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Dimension Compactification Naturally Follows from First Principles

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Dimension Compactification Naturally Follows from First Principles

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Abstract According to modern physics, space-time originally was of dimension 11 or higher, but then additional dimensions became compactified, i.e., size in these directions remains small and thus, not observable. As a result, at present, we only observed 4 dimensions of space-time. There are mechanisms that explain *how* compactification may have occurred, but the remaining question is *why* it occurred. In this paper, we provide two first-principles-based explanations for space-time compactification: based on Second Law of Thermodynamics and based on geometry and symmetries.

1 Formulation of the Problem

What is dimension compactification. According to modern physics (see, e.g., [5, 9, 11]), the requirement that the quantum field theory be consistent implies that the dimension of space-time should be at least 11. How can we combine this conclusion with the fact that the observed space-time is only 4-dimensional?

A usual explanation is that while in the beginning, space-time may have had 11 or more equally prominent dimensions, with time, most of these dimensions has been *compactified*: i.e., the size in the direction of these additional dimension remains as

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small as the Universe was in its first moments, while other dimensions expanded to the current astronomical sizes.

Compactification: how and why. There are several mechanisms that explain *how* compactification could have happened. However, these mechanisms do not explain *why* it happened.

In this paper, we provide arguments that compactification naturally follows from first principle. We actually provide *two* first-principles explanations for space-time compactification:

- an explanation based on the Second Law of Thermodynamics and
- an explanation based on geometry and symmetries.

2 Explanation Based on the Second Law of Thermodynamics

Second Law of Thermodynamics: a brief reminder. According to the Second Law of Thermodynamics (see, e.g., [2, 11]), the entropy of the Universe (and of any closed system) increases with time (or, in some cases, stays the same) – and there is no limit to such increase, eventually we get closer and closer to the state with the largest possible entropy.

What is entropy: a brief reminder. In general, the entropy is defined as [6, 10]:

$$S = -\int \rho(x) \cdot \ln(\rho(x))\, dx,$$

where $\rho(x)$ is the probability distribution of the set of all possible micro-states.

How is entropy depending on dimension. In general:

- close points or close particles are strongly correlated, while
- distant particles are independent.

A simplified description of this phenomenon can be obtained if we assume that all the points are divided into groups of nearby ones, so that:

- within each group there is a correlation, but
- between the groups there is no correlation.

It is known (see, e.g., [6]) that if we have several independent random processes, then the overall entropy is equal to the sum of the entropies of these processes. Thus, to find the overall entropy of the Universe in this approximation, it is sufficient to compute the entropy corresponding to each group, and then add up the resulting entropies.

How many points n are in each such group? Let us consider first the case when we only consider immediate neighbors – i.e., points whose all coordinates different from this one by no more then 1 appropriate unit of distance. In a coordinate system in which a central particle is at the point $(0, \ldots, 0)$, each of d coordinates of an immediate neighbor is equal to -1, 0, or 1 – three options. So overall, we have $n = 3^d$ points. If we consider neighbors of neighbors, we can have 5^d points – and, in general, $n = a^d$ for some $a > 1$.

This number clearly grows with the dimension d. So, when we go from a higher dimension d to a lower dimension $d' < d$, the number of neighbors decreases. This means that:

- instead of the original group of size n in which all particles were correlated,
- we have several subgroups of smaller size, and there is no longer correlation between different subgroups.

It is known – see, e.g., [6] – that if we know distributions corresponding to all the subgroups, then the entropy of the overall distribution for the whole group is the largest if and only if these subgroups are independent. Thus, when we divide a group in which all elements were correlated into smaller independent subgroups, we increase entropy.

Since, according to the usual interpretation of the Second Law of Thermodynamics, there are no limitations to the increase in entropy, eventually, we should also encounter a decrease in spatial dimension as a way to increase entropy – and this is exactly what compactification is about.

Comment. The above argument does not imply that compactification will stop at our 3 dimensions: it can go further, to having a 2- and even 1-dimensional space. Maybe this is what is already happening in the Universe, with 1D superclusters of Galaxies; see, e.g., [1, 7].

3 Explanation Based on Geometry and Symmetries

Our second explanation is based on a natural physical process. The original distribution of matter was uniform. However, the uniform distribution is not stable:

- if at some point, due to fluctuations, the density becomes larger than at the neighboring points,
- then this point start attracting matter from its neighbors – thus further increasing its density.

As a result, you get a large disturbance.

Symmetries and statistical physics: general idea. The original distribution in a d-dimensional space was invariant under shifts, rotations, and scaling (i.e., transformation $x_i \to \lambda \cdot x_i$).

According to statistical physics (see, e.g., [2, 11]):

- It is not very probably that from a highly symmetric state, we go straight into a completely asymmetric one.
- Usually, the most probably transition is to a state that preserves as many symmetries as possible.

So, we expect the shapes of the disturbances to have some symmetries.

Analysis of the problem. What is the shape that has the largest number of symmetries – i.e., for which the dimension of the corresponding symmetry group is the largest?

If the shape is invariant with respect to all rotations in the d-dimensional space, then it must consist of spheres, and a sphere has only rotations – so the dimension of the corresponding symmetry group is $\frac{d\cdot(d-1)}{2}$. Indeed, infinitesimal rotations are described by asymmetric matrices which have exactly as many parameters. So, in this case, the dimension of the symmetry group is

$$\frac{d^2-d}{2}.$$

If the shape includes a $(d-1)$-dimensional space, then we have $d-1$ independent shifts, $\frac{(d-1)\cdot(d-2)}{2}$ independent rotations, and 1 scaling, to the total of

$$d-1+\frac{(d-1)\cdot(d-2)}{2}+1=\frac{d^2-d+2}{2},$$

which is larger than for the sphere.

If we have all $(d-1)$-dimensional rotations but not all shifts or scaling, then we have fewer symmetries.

What if we only have rotations in a $(d-2)$-dimensional space, to the total of $\frac{(d-2)\cdot(d-3)}{2}$? We cannot have $d-1$ shifts, because this would lead to a $(d-1)$-dimensional space. Thus, we can have no more than $d-2$ independent shifts. Even if we have $d-2$ shifts and rotations, we will have

$$d-2+\frac{(d-2)\cdot(d-3)}{2}+1<d-1+\frac{(d-1)\cdot(d-2)}{2}+1$$

independent symmetries.

Conclusion. The most probable result of a natural spontaneous symmetry violation of a d-dimensional space is a $(d-1)$-dimensional space. Since fluctuations continue, we will then get space of dimension $d-2$, etc.

This provides another explanation of why the original space has lost many of its dimensions.

Comments.

- We have two explanations of the same phenomenon, but these explanations are not contradicting each other – both are based on statistical physics, we just took into account different aspects of it.
- The above idea of shapes motivated by symmetries has been used in physics – e.g., it explains the existing shapes of celestial bodies; see, e.g., [3, 4, 8].

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