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How Void Ratio Depends on Grain Size In Soil Mechanics: Theoretical Explanation

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Abstract

When designing a road, it is important to know how many voids are in the underlying soil – since these voids will affect the road stiffness. It is difficult to measure the voids ratio directly, so instead, we need to estimate it based on easier-to-measure characteristics such as grain size. There are empirical formulas for such estimation. In this paper, we provide a possible theoretical explanation for these empirical formulas.

Mathematics Subject Classification: 74L10

Keywords: Soil mechanics, pavement engineering, void ratio, grain size

1 Formulation of the Problem

In pavement engineering, it is important to estimate the volume of voids in the soil. A road is built on top of the soil. So, to properly design a

road, it is important to take into account the properties of the corresponding soil.

One of these properties is related to the fact that the soil is not a homogeneous solid body, it consists of grains and voids. If there are few voids, then the soil is more stiff. On the other hand, if there are many voids, then, when the road is built and becomes operational, the pressure from the traffic may cause these voids to be filled – and thus, may cause the road to sag.

To prevent this from happening, it is important to gauge the relative volume of voids – and, if it is too high, to strengthen the soil and/or to place a stiffer layer about this soil. This volume is usually gauged by the ratio $e \stackrel{\text{def}}{=} \frac{V_v}{V_s}$ between the total volume of the voids V_v and the total value of the solids V_s . This ratio e is known as the *void ratio*.

It is difficult to directly measure the volume of voids. An ideal situation would be if we could directly measure the volume of voids. The problem is that such measurements, while possible in the laboratory environment, are too complicated to be used in practical road design.

Since it is difficult to measure the voids volume directly, it is necessary to measure it indirectly, i.e., to estimate the relative volume of voids based on easier-to-measure characteristics. What is easier to measure – by simply mechanical filtering-by-size – is the size of the grains. Usually, the following three size-related characteristics are measured:

- the *median grain size* D_{50} , i.e., the size for which exactly 50% of the grains have larger size and exactly 50% have smaller size;
- the *fine content* F_c , the percentage of the grains whose size is smaller than 0.075 mm; and
- the *clay-size fraction* P_c , the percentage of the grains whose size is smaller than 0.005 mm.

These three values come with different accuracy, for the simple reason that the accuracy of each statistical estimate depends on the sample size.

- The value D_{50} is estimated based on the whole soil sample, while
- the values F_c and P_c are estimated based only on a portion of the grains – sometimes based on a very small sample.

As a result, while the median grain size D_{50} is usually measured with reasonable accuracy, the values F_c and P_c are usually only known with low accuracy. Thus, instead of the exact values of F_c and P_c , we know, in effect, only the ranges containing these values: e.g., below 5%, between 5% and 15%, etc.

For different ranges of F_c and P_c , we therefore need to estimate the void ratio e based on the median grain size D_{50} .

We need a range of possible values of void ratio. The void ratio depends not only on the values of the above three characteristics, it also depends on many other factors such as the grain shapes and on the other parameters describing the distribution of grain sizes. As a result, within the same ranges of possible values of F_c and P_c , and for the same median grain size D_{50} , we can have different values of the void ratio e .

To get a good understanding of the soil, it is therefore necessary to find, for each combination of ranges of F_c and P_c , and for each value of median grain size D_{50} , the range $[e_{\min}, e_{\max}]$ of possible values of the void ratio.

Empirical formulas. The following empirical formulas are known for determining e_{\min} and e_{\max} ; see, e.g., [1, 2, 3, 5]:

$$e_{\min} \approx a_{\min} + \frac{b_{\min}}{D_{50}}; \quad (1)$$

$$e_{\max} \approx a_{\max} + \frac{b_{\max}}{D_{50}}, \quad (2)$$

where a_{\min} , b_{\min} , a_{\max} , and b_{\max} are parameters which are, in general, different for different ranges of F_c and P_c .

