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# What Is the True Formula for Soil Permeability? Not Clear

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## Abstract

To design and maintain pavements, it is important to know how fast water will penetrate the underlying soil. The speed of this penetration is determined by a quantity called permeability. There are several seemingly very different empirical and semi-empirical formulas that predict permeability. A recent attempt to select the formula that best fits the experimental data ended up in an unexpected conclusion that all three formula provide a good fit for the data. But these formulas are very different, how come that all three of them fit the same data? In this paper, we explain this somewhat paradoxical result.

**Mathematics Subject Classification:** 74L10

**Keywords:** Soil mechanics, pavement engineering, permeability

## 1 Formulation of the Problem

**It is important to predict permeability.** One of the processes that can damage the pavement is that water seeps into the soil under the asphalt. To

properly design and maintain the pavements, it is therefore important to be able to predict this water flow.

In general, the ability of water to seep through the material is characterised by the velocity  $k$  with which the vertically flowing water comes through this material. This velocity is called *permeability*. For the road, water cannot penetrate the asphalt layer and thus, comes from the sides. For such flow, the speed  $v$  with which water flows into the soil is determined by the formula

$$v = k \cdot \frac{h}{L}, \quad (1)$$

where  $h$  is the change in the height of the water during the path of length  $L$ . This formula was first proposed by Darcy in [4] and is thus known as the Darcy law.

So, to predict how water will seep into the soil, we need to be able to predict the soil's permeability.

**Several formulas have been proposed for this prediction.** The ability of water to seep through the soil is caused by voids in the soil. The relative amount of voids is characterized by the *void ratio*  $e$ , which is defined as the ratio between the volume of voids and the volume of the solid parts of the soil. It is therefore important to find out how the permeability depends on the void ratio.

There are several different formulas describing the dependence of  $k$  on  $e$ ; see, e.g., [5]. There is the Casagrande formula [2, 3]:

$$k \approx c_1 \cdot e^2, \quad (2)$$

for some constant  $c_1$ .

There is the Kozeny-Carman equation [1, 6, 7]:

$$k \approx c_2 \cdot \frac{e^3}{1 + e}, \quad (3)$$

for some constant  $c_2$ .

There is also the third formula

$$k \approx c_3 \cdot \frac{e^2}{1 + e}, \quad (4)$$

for some constant  $c_3$ .

**All three formulas are equally good: how can it be?** Interestingly, experimental results described in [5] show that all three equations work equally well, with accuracy about 10%.

But how can it be? If the dependence is quadratic, it cannot be cubic? In this paper, we explain how this could happen.

## 2 Our Explanation

**Experimental results: range of  $e$ .** Experimental results described in [5] correspond to the values  $e$  from 0.4 to 0.5, i.e., to values

$$e = 0.45 \pm 0.05. \quad (5)$$

**Analysis of the problem.** Once the formula (4) is true, to get the formulas (2) and (3) we need to multiply both sides of this formula by the results of dividing (2) and (3) by the formula (4).

If we divide the formula (3) by the formula (4), we conclude that

$$1 \approx \frac{c_2}{c_3} \cdot e, \quad (6)$$

i.e., equivalently, that

$$e \approx \text{const} \stackrel{\text{def}}{=} \frac{c_3}{c_2}. \quad (7)$$

Similarly, if we divide the formula (2) by the formula (4), we conclude that

$$1 \approx \frac{c_1}{c_3} \cdot (1 + e), \quad (8)$$

i.e., equivalently, that

$$1 + e \approx \text{const} \stackrel{\text{def}}{=} \frac{c_3}{c_1}. \quad (9)$$

**How can we explain the experimental result.** Because of the above analysis, once the formula (4) is true, to get the formulas (2) and (3) with the needed accuracy, it is sufficient to prove that with the desired 10% accuracy, the values  $e$  and  $1 + e$  are indeed equal to constants.

This leads us to the desired explanation.

**Resulting explanation.** As we have mentioned, in the experimental results from [5], possible deviations of the value  $e$  from its central value  $e_0 = 0.45$  do not exceed  $\Delta e \stackrel{\text{def}}{=} 0.05$ .

Thus, the relative inaccuracy with which we know  $e$  does not exceed the ratio

$$\frac{\Delta e}{e_0} = \frac{0.05}{0.45} \approx 10\%. \quad (10)$$

Similarly, for the quantity  $1 + e$ , possible deviations of the value  $1 + e$  from the value  $1 + e_0 = 1 + 0.45 = 1.45$  do not exceed  $\Delta e = 0.05$ . Thus, the relative inaccuracy with which we know  $e$  does not exceed the ratio

$$\frac{\Delta e}{1 + e_0} = \frac{0.05}{1.45} \approx 3.5\% \ll 10\%. \quad (11)$$

In both cases, the ratio between the formulas (2) and (4) and the ratio between formulas (3) and (4) are indeed approximately constants – within the given 10% accuracy.

Thus, once the formula (4) holds, the formulas (2) and (3) hold as well.

The mysterious fitting by three different formulas is thus explained.

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