the overall intensity of light from the stars would be very large, and at night, it would be as light as in daytime. So, General Relativity was invented to avoid such paradoxes.

Even for different aspects of the same level, naive transitions rarely work: e.g., while Darwinism is a great way to describe evolution of species, attempts of Social Darwinism to explain social behavior the same way were not very successful.

1.2 But there is an important exception

Interestingly, there is an important case when, unexpectedly, equations that describe phenomena on one level are strangely successful in a completely different level: this is the case of *quantum economics*, a successful application of quantum physics to the description of economic (and, more generally, social) phenomena; see, e.g., see, e.g., [1, 3, 4, 5, 6, 7, 15, 16, 17, 18, 24, 25, 26, 28, 29] (see also [22, 20, 30]).

This success is even more puzzling if we take into account that – as we have mentioned earlier – attempts to use even closer-in-level phenomena like biological was much less successful.

1.3 We need a quantitative explanation for this important exception

It is easy to come up with *qualitative* explanations for the success of quantum methods in social studies. For example, when we study a social phenomenon, this study often changes the phenomenon itself: e.g., it is known that the very fact that a patient is visited by a doctor and has some tests done already makes many patients feel better. Of all other phenomena, only quantum processes have the same feature – that a measurement changes the state. So, it is reasonable to expect some analogies between social and quantum phenomena.

However, it is not just qualitative quantum ideas like this which are successful in studying social phenomena, it is also quantitative quantum equations which are successful. Why quantum methods are quantitatively successful in describing economic phenomena is a challenge. In this paper, we provide several explanations for this seemingly strange success – explanations that will hopefully make this success less puzzling.

2 Where Can Such an Explanation Come from: General Analysis

In general, we can distinguish between three aspects of a physical theory:

- First, there is a mathematical aspect: equations that the real-world phenomena satisfy. For Newtonian physics, there are Newton's equations. For quantum physics, there are Schrödinger's equations. For General Relativity, there are Einstein's equations.

- However, equations is not all. To be useful, we must have techniques for solving these equations – and coming up with such techniques is usually as difficult (or even more difficult) than coming up with equations themselves. Newton would not have been very famous if all he did was write a system of differential equations describing how the planets move – and then wait several centuries until computers would appear that could solve this system. Einstein would not have been that famous if all he did was write down a system of complex partial differential equations describing space-time geometry, with no clue on how to solve them and how to compare his predictions with observations. This actually almost happened: David Hilbert, the leading mathematician of his time, independently discovered the same equations – and submitted his paper two weeks after Einstein. If he submitted it two weeks before – would we value Einstein's contribution at all? Actually, yes: all Hilbert did was came us with equations, while Einstein also proposed some solutions – and a way to experimentally test these equations, which in a few years led to a spectacular success.

- Finally, for the theory to become widely used, it is not enough to just have techniques for solving the corresponding equations – this would have

limited this theory's use to academe where we have enough researchers and graduate students to apply these techniques and defend their theses and dissertations. We cannot hire a PhD student for every single practical problem. To be practically useful, we need to have a large corpus of already solved problems that practitioners can use. For example, all cell phones take relativistic effects into account when dealing with GPS signals – but the cell phone company does not have to hire a physicist every time a new model of a cell phone is designed – they can use known solutions.

This applies to all physical theories. This applies, in particular, to quantum physics – there are equations, there are techniques for solving these equations, and there are numerous solutions of these equations. We will show that each of the three aspects of quantum physics provides some explanation of why quantum equations can be successfully applied to social phenomena – and, taken together, all three explanations form a reasonably convincing case. So let us consider these aspects one by one.

3 First Explanation: Quantum Formulas Provide a Good Description for Many Phenomena in General

First, let us consider the mathematical aspect of quantum physics – the corresponding mathematical equations. We will show that the mathematical formulas of quantum physics provide a good approximate description for many phenomena – not only phenomena from micro-world and from economics.

3.1 Towards a general description of real-life phenomena

In most real-life situations, we have many objects of similar type:

- a galaxy consists of many stars,
- a species consists of many individuals,
- a macro-object consists of many molecules,
- a country or a firm is formed by many people, etc.

Each of these objects is characterized by the values of several quantities. The more quantities we study, the more accurate picture of this object we get.

