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Abstract Many phenomena like burnout are gauged by computing a linear combination of user-provided Likert-scale values. The problem with this traditional approach is that, while it makes sense to have linear combination of weights or other physical characteristics, a linear combination of Likert-scale values like "good" and "satisfactory" does not make sense. The only reason why linear combinations are used in practice is that the corresponding data processing tools are readily available. A more adequate approach would be to use fuzzy logic – a technique specifically designed to deal with Likert-scale values. We show that fuzzy logic actually leads to a linear combination – but not of the original degrees, but of their transformed values. The corresponding transformation function – as well as the coefficients of the corresponding linear combination – must be determined from the condition that the resulting expression best fits the available data.

1 Formulation of the Problem

Case study: burnout. In the past, most jobs were menial, tiring jobs. It is possible to get too tired on such a job – in which case the body is no longer capable of continuing, at least not with the same energy. These are natural barriers that evolution came up with – to prevent our bodies from situations when too much physical activity will induce a serious harm to our health.

At present, most jobs have a strong mental, intellectual component. This is a reasonably new phenomenon, for which evolution has not prepared us. In contrast to physical labor, nature does not impose any limits on our mental activities. Even after we are tired, we are still able to do mental work – and many of us continue to do it, especially if there are important deadlines ahead. As a result of this overworking,

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many people get into what is called burnout – they are stressed, they are unhappy, their productivity plummets. This phenomenon has been observed for many professions; in particular, software companies are affected by this phenomenon.

One of the problems with burnout is that, in contrast to physical tiredness, we ourselves often do not notice it at first, and others do not notice it either – until the situation becomes bad, and a serious intervention is needed.

Since there are no natural indications of a burnout, it is desirable to come up with tools that would allows us to monitor mental workers for such a burnout.

How burnout is measured. Burnout is all about the inner mental state of a person. So, the only reliable way to detect burnout is by asking this person questions – to which a person provides answers of a Likert scale (e.g., from 1 to 5) – and processing the results. This is a typical idea in psychological studies. In particular, for burnout, one of most widely used scheme is the Maslach Burnout Inventory first proposed in [4]. There are also later modifications of this test.

How Likert-scale data is usually processed in psychological analysis: a general description. Usually, a test consists of several (n) questions. For each question $i = 1, \ldots, n$, the patient provides an answer a_i – a number on the corresponding scale. Designers of the test try to find coefficients $c_1, \ldots, c_n$ for which the corresponding linear combination $c_1 \cdot a_1 + \ldots + c_n \cdot a_n$ is the best correlated with the phenomenon that we are trying to detect.

The resulting linear combination is then used as a numerical measure of the corresponding phenomenon – for example, the measure of burnout.

But why linear combination? Adding kilograms and kilograms makes sense, but adding "good" and "satisfactory" does not. Our understanding is that the only reason for using linear combinations is that this is an available computational tool. Honestly, this is not a very convincing argument.

So what shall we do? What are values on a Likert scale? These are exactly degrees that Lotfi Zadeh started with when he invented fuzzy logic; see, e.g., [2, 3, 5, 7, 8, 10]. So why not use fuzzy logic to process these degrees?

This is what we will explore in this paper.

2 Analysis of the Problem

First – naive – idea. Numerous questions on a test describe some symptoms of the analyzed phenomenon – in our case, of a burnout. The more symptoms a patient has, the more severe these symptoms, the more probable it is that this patient is approaching burnout. Each symptom by itself is already some indication of a burnout. So, it seems reasonable to say that a patient has a burnout if this patient has the first symptom or the second symptom, etc. In other words, it seems reasonable to associate the burnout with a disjunction $S_1 \vee S_2 \vee \ldots$, where S_i means that the i-th symptom is present.

In fuzzy logic, disjunction is represented by an appropriate "or"-operation $f_\vee(a, b)$ (also known as a t-conorm). So, it seems reasonable to estimate the degree of a burnout as the value $f_\vee(a_1, a_2, \ldots)$, where a_i is a degree to which the i-th symptom is present, and the "or"-operation $f_\vee(a, b)$ should be determined experimentally, so as to provide the best fit with the available data.

