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Zadeh's Vision, Modern Physics, and the Future of Computing

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ZADEH'S VISION, MODERN PHYSICS, AND THE FUTURE OF COMPUTING*

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Abstract. At first glance, Zadeh's ideas that everything is a matter of degree seem to be more appropriate for situations when we do not know the exact equations, when we only have expert rules for control and/or decision making. From this viewpoint, it may seem that in physics, where equations are ubiquitous and all the terms seem precise, there is not much place for fuzziness. But, as we show, in reality, fuzzy ideas can help – and help dramatically – in physics as well: in spite of the first impression, as physicists know well, many arguments in physics rely heavily on physical intuition, on imprecise terms like randomness, and a natural formalization of these terms makes them a matter of degree.

Keywords: fuzzy logic, physics, causality, randomness, Kolmogorov complexity, strings, quark confinement

AMS Subject Classification: 03B72 70A05 81P05 68Q12 68Q10

1. Introduction

Zadeh's fuzzy techniques (see, e.g., [5, 13, 28, 33, 35, 43, 44, 45, 46, 47, 48, 49, 50, 51]) originated in control.

From the purely mathematical viewpoint, control seems to be a precise discipline: we learn how exactly the system changes under different controls, we select an objective function, and we find the control which is optimal with respect to this objective function. In practice, we sometimes do not have full information about the system's dynamics. In this case, a reasonable idea is to identify the system better. This was the prevailing mood in control until 1960s, when Lotfi Zadeh realized that even without a precise knowledge of the system, we can drastically improve our control if we take into account expert knowledge – knowledge which is often formulated by using imprecise ("fuzzy") words from natural language like "small". The resulting fuzzy control techniques boosted many control applications.

Similarly, when applied to classification, clustering, and pattern recognition, fuzzy techniques helped to boost the corresponding applications; see, e.g., [5, 6, 13, 28, 33, 35].

In this paper, we argue that a similar boost is possible (and expected) in physics. There too, from the purely mathematical viewpoint, we have precise often-difficult-to-solve equations. However, in practice, a significant part of physicists's reasoning includes not-well-defined terms

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like randomness, causality, etc. This is a part that is difficult to formalize, difficult to use – that requires physical intuition. On several examples, we show that Zadeh's main idea – that everything is a matter of degree – can help formalize this reasoning. This idea can help both on the qualitative level – where it helps provide intuitive explanations for many complex physical ideas like strings and quark confinement, and even on the quantitative level – where it can lead to a new derivation of complex physical equations from intuitively clear first principles.

We also show how similar ideas can be used to identify physical processes which can potentially lead to future post-quantum-computing boosts in our computational abilities.

2. But Why Is Everything a Matter of Degree?

In modern physics, many things are a matter of degree. Many physical quantities are described by real numbers – be it coordinates, velocities, etc. However, there are also some properties of the physical world that seem to be best described on a "yes"–"no" basis: an object may be in one or another of several equilibrium states, an animal is either alive or dead, a particle either has an electric charge or it does not, etc.

This "yes"-"no" way is how such properties were described in traditional (pre-quantum) physics. However, it turns out that reality is described by quantum physics, and in quantum physics (see, e.g., [7, 42]), for every two possible states s and s' of an object, we can also have a *superposition* of these two states, i.e., as state $c\cdot s+c'\cdot s'$, where c and c' are complex numbers for which $|c|^2+|c'|^2=1$. If, in this state, we check whether the state is s or s', we will get the answer "s" with probability $|c|^2$ and the answer "s'" with the remaining probability $|c'|^2=1-|c|^2$. OK, a superposition of a dead and alive cat – example provided by Schroedinger, one of the founder of quantum physics – remains a paradoxical construction, but for micro-particles, superpositions are ubiquitous, they have been observed in many experiments.

For such states, the property of "being in the state s" is satisfied only to some degree – to be more precise, with some probability.

Comment. There are many other examples of everything-is-a-matter-of-degree phenomena in physics. For example, traditionally, it was believed that objects are either 1-dimensional, or 2-dimensional, or 3-dimensional – but it turned out that many physical objects are characterized by a real-valued (fractional) dimension– such objects are known as fractals; see, e.g., [26, 27].