The number of objects is usually very large, so it is not realistic to keep track of all these objects. A more realistic idea is to keep track of the corresponding distributions:

- what is the proportion of stars of given brightness,
- what is the proportion of employees whose salary is within a given range, etc.

Most practical situations are complex, each quantity is determined by many independent factors. For example, in a big multi-national corporation, a person's salary:

- depends on the person's skills,
- depends on the number of years with the company,
- depends on the geographic location – employees located in more expensive-to-live areas usually get higher salary – etc.

It is known that the distribution of a joint effect of a large number independent factors is close to Gaussian. This follows from the Central Limit Theorem, according to which, when the number of relatively small independent random variables increases, the distribution of their sum tends to Gaussian; see, e.g., [27]. So, we can conclude that the joint distribution of quantities $v_1, \ldots, v_n$ characterizing individual objects is (close to) Gaussian.

3.2 Need for an approximation

In general, a multi-D Gaussian distribution is uniquely determined by its first two moments, i.e., by its means $m_i \stackrel{\text{def}}{=} E[v_i]$ and by its covariance matrix $C_{ij} \stackrel{\text{def}}{=} E[(v_i - m_i) \cdot (v_j - m_j)]$. So, to describe the distribution, we need to know n values of the means and $\dfrac{n \cdot (n+1)}{2}$ values of the symmetric matrix C_{ij}, the total of $V = n + \dfrac{n \cdot (n+1)}{2}$ parameters. These values need to be determined experimentally, and herein lies a problem.

In general, according to statistics, based on N observations, we can estimate the value of a parameter with relative accuracy $\varepsilon \approx 1/\sqrt{N}$. So, to find the value of a parameter with given relative accuracy $\varepsilon > 0$, we need to perform $N(\varepsilon) \approx \varepsilon^{-2}$ observations. To find the values of V parameters, we therefore need to perform $V \cdot N(\varepsilon) \approx V \cdot \varepsilon^{-2}$. For large n, this becomes too large – e.g., if we are interested in comparing countries, and we want to characterize even $n = 3$ quantities with accuracy $\varepsilon \approx 20\%$, then we need a sample of 225 countries – and there are not so many countries in the world.

To be more precise, means m_i are not a problem, we can determine them, the problem is to determine the elements of the covariance matrix.

This simple argument shows that often, we cannot experimentally determine the actual Gaussian distribution – which depends on too many parameters. We therefore need to find a lower-parametric family of distributions that we will use for an approximate description of the phenomena of interest.

3.3 How can we find such an approximation?

How can we find a natural, intuitively clear approximation? Most of us do not have a good intuition about probability distributions, but we do have a good intuition about geometric descriptions. Good news is that there is a natural

geometric description of a multi-D Gaussian distribution. Namely, it is known that we can represent the components $\Delta v_i = v_i - m_i$ of a multi-D Gaussian distribution with 0 means ($E[\Delta v_i] = 0$) as linear combinations of standard independent Gaussian random variables $\xi_1, \ldots, \xi_n$ for which $E[\xi_k] = 0$, $E[\xi_k^2] = 1$, and $E[\xi_k \cdot \xi_\ell] = 0$ for all $k \neq \ell$: $\Delta v_i = \sum\limits_{k=1}^{n} v_{ik} \cdot \xi_k$. In this representation, the covariance $E[\Delta v_i \cdot \Delta v_j]$ takes the form $E[\Delta v_i \cdot \Delta v_j] = \sum\limits_{k=1}^{n} v_{ik} \cdot v_{jk}$.

This is exactly the formula for the dot (scalar) product of the two n-dimensional vectors. So, we conclude that each difference each difference Δv_i is represented by an n-dimensional vector $\vec{v}_i = (v_{i1}, \ldots, v_{in})$, and the covariance is equal to the dot products of these vectors: $E[\Delta v_i \cdot \Delta v_j] = \vec{v}_i \cdot \vec{v}_j$. In particular, the variance $V[\Delta v_i] \stackrel{\text{def}}{=} E[(\Delta v_i)^2]$ has the form $V[\Delta v_i] = (\vec{v}_i)^2 = \|\vec{v}_i\|^2$, where $\|\vec{a}\|$ denotes the length of the vector $\vec{a}$. This provides an exact n-dimensional representation of the situation.