From these formulas, it follows that there is an (approximate) linear relation between e_{\min} and e_{\max} :

$$e_{\max} \approx a + b \cdot e_{\min}. \quad (3)$$

Comment. Actually, while the original papers contain the formula (3), they do not explicitly contain the formulas (1) and (2). Instead, they contain a different formula

$$e_{\max} - e_{\min} \approx a_d + \frac{b_d}{D_{50}}. \quad (4)$$

However, one can easily check that in the presence of the formula (3), the formula (4) is equivalent to the formulas (1)-(2). Indeed:

- If (1) and (2) are true, then, by subtracting (2) from (1), we get the expression (4) with $a_d = a_{\max} - a_{\min}$ and $b_d = b_{\max} - b_{\min}$.
- Vice versa, if the formula (4) is true, then, by substituting, into the formula (4), the right-hand of the formula (3) instead of e_{\max} , we conclude that

$$a + b \cdot e_{\min} - e_{\min} \approx a_d + \frac{b_d}{D_{50}}, \quad (5)$$

hence

$$(b - 1) \cdot e_{\min} \approx (a_d - a) + \frac{b_d}{D_{50}}. \quad (6)$$

Dividing both sides of this approximate equality by $b - 1$, we get the expression (1), with

$$a_{\min} = \frac{a_d - a}{b - 1} \text{ and } b_{\min} = \frac{b_d}{b - 1}. \quad (7)$$

Finally, substituting the expression (1) into the formula (3), we get the formula (2), with

$$a_{\max} = a + b \cdot a_{\min} \text{ and } b_{\max} = b \cdot b_{\min}. \quad (8)$$

Challenge. How can we explain the empirical formulas (1) and (2) – or, equivalently, the empirical formulas (3) and (4)?

In this paper, we provide such an explanation.

2 Our Explanation

Basic physics of voids. In general, the gravitational force drags everything down. If this was the only force, all the molecules would get to their lowest possible positions, and there would be no voids at all.

What prevents this are the surface forces between the grains: while there are usually no voids inside a grain, friction between the grains prevents the molecules that form these grains to get as low as possible.

Resulting first-approximation model. As we have mentioned, the presence of grains is largely due to the surface forces. The larger the surface forces – in comparison to the gravitational forces F_g – the more voids we have. The void ratio e is therefore a function of the ratio $r \stackrel{\text{def}}{=} \frac{F_s}{F_g}$: $v = f(r)$.

The soil is not dust that floats in the air, it is mostly down all the time. This means that, in general, the gravitational force is much stronger than the surface force, i.e., that the ratio r is usually small. Thus, we can use the usual physics idea (see, e.g., [4, 6]) and in the first approximation, we can expand the dependence of v on r on Taylor series and keep only linear terms in this expansion – ignoring quadratic and higher order terms. In this linear approximation, we get

$$v \approx v_0 + v_1 \cdot r. \quad (9)$$

How these two forces depend on the grain size D . The gravitational force acting on a grain is proportional to its volume, i.e., to the cube D^3 of its linear size:

$$F_g \approx c_g \cdot D^3. \quad (10)$$

On the other hand, the surface forces are proportional to the grain surface, i.e., to the square D^2 of its linear size:

$$F_s \approx c_s \cdot D^2. \quad (11)$$

From the formulas (10) and (11), we conclude that

$$r = \frac{F_s}{F_g} \approx \frac{c_g/c_s}{D}. \quad (12)$$

Thus, from the formula (9), we conclude that

$$v \approx v_0 + \frac{v_1 \cdot (c_g/c_s)}{D}. \quad (13)$$

This is exactly the formulas (1) and (2), with $a_i = v_0$ and $b_i = v_1 \cdot (c_g/c_s)$.

Thus, the empirical formulas (1) and (2) have indeed been justified.

Comment. The above explanation of the formulas (1) and (2) is similar to the physics-based explanation of why there is a lower limit on the size of warm-blooded animals (see, e.g., [4]).

Indeed, in an animal, the amount of heat is proportional to the cube D^3 of the animal's size D , while the rate with which heat is being lost is proportional to the animal's surface, i.e., the square D^2 of the animal's size. To get the relative rate with which an animal loses heat, we need to divide the absolute heat loss by the amount of heat. If we divide D^2 by D^3 , we conclude that this relative rate is proportional to $\frac{1}{D}$. When the size D gets too small, this rate becomes so high that it is no longer possible to maintain the body to be warmer (or colder) than the environment.

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