So what? If all we said was that in quantum physics, everything is a matter of degree, this would be a good place to end the paper: equations of quantum physics are known, we know how to solve them in many situations, and speculations that this is one more example of fuzzy do not help to solve any practical physical problems.

Our point is different: we claim – and in this section, we will show – that this everything-is-a-matter-of-degree principle follows from the fundamental understanding of physics, and can thus apply to all physical theories: be it quantum physics or some future physics that come after it.

Comment. A detailed description of our arguments can be found in [32].

Why in physics, everything is a matter of degree: a fundamental explanation. Some processes we can predict, some we cannot. For example, we can predict the weather – and often reasonably accurately – but we cannot predict the exact shape of each ocean wave. Maybe in the future, we will be able to predict the shape of individual waves, but most probably, there will be always be some processes that cannot exactly predict. These not-easy-to-predict processes is what we call *random*.

Physics deals with random processes as well, but mostly it is interested in the processes and quantities that we *can* predict. This "can" means that we should have an algorithm for this prediction – and this algorithm must be practically feasible. Indeed, if prediction of tomorrow's weather takes millions of years, this "prediction" makes no sense – we will observe the next day's weather way before computations are over.

From the computational viewpoint, what does it mean that everything is a matter of degree? A "yes"-"no" situation would mean that the corresponding quantity characterizing a single object can only take values 1 (corresponding to "true") or 1 (corresponding to "'false") – and that the quantity characterizing many objects can take any non-negative integer values: 0 objects with this property, 1 object with this property, etc. That everything is matter of degree means that this quantity can take all intermediate real values as well.

From the computational viewpoint, this transition from discrete values and integers to real numbers makes all the difference:

- In general, when we consider all possible first-order arithmetic statements, i.e., statements obtained from $a + b = c$ and $a \cdot b = c$ by using "and", "or", "not", and quantifiers over all integers, then no algorithm can decide whether a given formula is true. This was, in effect, what Gödel proved in his famous 1931 theorem; see, e.g., [9].
- On the other hand, for first-order arithmetic formulas involving real numbers, there is a general algorithm that decides whether a given formula is true; see, e.g., [4, 29, 41].

A similar difference can be traced if we consider the question of algorithms feasibility:

- In the discrete (integer) case, even solving a system of linear (= simplest) equations is NP-hard, meaning that – unless P=NP (which most computer scientists believe not to be true), no feasible algorithm is possible for solving such systems; see, e.g., [36].
- On the other hand, in the real-valued case, there are known efficient algorithms for solving such systems.

Thus, predictable phenomena have to be described by all possible real numbers (if needed, within some bounds), and not just by some discrete values like "yes" and "no".

Another fundamental explanation. Our previous explanation was based on the need to predict future phenomena. Once we know that some phenomenon can be, in principle, predicted, a natural next question is: How can we predict it?

The main idea behind all predictions is that we look at similar situations in the past, and we use what we have observed then to predict what will happen now. For example, we know that the Sun will rise tomorrow because we have observed its rising many times in the past. The key word here is "similar". In physics, similarity in which the major properties are preserved is known as *symmetry*, and symmetry is indeed one of the most fundamental – and most fruitful – concepts of modern physics; see, e.g., [7, 42].

And, interestingly, symmetry leads to the need for values intermediate between "true" and "false". Let us illustrate it on the example of a *benzene* ring C_6H_6 – the fundamental unit from which all substances of organic chemistry, all the molecules of living beings are built.

In the traditional chemistry, a molecule is composed of atoms that exchange electrons with each other. If an atom borrows one electron from another atom, we say that they have a connection of valence 1; if it borrows two electrons, there is a connection of valence 2, etc.

From the analysis of benzene, it has been clear that it consists of 6 carbon and six hydrogen atoms, i.e., that its chemical formula is C_6H_6. However, for a long time, it was not clear how exactly they are connected to each other. The solution came in the 19th century to a chemist August Kekule in a dream. He dreamed of six monkeys that form a circle in which each monkey

holds to the previous monkey's tail. According to this solution, the six C atoms form a circle. To each of these atoms, a H atom is attached. Each C atom has a 1 valence connection to H, 1 valence connection to one of its neighbors, and 2 to another neighbor.