As we have mentioned, we often do not have enough experimental data to determine this exact n-dimensional representation. So, a natural idea is to have a lower-dimensional approximation. This is indeed natural: for example, when we do not have enough data to find a full 3D picture of some object, we often have enough data to determine its 2-D projection. In other words, instead of the original (ideal) multi-D vectors $\vec{v}_i$, we use lower-dimensional approximate vectors $\vec{V}_i = (V_{i1}, \ldots, V_{id})$, for $d \ll n$, and we try to find the vectors $\vec{V}_i$ so that the corresponding dot products are close to the ideal ones: $\vec{V}_i \cdot \vec{V}_j \approx \vec{v}_i \cdot \vec{v}_j$.

3.4 The simplest such approximation leads, in effect, to a quantum description

The most intuitively clear pictures are 2-D ones, with $d = 2$. In this case, each quantity v_j is represented as by a 2-D vector $\vec{V}_i = (V_{i1}, V_{i2})$, and its variance is approximately equal to $\|\vec{V}_i\|^2 = V_{i1}^2 + V_{i2}^2$.

How is all this related to complex numbers – one of the main techniques of quantum physics? The relation is straightforward: there is a natural geometric representation of complex numbers, where each number $a + b \cdot \mathrm{i}$ (and $\mathrm{i} \stackrel{\text{def}}{=} \sqrt{-1}$) is represented by a point (a, b) on a plane. In this representation, the absolute value $|a + b \cdot \mathrm{i}|$ of the complex number is equal to the length $\|(a, b)\| = \sqrt{a^2 + b^2}$ of the corresponding vector (a, b).

Thus, each vector $\vec{V}_i = (V_{i1}, V_{i2})$ characterising a quantity v_i can be naturally represented as a complex number $V_{i1} + V_{i2} \cdot \mathrm{i}$, and the standard deviation of the quantity v_i is equal to the absolute value of this complex number. So, in this approximation, indeed we naturally get a quantum-like description of general objects.

Comment. This idea was first described in [30]; see also [13].

4 Second Explanation: Quantum Formulas Are the Computationally Fastest Way to Describe Nonlinear Phenomena

Now let us concentrate on the computational, algorithmic aspect of quantum physics – and why namely the computational aspects of quantum physics (and not of any other area of physics) turn out to be helpful in describing social phenomena.

To provide this explanation, let us consider the computational aspects of physics from the most general viewpoint. From the purely theoretical viewpoint, once we have an equation, we can find its solution – worse comes to worse, we can try all possible values, all possible combinations of values, etc. Of course, on the abstract level, there are infinitely many possible values of each physical quantity, so it is not possible to try all infinitely many values – but in practice, since measurements are never absolutely accurate, we cannot distinguish between close values. For example, if our measuring instrument only works on the range from 0 to 10, and measures the value of the corresponding quantity with accuracy 0.1, then we can, in effect, only have values 0, 0.1, 0.2, ..., 9.9, 10 – any other value will be indistinguishable from one of these.

Of course, trying all possible combinations is not a practically feasible approach – for many unknowns, it can take an astronomical amount of time, up to the time larger than the lifetime of the Universe. So, from the practical viewpoint, the question is not to find *an* algorithm, the problem is to find a *reasonably fast* algorithm. This is a big challenge. For example, we have learned to predict tomorrow's weather reasonably well – it takes several hours on a high-performance computer. Almost the same algorithms can, in principle, predict where a devastating tornado will turn in the next 15 minutes – but, unfortunately, this prediction would also take several hours, which makes it useless. From this viewpoint, an important question is how to perform computations as fast as possible.

Computations consist of several elementary steps, steps on which we compute some elementary functions. For example, current computers use min, max, addition, and multiplication as such steps, everything else – whether it is computing the inverse $1/a$ or computing the value of the sine function – is implemented as a sequence of these operations.

So, to make computations faster, it is important to select elementary computational steps – i.e., corresponding functions – which are the fastest to compute. Which functions are the fastest to compute? The first idea is that functions are either linear or non-linear. Of course, linear functions are faster to compute than nonlinear ones, so we should use computing linear functions as elementary computational operations.

We cannot limit ourselves to linear functions only: if we only perform linear combinations, then, since the composition of linear function is linear, we will only get linear functions – but many real-life processes are nonlinear. So, some nonlinear elementary operations are needed – but, since we want to speed up

overall computations, we should have as few non-linear elements as possible.

