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Godel's Proof of Existence of God Revisited

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Abstract In his unpublished paper, the famous logician Kurt Gödel provided arguments in favor of the existence of God. These arguments are presented in a very formal way, which makes them difficult to understand to many interested readers. In this paper, we describe a simplifying modification of Gödel's proof which will hopefully make it easier to understand. We also describe, in clear terms, why Gödel's arguments are just that – arguments – and not a convincing proof.

1 Formulation of the Problem

What Gödel did. In his originally unpublished paper, the famous logician Kurt Gödel provides arguments in favor of the existence of an object that can be interpreted as God; see [5], see also [1, 2, 3, 4, 6, 7, 8, 9].

Problems with the original Gödel's proof. Gödel's proof is somewhat overcomplicated and, as a result, somewhat difficult to understand. It is therefore desirable to come up with a simplified version of this proof.

The fact that this proof is presented in a complicated way also makes it difficult to understand whether Gödel's arguments are simply arguments or a convincing proof.

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What we do. In this paper, we provide a modified (namely, simplified) version of Gödel's proof. This simplification, hopefully, makes it easier to understand the proof itself – and also to understand why this is not a fully convincing proof.

2 Intuitive Idea

Idea. Intuitively, God is an object that has all good properties and no bad properties.

We need to formalize this idea. Let us show how to formalize this idea.

3 Towards Formalizing This Idea

Possible worlds. Our knowledge about the world is incomplete. This means that we usually do not have full information about the world. Even if we have a reasonably full information about the current state of the world, we may not be sure about its future state. Thus, there are several possible descriptions of the world which are all consistent with our knowledge. Such descriptions are usually called *possible worlds*.

If a statement S is true in all possible worlds, we say that it is *necessarily true* and denote it by $\Box S$. If a statement holds in at least one of the possible worlds, then we say that this statement is *possibly true* and denote it by $\Diamond S$.

Good and bad properties. In each possible world, there are objects x, x', etc. that may have different properties φ, ψ, etc. Some properties are good; we will denote this by $g(\varphi)$. Other properties are bad; we will denote this by $b(\varphi)$.

It is reasonable to assume that goodness and badness are absolute – if a property is good in one world, it is good in every world – same for bad properties.

Intuitively, if a property is good, then this property cannot be bad, and its negation cannot be good:

$$g(\varphi) \to \neg b(\varphi) \text{ and } g(\varphi) \to \neg g(\neg\varphi). \quad (1)$$

Similarly, if a property is bad, then this property cannot be good, and its negation cannot be bad:

$$b(\varphi) \to \neg g(\varphi) \text{ and } b(\varphi) \to \neg b(\neg\varphi). \quad (2)$$

Formal implication vs. meaningful implication. An important part of our knowledge are if-then statements – knows as *implications*.

In mathematics, a statement "if A then B" if denoted by $A \to B$. The general meaning of such a statement in mathematics is that if A is true, then B is true too. If A is false, then the implication has no limitation on B, so the statement $A \to B$ is true. If A is true, then B should be true. Thus, $A \to B$ means that either A is false or B is true.

This sounds reasonable at first glance, but it leads to meaningless implications. For example, if it will not rain tomorrow in El Paso and a volcano Erebus in Antarctica will be active, then the implication "if it will rain tomorrow in El Paso, then Erebus will be inactive" is, in mathematical sense, true.

While this implication is mathematically true, from the commonsense viewpoint, it is meaningless. Indeed, intuitively, "if A then B" means that if we make A true, then B also becomes true. However, if we force rain to fall in El Paso – e.g., by seeding the clouds – if will not affect the Erebus volcano.

An intuitive meaning of a natural-language if-then statement is that once we make A true, B will always be true, i.e., that the implication $A \to B$ should be true in *all* possible worlds – and not just in our world as in the usual mathematical definition. This leads to the following formula:

$$\varphi \Rightarrow \psi \stackrel{\text{def}}{=} \Box(\forall x(\varphi(x) \to \psi(x))). \tag{3}$$

If this formula holds, we will say that φ *necessarily implies* ψ.