The resulting chemical structure is still routinely described in chemical textbooks – because a benzene loop is a basis of organic chemistry and life. However, now we understand that this formula is not fully adequate. Indeed, according to this formula, the connections between C atoms are of two different types: of valence 1 and of valence 2. In reality, the benzene molecule is completely symmetric, there is no difference between the strengths of different connections.

It is not possible to have a symmetric configuration if we require that valencies are integers. To equally split the remaining valence of 3 (1 is taken for H) between the two neighbors, we need a valence of 3/2. This is not possible in classical chemistry – but this is possible, in some sense, in quantum chemistry where, as we have mentioned, we have a continuum of intermediate states; see, e.g., [7].

3. The Idea That Everything Is a Matter of Degree Provides Intuitive Explanations for Many Physical Ideas – And Even Goes Beyond Explanations

Let us show that indeed, the idea that everything is a matter of degree can help physics.

How to avoid physically meaningless infinite answers (politely called divergence). Many non-physicists – especially those who are in (understandable) awe of physics – are not aware that in many cases, physics equations lead to physically meaningless infinite values. Let us give a simple example: what is the overall energy of the electric field generated by an electron? Mathematically, there is no suspense here: all the formulas are well known.

An electron is an elementary particle, which means that it cannot be represented as two independent parts. According to special relativity, the speed of light is an upper bound on all communication speeds. Thus, if an electron was not a single spatial point, two difference parts corresponding to different spatial locations would act independently. Thus, an electron is a point-wise particle.

The electric field $\overline{E}$ of a point-wise charged particle is proportional to r^{-2}, where r is the distance to the particle. The energy density ρ is proportional to the square $\overline{E}^2$ and thus, proportional to r^{-4}. We can get the overall energy by integrating this density over the whole 3D space. The problem is that the resulting integral $\int r^{-4}\,dV$ is infinite; see [7, 42].

There are many similar problem in classical physics – this is why quantum physics was invented in the first place, to resolve such problems – but for the above problem, infinities do not disappear in the quantum case. How do physicists deal with this problem? They use tricks – e.g., they start by having an electron of finite size ε – for which the overall energy is finite – and then subtract some appropriate terms and take a limit $\varepsilon \to 0$.

How can the idea that everything is a matter of degree help here? Very simply: whatever we measure comes with some accuracy; see, e.g., [37]. With the accuracy, we never get distance exactly 0: if the measurement accuracy is $\delta > 0$, the measurement results are always consistent with the assumption that all the distances are greater than or equal to δ, and if we limit ourselves to such distances, we get a finite value for the overall energy of the electron's electric field; see, e.g., [14, 15, 16].

Comments.

- The above argument is, in some sense, a formalization of similar arguments provided by physicists – except that they tried to described their arguments in crisp form, which made them mathematically inconsistent; see, e.g., [52].
- An important twist of this argument is related to the fact that in quantum physics, as we have mentioned, measurement results come with probabilities, and uncertainty means that we know these probabilities only with some accuracy. In the 3D space, the need to consider such imprecise probabilities does not just come from our technical imperfection: it is known that no probability measure is possible on the class of all subsets of the 3D space. The proof is in the famous Banach-Tarski paradox that shows that we can represent a unit ball as a union of several parts so that by shifting and rotating them, we can form two unit balls [3]! Interestingly, probability measures on the class of all subsets *are* possible in 1D and 2D spaces; see, e.g., [2, 30]. Since exact values lead to physically meaningless infinities, this explains why our proper space is at least 3-dimensional [16].

Why strings. The way we avoided infinities is by taking into account that an electron – and other elementary particles – are not really point-wise, i.e., not really 0-dimensional. The simplest non-0-dimensional object is 1-dimensional. This explains why many physical theories view elementary particles as 1D strings [7, 18, 42].