So, we end up with a computational scheme in which most operations are linear, but sometimes some nonlinear operations are needed. Which non-linear operations should we use?

To predict the results of deterministic processes, a natural idea is to use some nonlinear functions. So, we end up with a computation scheme in which we interchangingly apply linear transformations and some simple nonlinear ones. This is exactly what neural networks are doing (see, e.g., [14]): at each layer, we:

- first form a linear combination $s = w_0 + w_1 \cdot s_1 + \ldots + w_n \cdot s_n$ of signals coming from the previous layer, and
- then, we apply some nonlinear function $F(x)$ (known as *activation function*) to this result s, returning the value $F(s)$.

Many processes, however, are nondeterministic, in the sense that, based on the available information, we can only predict the results with some probabilities. In this case, it makes sense to have non-deterministic nonlinear components, that return different results with different probabilities. This is exactly what is happening according to the quantum physics, where interchanhingly, we have

- linear transformations – which correspond to normal dynamics as described by Schrödinger's equation – and
- measurement process – in which we get different results with different probabilities.

From this viewpoint, a quantum-type description is a natural way to describe nondeterministic phenomena – and social phenomena are, of course, largely nondeterministic – in the most computationally efficient way.

So, from the computational viewpoint, we also naturally arrive at a quantum-type description of social phenomena.

Comments.

- The main idea behind this explanation was first described in [19].
- In this section, we explained why some probabilistic quantum-like transformations are needed. In principle, we can have different probabilistic transformations. In [2], we explain why the transformations used in quantum physics are the most appropriate in this case.

5 Third Explanation: Quantum Physics Has Many Solved Problems

Finally, let us take into account that in quantum physics, we have a large corpus of solved complex problems.

Why is this important? It is known that all classes of sufficiently complex problems (they are known as *NP-hard*) can be reduced to each other; see, e.g., [21, 22, 23]. Thus, once we have come up with an efficient method of solving complex problems from one application area, we can reduce problems from other areas to these problems and thus, get a good algorithm for solving problems from other areas as well.

So, a natural way to solve complex problems in economics and finance is to reduce them to complex problems in other application areas – problems for which solutions are mostly known. One such area is physics – analysis of the physical world. Its equations are often very complex, and still, during several centuries of physics, researchers have found reasonably efficient algorithms for solving many of these equations. Thus, to solve complex problems in economics and finance, it makes sense to reduce them to solvable complex problems from physics.

It is known that to get an adequate description of a physical phenomenon, it is necessary to take quantum effects into account, i.e., to get into the domain of quantum physics; see, e.g., [12, 31]. Because of this, the most complex physical equations are equations of quantum physics. Thus, a natural idea is to reduce complex equations arising in economics and finance to complex quantum equations – namely, to complex quantum equations for solving which we have efficient algorithms. Then, by solving the corresponding quantum equation and translating the solutions back into the language of economics and finance, we can get efficient algorithms for solving complex economic and financial problems.

This is exactly what quantum econometrics is doing! So, we have yet another natural explanation of the empirical success of quantum techniques in economics and in social sciences in general.

Comment. This idea was first described in [22]; see also [20].

6 Beyond Quantum

In the previous sections, we explained why quantum techniques work reasonably well in economics. Of course, economics is different from quantum world. So quantum equations provide only a first approximation to economic situations. A natural question is: how can we get an even more accurate approximation – i.e., how can we modify quantum equations so that they will provide an even more adequate description of social phenomena?

In this section, we show that, based on the above explanations, we can come with some recommendations about these modifications. Indeed, in Section 3, we show that a quantum-style complex-valued description of different objects naturally appears if, as an approximation of the general geometric description – with n-dimensional vectors – we consider 2-D vectors. Of course, this is just an approximation. To make this approximation more accurate, the next natural way is to approximate the objects by 3-D, 4-D, etc. vectors.

We hope that this natural generalization of quantum-style techniques will, in particular, lead to a more accurate representation of social and economic

phenomena.

Acknowledgments

This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and HRD-1834620 and HRD-2034030 (CAHSI Includes), and by the AT&T Fellowship in Information Technology. It was also supported by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478.

The authors are greatly thankful to Dr. Emmanuel Haven for his encouragement.

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