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What Is the Logic Behind Cistercian Numbers?

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Abstract

In the 13-15 centuries, many European monasteries used an unusual number system developed originally by the Cistercian monks; later on, this system was used by winemakers. In this paper, we provide a possible explanation of why these particular symbols were used.

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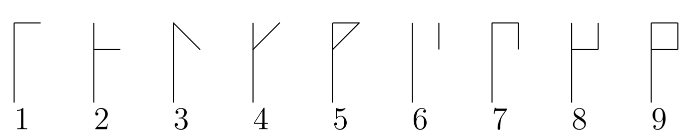
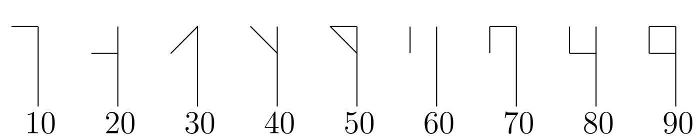
1 What Are Cistercian Numbers

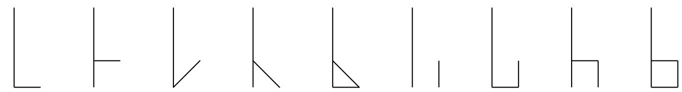
In the 13-15 centuries, many monks in Europe used a number system invented by Cistercian monks; see [6] and references therein. This system was later used by winemakers. In this system, digits from 1 to 9 are described as follows:



Tens are described by similar symbols placed to the left of the vertical line:

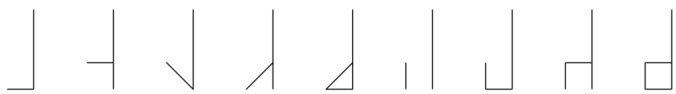


Hundreds are formed by placing similar symbols at the bottom right of the vertical line:



100 200 300 400 500 600 700 800 900

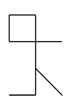
Finally, thousands are formed by placing similar symbols at the bottom left of the vertical line:



1000 2000 3000 4000 5000 6000 7000 8000 9000

A general 4-digit number is represented by placing the symbols corresponding to all 4 digits at the corresponding place near the vertical line. For example, 1492 can obtained by placing:

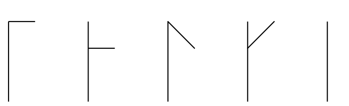
- symbols corresponding to 1 at the bottom left,
- symbols corresponding to 4 at the bottom right,
- symbols corresponding to 9 at the top left, and
- symbols corresponding to 2 at the top right:



Why? An interesting question is: why these strange notations? This is what we try to explain in this paper.

2 Our Explanation

First observation: basic digits vs. derivative digits. First, let us notice that 5 digits are formed by adding only one line to the main vertical line:



1 2 3 4 6

Other digits are formed by combining lines corresponding to these five basic digits:

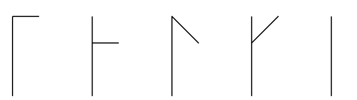
- 5 is formed by adding lines corresponding to 4 and 1: $5 = 4 + 1$;
- 7 is formed by adding lines corresponding to 6 and 1: $7 = 6 + 1$;
- 8 is formed by adding lines corresponding to 6 and 2: $8 = 6 + 2$; and
- 9 is formed by adding lines corresponding to 6, 2, and 1: $9 = 6 + 2 + 1$.

In all these cases, we add only numbers 1 and/or 2.

Remaining questions. Why select these 5 basic digits? Why assign the corresponding lines to each of these digits?

Let us provide possible answers to these questions.

Why 1, 2, 3, 4, and 6 are basic digits. In this scheme, there are exactly 5 possible locations of a short line: one location parallel to the vertical line, two locations orthogonal to this line, and two locations to the diagonal.

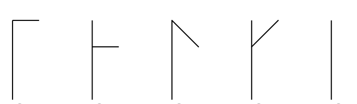


So, at first glance, the simplest idea is to have these expressions correspond to the first five digits 1 through 5.

However, in this case, if we only add 1 or 2, we can never reach 9: the largest that we can get by adding both 1 and 2 to 5 is 8. To get 9, we need to have a basic digit b for which $b + 1 + 2 \geq 9$, i.e., $b \geq 6$.

The minimal change in comparison to the original selection of the 5 digits is when we take 6 instead of 5. This explains why 1, 2, 3, 4, and 6 are the basic digits.

How to explain allocation of symbols to the basic digits. We have 5 possible patterns with one additional line.



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How do we allocate these patterns to the 5 basic digits?

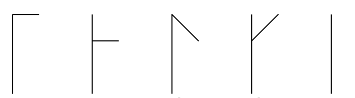
In real life, 1 is the most frequent digit, 2 is less frequent etc.; see, e.g., [1, 2, 3, 4, 5, 7, 8, 9]. To minimize the overall efforts, it is therefore reasonable to assign the easiest-to-write pattern to 1, the next-easiest to 2, etc.

It is easier to write a connected symbol then a symbol consisting of two parts. The only disconnected pattern – and thus, the most difficult-to-write one – is the pattern consisting of two parts, with a additional vertical line parallel to the main one. Thus, this pattern is assigned to the least frequent of the basic digits – the digit 6:

? ? ? ? 6

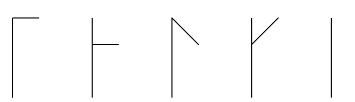
All other 5 patterns are connected. The easiest thing is usually to only use vertical and horizontal lines; these lines are, e.g., explicit in the squared paper. Such are the first two patterns in the above list, so they should be assigned to the two most frequent digits: 1 and 2.

Out of these two patterns, the first one is easier to write, since it can be naturally drawn in one movement, without having to go back or take the pen off the paper. Thus, this first easiest-to-write pattern should correspond to the most frequent digit 1, and the next one to the not-so-frequent digit 2:



1 2 ? ? 6

We have two remaining patterns and two remaining basic digits: 3 and 4. Out of the two remaining patterns, the first one can be drawn in one move, without the need to go back, so it is easier to write. Thus, we assign this easier-to-write pattern to the more frequent of the two remaining digits: the digit 3. The remaining pattern is then assigned to the remaining digit 4. Thus, we get exactly the arrangements originally proposed by the Cistercian monks:



1 2 3 4 6

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