Why quark confinement. After 1D the next in complexity is 2D. The interaction force F between two 2D charges surfaces does not depend on a distance, as a result of which the energy needed to separate two attracting surfaces linearly grows with distance r, and $F \cdot r$. As a result, such particles can never fully separate: they can only separate by a distance r for which $F \cdot r$ is smaller than or equal to their original kinetic energy E. Interestingly, this is exactly how quarks – parts of protons and neutrons – behave: they are visible inside a proton, but they can never separate; this is known as *quark confinement* [7, 18, 42].

Why 9D space. The next in complexity is 3D shape. What 3D shape should we consider? In quantum physics, processes are probabilistic. There are many different factors affecting each particle. According to the Central Limit Theorem (see, e.g., [38]), the distribution of a joint effect of many independent influences is close to Gaussian. A general 3D Gaussian distribution can be characterized by 3 parameters of its mean value and 6 parameters describing its symmetric covariance matrix – the total of 9. In other words, we need 9 parameters to describe each particle. This explains why in most modern physical theories, a particle is characterized by 9 spatial parameters [7, 14, 42].

Comment. To be more precise, 9 is the smallest number of spatial parameters at which the corresponding theory becomes consistent; there are also consistent theories with larger number of spatial parameters [7, 42].

A more fundamental question: how can we define causality? The notion of causality – the possibility of one process to influence another one – is one of the most fundamental concepts of physics. The big question is how to define it.

Sometimes, there is a good correlation between the two processes – but this does not necessarily mean that the earlier process is a cause of the later one: they may both be caused by some third past process. So, how do we determine causality?

In the macro-world of ours, there is a clear way to do it: we change A and if, as a result B changes, this means that A was one of the causes of B. For example, if injecting bacteria

into mice led to them getting a certain disease – which other mice did not get – this is a good indication that the tested bacteria causes this disease.

But what about micro- and mega-worlds where such experiments are difficult or even impossible? At first glance, it may seem that in these cases, there is no way to distinguish correlation from causation. But there is a way – if we take into account that, as we mentioned, not everything in the world is predictable, there are some random un-predictable processes. Intuitively, if a random process B can be partially explained when we know the random process A, this is a good indication that A can causally influence B. To describe this in precise terms, we need to recall how randomness is formally described now.

Traditional probability theory did not have the notion of a random sequence. However, every physicist, every practitioner understands that if when flipping a coin 1000 times and marking head as 1 and tail as 0 we get a sequence 010101...– 01 repeated 500 times – something is fishy. If we see such a pattern in a casino, this is a good reason to call the police. How can we distinguishing such a clearly non-random sequence from a sequence that can actually be obtained when flipping a coin? This was the question raised in the 1960s by Andrei N. Kolmogorov – a mathematician who in the 1930s pioneered the mathematical foundations of probability theory.

Kolmogorov's observation was that the sequence 010101... is not random because it can be generated by a short program, in which 01 is repeated 500 times in a loop. On the other hand, the actual sequence of random results 011... does not have any such regularities, so the only way to generate this sequence by a program is to write print(011...). To formalize this difference, he introduced a new notion $K(x)$ – later called *Kolmogorov complexity* – the shortest length of a program (in a given programming language) that generates the sequence x.

A non-random sequence like 010101... can be generated by a very short program, so for such sequences, $K(x) \ll \text{len}(x)$. On the other hand, for a truly random sequence, we have $K(x) \approx \text{len}(x)$. So, a natural idea is to select some constant C and call a sequence random if $K(x) \geq \text{len}(x) - C$; see, e.g., [25]. An even better idea is to take into account that the smaller C – i.e., the smaller the difference $\text{len}(x) - K(x)$ – the "more random" is the corresponding sequence – i.e., that randomness is also a matter of degree, and the size of the difference $\text{len}(x) - K(x)$ is an indication of this degree.

How does this help to define causality? If we observe two random sequences x and y, then, if y is largely based on x, then knowing x will make it easier to generate y – i.e., the shortest length $K(y|x)$ of a program that computes y by possibly using x is much smaller than the length $K(y)$ of the shortest program that generates y without access to x: $K(y|x) \ll K(y)$. The larger the difference $K(y) - K(y|x)$, the more confident we are that y was affected by x, i.e., that the event characterized by the sequence x was causally affecting the event characterized by the sequence y.

