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Is It Fair That Advanced Workers Get Paid Disproportionally More: Economic Analysis

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Abstract

On the one hand, everyone agrees that economics should be fair, that workers should get equal pay for equal work. Any instance of unfairness causes a strong disagreement. On the other hand, in many companies, advanced workers – who produce more than others – get paid dispropotionally more for their work, and this does not seem to cause any negative feelings. In this paper, we analyze this situation from the economic viewpoint. We show that from this viewpoint, additional payments for advanced workers indeed make economic sense, benefit everyone, and thus – in contrast to the naive literal interpretation of fairness – are not unfair. As a consequence of our analysis, we also explain why the labor share of the companies' income is, on average, close to 50% – an empirical fact that, to the best of our knowledge, was never previously explained.

1 Formulation of the problem

1.1 Fairness is important

We all want fairness. One of the important aspects of fairness is that all the workers should get equal pay for equal labor.

In the past, this was not the case in many countries: e.g., in the 19 century, in the US and in other industrialized countries, women were routinely paid less for the same result.

Good news is that, as time goes, more and more companies are providing such equal pay, more or more countries have legislation in place that prohibit discrimination of all kinds and require equal pay for equal work.

1.2 Disproportional payment for advanced workers – case of seeming unfairness

However, there is an aspect of the current pay system that seem to contradict this fairness idea: the fact that in many companies, the best workers get additional bonuses and other rewards – and in this sense, get paid disproportionally more.

Let us explain this on a simple example. Supposed that a company provides a 10% annual bonus to the best performer. Suppose that the two best workers perform at almost the same level, with a difference of 1%. The first worker's productivity was only 1% higher, but this worker gets the 10% bonus in addition to his/her pay – while naively understood fairness would mean that this worker be paid only 1% more.

1.3 But is this really unfair?

From the purely mathematical viewpoint, if we view the idea of equal pay for equal labor literally, this arrangement – when advanced workers get paid disproportinally more – is clearly not fair. However, in contrast to other cases of unfairness, no one seems to protest against this practice as an unfair one. But why?

In this paper, we analyze this situation from the economic viewpoint. We explain why the naive approach to fairness is not applicable to this situation, and why disporportionally higher pay to advanced workers makes perfect economic sense.

2 Analysis of the problem and the resulting explanation

2.1 Let us formulate the problem in precise terms

Suppose that the workers are paid amount s for each produced unit, and that, on average, the workers produce x_0 units a certain period of time: day, week, month, whatever. Then, the average worker's salary during this time period is $s \cdot x_0$. How can we encourage the workers to be more productive?

According to decision theory, when a rational person makes a decision, he/she maximizes quantity called *utility*; see, e.g., [1, 5, 6, 7, 8]. In these terms, the incentive works if the positive utility provided by this incentive is larger than the decrease in utility caused by the need to spend time and effort on producing additional units.

Let us denote by e the decrease in a person's utility caused by producing one unit. Then, when the worker produces a additional units, the time and effort needed for producing this unit decrease his/her utility by the amount $a \cdot e$.

We need to compensate this loss of utility by an additional monetary reward m. It is known that the utility $u(M)$ caused by the overall monetary gain M is proportional to the square root of the monetary amount: $u(M) = c \cdot \sqrt{M}$; see, e.g., [2] and references therein (see also [3]). This formula may sound strange at first glance, but it is actually in good accordance with common sense. Indeed, according to this empirical formula, the increase

$$u(M+1) - u(M) = c \cdot \sqrt{M+1} - c \cdot \sqrt{M}$$

in utility caused by an extra dollar decreases with M. This makes perfect sense:

- if a person has no money, getting a dollar is a big deal, but
- if a person already earned \$1000, having an extra dollar is practically unnoticeable.

The additional reward m increases the worker's salary from the original value $s \cdot x_0$ to the new value $s \cdot x_0 + m$. So, the additional reward m increases the worker's utility from the original value $c \cdot \sqrt{s \cdot x_0}$ to the new amount of $c \cdot \sqrt{s \cdot x_0 + m}$. We need to make sure that the resulting increase in utility

$$c \cdot \sqrt{s \cdot x_0 + m} - c \cdot \sqrt{s \cdot x_0}$$

is greater than or equal that the decrease in utility $a \cdot e$. In this case, the overall balance for the worker will be positive, and the worker will be incentivized to be more productive.

2.2 What if we provide equal pay for equal work

If the follow the idea of equal pay for equal work and pay the same amount s for each additional unit, then the additional pay should be equal to $m = s \cdot a$. In this case, the above condition takes the form

$$c \cdot \sqrt{s \cdot (x_0 + a)} - \sqrt{s \cdot x_0} \geq a \cdot e. \tag{1}$$

This condition is equivalent to

$$c \cdot \sqrt{s \cdot (x_0 + a)} \geq c \cdot \sqrt{s \cdot x_0} + a \cdot e. \tag{2}$$

By squaring both side of this inequality, we get an equivalent inequality

$$c^2 \cdot s \cdot (x_0 + a) \geq c^2 \cdot s \cdot x_0 + 2a \cdot e \cdot c \cdot \sqrt{s \cdot x_0} + a^2 \cdot e^2. \tag{3}$$

If we open parentheses, subtract $c^2 \cdot s \cdot x_0$ from both sides, and move the term $2a \cdot e \cdot c \cdot \sqrt{s \cdot x_0}$ to the left-hand side, we conclude that:

$$(c^2 \cdot s - 2e \cdot c \cdot \sqrt{s \cdot x_0}\,) \cdot a \geq a^2 \cdot e^2. \tag{4}$$

Dividing both sides by $a \cdot e^2$, we conclude that

$$a \leq \frac{c^2 \cdot s - 2e \cdot c \cdot \sqrt{s \cdot x_0}}{e^2}. \tag{5}$$

So, if we literally offer equal pay for equal work, we get a limitation on the amount of extra work that the workers will be willing to do under this incentive.