Limitations of the naive idea. The problem with the above idea is that it treats all symptoms equally. In reality, while some symptoms have a strong relation with the analyzed phenomena, the relation of other symptoms with this phenomenon is much weaker.

For example, tiredness is a clear indication of a burnout, while unhappiness may be caused by other reasons – e.g., low salary level, low morale, etc.

So what can we do. So, instead of using the same "or"-operation to combine all the degrees a_i, we need to take into account that these degrees have different strength: with respect to prediction of the analyzed phenomena, a very strong level of one of the degrees a_i may correspond to a much weaker level of another degree a_j. Thus, before combining, we need to bring these degrees to an equivalent form.

For example, we can bring all these degrees to the level of the degree a_1. This means that instead of simply combining the original degrees $a_2, \ldots$, we first transform them into the a_1-level, by applying some transformation $f_i(a_i)$, and then use the "or"-operation to combine the transformed degree, i.e., to compute the value

$$a = f_\vee(a_1, f_2(a_2), \ldots, f_i(a_i), \ldots). \tag{1}$$

Resulting question: what are these transformations $f_i(a_i)$? For each symptom i, we may (and often do) have several questions describing this symptom. Let us denote the corresponding degrees by $a_{i,1}, \ldots, a_{i,k}$. In principle, we can deal with this situation in two different ways:

- first idea is to combine these degrees into a single degree $a_i = f_\vee(a_{i,1}, \ldots, a_{i,k})$ and then apply the transformation function $f_i(a_i)$ to this combined degree, resulting in the value $f_i(a_i) = f_i(f_\vee(a_{i,1}, \ldots, a_{i,k}))$;
- another idea is to first apply the transformation f_i to all k degree, resulting in the values $f_i(a_{i,1}), \ldots, f_i(a_{i,k})$, and only then apply the "and"-operation, resulting in the value $f_\vee(f_i(a_{i,1}), \ldots, f_i(a_{i,k}))$.

The two resulting values describe the same situation and should, therefore, be equal. This leads to the following definition.

Definition 1. *Let $f_\vee(a, b)$ be an "or"-operation (t-conorm). We say that a function $f(a)$ is* consistent *with $f_\vee$ if for all possible values $a_1, \ldots, a_k$, we have*

$$f(f_\vee(a_1, \ldots, a_k)) = f_\vee(f(a_1), \ldots, f(a_k)). \tag{2}$$

Once we have found the appropriate "or"-operation and the appropriate consistent transformation functions $f_i(a_i)$, we can then estimate the desired degree a by using the formula (1).

How can we make this mathematical description more practical? It is known that for every "or"-operation $f_\vee(a, b)$, and for every $\varepsilon > 0$, there exists an ε-close strictly Archimedean "or"-operation, i.e., operation of the form

$$F^{-1}(F(a) + F(b)) \tag{3}$$

for some monotonic continuous function $F(a)$, where $F^{-1}(v)$ denotes the inverse function: $a = F^{-1}(v) \Leftrightarrow v = F(a)$; see, e.g., [6]. For very small ε, the difference between these two operations can be safely ignored. So, for all practical purposes, it makes sense to assume that the actual "or"-operation has the form (3).

Under this assumption, we can get a full description of all possible consistent transformations.

Proposition 1. *For each "or"-operation of type (3), every function $f(a)$ which is consistent with this operation has the form*

$$f(a) = F^{-1}(c \cdot F(a)) \tag{4}$$

for some constant $c > 0$.

Proof. Substituting the expression (3) into the formula (2), we conclude that

$$f(F^{-1}(F(a_1) + \ldots + F(a_k))) = F^{-1}(F(f(a_1)) + \ldots + F(f(a_k))). \tag{5}$$

By applying the function $F(a)$ to both sides of this equality, we conclude that

$$F(f(F^{-1}(F(a_1) + \ldots + F(a_k)))) = F(f(a_1)) + \ldots + F(f(a_k)). \tag{6}$$