In other words, we get a convincing definition of causality – but now causality becomes a matter of degree!

Comments.

- For technical details, see [23, 24].
- Will there be a problem if we make causality "fuzzy"? The simplest case of causality can be observed in special relativity theory, where an event (t, x) can affect the event (s, y) – we will denote it by $(t, x) \leq (y, s)$ – if and only the signal emitted at (t, x) with a speed not exceeding the speed of light c can reach (s, y), i.e., if the distance $d(x, y)$ does not exceed $c \cdot (s - t)$. This causality relation is known as Minkowski causality. It is known that every transformation of the 4D space-time that preserves the causality relation can be

represented as a composition of shifts, rotations, Lorentz transformations, and scalings $(t, x) \to (\lambda \cdot t, \lambda \cdot x)$. This result – which provides an important foundation for special relativity theory – assumes that we have the exact causality relation. Should we worry if the causality is only approximately described by the above Minkowski formula? We need not worry: it turns out that if we have a transformation that preserves (or even approximately preserves) a causality relation which is close to Minkowksi one, then the corresponding transformation is still close to the above-described composition; see [21].

4. The Idea That Everything Is a Matter of Degree Can Lead to a New Derivation of Physical Equations from First Principles

Is it even possible? Let us try. At first glance, fuzzy ideas lead to fuzzy conclusions – so all we can expect from them is approximate formulas. For example, fuzzy control may be good – if the corresponding expert rules are good – but it is not optimal. To get the optimal control, we need to explicitly write down the corresponding equations, and solve the resulting optimization problem.

But where do these equations come from? OK, some equations come from fundamental equations, but where did these fundamental equations – such as Newton's or Maxwell's – come from? Physicists start with imprecise qualitative ideas, and then transform them into precise equations. This transformation is done informally – since until fuzzy logic, there was no general way of transforming imprecise ("fuzzy") knowledge into precise terms. But now there is, so why not apply it to physics? Let us show, on the example of Newtonian mechanics, that this indeed leads to Newton's equations.

Comment. In this paper, we will describe the main ideas and results. Technical details can be found in [12].

Let us start with Newton's equations. In general, a fuzzy degree describes to what extent a state is possible – no wonder they are closely related with possibility theory. For a mechanical object, what is possible to do with it is determined by its potential energy V and kinetic energy K. If we have an immobile rock at the bottom, not much it can do, but if we have a rock at the top (with potential energy) – it can go down (and cause some damage going down). If we have a moving rock (with kinetic energy), it can smash into things.

How can we describe it in precise terms? Let us start with potential energy.

How does degree of possibility depend on potential energy? What is the membership function $\mu(V)$ that describes how the degree of possibility of a state depends on the potential energy V? To find this function, we can take into account that there is no fixed starting point for measuring potential level. We can consider the rock at sea level as having energy 0, or we can consider a rock at the level of the city (which is usually different from the sea level) as energy 0, etc. For example, when we analyzing a hydroelectric power station, it is reasonable to take the lower water level as 0 level, although it is still higher than the sea level.

In general, if we charge the level, the numerical value V changes to $V + V_0$, where V_0 is the difference in energy between the two levels. It is therefore reasonable to require that the corresponding fuzzy description does not change under this transformation, i.e., that the original membership function $\mu(V)$ and the transformed membership function $\mu(V + V_0)$ are, in some reasonable sense, equivalent.

What does "equivalent" mean? To understand what "equivalent" means, let us recall how we can get a numerical value of the membership function. One of the natural ways is by polling: we ask n experts, and if m of them consider the value V possible, we take $\mu(V) = m/n$.