2.3 How can we incentivize workers to be more productive

In view of the above analysis, the only way to incentivize workers to be even more productive is to offer them a disproportionally larger pay for extra work. This is exactly what happens when advanced workers get additional bonuses. From this viewpoint, the disproportional pay to advanced workers makes perfect economic sense and is, thus, perfectly fair:

- the whole society benefits from the increased productivity, and
- this disproportionally larger pay to advanced workers is the only way to increase productivity further.

This is also what happens when the company sometimes wants the workers to work overtime: they are paid disproportionally more per unit (or, alternatively, per hour). In the US, the economic aspect of this extra pay for overtime is not evident, since the larger payment for overtime is required by the labor laws, but our arguments show that it also makes perfect economic sense – and indeed, many companies had the same practice before it was required by law.

3 Further economic analysis leafs to a (somewhat unexpected) additional consequence

3.1 How can we determine the salary: formulation of the problem

In the above analysis, we assumed that the pay-per-unit amount s is fixed. A natural question is: how can we determine this amount?

A natural idea is to select the value s for which the overall positive effect on the company is the best. What does "the best" mean?

- If it is a private company, it needs to maximize its profit. Each produced unit can be sold for p monetary units. If overall, the company produces x units, then its profit P can be obtained if we subtract, from the overall revenue $p \cdot x$, the salary of the workers $s \cdot x$ and additional expenses $A \cdot x$ – which are also proportional to the overall production.
- If it is a state-owned company, then its goal is to maximize the benefit for the society as a whole. If the benefit from each unit is p, then to get the resulting overall gain in benefit, we also need to subtract from the overall benefit $p \cdot x$ the salary $s \cdot x$ and the additional expenses $A \cdot x$.

In both cases, we need to select the pay-per-unit s so as to maximize the overall benefit

$$(p - s - A) \cdot x. \tag{6}$$

In this formula, the values p and A are known, but the overall production x depends on the pay-per-unit s:

- the more we pay per unit,
- the more incentivized will the workers be, and
- the larger will be their production.

So, to find the optimal pay-per-unit s, let us analyze how the overall production depends on the pay-per-unit.

3.2 How production depends on the pay-per-unit: analysis

The overall production x is the sum of the amounts x_i produced by individual workers: $x = \sum x_i$. In line with the above-described general idea, each worker selects the production level x_i that maximizes his/her overall utility, i.e. – by using notations from the previous section – the level x_i that maximizes the difference

$$c_i \cdot \sqrt{s \cdot x_i} - e_i \cdot x_i. \tag{7}$$

In this formula, we took into account that both the value of money (as described by the coefficient c) and the effort needed to produce one unit (as described by the value e) are, in general, different for different workers. So, we denoted the values of c and e corresponding to the i-th worker by, correspondingly, c_i, and e_i.

To find the value x_i that maximizes the expression (7), we can differentiate this expression with respect to x_i and equate the derivative to 0. As a result, we get the equation

$$\frac{1}{2} \cdot \sqrt{\frac{s}{x_i}} - e_i = 0, \tag{8}$$

hence

$$x_i = \frac{1}{4} \cdot \frac{c_i^2}{e_i^2} \cdot s. \tag{9}$$

Thus, the overall production has the form

$$x = \sum_i x_i = \left(\frac{1}{4} \cdot \sum_i \frac{c_i^2}{e_i^2} \right) \cdot s, \qquad (10$$

i.e., the form

$$x = K \cdot s, \tag{11}$$

where we denoted

$$K \stackrel{\text{def}}{=} \frac{1}{4} \cdot \sum_i \frac{c_i^2}{e_i^2}. \tag{12}$$

3.3 So what is the optimal pay-per-unit?

Substituting the expression (11) for the dependence of production x on the pay-per-unit s into the formula (6) that describes the overall benefit (that we want to maximize), we end up with the following expression for the overall benefit:

$$(p - A) \cdot K \cdot s - K \cdot s^2. \tag{13}$$

To find the pay-per-unit s for which this benefit is the largest, we differentiate this expression with respect to s and equate the derivative to 0. As a result, we get

$$(p - A) \cdot K - 2K \cdot s = 0,$$

i.e.,

$$s = \frac{1}{2} \cdot (p - A). \tag{14}$$

3.4 This is exactly what is happening

The formula (14) describes the optimal value. How does this formula compare with the economic reality? As we will show, it compares perfectly.

Indeed, if we multiply both sides of the formula (14) by the overall production size x, we conclude that

$$s \cdot x = \frac{1}{2} \cdot (p \cdot x - A \cdot x). \tag{15}$$

In other words, the overall salary $s \cdot x$ of all the workers is equal exactly to one half of the overall difference between the revenue $p \cdot x$ and the needed expenses $A \cdot x$ – i.e., exactly to one half of the company's income.

In other words, in this arrangement, the labor share of the company's income should be equal to (or at least close to) 50%. And this is exactly what is observed in advanced economics; see, e.g., [4]. This empirical confirmation shows that our approximate model (of course, all economic models are approximate) adequately captures the phenomenon.

This empirical confirmation should be expected: we deduced the 50% value exactly by taking into account that companies want to maximize want to optimize their overall profit.

It should be mentioned that, to the best of our knowledge, ours is the first explanation of this empirical proportion.

3.5 This empirical confirmation strengthens our arguments about advanced workers' pay

This perfect fit with empirical data makes us confident that our explanation of disproportional pay for advanced workers – which is based on exactly the same model – is correct.

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