Let us denote $G(a) \stackrel{\text{def}}{=} F(f(F^{-1}(a)))$, then the left-hand side of the formula (6) takes the form $G(F(a_1) + \ldots + F(a_k))$, while each term in the right-hand side takes the form

$$F(f(a_i)) = F(f(F^{-1}(F(a_i)))) = G(F(a_i)). \tag{7}$$

Thus, the formula (6) takes the form

$$G(F(a_1) + \ldots + F(a_k)) = G(F(a_1)) + \ldots + G(F(a_k)). \tag{8}$$

For each set of values v_i, we can take $a_i = F^{-1}(v_i)$, then $F(a_i) = v_i$, so the formula (8) takes the form

$$G(v_1 + \ldots + v_k) = G(v_1) + \ldots + G(v_k). \tag{9}$$

For a monotonic function $G(v)$, this implies that the function $G(v)$ has the form $F(v) = c \cdot v$ for some constant c; see, e.g., [1]. By definition of the function $G(v)$, this means that $F(f(F^{-1}(v))) = c \cdot v$. So, for $a = f^{-1}(v)$, for which $v = F(a)$, we

get

$$F(f(a)) = c \cdot F(a). \tag{9}$$

By applying the inverse function F^{-1} to both sides of the formula (9), we get the desired expression (4).

The proposition is proven.

Resulting combination rule. If we substitute the expression (3) for the "or"-operation into the formula (1), we get

$$a = F^{-1}(F(a_1) + F(f_2(a_2)) + \ldots + F(f_i(a_i)) + \ldots),$$

i.e., equivalently,

$$F(a) = F(a_1) + F(f_2(a_2)) + \ldots + F(f_i(a_i)) + \ldots$$

To make this formula uniform, we can define $f_1(a_1) = a_1$, then this formula takes the form

$$F(a) = F(f_1(a_1)) + F(f_2(a_2)) + \ldots + F(f_i(a_i)) + \ldots \tag{10}$$

According to the formula (9) from the above Proposition, for each i, we have

$$F(f_i(a_i)) = c_i \cdot F(a_i) \tag{11}$$

for some constant c_i. Substituting the expression (11) into the formula (10), we conclude that

$$F(a) = c_1 \cdot F(a_1) + c_2 \cdot F(a_2) + \ldots + c_i \cdot F(a_i) + \ldots \tag{12}$$

Discussion. Our goal is to find the severity of the burnout – or of any other tested phenomenon.

From this viewpoint, what is important are not so much the numerical values themselves, but the relation between values corresponding to different situation: which is larger and which is smaller. Since the function $F(a)$ is monotonic, we can therefore use $A \stackrel{\text{def}}{=} F(a)$ instead of a.

So, we arrive at the following recommendation.

3 Resulting Recommendation

Recommendation. Once we have the degrees $a_1, \ldots, a_n$ corresponding to different questions, the overall degree A can be estimated as

$$A = c_1 \cdot F(a_1) + \ldots + c_n \cdot F(a_n), \tag{13}$$

where the values c_i and the function $F(a)$ should be determined based on the available data: they should provide the best fit for the cases when the value A is known.

Comments.

- We still get a linear combination – but a linear combination after some possibly non-linear transformation $F(a)$. The main advantage is that now we have a justification, so a linear combination is not just a heuristic idea.
- Our preliminary results show that by appropriately selecting a function $F(a)$, we can indeed get a better fit that simply by using a linear combination of values a_i.
- Similar results can be obtained if we allow patients to provide interval ranges $\left[\underline{a}_i, \overline{a}_i \right]$. In this case, instead of a single value A, we get the while range $\left[\underline{A}, \overline{A} \right]$ of values corresponding to different values a_i from the corresponding intervals. Due to monotonicity, the smallest value of A is attained when all the values a_i are the smallest possible, and the largest value of A is attained when all the values a_i are the largest possible:

$$\underline{A} = c_1 \cdot F\left(\underline{a}_1 \right) + \ldots + c_n \cdot F\left(\underline{a}_n \right) ; \tag{14}$$

$$\overline{A} = c_1 \cdot F\left(\overline{a}_1 \right) + \ldots + c_n \cdot F\left(\overline{a}_n \right) . \tag{15}$$

Acknowledgments

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