Ideally, we should only ask top experts – e.g., Nobel prize winners. However, there are not that many top experts, and for small n, polling results are not very informative. So, a natural idea is to ask more experts. But most of these experts will hardly contradict the opinion of the majority of their superiors. So, if m is a minority, even if we ask k more experts, we will still get only m positive replies: which means that the new value of the membership function will be $\mu'(V) = m/(n+k)$, i.e., $\mu'(V) = c \cdot \mu(V)$, where $c \stackrel{\text{def}}{=} n/(n+k)$. We can select different numbers of experts, the resulting membership functions describe the same situation and are, thus, equivalent in this sense. So, we can say that the two membership functions $\mu(V)$ and $\mu'(V)$ are equivalent if they differ by a multiplicative constant, i.e., if $\mu'(V) = c \cdot \mu(V)$ for some constant c.

How does degree of possibility depend on potential energy (cont-d). Let us get back to our question: what is $\mu(V)$? In view of the above description of equivalence, the fact that the functions $\mu(V)$ and $\mu(V + V_0)$ are equivalent means that $\mu(V + V_0) = c(V_0) \cdot \mu(V)$ for some constant c depending on V_0.

It is known that all definable (thus all continuous) solutions of this functional equation have the form $\mu(V) = c_0 \cdot \exp(-c_V \cdot V)$, for some constants c_0 and c_V; see, e.g., [1]. The requirement that $\mu(V) \le 1$ for all V implies that $c_V \ge 0$ – and the need to have some dependence on V excludes the case $c_V = 0$, so we have $c_V > 0$.

Which "and"-operation should we use? A similar formula can describe the possibility of different values of kinetic energy K. To describe the possibility of a given state (V, K), we need to describe the degree to which the value V is possible *and* the value K is possible. In fuzzy techniques, this degree is usually estimated by applying an appropriate "and"-operation (t-norm) to the degrees to which V is possible and to which K is possible, i.e., to compute $f_{\&}(\mu(V), \mu_K(K))$. However, there are many different "and"-operations. Which one should we choose?

To answer this question, we can take into account that many physical systems consists of practically independent subsystems. If two subsystems have potential energies V_1 and V_2, then the overall potential energy of the system is equal to the sum $V = V_1 + V_2$. If we consider the system as a whole, then the degree to which its potential energy is possible is $\mu(V) = c_0 \cdot \exp(-c_V \cdot (V_1 + V_2))$. On the other hand, we can consider these two subsystems separately, then for each subsystem, the degree is equal to $\mu(V_i) = c_0 \cdot \exp(-c_V \cdot V_i)$, and the overall degree of possibility is equal to $f_{\&}(c_0 \cdot \exp(-c_V \cdot V_1), c_0 \cdot \exp(-c_V \cdot V_2))$. It makes sense to require that the degree of possibility should not depend on whether we consider the system as a whole or as two subsystems, so we should get

$$c_0 \cdot \exp(-c_V \cdot (V_1 + V_2)) = f_{\&}(c_0 \cdot \exp(-c_V \cdot V_1), c_0 \cdot \exp(-c_V \cdot V_2)).$$

For each a_1 and a_2, substituting the values V_1 and V_2 for which $c_0 \cdot \exp(-c_V \cdot V_i) = a_i$ – i.e., the values $V_i = -(1/c_V) \cdot \ln(a_i/c_0)$, we conclude that $f_{\&}(a_1, a_2) = (1/c_0) \cdot a_1 \cdot a_2$.

Thus, the overall degree to which a particle has potential energy V and kinetic energy K is proportional to the product of the corresponding degrees, i.e., to $\exp(-c_V \cdot V) \cdot (-c_K \cdot K) = \exp(-c_V \cdot V - c_K \cdot K)$. To describe the probability of a trajectory, we need to similarly apply the "and"-operation to the degrees corresponding to different moments of time $t_1, \ldots, t_n$, so we

get the value proportional to the product

$$\prod_{i=1}^{n} \exp(-c_V \cdot V(t_i) - c_K \cdot K(t_i)) = \exp\left(-c_V \cdot \left(\sum_{i=1}^{n} (V(t_i) + c_1 \cdot K(t_i))\right)\right),$$

where $c_1 \stackrel{\text{def}}{=} c_K/c_V$. Some additional physical arguments lead to $c_1 = -1$ [12].

