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Why ∞ Is a Reasonable Symbol for Infinity

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Abstract The fact that ∞ is actively used as a symbol for infinity shows that this symbol is probably reasonable in this role, but why? In this paper, we provide a possible explanation for why this is indeed a reasonable symbol for infinity.

1 Formulation of the Problem

Fact. In mathematics, we use the symbol ∞ for infinity.

History. This symbol was first used to described infinity in 1655, by John Wallis in his book [6], on p. 4 of the section "De Sectionibus Conicis, Nova Methodo Expositis"; see also [1, 5].

Interestingly, this symbol was, at first, not universally accepted. For example, Leonard Euler, one of the most famous 18th century mathematicians (and probably the most productive mathematician of all ages), used a different symbol – similar to the current symbol $\sim$ for similarity; see, e.g., Euler's paper [2] published in the Proceedings of Russian Academy of Sciences. But:

- in spite of Euler's authority and fame as a mathematician, it was not his symbol that was eventually accepted as the symbol for infinity,
- it was the symbol proposed by a much less known and much less authoritative Wallis.

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A natural question. Why was the current infinity symbol adopted and other symbols not? Why is this a reasonable symbol for infinity – because if it was not, another more reasonable symbol would replace it.

What we do in this paper. In this paper, we provide a possible explanation.

2 Our Explanation

What is a natural way to represent infinity: a question. How can we represent infinity – i.e., a process without end?

First natural idea. The first natural idea that comes to mind when we think about infinity is a straight (or curved) line.

So why not use a straight line to represent infinity?

Problem with this idea. In each sheet of paper, we only have a limited space to put symbols in. As a result, it is not possible to place the whole straight line.

And if we cut it off and only draw a segment of the straight line, this segment does not have any association with infinity.

We need a closed curve. Since we cannot represent an infinite motion in which the body moves farther and farther from the original point, the next natural idea is to represent a never-ending motion that is confined to a limited space.

Of course, in a limited space, we can only represent a limited part of the infinite trajectory. To make sure that this part indeed represents the never-ending motion, we need to make sure that the trajectory does not end abruptly, that it is clear how it continues. The only way to do that is to make sure that the trajectory goes back to one of its previous points, i.e., in mathematical terms, that the trajectory is a *closed cycle*, a *closed curve*.

Which closed curve should we select? There are many different closed curves, with different number of self-intersections.

Which one should we select?

Second natural idea: let us select the simplest curve. A natural idea is to select the simplest of the closed curves, i.e., a closed curve without self-intersections.

Problem with this idea. The problem with this idea is that this is exactly a symbol 0 for zero – a closed curve with no self-intersections.

Final idea: let us select the next simplest curve. Since we cannot select the simplest closed curve, with no self-intersections, a natural idea is to select the next simplest curve, with exactly one self-intersection.

This is exactly the usual infinity symbol. This is exactly the usual infinity symbol!

Thus, this symbol is indeed explained.

3 Real-Life Examples of an ∞-Like Trajectory

Trajectories that form a closed curve with exactly one self-intersection are common. Let us give a few examples.

Astronomy. If we show how the position of the Sun in the sky – as seen from a fixed location on Earth at the same time of day – varies over the course of a year, we will end up with an ∞-shaped trajectory.

This fact was already known to the ancient Greeks, this is why this trajectory is known by its Greek name – *analemma*. Claudius Ptolemy, the most famous astronomer of the ancient times – whose system was used all the way until Copernicus – even had a book titled *Analemma*; see, e.g., [4].

Chaos. Now everyone has heard about chaos and chaotic systems, i.e., systems for which long-term prediction is not possible – since a tiny uncertainty in the original position will eventually lead to huge uncertainty in the future state.

Historically the first such system – a simplified version of a weather system – was discovered by Edward Lorenz in the 1960s and is, because of this, known as the *Lorenz system*. Its trajectories resemble the ∞ symbol; see, e.g., [3].

Space flights. This was the shape of the trajectories of all the missions during the 1960s Apollo missions to the Moon.

The fact that the selected trajectory was a closed curve made perfect sense: it made sure that even if the major engine fails near the Moon, the spaceship would return, by itself, to the near-Earth part of the orbit from which it started – and thus, be able to make a safe landing.

Out of all closed-curve trajectories, other considerations led to the selection of the trajectory with a single self-intersection; see, e.g., [7].

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