Relation between good, bad, and meaningful implication. Intuitively, if φ is a good property, and φ necessarily implies ψ, then the property ψ should also be good:

$$(g(\varphi)\,\&\,(\varphi \Rightarrow \psi)) \to g(\psi). \tag{4}$$

Similarly, if φ is a bad property, and φ necessarily implies ψ, then the property ψ should also be bad:

$$(b(\varphi)\,\&\,(\varphi \Rightarrow \psi)) \to b(\psi). \tag{5}$$

4 First (Preliminary) Result

Formulation of the result. The first result – proven by Gödel – is that for every good property φ, in some possible world, there is an object that satisfies this property.

Proof. Indeed, let us assume that this statement is not true. This means that in each world, for each object x, the statement $\varphi(x)$ is false. By definition of the usual implication, this means, in particular, that in every world, for every object x, we have $\varphi(x) \to \neg\varphi(x)$. By definition of necessary implication, this means that $\varphi \Rightarrow \neg\varphi$. Since the property φ is good, by formula (4), it implies that its negation $\neg\varphi$ is also good – but, according to formula (1), if a property is good, is negation cannot be good.

This contradiction shows that our assumption cannot be true. Thus, there must exist a possible world in which some object x satisfies the property φ.

5 It Is Good to Have Good Objects in All Possible Worlds

Here we are modifying (and simplifying) Gödel's proof. Up to now, we were following Gödel, but now, we will modify and simply his arguments.

Idea. It would be nice to have objects satisfying a good property in all possible worlds.

How to formalize this idea. The above condition can be described as follows:

$$c(x) \stackrel{\text{def}}{=} \forall\varphi((g(\varphi)\,\&\,\Diamond\exists y\varphi(y)) \to (\Box\exists z\,\varphi(z)). \tag{6}$$

Comment. The property $c(x)$ actually does not depend on x, the variable x is added solely for mathematical convenience.

This property is clearly good. The condition (6) is clearly good:

$$g(c). \tag{7}$$

6 Second Result

Formulation of the result. The second result is that for every good property φ, in *every* possible world, there is an object that satisfies this property.

Proof. Indeed, since the property c is good, according to the first result, it is true in *some* possible world. Thus, in some world, we have an implication

$$\forall\varphi((g(\varphi)\,\&\,\Diamond\exists y\varphi(y)) \to \Box\exists z\,\varphi(z)). \tag{8}$$

This implication does not depend on the world, thus it is just true.

According to the same first result, for every good property φ, there exists a world in which this property is true for some object, thus $\Diamond\exists y\varphi(y)$. Thus, due to the implication (8), we conclude that $\Box\exists z\,\varphi(z)$, i.e., that the object satisfying this property indeed exists in all possible worlds.

7 What Is God?

Now we can formalize the informal definition of God. We want to say that an object x is God (we will denote it by $G(x)$) is x has all good properties and no bad properties:

$$G(x) \stackrel{\text{def}}{=} \forall\varphi((g(\varphi) \to \varphi(x))\,\&\,(b(\varphi) \to \neg\varphi(x))). \tag{9}$$

God is good. Intuitively, being God is a good property:

$$g(G). \tag{10}$$

Conclusion. From the second result, we conclude that God exists in every possible world.

8 Word of Caution: Shall We All Run to a Place of Worship?

Does this result convincingly prove that God exist? Not necessarily.

The problem is in the definition of necessary implication. The way this notion is defined still enables us to make counterintuitive conclusions. Namely, if $\psi(x)$ is always true, then the implication $\forall x\,(\varphi(x) \to \psi(x))$ holds in all possible worlds, so, according to the above definition, we have $\varphi \Rightarrow \psi$.

For example, if $\psi(x)$ is "the Sun will rise tomorrow", then we get conclusion like "animal sacrifices necessarily imply that the Sun will rise tomorrow". This is *not* what we intuitively mean by if-then rules, since whether the Sum rises or not clearly does not depend on whether we make an animal sacrifice or not.

A more adequate description is to only conclude that φ necessarily imply ψ when, in addition, it is possible that ψ will be false:

$$\varphi \Rightarrow \psi \stackrel{\text{def}}{=} \Box(\forall x\,(\varphi(x) \to \psi(x)))\,\&\,\Diamond\exists x\,\neg\psi(x). \tag{11}$$

However, if we use this more adequate definition in formula (4), we can now longer prove the very first Gödel's result – and thus, we are no longer able to conclude that God exists.

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