The most probable trajectory is the one corresponding to the largest possible value of this degree, i.e., equivalently, to the smallest possible value of the sum $\sum\limits_{i=1}^{n}(V(t_i) - K(t_i))$. In the continuous limit, when we take into account that there are infinitely many possible moments of time, this sum tends to the integral $\int (V(t) - K(t))\, dt$. Interestingly, minimization of this integral – known as *action* – leads exactly to Newton's equations; see, e.g., [7, 12, 42].

From Newton's equations to the general case. According to modern physics, all interactions can be, in effect, reduced to mechanical ones. For example, electrical attraction/replusion can be explained by the fact that the charged particles emit photons (real or "virtual") that interact with other charged particles. Similarly, gravitational attraction means that each body emit gravitons which interact with other bodies, etc. Such interaction is the basis of the so-called Feynman diagram approach to physics, which leads to actions corresponding to all existing physical theories [7, 42].

Thus, the above derivation of Newton's equations can be naturally extended to the derivation of all other fundamental physical equations. For example, in [12], this derivation is explained on the example of gravity.

5. How All This Can Help Computing

Quantum computing. The most well-known example of how innovative physical ideas can speed up computations is quantum computing; see, e.g., [34]. With quantum computing:

- we can search in an unsorted list of n elements in time $\sqrt{n}$ – which is much faster than the time n which is needed on non-quantum computers [10, 11, 34]; and
- we can factor a large integer in time which does not exceed a polynomial of the length of this integer – and thus, we can break most existing cryptographic codes like widely used RSA codes which are based on the difficulty of such a factorization on non-quantum computers [34, 39, 40].

Randomness can also help. Physicists believe that if any phenomenon is never observed, there must be a physical reason why it is not observed. This applies to randomness too – all truly random sequences will sooner or later be observed. As we have mentioned, a natural formalization of randomness is $K(x) \geq \text{len}(x) - C$. If $K(x) < \text{len}(x) - C$, this means that there exists a program of length $< \text{len}(x) - C$ that generates x. So, for each sequence x, we can decide whether it is random or not: if it is random, we will eventually observe it, and if it is not random, we will generate it by trying all possible (finitely many) programs of length $< \text{len}(x) - C$.

On the other hand, one can prove that by using the usual algorithms, we cannot decide whether $K(x) \leq \text{len}(x) - C$, i.e., whether a given sequence is random; this is an easy consequence of known results about Kolmogorov complexity [25]. Thus, randomness provides us with computational abilities beyond the usual algorithms; see [19] for details.

That no physical theory is perfect can also help. Fuzziness – in the form that everything is a matter of degree – also means that no physical theory is perfect. This is actually a belief that many physicists hold – that no matter how good is a current physical theory, there will always

be some future observations and experiments that will require that this theory be modified. This happened when relativistic and quantum effects needed to be added, this will happen in the future.

Interesting, this very idea can already help computations – to the extent by using physical observations, we can potentially solve almost all complex (NP-complete) problems in reasonable time; see [20, 22] for technical details.

Fuzzy ideas can also help decide what is feasible. In the previous text, we mentioned that some algorithms are feasible – i.e., finish their computations in reasonable time for inputs of reasonable length – while some are not. How can we describe this notion in precise terms?

The current formalization of this notion is that an algorithm is feasible if its running time $t(x)$ on all inputs x is bounded by some polynomial of the length $\text{len}(x)$ of this input: $t(x) \le P(\text{len}(x))$; see, e.g., [36]. However, this definition is not perfect: for example, the time $t(x) = 10^{200}$ is a constant (thus a polynomial), but it is larger than the lifetime of the Universe, while the exponential time $\exp(10^{-30} \cdot \text{len}(x))$ grows faster than any polynomial – but is clearly practically feasible for all reasonable $\text{len}(x)$. Many attempts have been made to come up with a more adequate definition, but none have succeeded.

Solution: take into account that feasible is a matter of degree. If we take that into account, and use the usual fuzzy techniques to translate the rule "if $\text{len}(x)$ is reasonable, then $t(x)$ is reasonable" into precise terms, we get an adequate description of feasibility; see, e.g., [17, 31].

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